A tale of two curves and their influence on rocket and supersonic jet noise research

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JASA LETTER TO THE EDITOR

A tale of two curves and their influence on rocket and supersonic jet noise research (L)[a]

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ABSTRACT:
This letter describes how a landmark 1960s supersonic jet noise experiment influenced subsequent noise models. A discrepancy in other researchers' application of Potter and Jones's axial decomposition of the sound power generated from a laboratory-scale jet can be traced to an erroneous plot in the original report. Whereas most jet noise research indicates the dominant sound power is generated upstream of the supersonic core tip, propagation of this error in the ubiquitous NASA SP-8072 report has caused rocket noise modelers for five decades to disproportionately allocate sound power generation to the subsonic flow. *© 2021 Acoustical Society of America.*

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I. INTRODUCTION

After the initial papers by Lighthill (1952, 1954) on jet noise theory and early experiments by Westley and Lilley (1952), interest in developing high-speed jet aircraft and launch vehicles motivated researchers to understand and predict supersonic jet noise from turboengines and rockets. Studies from the mid-1950s to mid-1960s spanned laboratory-scale jets, turbojets, rockets, supersonic jet noise theory (e.g., see Cole *et al.*, 1957; Mayes *et al.*, 1959; Ffowcs Williams, 1963; Morgan and Young, 1963; Tedrick, 1964; Potter and Crocker, 1966), and this body of work laid the foundation for subsequent jet and rocket noise research.

Figure 1 shows the anatomy of a supersonic jet with nozzle exit diameter, D_e, and exit Mach number, M_e. Within the potential core, the jet velocity is relatively constant (though modulated by shock cells for nonideal expansion) and supersonic, i.e., the jet velocity is greater than its sound speed such that its local Mach number, M, is greater than one. The turbulent mixing layer, which begins at the nozzle lip and grows in width with downstream distance, is fully developed across the jet beyond the potential core length, L_c. However, the flow remains supersonic across part of the plume cross section until the supersonic core length, L_s, is reached. Beyond the supersonic core tip, or "sonic point," $M < 1$, and the jet is subsonic everywhere. Because of its complexity, the generation of sound from supersonic jets from different regions of the exhaust has been researched for more than 60 years.

Early supersonic jet noise research disagreed regarding the supersonic jet noise source, with theory suggesting it originated relatively near the nozzle (Ffowcs Williams, 1963) and early measurements of rockets and other jets indicating the dominant noise was located well downstream (Mayes, 1959; Morgan and Young, 1963). Because of this, Potter and Jones (1967) undertook an investigation to measure the sound power distribution along the length of a supersonic jet. The experiment is described in considerable detail in the report by Potter (1968), but its essential details are as follows. The sound power was measured from a Mach 2.5, ideally expanded and unheated jet by exhausting it into and out of a reverberation chamber. The nitrogen jet rig with a 2.54 cm diameter nozzle was mounted on a traverse and oriented so the jet exhausted through a small orifice in the reverberation chamber wall. By moving the jet in and out of the chamber, the overall sound power, W_{OA}, and associated power level, OAPWL, were measured by the reverberation method. By differentiating the sound power vs jet position curve, sound power per unit length, $W(x)$, and sound power level-per-length curves, $\mathrm{PWL}(x)$, were obtained. The mean for the power-per-length curves obtained for the jet firing in and out of the reverberation chamber was the experiment's primary result. With accompanying estimates for core lengths ($L_c \approx 13.1D_e$ and $L_s \approx 22.8D_e$), Potter and Jones's experiment helps to quantify noise production relative to the supersonic jet schematic in Fig. 1.

II. TWO MODELS AND A MYSTERY

At least two foundational jet noise studies built nearly concurrently on the Potter investigation. Nagamatsu *et al.* (1969) and Nagamatsu and Horvay (1970) used the $\mathrm{PWL}(x)$ result to develop and study a supersonic jet noise theory where the sound power radiated grows linearly with downstream distance until around L_s, after which it rapidly decays according to subsonic jet theory (e.g., see Lighthill, 1954; Ribner, 1958). The researchers applied their theory to data from laboratory-scale jets to rockets, but indicated the need

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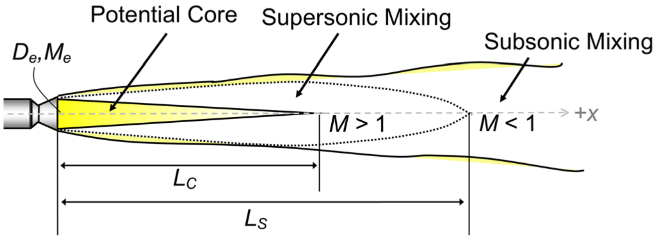

FIG. 1. (Color online) A schematic of a supersonic jet, identifying the potential core, supersonic mixing, and subsonic mixing regions, along with associated core tip lengths.

to further understand its applicability to the velocities and temperatures associated with rocket plumes. The Nagamatsu *et al.* findings influenced other important supersonic jet noise research and remain important today, at least in two senses. First, researchers credit Nagamatsu *et al.* for the understanding that the maximum sound power contributions originate between L_c and L_s (e.g., Greska *et al.*, 2008; Baars *et al.*, 2014). Second, investigators still refer to Nagamatsu *et al.* empirical expressions for L_c and L_s that rely solely on M_e (e.g., Greska *et al.*, 2008; Baars *et al.*, 2014; Gee *et al.*, 2017).

The second concurrent investigation that utilized Potter (1968) was by Eldred (1971). Eldred was responsible for compiling early supersonic jet noise research into empirical models for determining the acoustic loading produced by rocket plumes. One of the two distributed-source models in Eldred (1971) ("SP-8072" is how Eldred's work is commonly known) relies on Potter's result, which was directly extrapolated to become the total power per unit length for a "standard chemical rocket." Eldred's work has influenced the global launch vehicle community immeasurably; some researchers have applied it to different rockets and configurations, while others have studied its assumptions and refined certain aspects. However, through five decades, Potter's curve (Fig. 12 in SP-8072) has remained untouched.

The mystery surrounding these two important supersonic jet models is that Fig. 4 in Nagamatsu *et al.* (1969) and Fig. 12 in SP-8072, while supposedly being identical $\mathrm{PWL}(x)$ curves in that they originate from the same experiment, disagree when normalized to the same axes. The relative $\mathrm{PWL}(x/L_c)$ from both reports is shown in Fig. 2. After digitization of the Nagamatsu *et al.* version, it had to be plotted in terms of L_c and normalized by multiplying $W(x)$ by L_c/W_{OA}. Note that before normalization, integration of the digitized Nagamatsu *et al.* curve only differed from the absolute OAPWL reported by Potter by 0.5 dB. (Their use of the historical $10-13$ W decibel reference is irrelevant.)

Regarding the Eldred (1971) curve, it was already normalized as described above and should not have required additional manipulation. However, integration of the digitized version of Fig. 12 in SP-8072 revealed that, rather than integrating to yield 0 dB, the normalized OAPWL was equal to -0.35 dB. Consequently, the Eldred curve in Fig. 2 has been shifted up by 0.35 dB from that of SP-8072. Finally, note that to provide consistent graph boundaries, the curves shown in Fig. 2 have been extrapolated slightly from the original reports, which resulted in an OAPWL increase of less than 0.05 dB.

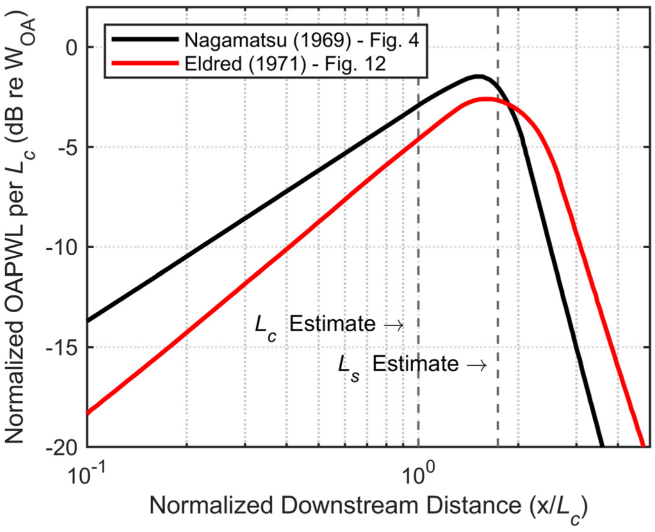

FIG. 2. (Color online) Potter's normalized overall power level per unit length, adapted from Nagamatsu *et al.* (1969) and Eldred (1971), plotted as a function of normalized downstream distance, x/L_c.

Within Fig. 2 lies the mystery. How can the same experimental result yield markedly different curves for the normalized power level per L_c?

III. A RESOLUTION

After investigation of the discrepancy, which included digitizing, normalizing, integrating, and comparing curves from several graphs, it was discovered that Eldred (1971) and Nagamatsu *et al.* (1969) and Nagamatsu and Horvay (1970) relied on different results from the Potter (1968) report. Eldred reproduced in SP-8072 almost exactly the shape of Fig. 12 from Potter (1968) and scaled it by L_c instead of D_e. On the other hand, Nagamatsu *et al.* appear to have slightly smoothed the absolute $\mathrm{PWL}(x)$ results in Fig. 13 of Potter (1968) in their Fig. 4 and then represented them on a logarithmic, rather than linear, distance scale. The different ways Potter presented the $\mathrm{PWL}(x)$ results made it difficult to spot the fact that they were physically inconsistent without quantitative comparison.

The $\mathrm{PWL}(x)$ curves in Fig. 2 are different, but is it possible to know which plot is correct? An important clue comes from Potter's Fig. 12 and accompanying text, where he describes the initial growth in the radiated power with downstream distance as having x^1 dependence, which translates to $+10$ dB/decade. With a $\mathrm{PWL}(x)$ growth of $\sim$13.7 dB/decade, Potter's Fig. 12 deviates appreciably from linear dependence. On the other hand, Potter's Fig. 13 (and Nagamatsu *et al.*, Fig. 4) much more closely approximate a linear increase in $\mathrm{PWL}(x)$, at $\sim$10.8/decade. Therefore, it appears that Fig. 13 in Potter (1968) is the correct version because it both matches the reported OAPWL within 0.5 dB and approximates the spatial dependence described in the report. Note that further experimental evidence for the linear growth of power upstream of L_s is described by Nagamatsu *et al.* (1969).

IV. PHYSICAL IMPLICATIONS

The curves in Fig. 2 are different, but is the difference physically meaningful in the context of supersonic jet noise radiation? The answer is yes. First, note the region of maximum power production. Although the location of the peak only changes marginally, occurring at $\sim 1.5\ L_c$ and $\sim 1.6\ L_c$ according to Nagamatsu *et al.* and Eldred, respectively, the width of the peak region is different. For the Nagamatsu *et al.* version of the Potter PWL (x) result, the full-width, 3-dB down region spans $0.7L_c$–$2L_c$, a total of $1.3\ L_c$. Eldred's version extends from $0.85\ L_c$ to $2.3\ L_c$, a distance of $1.65\ L_c$. Using the Nagamatsu *et al.* relation for L_c with $M_e = 3$ (typical of a rocket), $L_c = 14.2 D_e$. The $0.35\ L_c$ difference between the two curves represents a difference in peak sound power region of $5\ D_e$. For a space vehicle slowly lifting off a launch platform, that difference has the potential to significantly change pad and vehicle vibroacoustic loading.

Perhaps the most important physical implication of Eldred (1971) adopting an apparently erroneous plot is the region of the jet responsible for the greatest amount of sound generation. Because PWL (x) reaches a maximum at $L_c < x < L_s$ for both curves, it is useful to determine the power radiated from three spatial regions: upstream of the potential core tip, $x < L_c$; between the two core tips, $L_c < x < L_s$; and $x > L_s$, where PWL (x) decays sharply.

The relative radiated power from the three different regions was obtained for both curves in Fig. 2 by numerical integration of the normalized $W(x/L_c)$ using the trapz function in MATLAB® for each of the three regions. Note that because Fig. 2 is represented logarithmically with x/L_c and on a decibel scale, it is misleading to assess relative contributions visually. Table I shows the integration result for both the Nagamatsu *et al.* and Eldred versions of the Potter curve, presented both in terms of percentage of W_{OA} and relative OAPWL and accurate to within 0.02 dB. For the integrated Nagamatsu *et al.* curve, nearly 50% of the sound power comes from the region in between the potential and supersonic core lengths, with the remaining power coming slightly more from the downstream subsonic region than the upstream shear layer. On the other hand, the Eldred curve indicates the dominant sound power-producing region is the subsonic portion of the jet exhaust. It also ascribes little power radiation to $x < L_c$, which would imply that shear-layer Mach wave radiation is negligible for rockets. Table I ultimately points to the following conclusion: one curve indicating the maximum sound power is produced in the jet's fully turbulent, supersonic region, and the other, the subsonic region, is physically inconsistent for the same jet.

TABLE I. Percentage of sound power radiation, W_{OA}, and relative OAPWL, originating from different jet regions, according to the two graphs resulting from Potter's jet sound power localization experiment.

Region	Nagamatsu *et al.* (1969)	Eldred (1971)
$x < L_c$	24.9%/−6.0 dB	13.9%/−8.6 dB
$L_c < x < L_s$	46.4%/−3.3 dB	38.2%/−4.2 dB
$x > L_s$	29.0%/−5.4 dB	47.5%/−3.2 dB
Total	100.3%/0.01 dB	99.6%/−0.02 dB

The hypothesis that the curve Eldred adopted from Potter is incorrect is strengthened by analyses from Greska *et al.* (2008) and Baars *et al.* (2014), whose surveys of near-field sound levels showed that the apparent maximum source region was between L_c and L_s, and closer to L_c. Near-field vector intensity measurements of a Mach-1.8 unheated jet (Gee *et al.*, 2017) show that the dominant source region for the most energetic frequencies extends from $\sim L_c$ to some point upstream of L_s. An approximation of the peak source location of $1.5\ L_c$ with relatively little sound generation downstream of L_s also matches the measurements of Laufer *et al.* (1976), whose directional measurements and analysis were substantially influenced by Nagamatsu and Horvay (1970) and Potter (1968).

The preceding discussion references laboratory-scale supersonic jet research exclusively, where the accepted understanding that most of the radiated sound power originates upstream of L_s comes from Nagamatsu *et al.* However, within the rocket and launch vehicle noise community, the situation is less clear. Far less additional knowledge about noise generated specifically from rockets has been gained since the early 1970s (e.g., see McInerny, 1990), and researchers have mostly applied variants of SP-8072 (e.g., Fukuda *et al.*, 2009; Subramanyam and Natarajan, 2013; Casalino *et al.*, 2009) or studied its assumptions to refine aspects of the modeling, such as definitions of L_c in free (Varnier, 2001; James *et al.*, 2016) and deflected environments (Haynes and Kenny, 2009), directivity function definitions (Haynes and Kenny, 2009; James *et al.*, 2014), and the radiation efficiency of different-sized rockets (Sutherland, 1993). However, in these studies modifying other aspects of SP-8072, the PWL(x) curve shape assumed has remained unchanged.

The numerical integration of Fig. 2 found in Table I helps to finally resolve what was a puzzling schematic and accompanying statement made in an important study by Sutherland (1993). While citing both Potter (1968) and Nagamatsu and Horvay (1970) and discussing the maximum power as originating from near the supersonic core tip, Sutherland drew from Eldred's version of Potter's result to base his modified SP-8072-type model on the following assumption: "The dominant acoustic power comes from the subsonic flow downstream of the end of the supersonic core." Despite running counter to what others had learned about heated, supersonic jets, Sutherland was correct in interpreting the physical implications of Eldred's version of Potter's sound power result. Regardless of other uncertainties surrounding SP-8072, including core length definitions (e.g., James *et al.*, 2016), the launch vehicle noise modeling community has, for five decades, effectively assumed the dominant noise from a rocket exhaust plume is generated in the subsonic flow region and that the region upstream of L_c radiates little power.

V. CONCLUSION

Unless the physics of supersonic jets produced by rocket engines and motors are so fundamentally different from other supersonic jets, the conclusion of this letter is twofold. First, a likely plotting error made in 1968 led to a divergence in physical interpretation of the sound power generation from supersonic jets between the "jet noise" and "rocket noise" communities, and it appears that the Eldred (1971) NASA SP-8072 propagation of this error has been universally accepted until now. Second, those who employ Fig. 2's Eldred (1971) curve in their launch vehicle noise models should replace it with the version of Nagamatsu *et al.* (1969). This should also cause further examinations of empirical core length definitions that have been previously based, in part, on the assumption that much of the radiated power originates downstream of the supersonic core tip.

If the physics of rockets (and possibly afterburning tactical aircraft engines) is so different from that of other supersonic jets that the foundational work of Potter and Jones has no validity in this regime, then this letter further illustrates the markedly incomplete understanding we still have of highly heated supersonic jets. It also demonstrates the need for rigorous investigations to understand phenomena and scaling laws for these supersonic jets 70 years after the commencement of jet noise research.

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