

**Interactions of a stabilized radion and duality**Zackaria Chacko,<sup>1</sup> Rashmish K. Mishra,<sup>1</sup> Daniel Stolarski,<sup>2</sup> and Christopher B. Verhaaren<sup>1</sup><sup>1</sup>*Maryland Center for Fundamental Physics, Department of Physics, University of Maryland, College Park, Maryland 20742-4111, USA*<sup>2</sup>*Theory Division, Physics Department, CERN, CH-1211 Geneva 23, Switzerland*  
(Received 25 November 2014; published 11 September 2015)

We determine the couplings of the graviscalar radion in Randall-Sundrum models to Standard Model fields propagating in the bulk of the space, taking into account effects arising from the dynamics of the Goldberger-Wise scalar that stabilizes the size of the extra dimension. The leading corrections to the radion couplings are shown to arise from direct contact interactions between the Goldberger-Wise scalar and the Standard Model fields. We obtain a detailed interpretation of the results in terms of the holographic dual of the radion, the dilaton. In doing so, we determine how the familiar identification of the parameters on the two sides of the AdS/CFT correspondence is modified in the presence of couplings of the bulk Standard Model fields to the Goldberger-Wise scalar. We find that corrections to the form of the dilaton couplings from effects associated with the stabilization of the extra dimension are suppressed by the square of the ratio of the dilaton mass to the Kaluza-Klein scale, in good agreement with results from the CFT side of the correspondence.

DOI: [10.1103/PhysRevD.92.056004](https://doi.org/10.1103/PhysRevD.92.056004)

PACS numbers: 04.50.-h, 12.40.Nn

**I. INTRODUCTION**

The unambiguous discovery of a new scalar resonance with the properties expected of the Standard Model (SM) Higgs represents a milestone in the history of elementary particle physics. A careful study of the properties of this Higgs particle is expected to shed light on the dynamics that drives electroweak symmetry breaking. At present, an important open question is whether this state is an elementary particle, or a composite made up of more fundamental constituents held together by some form of new strong dynamics. Compositeness of the Higgs would allow a simple resolution of the hierarchy problem, provided the new strong dynamics kicks in at energies close to the weak scale, and therefore constitutes a very compelling theoretical possibility. However, the generation of fermion masses in composite Higgs scenarios is a challenge. The simplest models involve new sources of flavor violation close to the weak scale and are therefore disfavored by experiment.

An interesting class of composite Higgs models that can resolve this flavor problem are those where the new strong dynamics is conformal in the ultraviolet (UV). Strong conformal dynamics allows the flavor scale in these theories to be well separated from the weak scale, allowing the stringent experimental limits on flavor changing neutral currents to be satisfied. This scenario is closely related to earlier proposals for suppressing flavor violation in technicolor models [1], (see also [2–5]). In this class of theories

the conformal symmetry is spontaneously broken at low energies. As a consequence, if the conformal symmetry were exact, the low energy spectrum would contain a massless Nambu-Goldstone boson (NGB), the dilaton [6–10]. In this limit the form of the dilaton couplings to the SM fields can be completely determined from the requirement that the conformal symmetry be realized nonlinearly.

In the theories of phenomenological interest, however, the conformal symmetry is only approximate. It is explicitly violated by operators that are small in the UV but grow large in the infrared (IR), thereby driving the breaking of conformal symmetry. Provided the operator primarily responsible for this breaking has a scaling dimension close to marginal, the theory can remain approximately conformal for enough decades in scale for the flavor problem to be addressed. However, as a consequence of the explicit breaking, the dilaton is not massless and its couplings receive corrections. It is important, therefore, to understand the exact circumstances under which the dilaton can remain light and to determine the size and form of the corrections to its couplings.

Recently, several authors have studied the conditions under which the low energy spectrum contains a light dilaton [11–13]. The general picture that has emerged is that if the operator  $\mathcal{O}$  primarily responsible for the breaking of conformal symmetry is close to marginal at the breaking scale, the mass of the dilaton can naturally lie below the scale of the strong dynamics. This result is explained by the fact that the extent of explicit conformal symmetry violation at the breaking scale depends not just on the size of the deformation associated with  $\mathcal{O}$ , but also on the deviation from marginality of the operator  $\mathcal{O}$ . In particular, the theory will retain an approximate conformal symmetry if the

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operator  $\mathcal{O}$  is very close to marginal, independent of the size of the deformation. In such a scenario, even if the deformation is large, the dilaton can naturally be light provided  $\mathcal{O}$  is close to marginal at the breaking scale. Unfortunately, unless the theory possesses some special feature, this condition is not expected to be satisfied and the dilaton is not light. The underlying reason for this is that, even if the operator  $\mathcal{O}$  that drives the breaking of conformal symmetry is indeed close to marginal in the UV, as in the theories of phenomenological interest, its scaling behavior is expected to receive big corrections when the deformation grows large. Therefore, in general  $\mathcal{O}$  does not remain marginal near the breaking scale where the deformation is large. As a consequence, the presence of a light dilaton in the spectrum is not a robust prediction of the class of theories of interest for electroweak symmetry breaking.

One special class of theories where the dilaton can naturally remain light are those which possess not just a single isolated fixed point, but an entire line of fixed (or quasifixed) points. This feature, which is quite common in supersymmetric theories, allows the deformation to remain marginal at the breaking scale. Other constructions which admit the possibility of a naturally light dilaton are gauge theories that lie near the edge of the conformal window [14]. One scenario which allows the spectrum of light states to contain a dilaton, albeit at the expense of mild tuning, arises if the breaking of conformal symmetry occurs before the deformation associated with  $\mathcal{O}$  reaches its natural strong coupling value. In this limit, because the size of the deformation is small, the corresponding corrections to the scaling behavior of the operator  $\mathcal{O}$  at the breaking scale are also suppressed, allowing it to remain close to marginal. Then, the limited extent to which conformal symmetry is violated allows the dilaton to remain light. In general, however, the conformal symmetry is not expected to break until the deformation becomes large, so this scenario is associated with tuning. This tuning is mild, however, scaling only linearly with the mass of the dilaton [11,12]. Therefore, the presence in the low energy spectrum of a dilaton with a mass just a factor of a few below the compositeness scale is associated with only modest tuning. From this discussion we see that a light dilaton can arise in several different realistic scenarios, and therefore the dynamics of theories with a light dilaton remains a problem of phenomenological interest.<sup>1</sup>

The form of the dilaton couplings to the SM states has been determined in the limit that effects that explicitly violate conformal symmetry are neglected. Both the case when the SM matter and gauge fields are composites emerging from the strong dynamics [19,20], and the case when they are external elementary states [11,12,21], have been studied. Corrections to the form of the dilaton

couplings arising from explicit conformal symmetry violating effects have also been studied [11], and found to scale as the square of the ratio of the mass of the dilaton to the strong coupling scale. A physical understanding of this result may be obtained by noting that in the theories of interest with a light dilaton, the operator  $\mathcal{O}$  is close to marginal at the breaking scale, even though the deformation associated with  $\mathcal{O}$  may be large. If  $\mathcal{O}$  were exactly marginal the conformal symmetry would be exact, and independently of the size of the deformation, the dilaton couplings would be of the form dictated by nonlinearly realized conformal invariance. In this limit, the corrections to the dilaton couplings that arise from the deformation do not, in general, vanish. However, these effects can be exactly absorbed into corresponding changes in the low energy parameters, leaving the form of the interactions unchanged. The size of the corrections to the form of the dilaton couplings is therefore dictated not just by the size of the deformation, but also by the deviation from marginality of the operator  $\mathcal{O}$  at the breaking scale. However, as noted above, it is precisely these two effects that also determine the dilaton mass. Therefore, the size of the corrections to the form of the dilaton couplings is correlated with the mass of the dilaton. These corrections are therefore small and under good theoretical control if the dilaton is light. If, however, the deformation is large and the scaling behavior of the operator  $\mathcal{O}$  deviates significantly from marginality, the dilaton mass is raised to the strong coupling scale, and the corrections to the form of the dilaton couplings become of order one.

The AdS/CFT correspondence [22–25] relates theories of strong conformal dynamics to theories of gravity in higher dimensions. Theories of phenomenological interest where the strong conformal dynamics is spontaneously broken giving rise to a composite Higgs are dual [26,27] to Randall-Sundrum (RS) models [28] where the extra dimension is negatively curved and finite, with a brane at either end. In this correspondence, the dilaton is dual to the radion, the excitation corresponding to fluctuations in the size of the extra dimension [26,27]. In the original RS model, the hierarchy between the Planck and weak scales depends on the brane spacing, which is a free parameter. In this limit the radion is massless. The brane spacing, and the associated Planck-weak hierarchy, can be stabilized using the Goldberger-Wise (GW) mechanism [29]. This mechanism introduces a bulk scalar field  $\Phi$  which is sourced on the two branes, and has a potential in the bulk. It therefore acquires a vacuum expectation value (VEV) which varies as a function of position in the extra dimension, and contributes to the vacuum energy. Consequently, the brane spacing is stabilized and the radion acquires a mass. Since the RS model is one of the most promising candidates for physics beyond the SM, it is important to obtain an understanding of the mass and couplings of the radion in this framework.

<sup>1</sup>String motivated constructions that can give rise to a light dilaton have been considered, for example, in [15–18].

By holography the GW scalar  $\Phi$  is dual to the CFT operator  $\mathcal{O}$ , whose dynamics drives the breaking of conformal symmetry. Sourcing the GW field corresponds to a deformation of the CFT by this operator, with the VEV of the GW field corresponding to the size of the deformation. The bulk mass term for the GW field is related to the scaling dimension of  $\mathcal{O}$ , while the self-interaction terms in the bulk potential for  $\Phi$  correspond to corrections to the scaling behavior of  $\mathcal{O}$  that are important when the deformation grows large.

The conditions under which the low energy spectrum of the RS model contains a light radion after stabilization have been studied, and found to agree with the results for the dilaton from the CFT side of the correspondence [13,30,31]. The desired large hierarchy of scales can naturally arise if the mass term for the GW scalar is small. This corresponds to the scaling dimension of the dual operator  $\mathcal{O}$  being close to marginal. However, for the spectrum to naturally contain a light dilaton, the coefficients of the self-interaction terms for the GW scalar must also lie below their natural strong coupling values. From the dual perspective this ensures that the corrections to the scaling behavior of  $\mathcal{O}$  from the deformation remain small, even when the deformation itself is large, so that  $\mathcal{O}$  remains close to marginal at the breaking scale. However, unless the 5D construction possesses some special feature, in general the self-interaction terms are not small and this condition is not satisfied. Therefore, the presence of a light radion in the low energy spectrum below the Kaluza-Klein (KK) scale is not a robust feature of RS models [30].

One special class of theories where the radion can naturally remain light are those where the GW scalar arises as the pseudo-Nambu Goldstone boson (pNGB) of an approximate global symmetry. In this case the mass and self-interaction terms in the potential for the GW scalar can naturally be small, thereby allowing the radion mass to lie below the KK scale. Several authors have considered this limit and found that the radion is indeed light, its mass scaling as the mass of the GW scalar [30,32,33]. Careful studies have shown that the inclusion of gravitational backreaction does not alter this conclusion [13,31,34,35]. This corresponds in the dual theory to the case when the CFT possesses a line of quasifixed points. An alternative scenario which allows the spectrum of light states to contain a radion, albeit at the expense of mild tuning, arises if, after stabilization, the VEV of the GW scalar in the neighborhood of the IR brane lies below its natural strong coupling value. This is dual to the 4D operator corresponding to  $\mathcal{O}$  being below its strong coupling value at the breaking scale. In this limit, the overall contribution of the GW field to the potential for the radion and the effects of the self-interaction terms are both suppressed, allowing the radion to remain light. Although such a scenario is associated with tuning, the tuning is mild, scaling only linearly with the ratio of the mass of the radion to the KK scale [30].

Since the radion is the graviscalar excitation of the metric [28], the form of its interactions follows from general covariance [29]. The radion couplings to SM fields have been determined, both in the case of brane-localized matter [36–39], and in the case of matter in the bulk [40,41]. The dynamics associated with stabilization of the extra dimension leads to corrections to these couplings. Previous work to determine the form of these corrections was restricted to the technically simpler case of brane-localized fields [30]. In the dual picture, this corresponds to the case when all the SM fields are composites of the strong dynamics. The results obtained are in good agreement with those from the CFT side of the correspondence. The goal of this paper is to extend this analysis to the case when the SM matter and gauge fields reside in the bulk of the space. This scenario, which admits an elegant solution to the SM flavor problem [42–45], corresponds in the dual picture to the SM fermions arising as partial composites of elementary particles and CFT states [46].

In what follows, we consider a scenario where the SM gauge bosons and fermions propagate in the bulk of the RS geometry, but the Higgs is localized to the IR brane. We stabilize the brane spacing by employing a GW scalar  $\Phi$  that is sourced on the branes and determine the radion couplings to the bulk SM fields. This construction allows direct couplings of the GW scalar to SM fields in the bulk. To leading order in  $\Phi$ , these couplings take the schematic form

$$\sqrt{|G|} \mathcal{O}_{\text{SM}} \Phi. \quad (1.1)$$

Here  $G$  is the determinant of the 5D RS metric, and  $\mathcal{O}_{\text{SM}}$  is a gauge invariant operator composed of bulk SM fields. Brane localized interactions between the GW scalar and the SM fields are also expected to be present. The operators in (1.1) affect the masses and interactions of the fields in the low energy effective theory. We find that the leading corrections to the radion couplings to the SM fields arise from such terms, and perform a careful calculation to determine their effects. One might expect that the effects of the stabilization mechanism on the radion profile would lead to corrections to the radion couplings, even in the absence of direct couplings of the GW scalar to the SM particles. However, we show in Appendix A that these effects are much smaller than the corrections obtained from operators of the form (1.1).

We obtain a detailed interpretation of our results in terms of the holographic dual of the radion, the dilaton. In doing so, it is important to take into account the fact that the familiar identification of the parameters on the two sides of the AdS/CFT correspondence is modified in the presence of couplings of the bulk SM fields to the GW scalar. This is because one class of corrections to the radion couplings can be completely absorbed into changes in the parameters of the dual theory, and do not affect the form of the dilaton



interactions. As in the case of brane-localized SM fields, we find that all corrections to the form of the dilaton couplings are suppressed by the square of the ratio of the dilaton mass to the KK scale, in good agreement with results from the CFT side of the correspondence.

These results have implications for phenomenological studies of the radion. Several authors have investigated the possibility that the resonance observed at 125 GeV is not the SM Higgs, but a dilaton/radion, for example [47–54]. Studies have also been performed using LHC data to place limits on the mass of the radion in RS models [55–59], and investigating the prospects for detecting the radion at the LHC [60] and future colliders [61]. The dilaton has been investigated as a possible mediator of the interactions of dark matter with the SM [62–65]. It has been shown that in certain theories the presence of a light radion can help explain the baryon asymmetry [66]. In all these cases, an understanding of the size of the corrections to the radion couplings is necessary to understand the robustness of the conclusions.

The outline of this paper is as follows. In Sec. II we provide the details of the GW mechanism that stabilizes the extra dimension and results in the radion acquiring a mass. We also explain the origin of the corrections to the radion couplings. In subsequent sections, we consider in turn the massless gauge bosons, massive gauge bosons, and fermions of the SM. For each case we determine the radion couplings and interpret the results from a holographic point of view. Details of the calculation are presented in the appendices.

## II. RADION DYNAMICS

In this section, we outline the steps involved in obtaining the mass and couplings of the radion in the presence of the GW mechanism. The discussion in this section closely follows [30], and only the most relevant features are presented here. We begin with the 5D action for the RS model in the absence of stabilization,

$$\mathcal{S} = \int d^4x d\theta [\sqrt{G}(-2M_5^3 \mathcal{R}[G] - \Lambda_b) - \sqrt{-G_{\text{UV}}}\delta(\theta)T_{\text{UV}} - \sqrt{-G_{\text{IR}}}\delta(\theta - \pi)T_{\text{IR}}]. \quad (2.1)$$

Here  $M_5$  is the 5D Planck mass,  $\Lambda_b$  is the bulk cosmological constant, and  $T_{\text{UV}}, T_{\text{IR}}$  are the brane tensions on the UV and IR branes. The extra dimensional coordinate  $\theta$  is compactified over  $S^1$  and the region  $[-\pi, 0)$  is identified with  $[0, \pi)$  by a  $\mathbb{Z}_2$  symmetry. The locations  $\theta = 0, \pi$  correspond to the locations of the UV and IR branes respectively. The static metric<sup>2</sup> that describes the geometry of the two brane RS model is obtained as the solution to the 5D Einstein equations and can be written as

$$ds^2 = e^{-2kr_c|\theta|} \eta_{\mu\nu} dx^\mu dx^\nu - r_c^2 d\theta^2 \quad -\pi \leq \theta < \pi. \quad (2.2)$$

Here  $k$  is the inverse curvature and the constant  $r_c$  is proportional to the distance between the two branes. The parameter  $k$  is related to the bulk cosmological constant and 5D Planck scale by

$$\Lambda_b = -24M_5^3 k^2. \quad (2.3)$$

A condition for the existence of a static solution of this form is that the brane and bulk cosmological constants satisfy the relation  $\Lambda_b = kT_{\text{IR}} = -kT_{\text{UV}}$ . The value of  $r_c$  is a free parameter, corresponding to the fact that the brane spacing in the RS solution is undetermined. When we include a stabilization mechanism for the size of the extra dimension, we can detune the tension of the IR brane away from the RS value and still obtain a static solution [29].

When fluctuations about this background are considered, the low energy spectrum is found to contain, in addition to the massless 4D graviton, a massless radion field associated with the fluctuations of the brane spacing. To obtain the low energy effective theory for the light fields, we replace in the 5D metric  $\eta_{\mu\nu}$  by the dynamical field  $g_{\mu\nu}(x)$  and  $r_c$  by  $r(x)$ . These fields are identified with the 4D graviton and the radion fields respectively. The metric is then substituted back into the 5D action. After integrating over the extra dimension, the resulting 4D action describes the low energy effective theory of the graviton and the radion,

$$\mathcal{S} = \int d^4x \sqrt{-g} \left( \frac{2M_5^3}{k} \mathcal{R}[g_{\mu\nu}] + \frac{1}{2} \partial_\mu \varphi \partial^\mu \varphi \right). \quad (2.4)$$

Here  $\varphi$  represents the canonically normalized radion field and is related to  $r(x)$  by

$$\varphi(x) = \sqrt{\frac{24M_5^3}{k}} e^{-k\pi r(x)}. \quad (2.5)$$

The absence of a potential for  $\varphi$  reflects the fact that the value of  $r_c$  is undetermined. Stabilization of the extra dimension is accomplished by adding a bulk GW scalar  $\Phi$  to the theory. This scalar acquires a  $\theta$  dependent VEV,  $\hat{\Phi}(\theta)$ , from potentials on the branes and in the bulk. Its VEV is also a function of  $r_c$ . The Lagrangian for the 4D effective theory, including the contribution of the GW field, may be obtained in the same manner as before. Specifically, after replacing  $r_c$  by  $r(x)$ ,  $\hat{\Phi}$  is substituted back into the action and the integration over the extra dimension is performed. The resulting 4D action includes the contribution of the GW scalar to the low energy theory. This effect generates a potential for  $\varphi$  that, when minimized, fixes  $r_c$  and gives mass to the physical radion field.

To understand this in more detail, consider the action for the GW scalar,

<sup>2</sup>We use (+ - - - -) signature.

$$\mathcal{S}_{\text{GW}} = \int d^4x d\theta \left[ \sqrt{G} \left( \frac{1}{2} G^{AB} \partial_A \Phi \partial_B \Phi - V_b(\Phi) \right) - \sum_{i=\text{IR,UV}} \delta(\theta - \theta_i) \sqrt{-G_i} V_i(\Phi) \right]. \quad (2.6)$$

Here  $V_{\text{UV}}$  and  $V_{\text{IR}}$  are the potentials on the UV and IR branes and  $V_b$  is the potential in the bulk. For simplicity, we choose to work with a linear potential on the IR brane,

$$V_{\text{IR}} = 2\alpha k^{5/2} \Phi. \quad (2.7)$$

This is a consistent choice if  $\Phi$  is not charged under any symmetries, and the qualitative features of our results do not depend on the specific form of this potential. On the UV brane we do not specify a form of the potential but require that the value of  $\Phi$  is  $k^{3/2}v$ . This requirement is satisfied for many choices of potentials including the one considered in the original GW proposal [29]. In order to generate a sizable hierarchy, the size of the extra dimension must be large in units of the curvature. To accomplish this, we require that  $v$  be somewhat smaller than its natural strong coupling value.

The bulk potential for  $\Phi$  is of the general form

$$V_b(\Phi) = \frac{1}{2} m^2 \Phi^2 + \frac{1}{3!} \eta \Phi^3 + \frac{1}{4!} \zeta \Phi^4 + \dots \quad (2.8)$$

The bulk mass parameter  $m^2$  of the GW scalar must be small in units of the inverse curvature  $k$  to address the large Planck-weak hierarchy. However, there are no such requirements on the cubic and higher order terms. This can be understood from the holographic perspective. AdS/CFT relates the extra dimensional coordinate  $\theta$  to the renormalization scale  $\mu$  in the dual 4D theory,  $\log(k/\mu) \sim kr_c \theta$ . The duality also relates the value of the GW field  $\Phi(kr_c \theta)$  at any point  $\theta$  in the bulk to the size of the coefficient of the operator that deforms the dual CFT at the corresponding scale  $\mu$ . Therefore, requiring that the value of  $v$  on the UV brane be small corresponds to requiring that the size of the deformation be small at high scales  $\mu \sim k$ . Then, if the bulk mass term is also small, the initial growth in the value of  $\Phi$  is slow, allowing a large hierarchy to develop. In the dual picture, the mass of the GW scalar is related to the scaling dimension of the dual operator. A massless scalar corresponds to an exactly marginal deformation, while a negative mass squared for  $\Phi$  corresponds to a relevant operator in the dual CFT. Note that a negative mass squared for  $\Phi$  in AdS space is free from any instabilities for  $|m^2| \leq 4k^2$  [67] and corresponds to the scaling dimension of the operator in the dual theory being relevant. If the mass term is small and negative, the deformation is relevant, but close to marginal. This allows the coefficient of this relevant operator to start at a small value at high energies and grow slowly, leading to a large hierarchy before it eventually

becomes strong enough to trigger breaking of the conformal symmetry. This is the scenario we shall focus on.

Higher order terms in the bulk potential correspond to corrections to the scaling behavior of the dual operator that become important when the deformation grows large. As the value of  $\Phi$  becomes large close to the IR brane, the higher order interaction terms are expected to dominate over the suppressed mass term unless they are also small from symmetry considerations, as in the case where  $\Phi$  is a pNGB. For simplicity, we consider a scenario where the detuning of the IR brane tension away from the pure RS solution is slightly below its natural strong coupling value by a factor that could be as small as a few [30]. This allows the extra dimension to be stabilized when the VEV of the GW field in the neighborhood of the IR brane is also slightly below its natural strong coupling value. This limit captures the qualitative features we are interested in, but allows the gravitational backreaction to be neglected. In the dual picture, this corresponds to the assumption that the breaking of conformal symmetry is triggered when the deformation is still slightly below its strong coupling value. For this choice of parameters the cubic self-interaction term in the GW potential is expected to dominate over the other higher order terms. Therefore, in what follows, we keep only the mass and cubic terms in the bulk potential for  $\Phi$  and neglect the higher order terms.

This limit also allows an approximate solution to the equations of motion for  $\hat{\Phi}$ . The equations and the boundary conditions are given by

$$\begin{aligned} \partial_\theta^2 \hat{\Phi} - 4kr_c \partial_\theta \hat{\Phi} - r_c^2 m^2 \hat{\Phi} - r_c^2 \frac{\eta}{2} \hat{\Phi}^2 &= 0 \\ \theta = 0: \hat{\Phi} &= k^{3/2} v, \quad \theta = \pi: \partial_\theta \hat{\Phi} = -\alpha k^{3/2} kr_c. \end{aligned} \quad (2.9)$$

For notational simplicity, we trade the parameters  $m$  and  $\eta$  in the bulk potential of  $\Phi$  for  $\epsilon$  and  $\xi$ , which are given by

$$\epsilon \equiv \frac{m^2}{4k^2}, \quad \xi \equiv \frac{\eta v}{8\sqrt{k}}. \quad (2.10)$$

In the limit that the hierarchy is large,  $kr_c \gg 1$ , the solution of this equation exhibits boundary layer structure [30]. This allows an approximate solution to be obtained using boundary layer analysis. Using these methods, the solution for  $\hat{\Phi}$  is found to be of the form

$$\begin{aligned} \hat{\Phi}(\theta) &= -\frac{k^{3/2} \alpha}{4} e^{-4kr_c(\pi-\theta)} + \hat{\Phi}_{\text{OR}}(kr_c \theta) \\ &= -\frac{k^{3/2} \alpha}{4} e^{-4kr_c(\pi-\theta)} + \frac{k^{3/2} v e^{-\epsilon kr_c \theta}}{1 + \xi(1 - e^{-\epsilon kr_c \theta})/\epsilon}. \end{aligned} \quad (2.11)$$

While we have been specific about the  $kr_c \theta$  dependence of  $\hat{\Phi}_{\text{OR}}$  in the above expression, we shall usually just write

$\hat{\Phi}_{\text{OR}}(\theta)$ . Several features of this classical solution are now apparent:

- (i) The boundary region term, proportional to  $\alpha$ , is exponentially suppressed as long as one is away from the region  $\pi - \theta \lesssim \epsilon$ . This region is the “boundary layer” where the  $\alpha$  term becomes important in the classical solution and  $\hat{\Phi}$  changes very quickly. In the dual 4D theory, this region corresponds to the energy scales at which the phase transition associated with the spontaneous breaking of conformal symmetry occurs.
- (ii) The second term, or outer region solution  $\hat{\Phi}_{\text{OR}}(\theta)$ , depends on the mass  $m^2$  and the cubic coupling  $\eta$ . If we make the cubic coupling small by setting  $\xi$  to zero and work in the limit  $\epsilon < 0$ ,  $|e| \ll 1$ ,  $\hat{\Phi}$  grows slowly with  $\theta$ , allowing a large hierarchy to be realized. As discussed above, a negative mass squared corresponds to the operator in the dual theory having a relevant scaling dimension.
- (iii) In the presence of a nonzero  $\xi$  in  $\hat{\Phi}_{\text{OR}}$ , the VEV again starts small and grows slowly, its growth controlled by the small parameter  $\epsilon$ . For  $\theta$  away from  $\pi$ , the term multiplying  $\xi$  is small and shields the effect of a nonzero  $\xi$ . As  $\theta$  approaches  $\pi$ , however, the presence of  $\xi$  cannot be ignored. Choosing a negative  $\xi$  (and equivalently  $\eta$ ) leads to a faster growth of  $\hat{\Phi}$  as  $\theta$  approaches  $\pi$ . The cubic term is dual to the leading correction to the scaling behavior of the dual operator.

We see that the qualitative features of this classical solution can be understood from holography and allow us to identify the range and sign of parameters in the AdS side of the correspondence. A plot comparing the classical solutions in the presence and absence of the cubic term is shown in Fig. 1. We see that in the presence of the additional cubic interaction,  $\hat{\Phi}$  starts out the same but then grows faster with increasing  $\theta$ .

Once  $r_c$  is made dynamical,  $\hat{\Phi}$  generates a contribution to the radion potential leading to a mass for the radion.<sup>3</sup> The dynamics associated with radion stabilization affects the couplings of the radion field. In general, the GW scalar has contact interactions with the SM fields. Once  $\Phi$  acquires a VEV, these interactions alter the parameters in the low energy theory and correct the radion couplings to SM states. To understand schematically how these effects arise, consider the following operator which couples  $\Phi$  to the SM fields,

<sup>3</sup>The radion potential also receives contributions from the SM gauge bosons and fermions in the bulk through the Casimir effect [68,69]. Since this contribution is loop suppressed, it is much smaller than the classical effect associated with the GW scalar, and can be neglected.

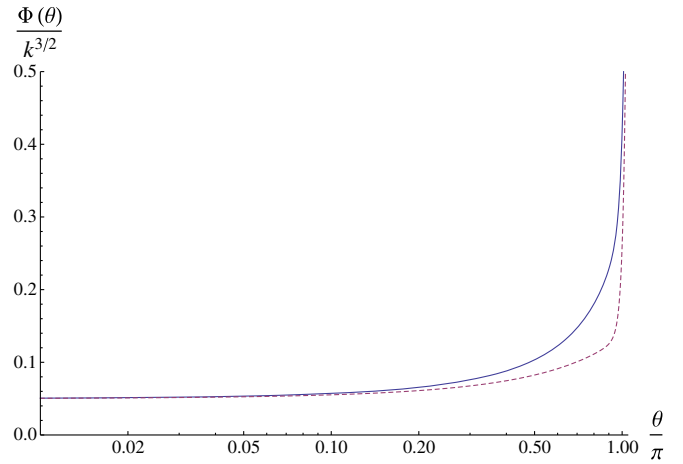


FIG. 1 (color online). A comparison of the classical solution for  $\hat{\Phi}$  in the presence of an additional cubic bulk interaction term (solid line) compared to only having a bulk mass term (dashed), on a log scale for  $\theta$ . The choice of parameters are  $\epsilon = -0.1$ ,  $k\pi r_c = 10$ ,  $v = 0.05$ ,  $\alpha = -0.5$ ,  $\xi = -0.03$ . The effect of the cubic term is negligible near the  $\theta = 0$  boundary, but becomes important for larger  $\theta$ .

$$\mathcal{L} \supset \sqrt{G} \mathcal{O}_{\text{SM}}(x, \theta) \frac{\Phi(\theta)}{k^{3/2}}. \quad (2.12)$$

Here  $\mathcal{O}_{\text{SM}}$  is a gauge invariant operator made of SM fields. The operator in Eq. (2.12) is expected to be present in the absence of any symmetries that prohibit it. In the limit of small backreaction,  $\hat{\Phi}$  is slightly below its natural strong coupling value, so that this operator is expected to dominate over similar terms involving higher powers of  $\hat{\Phi}$ . Replacing  $r_c$  by  $r(x)$  and working in terms of the canonical radion  $\varphi$ ,  $\hat{\Phi}$  can be expanded as

$$\hat{\Phi}(\theta, \varphi(x)) = \hat{\Phi}(\theta, f) + \partial_\varphi \hat{\Phi}(\theta, f)(\varphi - f) + \dots, \quad (2.13)$$

where  $f = \langle \varphi \rangle$ . After substituting this expansion into Eq. (2.12), performing a KK expansion and integrating over the extra dimension, we find that the first term in the  $\hat{\Phi}$  expansion can be absorbed into redefinitions of the parameters of the theory, while the second and subsequent terms generate corrections to the coupling of the radion to the zero modes of the SM fields contained in  $\mathcal{O}_{\text{SM}}$ .

In the next few sections, we determine the couplings of the radion to bulk fields such as SM gauge bosons and fermions, focusing on how operators like Eq. (2.12) modify the leading order story. To write the corrections to radion couplings in a meaningful form, we define  $\frac{d}{d(kr_c, \theta)} \hat{\Phi} \equiv \hat{\Phi}'$ . Then, to leading order in  $\hat{\Phi}'_{\text{OR}}$  and  $e^{-k\pi r_c}$ , we find

$$\frac{m_\varphi^2}{\Lambda_{\text{IR}}^2} = -\frac{\alpha k^3}{6M_5^3} k^{-3/2} \hat{\Phi}'_{\text{OR}}(kr_c \pi), \quad (2.14)$$

where  $m_\varphi$  is the mass of the radion and  $\Lambda_{\text{IR}} \sim ke^{-k\pi r_c}$  is the KK scale. We note that in the two physical limits where the interactions are suppressed ( $\eta \rightarrow 0$ ) or the mass is very small ( $m^2 \rightarrow 0$ ), the expression for  $\hat{\Phi}_{\text{OR}}$  simplifies considerably,

$$\hat{\Phi}(\theta) = -\frac{k^{3/2}\alpha}{4}e^{4kr_c(\theta-\pi)} + \begin{cases} k^{3/2}ve^{-\epsilon kr_c\theta} & , \quad \xi \rightarrow 0 \\ (k^{3/2}v)/(1 + \xi kr_c\theta) & , \quad \epsilon \rightarrow 0 \end{cases}, \quad (2.15)$$

and Eq. (2.14) can be simplified to

$$\begin{aligned} \eta \rightarrow 0: \quad \frac{m_\varphi^2}{\Lambda_{\text{IR}}^2} &= \frac{\alpha k^3}{6M_5^3} \epsilon v e^{-\epsilon kr_c\pi} \\ m^2 \rightarrow 0: \quad \frac{m_\varphi^2}{\Lambda_{\text{IR}}^2} &= \frac{\alpha k^3}{6M_5^3} \frac{\xi v}{(1 + \xi k\pi r_c)^2}. \end{aligned} \quad (2.16)$$

These results may be obtained by analyzing the minimization condition for the radion potential and the expressions for the mass of the radion in each case [30].

### III. MASSLESS GAUGE BOSONS

In this section, we determine the radion couplings to the massless gauge bosons of the SM, the photon and the gluon. We begin by considering the theory in the absence of a stabilization mechanism. The relevant part of the action is given by

$$\mathcal{S} = \int d^4x d\theta \left[ -\frac{\delta(\theta)\sqrt{-G_{\text{UV}}}}{4g_{\text{UV}}^2} F^2 - \frac{\sqrt{G}}{4g_5^2} F^2 - \frac{\delta(\theta-\pi)\sqrt{-G_{\text{IR}}}}{4g_{\text{IR}}^2} F^2 \right], \quad (3.1)$$

where  $F^2 = G^{MK}G^{NL}F_{MN}F_{KL}$  and  $g_{\text{UV}}$ ,  $g_5$ , and  $g_{\text{IR}}$  represent the gauge couplings on the UV brane, in the bulk, and on the IR brane.

After KK decomposition of the 5D action, we find that the spectrum of gauge bosons consists of a massless zero mode and heavier KK modes. The zero mode, which is identified with the corresponding massless gauge boson of the SM, has a flat profile in the extra dimension. To obtain the effective theory involving the massless mode, which we denote by  $A_\mu(x)$ , we simply replace  $A_\mu(x, \theta)$  by  $A_\mu(x)$  in the action and integrate over the extra dimension. Then the Lagrangian for the massless gauge bosons in the 4D effective theory takes the form

$$-\frac{1}{4} \frac{1}{g_4^2} F_{\mu\nu} F^{\mu\nu}, \quad (3.2)$$

where the 4D gauge coupling  $g_4$  at the KK mass scale is related to the underlying parameters of the 5D theory by

$$\frac{1}{g_4^2} = \frac{1}{g_{\text{UV}}^2} + \frac{2\pi r_c}{g_5^2} + \frac{1}{g_{\text{IR}}^2}. \quad (3.3)$$

To obtain the coupling of the zero mode to the radion, we substitute the metric from Eq. (2.2) into Eq. (3.1) and promote  $r_c$  to a dynamical field  $r(x)$ . Expressing the result in terms of the canonically normalized radion field  $\varphi$  and expanding about its VEV  $\langle \varphi \rangle = f$ , we obtain the coupling of the zero mode to the physical radion  $\tilde{\varphi} = \varphi - f$ . The result, in a basis where the gauge kinetic term is normalized as in Eq. (3.2), takes the form [40,41]

$$\frac{1}{2kg_5^2} \tilde{\varphi} F_{\mu\nu} F^{\mu\nu}, \quad (3.4)$$

where indices are raised and lowered using the Minkowski metric  $\eta^{\mu\nu}$ . In contrast to the case of massless gauge bosons localized on the IR brane [38], we see that in this scenario the classical contribution to the coupling does not vanish. In Appendix D we estimate the natural size of the bulk gauge  $g_5$  coupling in units of  $k$ . We find that  $1/2kg_5^2$  is expected to be small,  $1/2kg_5^2 \ll 1$ .

The one-loop quantum contribution to the radion coupling to the massless gauge bosons is also important, potentially comparable in size to the effect in Eq. (3.4). To determine this effect, note that the value of the 4D gauge coupling below the KK scale is in general a function of the background radion field. At low energies, the 4D gauge coupling satisfies a one-loop renormalization group (RG) equation of the form

$$\frac{d}{d \log \mu} \frac{1}{g^2(\mu)} = \frac{b_{<}}{8\pi^2}, \quad \Lambda_{\text{IR}} \geq \mu \geq 0, \quad (3.5)$$

where  $\Lambda_{\text{IR}}$  represents the cutoff of the 4D effective theory and scales with  $r_c$  as

$$\Lambda_{\text{IR}} \sim m_{\text{KK}} \sim ke^{-k\pi r_c}. \quad (3.6)$$

The value of the gauge coupling at the cutoff  $g(\Lambda_{\text{IR}})$  is identified with  $g_4$  in Eq. (3.3). The quantity  $b_{<}$  receives contribution from the particles in the spectrum below  $\Lambda_{\text{IR}}$  that run in the loops that renormalize the gauge coupling. We can solve Eq. (3.5) to obtain the 4D gauge coupling at scales  $\mu < \Lambda_{\text{IR}}$ :

$$\frac{1}{g^2(\mu)} = \frac{1}{g_4^2} - \frac{b_{<}}{8\pi^2} \log\left(\frac{\Lambda_{\text{IR}}}{\mu}\right). \quad (3.7)$$

To compute the corresponding one-loop contribution to the radion-gauge boson vertex, we promote the parameter  $r_c$  contained in  $\Lambda_{\text{IR}}$  in Eq. (3.7) to a dynamical field and expand about its VEV. The kinetic term in the low energy theory



$$-\frac{1}{4g^2(\mu)}F_{\mu\nu}F^{\mu\nu} \quad (3.8)$$

then generates a coupling to the normalized radion that is given by

$$\frac{b_{<}}{32\pi^2} \frac{\tilde{\varphi}}{f} F_{\mu\nu}^2. \quad (3.9)$$

Combining this with (3.4), the full radion coupling is given by

$$\left(\frac{1}{2kg_5^2} + \frac{b_{<}}{32\pi^2}\right) \frac{\tilde{\varphi}}{f} F_{\mu\nu}F^{\mu\nu}. \quad (3.10)$$

To understand this result from a holographic point of view, recall that the AdS/CFT dictionary relates the bulk coordinate  $\theta$  to the RG scale  $\mu$  in the dual theory. The position of the UV brane corresponds to the cutoff  $\Lambda_{\text{UV}} \sim k$  of the CFT, while the position of the IR brane is associated with the scale  $\Lambda_{\text{IR}} \sim ke^{-k\pi r_c}$ , where the CFT is spontaneously broken. Holography also relates a bulk gauge symmetry in the two brane AdS space to the weak gauging of a global symmetry in the dual CFT [26,27]. In general, this gauge coupling is expected to run with the RG scale:

$$\frac{d}{d \log \mu} \frac{1}{g^2(\mu)} = \frac{b_{>}}{8\pi^2}, \quad \Lambda_{\text{UV}} \geq \mu \geq \Lambda_{\text{IR}}. \quad (3.11)$$

To relate  $b_{>}$  to the parameters of the dual AdS theory, we take the following approach. Consider moving the UV brane from  $\theta = 0$  to an arbitrary point  $\theta = \theta_0$  in the bulk. This corresponds to lowering the cutoff of the theory from  $\Lambda_{\text{UV}} \sim k$  to the scale  $\Lambda_0$ , given by

$$\Lambda_{\text{UV}} \exp(-k\theta_0 r_c) = \Lambda_0. \quad (3.12)$$

The parameter  $b_{>}$  can be determined by studying the corresponding change in the gauge coupling. We split the  $\theta$  integral in the 5D action Eq. (3.1) into two parts, one from 0 to  $\theta_0$  and another from  $\theta_0$  to  $\pi$ ,

$$\mathcal{S} = \mathcal{S}_{\theta < \theta_0} + \mathcal{S}_{\theta > \theta_0}. \quad (3.13)$$

Substituting the zero mode back into the action, we evaluate the contribution to the  $\theta$  integral from  $\theta < \theta_0$  and match to the theory with the lower cutoff. This determines the correction to the brane localized kinetic term localized at  $\theta_0$ ,

$$\frac{1}{g_{\text{UV}}^2(\theta_0)} = \frac{1}{g_{\text{UV}}^2} + \frac{2\theta_0 r_c}{g_5^2}. \quad (3.14)$$

The effective 4D gauge coupling at the scale  $\Lambda_0$ , which we denote by  $g_4(\Lambda_0)$ , is equal to  $g_{\text{UV}}(\theta_0)$  (up to small corrections of order  $g_{\text{UV}}^2/kg_5^2$ ). This allows us to obtain the beta function at the scale  $\Lambda_0$ ,

$$\begin{aligned} \frac{b_{>}}{8\pi^2} &\equiv \frac{d}{d \log \Lambda_0} \frac{1}{g_{\text{UV}}^2(\Lambda_0)} \\ &= -\frac{1}{kr_c} \frac{d}{d\theta_0} \frac{1}{g_{\text{UV}}^2(\theta_0)} = -\frac{2}{kg_5^2}. \end{aligned} \quad (3.15)$$

Notice that the expression for  $b_{>}$  is independent of  $\Lambda_0$ . Using this, we can rewrite Eq. (3.10) as<sup>4</sup>

$$\frac{b_{<} - b_{>}}{32\pi^2} \frac{\tilde{\varphi}}{f} F_{\mu\nu}F^{\mu\nu}. \quad (3.16)$$

This expression agrees with results obtained directly from the CFT side of the correspondence [11,12].

We now include the effects of stabilization. In the dilaton case, the corrections to the form of Eq. (3.16) arising from the explicit breaking of the CFT are one-loop suppressed and scale as  $m_\varphi^2/\Lambda_{\text{IR}}^2$ , where  $m_\varphi$  is now the dilaton mass and  $\Lambda_{\text{IR}}$  is the cutoff of the effective theory where we expect composite states to appear. For the radion, the leading corrections arise from direct couplings of the GW scalar to gauge bosons in the bulk and on the branes. To leading order in  $\Phi$ , the effect is captured by

$$\begin{aligned} \mathcal{L}_{\text{int}} = \frac{\Phi}{k^{3/2}} &\left[ -\beta_{\text{UV}} \frac{\delta(\theta)\sqrt{-G_{\text{UV}}}}{4g_{\text{UV}}^2} F^2 \right. \\ &\left. - \beta \frac{\sqrt{G}}{4g_5^2} F^2 - \beta_{\text{IR}} \frac{\delta(\theta - \pi)\sqrt{-G_{\text{IR}}}}{4g_{\text{IR}}^2} F^2 \right]. \end{aligned} \quad (3.17)$$

Here  $\beta_{\text{UV}}, \beta$  and  $\beta_{\text{IR}}$  are dimensionless numbers. When we replace  $\Phi$  by its VEV and consider fluctuations of the radion about its background value, these interaction terms generate corrections to the 4D gauge coupling in the low energy effective theory, and to the radion coupling to the gauge bosons. Gauge invariance requires that the zero mode  $A_\mu(x)$  continue to have a flat profile even in the presence of the  $\Phi$  terms, but the relationship between the 4D gauge coupling  $g_4$  and the underlying 5D parameters of Eq. (3.3) now becomes

$$\begin{aligned} \frac{1}{g_4^2} &= \frac{1}{g_{\text{UV}}^2} \left( 1 + \beta_{\text{UV}} \frac{\hat{\Phi}(0)}{k^{3/2}} \right) \\ &+ \frac{2r_c}{g_5^2} \int_0^\pi d\theta \left( 1 + \beta \frac{\hat{\Phi}(\theta)}{k^{3/2}} \right) + \frac{1}{g_{\text{IR}}^2} \left( 1 + \beta_{\text{IR}} \frac{\hat{\Phi}(\pi)}{k^{3/2}} \right). \end{aligned} \quad (3.18)$$

When determining the corrections to the couplings of the radion arising from the GW field it is useful to employ the identity

<sup>4</sup>In determining  $b_{>}$  in Eqs. (3.14) and (3.15), we have not taken into account the quantum contributions from states localized on or toward the UV brane. However, because these states contribute equally to  $b_{<}$  in Eq. (3.5), their net contribution to  $b_{>} - b_{<}$  in Eq. (3.16) vanishes. As a consequence, they do not affect the final result.



$$\hat{\Phi}(\theta)|_{r=r_c+\delta r} = \hat{\Phi}(\theta)|_{r=r_c} + \delta r k^{5/2} \alpha(\pi - \theta) e^{-4kr_c(\pi - \theta)} + \delta r \theta k \hat{\Phi}'_{\text{OR}}(\theta kr_c), \quad (3.19)$$

with  $\hat{\Phi}'_{\text{OR}} \equiv \frac{d}{d(kr_c\theta)} \hat{\Phi}_{\text{OR}}$ . After integrating over the extra dimension the contribution to the radion coupling from classical effects is obtained as

$$\left[ \frac{1}{2kg_5^2} \left( 1 + \frac{\beta}{k^{3/2}} \hat{\Phi}_{\text{OR}}(\pi) \right) + \frac{\beta_{\text{IR}}}{4g_{\text{IR}}^2 k^{3/2}} \hat{\Phi}'_{\text{OR}}(\pi) \right] \frac{\tilde{\varphi}}{f} F_{\mu\nu} F^{\mu\nu}. \quad (3.20)$$

In this expression we have dropped the negligible contribution proportional to the  $\alpha$  term in  $\hat{\Phi}$  that only receives support from the boundary region. This classical contribution must be added to the contribution arising from quantum effects, which remains of the same form as Eq. (3.9).

From Eq. (2.14) we see that the final term in brackets in Eq. (3.20) scales as  $m_\phi^2/\Lambda_{\text{IR}}^2$ . However, the other correction term proportional to  $\hat{\Phi}'_{\text{OR}}(\pi)$  does not appear to scale in a simple way with the radion mass. In order to understand the presence of this term, it is useful to consider the holographic interpretation of this scenario. In the dual description, sourcing the GW scalar on the UV brane corresponds to a deformation of the CFT by a primary operator. This deformation affects the RG evolution of the gauge coupling. To understand this in detail, we again need to relate the beta function for the gauge theory above the scale  $\Lambda_{\text{IR}}$ , where the conformal symmetry is broken, to the parameters of the extra dimensional theory. When  $\Phi$  acquires a VEV, the coupling of the GW field to the gauge bosons affects the gauge kinetic terms in the 5D construction, and hence the 4D gauge coupling in the dual theory. Since  $\hat{\Phi}$  depends on the location in the extra dimension, the beta function coefficient  $b_>$  in the dual theory is affected, and now depends on the energy scale.

To determine the new  $b_>$ , we must once again obtain the correction to the brane localized gauge kinetic term as the location of the UV brane is moved. We separate the 5D integral over  $\theta$  into two parts, one from 0 to  $\theta_0$  and another from  $\theta_0$  to  $\pi$ . After integrating out the part of the extra dimension corresponding to  $\theta < \theta_0$ , we match to the theory with the lower cutoff. Then the gauge coupling at the scale  $\Lambda_0$  corresponding to  $\theta = \theta_0$  is given by

$$\frac{1}{g_{\text{UV}}^2(\Lambda_0)} = \frac{1}{g_{\text{UV}}^2} + \frac{2r_c}{g_5^2} \int_0^{\theta_0} d\theta \left[ 1 + \beta \frac{\hat{\Phi}}{k^{3/2}} \right]. \quad (3.21)$$

The beta function in 4D dual theory is given by

$$\frac{b_>}{8\pi^2} \equiv \frac{d}{d \log \Lambda_0} \frac{1}{g_{\text{UV}}^2(\Lambda_0)} = -\frac{2}{kg_5^2} \left( 1 + \frac{\beta}{k^{3/2}} \hat{\Phi}(\theta_0) \right). \quad (3.22)$$

We notice that  $b_>$  now depends on  $\theta_0$ , and hence on  $\Lambda_0$ . The form of the contribution from scales below  $\Lambda_{\text{IR}}$  remains unaffected by the addition of the GW scalar. Therefore, the form of the term proportional to  $b_<$  is unchanged.

By taking the limit  $\theta_0 \rightarrow \pi - \frac{1}{kr_c}$  in Eq. (3.22) we can, in the limit of large  $kr_c$ , neglect the effects of boundary region of  $\hat{\Phi}$  and obtain the value of  $b_>$  just above the breaking scale. The full radion coupling is then obtained by combining this result with Eq. (3.20) and Eq. (3.9) as

$$\left[ \frac{b_< - b_>}{32\pi^2} + \frac{k^{-3/2}}{2} \hat{\Phi}'_{\text{OR}}(\pi) \left( \frac{\beta}{kg_5^2} + \frac{\beta_{\text{IR}}}{2g_{\text{IR}}^2} \right) \right] \frac{\tilde{\varphi}}{f} F_{\mu\nu}^2. \quad (3.23)$$

It follows from Eq. (3.20) that, in general, the correction to the radion couplings from effects associated with stabilization of the extra dimension can be large. However, we see from Eq. (3.22) that in the presence of the GW scalar, the identification of  $b_>$  on the CFT side of the correspondence in terms of parameters on the AdS side is also modified. As can be seen from Eq. (3.23), when this effect is incorporated the correction to the form of the radion coupling is proportional to  $\hat{\Phi}'_{\text{OR}}(\pi)k^{-3/2}$ . From Eq. (2.14) it follows that this scales as  $m_\phi^2/\Lambda_{\text{IR}}^2$ , in agreement with results from the CFT side of the correspondence. It also follows from naive dimensional analysis (NDA) estimates of the sizes of the brane and bulk gauge couplings (see Appendix D for details) that the overall size of the correction is parametrically one-loop suppressed. This differs from the case of brane localized gauge bosons [30], but agrees with the dual result for elementary gauge bosons in the 4D CFT [11].

#### IV. MASSIVE GAUGE BOSONS

In this section we determine the corrections to the radion couplings to the massive gauge bosons of the SM, the  $W^\pm$  and the  $Z$ . As in the previous section, we consider SM gauge bosons residing in the bulk of the space and the SM Higgs field  $H$  localized on the IR brane. For simplicity, we assume that Higgs-radion mixing is absent, which is natural if, for example, the Higgs is a pNGB. When the Higgs acquires a VEV, the  $W^\pm$  and  $Z$  gauge bosons become massive. The coupling of the radion to the field strength tensor squared can be determined just as in Sec. III. In this case, however, because the gauge symmetry is broken, the radion can also have a nonderivative coupling to the gauge fields of the form  $\varphi W_\mu W^\mu$ . Since this is an operator of lower dimension than  $\varphi F_{\mu\nu} F^{\mu\nu}$ , it constitutes the dominant effect at low energies. In this section, we focus on couplings of this form.

We first determine the couplings in the absence of stabilization. The action, in addition to gauge kinetic terms of Eq. (3.1), includes the brane localized operator

$$\mathcal{S} \supset \int d^4x d\theta \delta(\theta - \pi) \sqrt{-G_{\text{IR}}} G_{\text{IR}}^{\mu\nu} (\mathcal{D}_\mu H) (\mathcal{D}_\nu H)^\dagger, \quad (4.1)$$

where  $\mathcal{D}_\mu = \partial_\mu - iW_\mu$  represents the gauge covariant derivative, and  $W_\mu$  represents any massive gauge boson. After replacing  $H$  by its VEV, the operator in Eq. (4.1) generates a mass  $m_W$  for the zero mode gauge bosons  $W_\mu$ . To zeroth order in  $m_W^2/\Lambda_{\text{IR}}^2$ , the profile for the zero mode gauge boson is a constant [70,71]. Therefore, to obtain the couplings of the zero mode  $W_\mu(x)$ , we can simply replace  $W_\mu(x, \theta)$  by  $W_\mu(x)$  in the action and integrate over the extra dimension.

To determine the coupling of the radion to the zero mode, we follow the same steps as in Sec. III. The leading coupling in this case comes from the operator Eq. (4.1) itself and is given by [40,41]

$$\frac{2m_W^2 \tilde{\varphi}}{g_4^2 f} W_\mu W^\mu, \quad (4.2)$$

where the index on  $W$  is raised by  $\eta_{\mu\nu}$ . We see that this coupling has the same form as for the case when  $W_\mu$  is localized on the visible brane.

In the presence of the GW scalar  $\Phi$ , there are additional operators in the action involving couplings between the gauge bosons and  $\Phi$ . In addition to operators of the form Eq. (3.17) that lead to corrections to the 4D gauge coupling as in Eq. (3.18), we consider the operator

$$\mathcal{L}_{\text{int}} = \beta_W \sqrt{-G_{\text{IR}}} \delta(\theta - \pi) G_{\text{IR}}^{\mu\nu} (\mathcal{D}_\mu H) (\mathcal{D}_\nu H)^\dagger \frac{\Phi}{k^{3/2}} \quad (4.3)$$

where  $\beta_W$  is a dimensionless number. When  $\Phi$  gets a VEV, this term corrects the mass  $m_W$  of the bulk gauge boson, which is now given by

$$m_W^2 = \hat{m}_W^2 \left( 1 + \beta_W \frac{\hat{\Phi}(\pi)}{k^{3/2}} \right). \quad (4.4)$$

In this expression  $\hat{m}_W$  represents the gauge boson mass that arises from Eq. (4.1) in the absence of the correction term Eq. (4.3). In the presence of this operator, the coupling of the radion also receives corrections taking the form

$$\frac{m_W^2 \tilde{\varphi}}{g_4^2 f} W^2 \left[ 2 - \frac{\beta_W \hat{\Phi}'_{\text{OR}}(\pi)}{k^{3/2} + \beta_W \hat{\Phi}(\pi)} \right]. \quad (4.5)$$

In this expression  $m_W$  and  $g_4$  are the corrected mass and gauge coupling. From Eq. (2.14) we see that the correction term scales as  $m_\varphi^2/\Lambda_{\text{IR}}^2$ , and is small if the radion is light.

## V. BULK FERMIONS

In this section we determine the couplings of the radion to SM fermions. For concreteness we focus on the interactions of the radion with the up-type quarks, the generalization to other SM fermions being straightforward. We consider a scenario where these fields emerge from bulk

fermions  $\mathcal{Q}$  and  $\mathcal{U}$ , and obtain their masses from a brane-localized Higgs  $H$ . As in the previous sections, we first obtain the radion couplings in the absence of a stabilization mechanism, and then we show how these results are modified in the presence of the GW field. We also obtain the holographic interpretation of the results.

### A. Radion couplings in the absence of a stabilization mechanism

In the absence of any dynamics that fixes the brane spacing, the relevant part of the action takes the form

$$\int d^4x \int_0^\pi d\theta \left[ \sqrt{G} \left( \frac{i}{2} e_a^M \bar{\mathcal{Q}} \Gamma^a \overleftrightarrow{\partial}_M \mathcal{Q} - k c_q \bar{\mathcal{Q}} \mathcal{Q} + \mathcal{Q} \rightarrow \mathcal{U} \right) + \sqrt{-G_{\text{IR}}} \delta(\theta - \pi) \left( \frac{Y}{k} \bar{\mathcal{Q}} H \mathcal{U} + \text{H.c.} \right) \right], \quad (5.1)$$

where  $\overleftrightarrow{\partial} \equiv \overrightarrow{\partial} - \overleftarrow{\partial}$ . The dimensionless parameters  $Y$  and  $c_q, c_u$  represent the brane localized Yukawa coupling and the bulk mass parameters for the 5D fermions respectively. For simplicity, we suppress all flavor indices. The  $e_a^M$  represent the vielbein and  $\Gamma^a$  the matrices that realize the 5D Clifford algebra.

In the absence of a VEV for the brane-localized Higgs, the boundary conditions on the 5D fermions  $\mathcal{Q}$  and  $\mathcal{U}$  are chosen such that each has a zero mode with the appropriate chirality. These zero modes are identified with the corresponding massless quarks in the SM before electroweak symmetry breaking. Once the Higgs gets a VEV, these modes acquire a mass. At the same time, a mixing is induced between the zero mode of  $\mathcal{Q}$  and the KK modes of  $\mathcal{U}$  and vice versa. As a result, the zero modes are not mass eigenstates. To work in a mass diagonal basis, a ‘‘mixed’’ KK decomposition can be performed [72]. In this basis the 5D fermions are expanded as

$$\begin{aligned} \mathcal{Q}(x, \theta) &= \begin{pmatrix} \mathcal{Q}_L(x, \theta) \\ \mathcal{Q}_R(x, \theta) \end{pmatrix} \\ &= \begin{pmatrix} \mathcal{Q}_L^0(\theta) q_L^0(x) + \mathcal{Q}_L^1(\theta) q_L^1(x) + \dots \\ \mathcal{Q}_R^0(\theta) u_R^0(x) + \mathcal{Q}_R^1(\theta) q_R^1(x) + \dots \end{pmatrix} \\ \mathcal{U}(x, \theta) &= \begin{pmatrix} \mathcal{U}_L(x, \theta) \\ \mathcal{U}_R(x, \theta) \end{pmatrix} \\ &= \begin{pmatrix} \mathcal{U}_L^0(\theta) q_L^0(x) + \mathcal{U}_L^1(\theta) u_L^1(x) + \dots \\ \mathcal{U}_R^0(\theta) u_R^0(x) + \mathcal{U}_R^1(\theta) u_R^1(x) + \dots \end{pmatrix}, \quad (5.2) \end{aligned}$$

where the subscripts  $L, R$  refer to 4D chiralities. In our notation the lower case letters represent the 4D fields, while the upper case letters represent their profiles in the bulk. The superscripts 0, 1, ... refer to the mode number in the

KK expansion. Using this expansion, the 4D fields in the spectrum are

$$(q_L^0, u_R^0), (q_L^1, q_R^1), (u_L^1, u_R^1), \dots \quad (5.3)$$

Notice that the first term in the KK expansion of  $\mathcal{Q}_R$  contains the 4D field  $u_R$  which is where the mixed nature of KK decomposition manifests itself. The profiles  $Q_{L,R}^i, U_{L,R}^i$  can be solved for in this decomposition, and the details are given in Appendix B. The calculation of the profiles fixes the mass  $m_f$  for the pair  $(q_L^0, u_R^0)$  and the masses of the other KK modes in terms of the parameters of the 5D theory. We will take the KK scale  $m_{\text{KK}} \sim ke^{-k\pi r_c}$  to be parametrically larger than the zero mode fermion masses  $m_f$  and work to lowest order in  $m_f/m_{\text{KK}}$ .

To obtain the coupling of the radion to the zero modes, we write the 5D metric  $G_{MN}$  and the vielbein  $e_a^M$  in terms of  $\varphi$  and expand about the VEV  $\langle \varphi \rangle = f$ . Using the expressions for the profiles, the couplings of the radion can be determined as shown in Appendix B. The final result takes the form

$$\mathcal{L} \supseteq -m_f(I_q + I_u) \frac{\tilde{\varphi}}{f} (q_L^\dagger u_R + \text{H.c.}), \quad (5.4)$$

where  $I_q, I_u$  are dimensionless numbers given by an overlap integral involving the profiles, and depend on the dimensionless 5D mass parameters  $c_q, c_u$  respectively.<sup>5</sup> In what follows we will choose positive chirality for  $\mathcal{Q}$  and negative chirality for  $\mathcal{U}$ , so the expression for  $I_u$  may be obtained from that for  $I_q$  by making the replacement  $c_q \rightarrow -c_u$ . To leading order in  $e^{-k\pi r_c}$ , the quantity  $I_q$  is

$$I_q = \frac{1/2 - c_q}{1 - e^{-(1-2c_q)kr_c\pi}} + c_q \approx \begin{cases} c_q, & c_q > \frac{1}{2} \\ \frac{1}{2}, & c_q < \frac{1}{2} \end{cases} \quad (5.5)$$

where we have taken the two limits in which the expression simplifies considerably. Therefore, if  $c_q < 1/2$  and  $c_u > -(1/2)$ , the radion coupling scales as  $-m_f(c_q - c_u)$ . In the opposite regime,  $c_q > 1/2$  and  $c_u < -(1/2)$ , the coupling scales as  $-m_f$ . This agrees with the existing results in the literature [41].

How do we understand this result from the dual point of view? Recall that AdS/CFT relates the extra-dimensional coordinate  $\theta$  to the RG scale  $\mu$  in the dual theory. A 5D fermion  $\Psi$  in AdS space corresponds to a fermionic CFT operator  $\mathcal{O}_\Psi$ . The value of the fermion field  $\Psi$  at the boundary of AdS space, which we denote by  $q_s(x)$ , is

identified with the source for the operator  $\mathcal{O}_\Psi$ . Therefore, the 4D CFT Lagrangian contains the term

$$\delta\mathcal{L} = \Psi(x)|_{\text{AdS boundary}} \mathcal{O}_\Psi(x) \equiv q_s(x) \mathcal{O}_\Psi(x). \quad (5.6)$$

Because the 4D Dirac equation is first order, the boundary condition for the 5D field  $\Psi$  must be subject to a chiral projection relating the left- and right-handed chiralities. Therefore only one of the two chiralities can be identified with the source. The 5D (dimensionless) mass parameter  $c_\Psi$  is related to the scaling dimension  $\Delta_\Psi$  of  $\mathcal{O}_\Psi$  by

$$\Delta_\Psi = \left| c_\Psi \pm \frac{1}{2} \right| + \frac{3}{2}, \quad (5.7)$$

where  $\Psi = \mathcal{Q}, \mathcal{U}$  and the  $\pm$  denotes the two choices for the chirality of the source [73].

The correspondence can be extended to the case of AdS space with two branes, thereby allowing a holographic interpretation of our results. The source  $q_s(x)$  now becomes dynamical, being promoted to an elementary field that couples weakly to the CFT. Since, in general, the coupling to an elementary field constitutes an explicit breaking of the CFT, we expect that other conformal symmetry violating operators will be generated and will be present in the theory at an arbitrary renormalization scale  $\mu$ . These are represented by higher dimensional operators on the UV brane that are suppressed by powers of  $k$ . The operator in Eq. (5.6) generates a mixing between the CFT states and elementary field  $q_s(x)$ . The presence of the IR brane in AdS corresponds to the spontaneous breaking of the CFT, and leads to a mass gap in the spectrum. As a consequence of Eq. (5.6), the mass eigenstates are mixtures of the elementary state  $q_s(x)$  and composites that arise from the CFT dynamics.

In the mass diagonal basis prior to electroweak symmetry breaking, the spectrum in the 4D theory contains a massless chiral fermion corresponding to the zero mode of the 5D field. The localization of the zero mode in AdS space is governed by the mixing between  $q_s$  and  $\mathcal{O}_\mathcal{Q}$  in the dual picture. If the scaling dimension  $\Delta_\mathcal{Q}$  is less than  $5/2$ , the operator in Eq. (5.6) is relevant and therefore large at low energies. As a result, the massless mode is mostly composite. Using Eq. (5.7), this corresponds to the case of the corresponding 5D mass parameter being less than  $1/2$  (we are focusing on the case of  $\mathcal{Q}$  which has positive chirality) and results in the zero mode being localized toward the IR brane. Similarly, if the scaling dimension  $\Delta_\mathcal{Q}$  is more than  $5/2$ , the operator in Eq. (5.6) becomes irrelevant, and, as a result, the mixing is small at low energies. Consequently, the massless mode is mostly elementary and corresponds to the 5D mass parameter  $c_\Psi$  being greater than  $1/2$  using Eq. (5.7). This translates into the zero mode being localized toward the UV brane.

<sup>5</sup>Throughout our analysis, we work in the regime where  $c_q > -1/2$  and  $c_u < 1/2$ , because in the opposite regime, the behavior of the zero mode spectrum is qualitatively different [73].

The form of the coupling of the dilaton to light fermions has been obtained [11,12], both for the case when the fermions are mostly composite and the case when they are mostly elementary. In the first case, the dilaton coupling simply scales like  $m_f$ , which agrees with the result from Eq. (5.5). In the other case, the coupling scales as  $m_f(\Delta_Q + \Delta_U - 4)$ . This coefficient can be rewritten as

$$\Delta_Q + \Delta_U - 4 = c_q - c_u = I_q + I_u \quad (5.8)$$

for  $c_q > 1/2$ ,  $c_u < -1/2$

where the first equality employs Eq. (5.7), and the second Eq. (5.5). The case of one composite and one elementary fermion is a straightforward generalization. From this analysis, we see that in each case the radion coupling agrees with the corresponding result for the dilaton in the literature.

### B. Corrections arising from stabilization

We now include the effects of stabilizing the extra dimension. In general, the fields  $\mathcal{Q}$  and  $\mathcal{U}$  will couple to the GW field  $\Phi$  in the bulk, resulting in corrections to the radion couplings. To leading order in  $\Phi$ , the interactions of the bulk fermions with the GW scalar are of the form

$$\frac{\Phi}{k^{3/2}} \left[ \sqrt{G} \left( \alpha_q \frac{i}{2} e_a^M \bar{\mathcal{Q}} \Gamma^a \overleftrightarrow{\partial}_M \mathcal{Q} - \beta_q k c_q \bar{\mathcal{Q}} \mathcal{Q} + \mathcal{Q} \rightarrow \mathcal{U} \right) + \sqrt{-G_{\text{IR}}} \delta(\theta - \pi) \alpha_y \left( \frac{Y}{k} \bar{\mathcal{Q}} H U + \text{H.c.} \right) \right] \quad (5.9)$$

where  $\alpha_q$ ,  $\alpha_u$ ,  $\beta_q$ ,  $\beta_u$  and  $\alpha_y$  are dimensionless couplings whose natural sizes are estimated in Appendix D. To calculate the coupling of the radion to the zero modes in the presence of these terms, we follow the same steps as before. First, we replace  $\Phi$  by its VEV and perform the mixed KK decomposition. This fixes the mass  $m_f$  for the zero mode pair  $(q_L^0, u_R^0)$  and the KK modes and determines the profiles in terms of the other theory parameters. Next, we consider fluctuations of  $\Phi$  about its VEV associated with fluctuations of the background radion field. The operators in Eq. (5.9) generate corrections to the radion coupling of Eq. (5.4). The details of the calculation are in Appendix B. Including these effects, the coupling has the form

$$\mathcal{L} \supseteq -m_f (I_q + I_u + I_h) \frac{\tilde{\varphi}}{f} (q_L^\dagger u_R + \text{H.c.}). \quad (5.10)$$

The quantities  $I_q$  and  $I_u$  again arise from overlap integrals involving the profiles and reduce to the results in Eq. (5.5) when  $\hat{\Phi}$  is set to zero. The quantity  $I_h$  originates from the brane localized term involving both  $\Phi$  and  $H$  and also vanishes if the  $\hat{\Phi}$  is set to zero. It is given by

$$I_h = -\frac{\hat{\Phi}'_{\text{OR}}(\pi)}{2} \left[ \frac{2\alpha_y}{k^{3/2} + \alpha_y \hat{\Phi}(\pi)} - \frac{\alpha_q}{k^{3/2} + \alpha_q \hat{\Phi}(\pi)} - \frac{\alpha_u}{k^{3/2} + \alpha_u \hat{\Phi}(\pi)} \right] \equiv -\frac{\hat{\Phi}'_{\text{OR}}(\pi)}{2k^{3/2}} X_h \quad (5.11)$$

where  $X_h$  is expected to be of order one by NDA as shown in Appendix D.

We next compute  $I_q$ , and, as before, the expression for  $I_u$  is obtained from  $I_q$  by making the replacement  $c_q \rightarrow -c_u$ ,  $\alpha_q \rightarrow \alpha_u$ ,  $\beta_q \rightarrow \beta_u$ . It is important to take into account the fact that, in addition to inducing the direct coupling of the radion to the fermions,  $\hat{\Phi}$  also modifies the leading order fermion bulk profiles. In Appendix B we obtain the solution for  $I_q$  taking all these effects into account. The result may be found in Eq. (B31).

We focus our attention on the phenomenologically interesting cases where the fermion profiles are peaked toward either the UV or IR brane. These represent fermions that are either mostly elementary or mostly composite, and correspond to generalizations of the unstabilized analysis considered previously. We shall refer to these cases as being UV localized or IR localized respectively. We define the quantity

$$\tilde{c}_q \equiv c_q + c_q \frac{(\beta_q - \alpha_q) \hat{\Phi}'_{\text{OR}}(\pi)}{k^{3/2} + \alpha_q \hat{\Phi}_{\text{OR}}(\pi)}. \quad (5.12)$$

To leading order in  $\frac{d}{d(kr_c\theta)}$   $\hat{\Phi} \equiv \hat{\Phi}'$  and  $e^{-k\pi r_c}$ ,  $I_q$  is given by

$$I_q = \begin{cases} \tilde{c}_q & , \text{UV Localized} \\ \frac{1}{2} + \frac{\tilde{c}_q (\beta_q - \alpha_q) \hat{\Phi}'_{\text{OR}}(\pi) k^{3/2}}{(1 - 2\tilde{c}_q)(k^{3/2} + \alpha_q \hat{\Phi}_{\text{OR}}(\pi))(k^{3/2} + \beta_q \hat{\Phi}_{\text{OR}}(\pi))} & , \text{IR Localized} \end{cases} \quad (5.13)$$

Now that the functions  $I_q$ ,  $I_u$  and  $I_h$  have been determined, the fermion coupling to the radion can be determined from Eq. (5.10). In the case of IR localized profiles, the sum of the  $I$  functions is given by

$$1 - \frac{\hat{\Phi}'_{\text{OR}}(\pi)}{k^{3/2}} \times \left[ \frac{X_h}{2} - \frac{\tilde{c}_q (\beta_q - \alpha_q) k^3}{(1 - 2\tilde{c}_q)(k^{3/2} + \alpha_q \hat{\Phi}_{\text{OR}}(\pi))(k^{3/2} + \beta_q \hat{\Phi}_{\text{OR}}(\pi))} + \frac{\tilde{c}_u (\beta_u - \alpha_u) k^3}{(1 + 2\tilde{c}_u)(k^{3/2} + \alpha_u \hat{\Phi}_{\text{OR}}(\pi))(k^{3/2} + \beta_u \hat{\Phi}_{\text{OR}}(\pi))} \right]. \quad (5.14)$$



We see that the corrections to the unstabilized result of Eqs. (5.4) and (5.5) scale as  $\hat{\Phi}'_{\text{OR}}(\pi)k^{-3/2}$ . From Eq. (2.14), it follows that the corrections to the leading result are proportional to  $m_\phi^2/\Lambda_{\text{IR}}^2$ , in good agreement with the CFT side of the correspondence.

We now turn to UV localized profiles. The sum of the  $I$  functions in this case is given by

$$(\tilde{c}_q - \tilde{c}_u) - \frac{\hat{\Phi}'_{\text{OR}}(\pi)}{2k^{3/2}} X_h \quad (5.15)$$

This contains two distinct types of corrections to the unstabilized result. The term proportional to  $\hat{\Phi}'_{\text{OR}}(\pi)k^{-3/2}$  scales as  $m_\phi^2/\Lambda_{\text{IR}}^2$ , in line with our expectations from holography. On the other hand, the difference between  $c$  and  $\tilde{c}$  contains a correction term proportional to  $\hat{\Phi}_{\text{OR}}(\pi)$ . This term is expected to be somewhat large, and does not scale in a simple way with the radion mass. To understand this result, we consider the holographic dual of this scenario. In the presence of operators such as Eq. (5.9), the relation between  $c_\Psi$  and  $\Delta_\Psi$  is modified from Eq. (5.7). Specifically, the effective scaling dimension  $\Delta_\Psi$  changes with the RG scale and the corrections to its value become large close to the breaking scale. This effect must be taken into account when relating  $\Delta_\Psi$  at the breaking scale to  $c_\Psi$ . The details of the calculation are presented in Appendix C and follow the approach presented in [73]. We find that to leading order in  $\hat{\Phi}'_{\text{OR}}$ , Eq. (5.7) generalizes to

$$\begin{aligned} \Delta_\Psi &= \left| c_\Psi \left[ 1 + \frac{(\beta_\Psi - \alpha_\Psi)\hat{\Phi}_{\text{OR}}(\pi)}{k^{3/2} + \alpha_q\hat{\Phi}'_{\text{OR}}(\pi)} \right] \pm \frac{1}{2} \right| + \frac{3}{2} \\ &\quad + \mathcal{O}(k^{-3/2}\hat{\Phi}'_{\text{OR}}) \\ &= \left| \tilde{c}_\Psi \pm \frac{1}{2} \right| + \frac{3}{2} + \mathcal{O}(k^{-3/2}\hat{\Phi}'_{\text{OR}}) \end{aligned} \quad (5.16)$$

where  $\Psi = \mathcal{Q}, \mathcal{U}$  and the operator dimension  $\Delta_\Psi$  in this expression is understood to be evaluated close to the symmetry breaking scale. As before, the  $\pm$  denotes the two choices of chirality, and we choose it to be positive for  $\mathcal{Q}$  and negative for  $\mathcal{U}$ .

Using this modified relation and neglecting terms of order  $k^{-3/2}\hat{\Phi}'_{\text{OR}}$ , we find that

$$\begin{aligned} c_q \left[ 1 + \frac{(\beta_q - \alpha_q)\hat{\Phi}_{\text{OR}}(\pi)}{k^{3/2} + \alpha_q\hat{\Phi}'_{\text{OR}}(\pi)} \right] - c_u \left[ 1 + \frac{(\beta_u - \alpha_u)\hat{\Phi}_{\text{OR}}(\pi)}{k^{3/2} + \alpha_u\hat{\Phi}'_{\text{OR}}(\pi)} \right] \\ = \tilde{c}_q - \tilde{c}_u = \Delta_{\mathcal{Q}} + \Delta_{\mathcal{U}} - 4. \end{aligned} \quad (5.17)$$

As a result, the large term that scales as  $\hat{\Phi}_{\text{OR}}(\pi)$  for UV profiles in Eq. (5.15) is absorbed into  $\Delta_{\mathcal{Q}}$  and  $\Delta_{\mathcal{U}}$  when the dilaton coupling is written in terms of the operator dimensions at the breaking scale. The remaining

corrections scale as  $\hat{\Phi}'_{\text{OR}}(\pi)k^{-3/2} \sim m_\phi^2/\Lambda_{\text{IR}}^2$ , as expected from the CFT side of the correspondence [11].

In summary, we see that the leading order radion couplings to bulk SM fermions correspond to dilaton interactions that scale either as  $m_f$  or as  $m_f(\Delta_{\mathcal{Q}} + \Delta_{\mathcal{U}} - 4)$ , depending on whether the SM fermions are mostly composite or mostly elementary. In the presence of the GW field, the identification of  $\Delta_{\mathcal{Q}}$  and  $\Delta_{\mathcal{U}}$  with parameters in the dual 5D theory receives corrections. When this effect is taken into account, the leading corrections to the form of the dilaton interaction are found to scale as  $m_\phi^2/\Lambda_{\text{IR}}^2$ , in good agreement with results from the CFT side of the correspondence.

## VI. CONCLUSION

The AdS/CFT correspondence is a powerful tool that can help us understand the dynamics of strongly coupled 4D theories by studying their weakly coupled higher dimensional duals. A particularly interesting laboratory to study the duality is in the context of the explicit and spontaneous breaking of the isometries of the extra dimensions, corresponding to the spontaneous breaking of an approximate conformal symmetry in the 4D theory. The spontaneous breaking gives rise to an associated Goldstone boson, the radion in the extra dimension and dilaton in the CFT. In this work we have studied the interactions of a radion in a class of theories of phenomenological interest, specifically RS models with the SM gauge and matter fields in the bulk. We have compared the results against those in the literature for the dilaton, finding good agreement.

In the absence of a stabilization mechanism for the extra dimension such as the GW framework [29], the form of the radion couplings is determined by diffeomorphism invariance. Here, we have computed the corrections to these couplings that arise from the stabilization mechanism. We have focused on the phenomenologically interesting case where the radion is somewhat lighter than the KK states associated with the extra dimension. We have extended the analysis of [30], which was restricted to the scenario when all the SM fields were localized to the IR brane, to the case when the gauge bosons and fermions of the SM reside in the bulk of the extra dimension. These corrections primarily arise from direct couplings of the GW scalar to the SM fields of the form shown schematically in Eq. (1.1).

We have obtained a detailed interpretation of our results in terms of the holographic dual of the radion, the dilaton. In doing so, we have taken into account the fact that the familiar identification of the parameters on the two sides of the AdS/CFT correspondence is modified in the presence of couplings of the bulk SM fields to the GW scalar. As in the case of brane-localized SM fields, we find that corrections to the form of the dilaton couplings to these states are suppressed by the square of the ratio of the dilaton mass to the KK scale. These effects are therefore parametrically

small in the limit of a light radion, in good agreement with the corresponding results for the dilaton [11].

### ACKNOWLEDGMENTS

Z. C., R. K. M., and C. B. V. are supported by the NSF under Grant No. PHY-1315155.

### APPENDIX A: RADION MIXING WITH THE NEW GW FIELD

In general, the GW stabilization mechanism will affect the radion profile, leading to corrections to its couplings. In the KK picture, these changes in the radion wave function arise from mixing between the radion and other states after stabilization. In the limit of small backreaction, the leading corrections to the radion profile are expected to arise from mixing with the KK modes of the GW scalar, rather than from mixing with the graviton or its KK modes. The physical radion state is then a linear combination of the graviscalar and these heavy scalar fields. Consequently, the radion couplings to the SM fields receive corrections.

In this appendix we determine the size of these effects. In particular, we show that they are smaller than the corrections that arise from direct couplings of the GW scalar to SM fields.

We begin from the action for the GW scalar

$$\mathcal{S}_{\text{GW}} = \int d^4x d\theta \left[ \sqrt{G} \left( \frac{1}{2} G^{AB} \partial_A \Phi \partial_B \Phi - V_b(\Phi) \right) - \sum_{i=\text{IR,UV}} \delta(\theta - \theta_i) \sqrt{-G_i} V_i(\Phi) \right]. \quad (\text{A1})$$

We now make a change of variables from  $\Phi(x, \theta)$  to a new variable  $\phi(x, \theta)$ , by making the separation  $\Phi(x, \theta) = \hat{\Phi}(r(x), \theta) + \phi(x, \theta)$ . Here  $\hat{\Phi}(r(x), \theta)$  corresponds to the VEV of  $\Phi$  at the minimum, but with  $r_c$  promoted to the dynamical field  $r(x)$ . Having made this change of variables, we substitute for  $\Phi(x, \theta)$  in the action. Because  $\hat{\Phi}$  satisfies the classical equations of motion, several terms in the action cancel. We are left with

$$\mathcal{S}_{\text{GW}} = \int d^4x d\theta \left[ -\frac{e^{-4kr\theta}}{2r} \partial_\theta \phi \partial_\theta \phi + \frac{r e^{-2kr\theta}}{2} (\partial_\mu \hat{\Phi} \partial^\mu \hat{\Phi} + 2\partial_\mu \hat{\Phi} \partial^\mu \phi + \partial_\mu \phi \partial^\mu \phi) - r e^{-4kr\theta} \left( \frac{1}{2} \phi^2 \frac{\partial^2}{\partial \hat{\Phi}^2} V_b(\hat{\Phi}) + \dots \right) - \delta(\theta) \left( \frac{1}{2} \phi^2 \frac{\partial^2}{\partial \hat{\Phi}^2} V_{\text{UV}}(\hat{\Phi}) + \dots \right) - \delta(\theta - \pi) e^{-4kr\pi} \left( \frac{1}{2} \phi^2 \frac{\partial^2}{\partial \hat{\Phi}^2} V_{\text{IR}}(\hat{\Phi}) + \dots \right) \right], \quad (\text{A2})$$

where the  $+\dots$  represent terms higher order in  $\phi$ . We neglect these higher order terms, since their effects are subleading. In addition, we replace  $r(x)$  with  $r_c$  in terms that are quadratic in  $\phi$ , or that involve  $\partial_\mu \hat{\Phi} \sim \partial_\mu r(x)$ , since the effects being neglected are small. After these simplifications, the relevant part of the action takes the form

$$\mathcal{S}_{\text{GW}} = \int d^4x d\theta \left[ -\frac{e^{-4kr_c\theta}}{2r_c} \partial_\theta \phi \partial_\theta \phi + \frac{r_c e^{-2kr_c\theta}}{2} (\partial_\mu \hat{\Phi} \partial^\mu \hat{\Phi} + 2\partial_\mu \hat{\Phi} \partial^\mu \phi + \partial_\mu \phi \partial^\mu \phi) - \frac{r_c}{2} e^{-4kr_c\theta} \phi^2 \frac{\partial^2}{\partial \hat{\Phi}^2} V_b(\hat{\Phi}) - \delta(\theta) \frac{1}{2} \phi^2 \frac{\partial^2}{\partial \hat{\Phi}^2} V_{\text{UV}}(\hat{\Phi}) - \delta(\theta - \pi) e^{-4kr_c\pi} \frac{1}{2} \phi^2 \frac{\partial^2}{\partial \hat{\Phi}^2} V_{\text{IR}}(\hat{\Phi}) \right]. \quad (\text{A3})$$

We see from the form of the action that the only mixing between the light graviscalar and the heavy modes contained in  $\phi$  arises from the kinetic terms. To determine the size of this effect we employ the KK decomposition,  $\phi(x, \theta) = \sum f_n(\theta) \phi_n(x)$  in the classical background,  $r = r_c$ . The profiles  $f_n$  satisfy the equation

$$\partial_\theta (e^{-4kr_c\theta} \partial_\theta f_n) - \frac{r_c^2}{2} e^{-4kr_c\theta} \frac{\partial^2 V_b}{\partial \hat{\Phi}^2} f_n = -m_n r_c^2 e^{-2kr_c\theta} f_n, \quad (\text{A4})$$

subject to the boundary conditions

$$\partial_\theta f_n = r_c f_n \frac{\partial^2 V_{\text{UV}}}{\partial \hat{\Phi}^2} \quad \theta = 0, \quad -\partial_\theta f_n = r_c f_n \frac{\partial^2 V_{\text{IR}}}{\partial \hat{\Phi}^2} \quad \theta = \pi. \quad (\text{A5})$$

It is convenient to normalize these profiles as

$$\int d\theta r_c e^{-2kr_c\theta} f_n f_m = \delta_{nm}. \quad (\text{A6})$$

Then the action reduces to

$$\mathcal{S}_{\text{GW}} = \int d^4x \left[ \sum_n \left( \frac{1}{2} \partial_\mu \phi_n \partial^\mu \phi_n - \frac{m_n^2}{2} \phi_n^2 \right) + \int d\theta \left( \frac{r_c}{2} e^{-2kr_c\theta} \partial_\mu \hat{\Phi} \partial^\mu \hat{\Phi} + \sum_n r_c e^{-2kr_c\theta} f_n \partial_\mu \hat{\Phi} \partial^\mu \phi_n \right) \right]. \quad (\text{A7})$$

At this point we recall that the  $x$  dependence of  $\hat{\Phi}$  arises through  $r(x)$ ,

$$\partial_\mu \hat{\Phi} = \frac{\partial \hat{\Phi}}{\partial r} \partial_\mu r = -\frac{1}{k\pi\varphi} \frac{\partial \hat{\Phi}}{\partial r} \partial_\mu \varphi, \quad (\text{A8})$$

where we have made the change of variable from  $r(x)$  to the canonically normalized radion field  $\varphi = \sqrt{24M_3^3/k} \exp(-k\pi r(x))$ . We see that Eq. (A7) contains, in addition to a correction to the radion kinetic term, a term that generates kinetic mixing between the radion and the KK states of the GW field. Now, from (2.11) we have

$$\frac{\partial \hat{\Phi}}{\partial r} = k^{5/2} \alpha (\pi - \theta) e^{-4kr(\pi - \theta)} + k\theta \hat{\Phi}'_{\text{OR}}. \quad (\text{A9})$$

Under the  $\theta$  integrals the  $\alpha$  term is exponentially suppressed except in the region close to  $\theta = \pi$ , where its coefficient is small. In what follows we neglect this term, since its contribution is small.

The coefficient of the correction to the radion kinetic term is given by

$$\frac{r_c k^3}{\pi^2 \langle \varphi \rangle^2} \int d\theta e^{-2kr_c\theta} \theta^2 k^{-3} \hat{\Phi}'_{\text{OR}}{}^2. \quad (\text{A10})$$

Now, the VEV of  $\Phi$  grows from the UV to the IR, and, in general, so does  $\hat{\Phi}'_{\text{OR}}$ . Therefore, we expect that at some arbitrary point  $\theta$  in the bulk, we have that  $\hat{\Phi}'_{\text{OR}}(kr_c\theta) \lesssim \hat{\Phi}'_{\text{OR}}(kr_c\pi)$ . This allows us to bound (A10) as

$$\begin{aligned} & \frac{r_c k^3}{\pi^2 \langle \varphi \rangle^2} \int d\theta e^{-2kr_c\theta} \theta^2 k^{-3} \hat{\Phi}'_{\text{OR}}{}^2 \\ & \lesssim \frac{r_c k^3}{\pi^2 \langle \varphi \rangle^2} k^{-3} \hat{\Phi}'_{\text{OR}}(kr_c\pi)^2 \int d\theta e^{-2kr_c\theta} \theta^2. \end{aligned} \quad (\text{A11})$$

Up to exponentially suppressed terms the  $\theta$  integral evaluates to  $(2kr_c)^{-2}$ . Noting that  $k^{-3/2} \hat{\Phi}'_{\text{OR}}(kr_c\pi) \sim m_\varphi^2 / \Lambda_{\text{IR}}^2$ , we see that the correction to the radion kinetic term satisfies

$$\frac{r_c k^3}{\pi^2 \langle \varphi \rangle^2} \int d\theta e^{-2kr_c\theta} \theta^2 k^{-3} \hat{\Phi}'_{\text{OR}}{}^2 \lesssim \left( \frac{1}{2\pi \langle \varphi \rangle r_c} \right)^2 \frac{m_\varphi^4}{\Lambda_{\text{IR}}^4}. \quad (\text{A12})$$

Since this correction scales as  $m_\varphi^4 / \Lambda_{\text{IR}}^4$ , we see that its effect on the radion interactions is smaller than the corrections that arise from direct couplings of the GW field to the SM, which scale as  $m_\varphi^2 / \Lambda_{\text{IR}}^2$ .

The mixing term takes the form

$$\begin{aligned} & - \int d^4x \frac{r_c k^{3/2}}{\pi \langle \varphi \rangle} \sum_n f_n \left[ \int d\theta e^{-2kr_c\theta} \theta k^{-3/2} \hat{\Phi}'_{\text{OR}} \right] \partial_\mu \varphi \partial^\mu \phi_n \\ & \equiv \int d^4x \sum_n \kappa_n \partial_\mu \varphi \partial^\mu \phi_n. \end{aligned} \quad (\text{A13})$$

Employing the same methods as in the previous case, we find that the coefficients  $\kappa_n$  of the mixing terms satisfy  $\kappa_n \lesssim m_\varphi^2 / \Lambda_{\text{IR}}^2$ . Upon transforming to a basis where the kinetic terms are diagonal and canonically normalized, we find that

$$\varphi \rightarrow \varphi - \kappa_n \frac{m_n^2}{m_n^2 - m_\varphi^2} \phi_n, \quad \phi_n \rightarrow \phi_n + \kappa_n \frac{m_\varphi^2}{m_n^2 - m_\varphi^2} \varphi. \quad (\text{A14})$$

The mass of the KK states of  $\phi$  is of the order of the IR scale,  $m_n \sim \Lambda_{\text{IR}}$ . Then it follows that the corrections to the radion couplings that arise from mixing with the KK states of the GW scalar scale as  $m_\varphi^4 / \Lambda_{\text{IR}}^4$ , and are, therefore, smaller than the effects from direct couplings to the GW field.

## APPENDIX B: COUPLINGS OF BULK FERMIONS TO THE RADION

In this appendix we determine the form of the radion coupling to bulk fermions in the presence of the GW field, filling in many of the steps outlined in Sec. V. Consistent with the metric Eq. (2.2), we define the vielbein

$$e_a^M = \delta_a^\mu \delta_\mu^M e^{kr_c|\theta|} + \frac{1}{r_c} \delta_a^5 \delta_5^M. \quad (\text{B1})$$

Here  $M$  is the 5D curved index and  $a$  is the 5D index in the tangent space. We choose the gamma matrices to be

$$\begin{aligned}
\gamma^a &= (\gamma^\mu, -i\gamma^5), \\
\gamma^\mu &= \begin{pmatrix} 0 & \sigma^\mu \\ \bar{\sigma}^\mu & 0 \end{pmatrix}, \\
\gamma^5 &= \begin{pmatrix} -\mathbb{1} & 0 \\ 0 & \mathbb{1} \end{pmatrix}.
\end{aligned} \tag{B2}$$

For later convenience, we choose to write a 5D fermion  $\Psi$  as

$$\Psi = \begin{pmatrix} \Psi_L \\ \Psi_R \end{pmatrix}, \tag{B3}$$

which fixes the form of  $\bar{\Psi} = \Psi^\dagger \gamma^0 = (\Psi_R^\dagger \quad \Psi_L^\dagger)$ . For concreteness we focus on radion couplings to the up-type quarks. The generalization to the cases of the other SM fermions is straightforward.

In the presence of the GW scalar  $\Phi$ , the 5D fermion action is given by

$$\begin{aligned}
S &= \int d^4x \int_0^\pi d\theta \sqrt{G} \left[ \frac{i}{2} (\bar{\mathcal{Q}} \Gamma^M \partial_M \mathcal{Q} - (\partial_M \bar{\mathcal{Q}}) \Gamma^M \mathcal{Q}) \left( 1 + \frac{\alpha_q}{k^{3/2}} \Phi \right) - m_{\mathcal{Q}} \bar{\mathcal{Q}} \mathcal{Q} \left( 1 + \frac{\beta_q}{k^{3/2}} \Phi \right) \right. \\
&\quad + \frac{i}{2} (\bar{\mathcal{U}} \Gamma^M \partial_M \mathcal{U} - (\partial_M \bar{\mathcal{U}}) \Gamma^M \mathcal{U}) \left( 1 + \frac{\alpha_u}{k^{3/2}} \Phi \right) - m_{\mathcal{U}} \bar{\mathcal{U}} \mathcal{U} \left( 1 + \frac{\beta_u}{k^{3/2}} \Phi \right) \\
&\quad \left. + \frac{\delta(\theta - \pi)}{r_c} \left( \frac{Y}{k} \bar{\mathcal{Q}} H \mathcal{U} + \text{H.c.} \right) \left( 1 + \frac{\alpha_y}{k^{3/2}} \Phi \right) \right],
\end{aligned} \tag{B4}$$

where  $\Gamma^M = e_a^M \gamma^a$ . For simplicity we take  $Y$  to be real and consider the Higgs field  $H$  to be localized to the visible brane at  $\theta = \pi$ . We denote the VEV of  $H$  by  $v_h$ .

The dynamics in the  $\Phi$  sector leads to a background value  $\hat{\Phi}(\theta)$ . The excitations of the GW field are generically heavy, being of order  $m_{\text{KK}}$ . This allows us to integrate out the GW field in a dynamical radion background, thereby obtaining the low energy effective theory for the radion. We do this by promoting  $r_c$  to a dynamical field, which we denote by  $r(x)$ , and expanding  $r(x)$  about  $r_c$  as  $r(x) = r_c + \delta r(x)$ . Using Eq. (2.5), the canonically normalized physical radion  $\tilde{\varphi}$  is related to the other parameters by

$$\varphi = \langle \varphi \rangle + \tilde{\varphi} = \sqrt{\frac{24M_5^3}{k}} e^{-kr_c\pi} (1 - k\pi\delta r) \tag{B5}$$

which leads to the relation

$$\delta r = -\frac{\tilde{\varphi}}{fk\pi} \tag{B6}$$

where  $\langle \varphi \rangle = f$ .

To proceed, we take the following approach. We first set  $r = r_c$  and determine the equations of motion for the 5D fermions. We then set the Higgs field to its VEV and perform a KK decomposition to obtain the 4D fermion spectrum. We then write out the action to linear order in  $\delta r$ , expand out the bulk fermions in terms of their KK modes, and integrate over the extra dimension to obtain the radion coupling to the zero modes. This approach is consistent within the effective theory and yields the leading contribution to the operators that couple a single radion to the SM fermions.

To be consistent with phenomenology, we choose  $\mathcal{Q}_L$  and  $\mathcal{U}_R$  to be even about  $\theta = 0$  and  $\mathcal{Q}_R$  and  $\mathcal{U}_L$  to be odd. As usual, the mass parameter  $c_i$  is taken to be odd. We also take  $\hat{\Phi}_c$ , the VEV of  $\Phi$  at  $r = r_c$ , to be even at  $\theta = 0$  and  $\theta = \pi$ . Using the orbifold symmetry, we restrict the limits on the  $\theta$  integral in the action to be from 0 to  $\pi$ .

Minimizing the action (at  $r = r_c$ ), we find the equations of motion satisfied by the fermion fields. For instance, considering the variation  $\delta \mathcal{Q}_R^\dagger$  gives

$$\begin{aligned}
&ir_c e^{kr_c\theta} \sigma^\mu \partial_\mu \mathcal{Q}_R - \partial_\theta \mathcal{Q}_L + 2kr_c \mathcal{Q}_L - kr_c c_q \frac{1 + (\beta_q/k^{3/2})\hat{\Phi}_c}{1 + (\alpha_q/k^{3/2})\hat{\Phi}_c} \mathcal{Q}_L - \frac{(\alpha_q/2k^{3/2})\partial_\theta \hat{\Phi}_c}{1 + (\alpha_q/k^{3/2})\hat{\Phi}_c} \mathcal{Q}_L \\
&+ \frac{v_h Y}{2k} \delta(\theta - \pi) \frac{1 + (\alpha_y/k^{3/2})\hat{\Phi}_c}{1 + (\alpha_q/k^{3/2})\hat{\Phi}_c} \mathcal{U}_L = 0.
\end{aligned} \tag{B7}$$

The boundary terms in the action fix the boundary conditions to be



$$\left[ \delta \mathcal{Q}_R^\dagger \mathcal{Q}_L e^{-4kr_c\theta} \left( 1 + \frac{\alpha_q}{k^{3/2}} \hat{\Phi}_c \right) + \frac{Yv_h}{2r_c k} \delta \mathcal{Q}_R^\dagger \mathcal{U}_L e^{-4kr_c\theta} \left( 1 + \frac{\alpha_Y}{k^{3/2}} \hat{\Phi}_c \right) \right] \Big|_{\theta=\pi} - \left[ \delta \mathcal{Q}_R^\dagger \mathcal{Q}_L e^{-4kr_c\theta} \left( 1 + \frac{\alpha_q}{k^{3/2}} \hat{\Phi}_c \right) \right] \Big|_{\theta=0} = 0. \quad (\text{B8})$$

We proceed by employing the mixed KK decomposition described in Eq. (5.2) and require the zero modes  $q_L^0$  and  $u_R^0$  to satisfy the 4D Dirac equations

$$i\bar{\sigma}^\mu \partial_\mu q_L^0 - m_f u_L^0 = 0, \quad i\sigma^\mu \partial_\mu u_R^0 - m_f q_L^0 = 0, \quad (\text{B9})$$

where  $m_f$  is the mass of the zero mode generated by the Higgs VEV. In what follows, we work to leading order in  $m_f/(ke^{-k\pi r_c})$ . Because we have chosen  $\mathcal{Q}_R$  and  $\mathcal{U}_L$  to be odd about  $\theta = 0$ , they vanish at the boundary. This ensures that the boundary condition at  $\theta = 0$  is satisfied in Eq. (B8). To leading order in  $m_f/(ke^{-k\pi r_c})$ , this completely fixes the profiles  $Q_L^0$  and  $U_R^0$  up to an overall normalization that is determined by the requirement of a canonical kinetic term for the 4D field. The boundary condition at  $\theta = \pi$ , to this order, fixes the mass  $m_f$  in terms of other parameters.

Using Eqs. (B9) and (B7), the profile  $Q_L^0$  satisfies

$$\partial_\theta Q_L^0 - \left( 2kr_c - kr_c c_q \frac{1 + (\beta_q/k^{3/2})\hat{\Phi}_c}{1 + (\alpha_q/k^{3/2})\hat{\Phi}_c} - \frac{(\alpha_q/2k^{3/2})\partial_\theta \hat{\Phi}_c}{1 + (\alpha_q/k^{3/2})\hat{\Phi}_c} \right) Q_L^0 - m_f r_c e^{kr_c\theta} Q_R^0 = 0. \quad (\text{B10})$$

Similar equations can be derived for the other three fermion zero mode profiles  $Q_R^0, U_{L,R}^0$ . By our choice of boundary conditions, the even profiles  $Q_L^0$  and  $U_R^0$  correspond to the chiral fermions in the effective theory and hence survive in the  $m_f \rightarrow 0$  limit. The odd profiles  $Q_R^0$  and  $U_L^0$  vanish at  $\theta = 0$  and are forced by the equations of motion to begin at order  $m_f/(ke^{-kr_c\pi})$ . As a result, we can drop the term proportional to  $Q_R^0$  in Eq. (B10). To make the notation simpler, we define the functions

$$T(\alpha, \theta) = \int_0^\theta d\theta' \frac{\hat{\Phi}_c(\theta')}{k^{3/2} + \alpha \hat{\Phi}_c(\theta')},$$

$$G(c, \alpha, \beta, \theta) = \exp \left[ kr_c\theta \left( \frac{1}{2} - c \right) - kr_c c (\beta - \alpha) T(\alpha, \theta) \right]. \quad (\text{B11})$$

The even profiles, to leading order in  $m_f/(ke^{-kr_c\pi})$ , are given by

$$Q_L^0 = \frac{N_{Q_L}}{\sqrt{1 + (\alpha_q/k^{3/2})\hat{\Phi}_c}} \exp \left( \frac{3}{2} kr_c\theta \right) G(c_q, \alpha_q, \beta_q, \theta) \quad (\text{B12})$$

$$U_R^0 = \frac{N_{U_R}}{\sqrt{1 + (\alpha_u/k^{3/2})\hat{\Phi}_c}} \exp \left( \frac{3}{2} kr_c\theta \right) G(-c_u, \alpha_u, \beta_u, \theta). \quad (\text{B13})$$

In the limit where  $\hat{\Phi}_c$  goes to zero, these agree with the results for the profiles in the absence of stabilization [43,44]. The constants  $N_{Q_L}$  and  $N_{U_R}$  are determined by normalizing the kinetic terms for  $q_L^0$  and  $u_R^0$  and are given by

$$N_{Q_L}^{-2} = 2r_c \int_0^\pi d\theta G^2(c_q, \alpha_q, \beta_q, \theta), \quad N_{U_R}^{-2} = 2r_c \int_0^\pi d\theta G^2(-c_u, \alpha_u, \beta_u, \theta). \quad (\text{B14})$$

To leading order in  $m_f/(ke^{-kr_c\pi})$  the odd profiles are given by

$$\begin{aligned} Q_R^0(\theta) &= -\frac{m_f r_c N_{Q_L}}{\sqrt{1 + (\alpha_q/k^{3/2})\hat{\Phi}_c}} \exp\left(\frac{3}{2}kr_c\theta\right) G(-c_q, \alpha_q, \beta_q, \theta) \int_0^\theta d\theta' G^2(c_q, \alpha_q, \beta_q, \theta'), \\ U_L^0(\theta) &= \frac{m_f r_c N_{U_R}}{\sqrt{1 + (\alpha_u/k^{3/2})\hat{\Phi}_c}} \exp\left(\frac{3}{2}kr_c\theta\right) G(c_u, \alpha_u, \beta_u, \theta) \int_0^\theta d\theta' G^2(-c_u, \alpha_u, \beta_u, \theta'). \end{aligned} \quad (\text{B15})$$

The brane localized Higgs term contributes to the boundary condition at  $\theta = \pi$ . Since the Yukawa operator is associated with effects suppressed by  $m_f/(ke^{-kr_c\pi})$ , to the order we are working this only affects the odd profiles. More specifically, the boundary conditions in Eq. (B8) require

$$Q_R^0(\pi) = \frac{v_h Y}{2k} U_R^0(\pi) \left(1 + \frac{\alpha_y}{k^{3/2}} \hat{\Phi}_c(\pi)\right) \left(1 + \frac{\alpha_q}{k^{3/2}} \hat{\Phi}_c(\pi)\right)^{-1}, \quad (\text{B16})$$

$$U_L^0(\pi) = -\frac{v_h Y}{2k} Q_L^0(\pi) \left(1 + \frac{\alpha_y}{k^{3/2}} \hat{\Phi}_c(\pi)\right) \left(1 + \frac{\alpha_u}{k^{3/2}} \hat{\Phi}_c(\pi)\right)^{-1}, \quad (\text{B17})$$

which fixes the mass  $m_f$  in terms of other parameters of the theory as

$$m_f = -\frac{v_h Y}{k} \frac{[1 + (\alpha_y/k^{3/2})\hat{\Phi}_c(\pi)]N_{Q_L}N_{U_R}}{\sqrt{1 + (\alpha_u/k^{3/2})\hat{\Phi}_c(\pi)}\sqrt{1 + (\alpha_q/k^{3/2})\hat{\Phi}_c(\pi)}} \frac{G(-c_u, \alpha_u, \beta_u, \pi)}{G(-c_q, \alpha_q, \beta_q, \pi)}. \quad (\text{B18})$$

To derive the coupling of the radion to the zero modes, we restrict  $\hat{\Phi}$  to its background value in the action and expand the action to linear order in  $\delta r = r(x) - r_c$ . We then plug in the profiles for the zero modes and integrate over the extra dimension. As  $r$  varies from its background value, the leading terms in the action can be written  $S = S_c + \delta S$  with  $S_c$  independent of  $\delta r$  and  $\delta S$  linear in  $\delta r$ . Before doing so we note that to linear order in the fluctuation of the radius  $\delta r$ ,  $\hat{\Phi}$  satisfies

$$\hat{\Phi}(\theta) = \hat{\Phi}_c(\theta) + \delta r \partial_r \hat{\Phi}_c = \hat{\Phi}_c(\theta) + \delta r \left( k^{5/2} \alpha(\pi - \theta) e^{-4kr_c(\pi - \theta)} + \frac{\theta}{r_c} \partial_\theta \hat{\Phi}_{\text{OR}}(\theta) \right), \quad (\text{B19})$$

where we have used (2.11). We then find

$$\begin{aligned} \delta S &= \int d^4x \int_0^\pi d\theta \delta r e^{-4kr_c\theta} \left\{ i e^{kr_c\theta} \left[ \left(1 - 3kr_c\theta\right) \left(1 + \frac{\alpha_q}{k^{3/2}} \hat{\Phi}_c\right) + \frac{r_c \alpha_q}{k^{3/2}} \partial_r \hat{\Phi}_c \right] \left[ Q_R^\dagger \overleftrightarrow{\partial}_\mu Q_R + Q_L^\dagger \overleftrightarrow{\partial}_\mu Q_L \right] \right. \\ &\quad + \left[ \frac{\alpha_q}{r_c k^{3/2}} \partial_r \hat{\Phi}_c - 4k\theta \left(1 + \frac{\alpha_q}{k^{3/2}} \hat{\Phi}_c\right) \right] \left[ Q_L^\dagger \overleftrightarrow{\partial}_\theta Q_R - Q_R^\dagger \overleftrightarrow{\partial}_\theta Q_L \right] \\ &\quad - 2kc_q \left[ \left(1 - 4kr_c\theta\right) \left(1 + \frac{\beta_q}{k^{3/2}} \hat{\Phi}_c\right) + \frac{r_c \beta_q}{k^{3/2}} \partial_r \hat{\Phi}_c \right] \left[ Q_L^\dagger Q_R + Q_R^\dagger Q_L \right] + (Q \rightarrow U) \\ &\quad \left. + \frac{v_h Y}{k} \delta(\theta - \pi) \left[ \frac{\alpha_y}{k^{3/2}} \partial_r \hat{\Phi}_c - 4k\theta \left(1 + \frac{\alpha_y}{k^{3/2}} \hat{\Phi}_c\right) \right] \left[ Q_R^\dagger U_L + Q_L^\dagger U_R + U_R^\dagger Q_L + U_L^\dagger Q_R \right] \right\}. \end{aligned} \quad (\text{B20})$$

Before inserting the profiles into  $\delta S$  to derive the coupling of the 4D fields to the radion, we note that the following identities hold to first order in the small parameter  $m_f/(ke^{-kr_c\pi})$ :

$$i(Q_R^\dagger \overleftrightarrow{\partial}_\mu Q_R + Q_L^\dagger \overleftrightarrow{\partial}_\mu Q_L) = m_f (u_R^{0\dagger} q_L^0 + q_L^{0\dagger} u_R^0) (Q_R^0 Q_R^0 + Q_L^0 Q_L^0), \quad (\text{B21})$$

$$Q_L^\dagger Q_R + Q_R^\dagger Q_L = (u_R^{0\dagger} q_L^0 + q_L^{0\dagger} u_R^0) Q_L^0 Q_R^0, \quad (\text{B22})$$

$$\mathcal{Q}_L^\dagger \overleftrightarrow{\partial}_\theta \mathcal{Q}_R - \mathcal{Q}_R^\dagger \overleftrightarrow{\partial}_\theta \mathcal{Q}_L = (u_R^{0\dagger} q_L^0 + q_L^{0\dagger} u_R^0)(\mathcal{Q}_L^0 \partial_\theta \mathcal{Q}_R^0 - \mathcal{Q}_R^0 \partial_\theta \mathcal{Q}_L^0). \quad (\text{B23})$$

Similar relations exist for the  $\mathcal{U}$  field. The last of these can be further simplified by using the equation satisfied by the profiles, Eq. (B10). To take the boundary conditions into account, we write the boundary terms as  $\delta$ -functions in the equations of motion. A partial cancellation of terms results in the simplification

$$\begin{aligned} (\mathcal{Q}_L^0 \partial_\theta \mathcal{Q}_R^0 - \mathcal{Q}_R^0 \partial_\theta \mathcal{Q}_L^0) &= 2\mathcal{Q}_L^0 \mathcal{Q}_R^0 k c_q r_c \frac{1 + (\beta_q/k^{3/2})\hat{\Phi}_c}{1 + (\alpha_q/k^{3/2})\hat{\Phi}_c} - m_f r_c e^{kr_c\theta} (\mathcal{Q}_R^0 \mathcal{Q}_R^0 + \mathcal{Q}_L^0 \mathcal{Q}_L^0) \\ &\quad - \frac{v_h Y}{2k} \delta(\theta - \pi) \frac{1 + (\alpha_y/k^{3/2})\hat{\Phi}_c}{1 + (\alpha_q/k^{3/2})\hat{\Phi}_c} (\mathcal{Q}_R^0 U_L^0 + \mathcal{Q}_L^0 U_R^0). \end{aligned} \quad (\text{B24})$$

Similarly, the term in Eq. (B20) for  $\delta S$  localized at  $\theta = \pi$  can be expressed as

$$\mathcal{Q}_R^\dagger \mathcal{U}_L + \mathcal{Q}_L^\dagger \mathcal{U}_R + \mathcal{U}_R^\dagger \mathcal{Q}_L + \mathcal{U}_L^\dagger \mathcal{Q}_R = (u_R^{0\dagger} q_L^0 + q_L^{0\dagger} u_R^0)(\mathcal{Q}_R^0 U_L^0 + U_R^0 \mathcal{Q}_L^0). \quad (\text{B25})$$

Inserting these relations into Eq. (B20) we find

$$\begin{aligned} \delta S &= \int d^4x (u_R^{0\dagger} q_L^0 + q_L^{0\dagger} u_R^0) \delta r \int_0^\pi d\theta e^{-4kr_c\theta} \left[ m_f e^{kr_c\theta} (1 + kr_c\theta) \left( 1 + \frac{\alpha_q}{k^{3/2}} \hat{\Phi}_c \right) (\mathcal{Q}_R^0 \mathcal{Q}_R^0 + \mathcal{Q}_L^0 \mathcal{Q}_L^0) \right. \\ &\quad - 2kc_q \left( 1 + \frac{\beta_q}{k^{3/2}} \hat{\Phi}_c + \frac{\beta_q - \alpha_q}{k^{3/2} + \alpha_q \hat{\Phi}_c} r_c \partial_r \hat{\Phi}_c \right) \mathcal{Q}_L^0 \mathcal{Q}_R^0 + (\mathcal{Q}^0 \rightarrow U^0) + \frac{v_h Y}{k} \delta(\theta - \pi) \frac{\partial_r \hat{\Phi}}{k^{3/2}} \\ &\quad \left. \times \left\{ \alpha_y - \frac{1}{2} \left( \frac{\alpha_q (1 + (\alpha_y/k^{3/2})\hat{\Phi}_c)}{1 + (\alpha_q/k^{3/2})\hat{\Phi}_c} + \frac{\alpha_u (1 + (\alpha_y/k^{3/2})\hat{\Phi}_c)}{1 + (\alpha_u/k^{3/2})\hat{\Phi}_c} \right) \right\} (\mathcal{Q}_R^0 U_L^0 + U_R^0 \mathcal{Q}_L^0) \right]. \end{aligned} \quad (\text{B26})$$

To proceed further, we insert the functional form of the profiles, use the relationship between  $m_f$  and  $v_h$ , and work to linear order in  $m_f/(ke^{-kr_c\pi})$ . To this order, we can drop terms like  $\mathcal{Q}_R^0 \mathcal{Q}_R^0$  and  $U_L^0 U_L^0$  in the above. This results in

$$\begin{aligned} \delta S &= \int d^4x (u_R^{0\dagger} q_L^0 + q_L^{0\dagger} u_R^0) \delta r m_f \int_0^\pi d\theta \left\{ \left[ N_{\mathcal{Q}_L}^2 (1 + kr_c\theta) G^2(c_q, \alpha_q, \beta_q, \theta) \right. \right. \\ &\quad \left. \left. + \frac{2c_q kr_c N_{\mathcal{Q}_L}^2}{1 + (\alpha_q/k^{3/2})\hat{\Phi}_c} \left( 1 + \frac{\beta_q}{k^{3/2}} \hat{\Phi}_c + \frac{\beta_q - \alpha_q}{k^{3/2} + \alpha_q \hat{\Phi}_c} r_c \partial_r \hat{\Phi}_c \right) \int_0^\theta d\theta' G^2(c_q, \alpha_q, \beta_q, \theta') \right] \right. \\ &\quad \left. + \left[ N_{U_R}^2 (1 + kr_c\theta) G^2(-c_u, \alpha_u, \beta_u, \theta) \right. \right. \\ &\quad \left. \left. - \frac{2c_u kr_c N_{U_R}^2}{1 + (\alpha_u/k^{3/2})\hat{\Phi}_c} \left( 1 + \frac{\beta_u}{k^{3/2}} \hat{\Phi}_c + \frac{\beta_u - \alpha_u}{k^{3/2} + \alpha_u \hat{\Phi}_c} r_c \partial_r \hat{\Phi}_c \right) \int_0^\theta d\theta' G^2(-c_u, \alpha_u, \beta_u, \theta') \right] \right. \\ &\quad \left. - \delta(\theta - \pi) \frac{\partial_r \hat{\Phi}_c}{2k^{3/2}} \left( \frac{2\alpha_y}{1 + (\alpha_y/k^{3/2})\hat{\Phi}_c} - \frac{\alpha_q}{1 + (\alpha_q/k^{3/2})\hat{\Phi}_c} - \frac{\alpha_u}{1 + (\alpha_u/k^{3/2})\hat{\Phi}_c} \right) \right\}. \end{aligned} \quad (\text{B27})$$

For compactness, we write  $\delta S$  as

$$\delta S = - \int d^4x (u_R^{0\dagger} q_L^0 + q_L^{0\dagger} u_R^0) \frac{\tilde{\varphi}}{f} m_f (I_q + I_u + I_h), \quad (\text{B28})$$

where  $I_q$  ( $I_u$ ) is associated with the term in the first (second) set of square brackets and  $I_h$  arises from the final boundary term,

$$I_h = -\frac{\hat{\Phi}_{\text{OR}}(\pi)}{2} \left[ \frac{2\alpha_y}{k^{3/2} + \alpha_y \hat{\Phi}_c(\pi)} - \frac{\alpha_q}{k^{3/2} + \alpha_q \hat{\Phi}_c(\pi)} - \frac{\alpha_u}{k^{3/2} + \alpha_u \hat{\Phi}_c(\pi)} \right], \quad (\text{B29})$$

where we have used Eq. (B19) and  $\frac{d}{d(kr_c\theta)} \hat{\Phi} \equiv \hat{\Phi}'$ .

The quantity  $I_q$  is related to  $I_u$  by taking  $c_q \rightarrow -c_u$  and replacing the  $q$  labels with  $u$  labels on  $\alpha_q$  and  $\beta_q$ . This holds quite generally for the rest of this appendix, and so we limit our attention to  $I_q$ . Using the definition of  $N_{QL}$  from Eq. (B14) and integrating by parts, the expression for  $I_q$  simplifies to

$$I_q = \frac{1}{2kr_c\pi} + c_q + \frac{c_q}{\pi k^{3/2}} (\beta_q - \alpha_q) \int_0^\pi \frac{d\theta}{1 + \frac{\alpha_q}{k^{3/2}} \hat{\Phi}_c} \left( \hat{\Phi}_c + \frac{r_c \partial_r \hat{\Phi}_c}{1 + \frac{\alpha_q}{k^{3/2}} \hat{\Phi}_c} \right) + N_{QL}^2 \frac{r_c}{\pi} \int_0^\pi d\theta e^{kr_c\theta(1-2c_q)} e^{-2c_q kr_c(\beta_q - \alpha_q)T(\alpha_q, \theta)} \\ \times \left[ \theta(1-2c_q) - \frac{2c_q}{k^{3/2}} (\beta_q - \alpha_q) \int_0^\theta \frac{d\theta'}{1 + \frac{\alpha_q}{k^{3/2}} \hat{\Phi}_c} \left( \hat{\Phi}_c + \frac{r_c \partial_r \hat{\Phi}_c}{1 + \frac{\alpha_q}{k^{3/2}} \hat{\Phi}_c} \right) \right], \quad (\text{B30})$$

where we have used the fact that  $\hat{\Phi}(\theta)$  is even about  $\theta = 0$ . From Eqs. (2.11) and (B19), this expression depends on both  $\hat{\Phi}_{\text{OR}}$  and on  $\alpha \exp[-4kr_c(\pi - \theta)]$ . The effects of this second term, however, are small and can be neglected. This is because this term is only significant in a small region close to the IR brane, and so the region of integration where it has support is parametrically small. Therefore its contribution is suppressed by the size of this region,  $\mathcal{O}(1/kr_c)$ . Therefore, in the rest of this section, we drop all  $\alpha \exp[-4kr_c(\pi - \theta)]$  terms and replace  $\hat{\Phi}_c$  with  $\hat{\Phi}_{\text{OR}}$ .

Integrating Eq. (B30) by parts we then find

$$I_q = c_q + c_q \frac{(\beta_q - \alpha_q) \hat{\Phi}_{\text{OR}}(\pi)}{k^{3/2} + \alpha_q \hat{\Phi}_{\text{OR}}(\pi)} + \frac{1}{2kr_c} \left[ \int_0^\pi d\theta e^{kr_c(\theta-\pi)(1-2c_q)} \exp \left( 2c_q kr_c(\beta_q - \alpha_q) \int_\theta^\pi d\theta' \frac{\hat{\Phi}_{\text{OR}}(\theta')}{k^{3/2} + \alpha_q \hat{\Phi}_{\text{OR}}(\theta')} \right) \right]^{-1} \\ = c_q + c_q \frac{(\beta_q - \alpha_q) \hat{\Phi}_{\text{OR}}(\pi)}{k^{3/2} + \alpha_q \hat{\Phi}_{\text{OR}}(\pi)} + \frac{G^2(c_q, \alpha_q, \beta_q, \pi)}{2kr_c \int_0^\pi d\theta G^2(c_q, \alpha_q, \beta_q, \theta)} \quad (\text{B31})$$

where we have used the notation of Eq. (B11). This expression can be further simplified in the cases of phenomenological interest. Recall that these  $G$  functions are tied to the fermion profiles in the  $\theta$  dimension. In the unstabilized case, the fermion profiles are peaked towards one brane and exponentially small near the other. If  $\hat{\Phi}/k^{3/2}$  is large or rapidly varying, then the fermion profiles could in principle have much more complicated behavior, such as local extrema in the bulk. In the rest of the analysis, we will focus on the phenomenologically interesting case when the profiles are peaked towards either  $\theta = 0$  or  $\theta = \pi$ . Because the  $G$  function is an exponential, in general when it is peaked near the one brane, it is exponentially small near the other. We can use this fact to simplify the integral in Eq. (B31). When the fermion profile is peaked near  $\theta = 0$ , we can immediately see that  $G(c_q, \alpha_q, \beta_q, \pi)$  is exponentially suppressed, making the third term in Eq. (B31) negligible.

When the fermion profile is peaked near  $\theta = \pi$  we make the change of variables  $\theta \rightarrow \pi - \vartheta$  in the integral in the denominator of the third term to obtain

$$I_q = c_q + c_q \frac{(\beta_q - \alpha_q) \hat{\Phi}_{\text{OR}}(\pi)}{k^{3/2} + \alpha_q \hat{\Phi}_{\text{OR}}(\pi)} + \frac{1}{2kr_c} \left[ \int_0^\pi d\vartheta e^{f(\vartheta)} \right]^{-1} \quad (\text{B32})$$

where

$$f(\vartheta) = -kr_c\vartheta(1-2c_q) + 2kr_c c_q (\beta_q - \alpha_q) \int_{\pi-\vartheta}^\pi d\theta \frac{\hat{\Phi}_{\text{OR}}(\theta)}{k^{3/2} + \alpha_q \hat{\Phi}_{\text{OR}}(\theta)}. \quad (\text{B33})$$

The integral is now dominated by values of the integrand close to  $\vartheta = 0$ . Now, notice that the leading terms in the Taylor series expansion of  $f(\vartheta)$  about  $\vartheta = 0$  are

$$\vartheta \left[ -kr_c(1-2c_q) + 2kr_c c_q (\beta_q - \alpha_q) \frac{\hat{\Phi}_{\text{OR}}(\pi)}{k^{3/2} + \alpha_q \hat{\Phi}_{\text{OR}}(\pi)} \right] - \vartheta^2 \frac{k^2 r_c^2 c_q (\beta_q - \alpha_q) k^{3/2} \hat{\Phi}'_{\text{OR}}(\pi)}{(k^{3/2} + \alpha_q \hat{\Phi}_{\text{OR}}(\pi))^2}. \quad (\text{B34})$$



Close to  $\vartheta = 0$  the linear term dominates and we treat the quadratic term as a small correction. We therefore write the integral as

$$\int_0^\pi d\vartheta \exp \left[ kr_c \vartheta \left( -1 + 2c_q + 2c_q(\beta_q - \alpha_q) \frac{\hat{\Phi}_{\text{OR}}(\pi)}{k^{3/2} + \alpha_q \hat{\Phi}_{\text{OR}}(\pi)} \right) \right] \left[ 1 - \vartheta^2 \frac{k^2 r_c^2 c_q (\beta_q - \alpha_q) k^{3/2} \hat{\Phi}'_{\text{OR}}(\pi)}{(k^{3/2} + \alpha_q \hat{\Phi}_{\text{OR}}(\pi))^2} \right] \quad (\text{B35})$$

which is evaluated exactly as

$$\frac{1}{2} (1 - 2\tilde{c}_q) \left[ 1 - \frac{2\tilde{c}_q(\beta_q - \alpha_q) \hat{\Phi}'_{\text{OR}}(\pi) k^{3/2}}{(1 - 2\tilde{c}_q)^2 (k^{3/2} + \alpha_q \hat{\Phi}_{\text{OR}}(\pi)) (k^{3/2} + \beta_q \hat{\Phi}_{\text{OR}}(\pi))} \right]^{-1} \quad (\text{B36})$$

where we have defined

$$\tilde{c}_q \equiv c_q + c_q \frac{(\beta_q - \alpha_q) \hat{\Phi}_{\text{OR}}(\pi)}{k^{3/2} + \alpha_q \hat{\Phi}_{\text{OR}}(\pi)}. \quad (\text{B37})$$

In the limit  $k^{-3/2} \hat{\Phi}'(\pi) \ll 1$ , corresponding to a light radion, we therefore find

$$I_q = \frac{1}{2} + \frac{\tilde{c}_q(\beta_q - \alpha_q) \hat{\Phi}'_{\text{OR}}(\pi) k^{3/2}}{(1 - 2\tilde{c}_q)(k^{3/2} + \alpha_q \hat{\Phi}_{\text{OR}}(\pi))(k^{3/2} + \beta_q \hat{\Phi}_{\text{OR}}(\pi))}. \quad (\text{B38})$$

Combining the two cases, we obtain

$$I_q = \begin{cases} \tilde{c}_q & , \quad G \text{ peaked at } \theta = 0 \\ \frac{1}{2} + \frac{\tilde{c}_q(\beta_q - \alpha_q) \hat{\Phi}'_{\text{OR}}(\pi) k^{3/2}}{(1 - 2\tilde{c}_q)(k^{3/2} + \alpha_q \hat{\Phi}_{\text{OR}}(\pi))(k^{3/2} + \beta_q \hat{\Phi}_{\text{OR}}(\pi))} & , \quad G \text{ peaked at } \theta = \pi \end{cases}. \quad (\text{B39})$$

### APPENDIX C: EFFECTS OF RADION STABILIZATION ON OPERATOR SCALING DIMENSIONS

In this appendix we determine how the scaling dimension  $\Delta_{\mathcal{Q}}$  of the dual CFT operator associated with the fermion field  $\mathcal{Q}$  is affected by the dynamics that stabilizes the radion. We follow closely the approach of [73]. The central idea is to relate the bulk physics to that of a CFT by treating a bulk field and its boundary value as separate fields, and then integrating out the bulk physics.

Using Eq. (B1) and Eq. (B2) we begin with the fermion action

$$S = \int d^4x \int_0^\pi d\theta \left\{ \frac{1}{2} [ir_c e^{-3kr_c \theta} (\mathcal{Q}_R^\dagger \overrightarrow{\partial}_\mu \mathcal{Q}_R + \mathcal{Q}_L^\dagger \overrightarrow{\partial}_\mu \mathcal{Q}_L) + e^{-4kr_c \theta} (\mathcal{Q}_L^\dagger \overleftarrow{\partial}_\theta \mathcal{Q}_R - \mathcal{Q}_R^\dagger \overleftarrow{\partial}_\theta \mathcal{Q}_L)] \left( 1 + \frac{\alpha_q}{k^{3/2}} \hat{\Phi}_c \right) - kc_q r_c e^{-4kr_c \theta} [\mathcal{Q}_L^\dagger \mathcal{Q}_R + \mathcal{Q}_R^\dagger \mathcal{Q}_L] \left( 1 + \frac{\beta_q}{k^{3/2}} \hat{\Phi}_c \right) \right\} \quad (\text{C1})$$

where  $\overleftarrow{\partial}_\mu \equiv \overrightarrow{\partial}_\mu - \overleftarrow{\partial}_\mu$  and  $\hat{\Phi}_c(\theta)$  is the VEV of the GW scalar. In this appendix we focus on the  $\mathcal{Q}$  field. The end result can be mapped to the  $\mathcal{U}$  by simply taking  $c_q \rightarrow -c_u$  while changing all other  $q$  labels to  $u$  labels. The scaling dimension is associated with physics above the conformal symmetry breaking scale, and so in this appendix we can safely ignore details of the IR brane dynamics such as couplings to the Higgs.

In Appendix B, we took the  $\mathcal{Q}_L$  field and  $\hat{\Phi}_c$  to be even about  $\theta = 0$  and  $\theta = \pi$ , and  $\mathcal{Q}_R$  to be odd. In this appendix, solely for the purpose of determining the scaling dimensions of bulk fields, we relax those restrictions. We now associate the value of  $\mathcal{Q}_L$  on the UV ( $\theta = 0$ ) brane with the source  $q_s$  for some fermionic operator  $\mathcal{O}_{\mathcal{Q}}$  in the CFT on the boundary with scaling dimension  $\Delta_{\mathcal{Q}}$ . Specifically,

$$\mathcal{Q}_L(x, \theta)|_{\theta=0} = q_s(x), \Rightarrow \mathcal{L}_{\text{CFT}} \supset q_s \mathcal{O}_{\mathcal{Q}}. \quad (\text{C2})$$

This function is fixed, or  $\delta\mathcal{Q}_L = 0$ , on the UV boundary. Because the equation of motion for fermions is first order, we cannot fix the boundary conditions for both chiralities of  $\mathcal{Q}$  so we leave  $\mathcal{Q}_R$  free to vary on the boundary.

When we take the variation of the action,<sup>6</sup> we generate the equations of motion such as Eq. (B7) and a boundary term such as led to Eq. (B8). The total boundary term is

$$\delta S \supset \frac{1}{2} \int d^4x e^{-4kr_c\theta} \left( 1 + \frac{\alpha_q}{k^{3/2}} \hat{\Phi}_c \right) \times [\mathcal{Q}_L^\dagger \delta\mathcal{Q}_R - \delta\mathcal{Q}_L^\dagger \mathcal{Q}_R - \mathcal{Q}_R^\dagger \delta\mathcal{Q}_L + \delta\mathcal{Q}_R^\dagger \mathcal{Q}_L] \Big|_0^\pi. \quad (\text{C3})$$

Now we choose  $\mathcal{Q}_L|_\pi = 0$  to eliminate the boundary term at  $\theta = \pi$ .<sup>7</sup> The UV boundary, where  $\delta\mathcal{Q}_L = 0$ , remains because  $\mathcal{Q}_L \neq 0$  and  $\delta\mathcal{Q}_R \neq 0$ . Thus, in order for  $\delta S = 0$  to hold we must add a term on the UV boundary to cancel this remainder. This term is

$$S_4 = \frac{1}{2} \int_{\text{UV}} d^4x \left( 1 + \frac{\alpha_q}{k^{3/2}} \hat{\Phi}_c \right) (\mathcal{Q}_L^\dagger \mathcal{Q}_R + \mathcal{Q}_R^\dagger \mathcal{Q}_L) \quad (\text{C4})$$

where all the fields are evaluated at the UV brane.

Because  $\delta\mathcal{Q}_L = 0$  on the UV brane, we can also add to the boundary Lagrangian any term  $\mathcal{L}_{\text{UV}}$  which is only a function of  $\mathcal{Q}_L$  without changing the equations of motion. For instance

$$S_{\text{UV}} = \int_{\text{UV}} d^4x \mathcal{L}_{\text{UV}} = \int_{\text{UV}} d^4x \left[ \left( \hat{\beta}_q + \frac{\hat{\alpha}_q}{k^{3/2}} \hat{\Phi}_c \right) i\mathcal{Q}_L^\dagger \not{\partial} \mathcal{Q}_L + \dots \right] \quad (\text{C5})$$

where  $\hat{\beta}_q$  and  $\hat{\alpha}_q$  are arbitrary coefficients. We are now ready to integrate out the bulk by substituting the solutions to the 5D equations of motion back into the action. By design, the bulk action vanishes when the variation vanishes, so we are left with only the UV boundary terms.

It is useful to Fourier transform the 4D coordinates of the 5D fields and parametrize their  $\theta$  dependence by

$$\begin{aligned} \mathcal{Q}_L(p, \theta) &= \frac{f_L(p, \theta)}{f_L(p, 0)} q_s(p), \\ \mathcal{Q}_R(p, \theta) &= \frac{f_R(p, \theta)}{f_R(p, 0)} q_R(p) \end{aligned} \quad (\text{C6})$$

where we have made the definition  $\mathcal{Q}_R(p, 0) = q_R(p)$ . The 4D fermions  $q_R$  and  $q_s$  are related by the Dirac equation, and we can fix the relative normalization by taking

<sup>6</sup>We treat  $\hat{\Phi}_c$  as a background field, so  $\delta\hat{\Phi}_c = 0$ .

<sup>7</sup>It would also be consistent to choose  $\mathcal{Q}_R|_\pi = 0$ . This has no effect on the final result.

$$\not{p}q_s = p \frac{f_L(p, 0)}{f_R(p, 0)} q_R. \quad (\text{C7})$$

The bulk equations of motion for  $\mathcal{Q}_L$  and  $\mathcal{Q}_R$  then imply

$$\begin{aligned} \partial_\theta f_{L,R} \mp kr_c \left( \frac{p}{k} e^{kr_c\theta} f_{R,L} - 2f_{L,R} \pm c_q \frac{k^{3/2} + \beta_q \hat{\Phi}_c}{k^{3/2} + \alpha_q \hat{\Phi}_c} f_{L,R} \right. \\ \left. + \frac{\alpha_q}{2} \frac{\hat{\Phi}'_c}{k^{3/2} + \alpha_q \hat{\Phi}_c} f_{L,R} \right) = 0 \end{aligned} \quad (\text{C8})$$

where the first and second labels on  $f_{i,j}$  correspond to the upper and lower signs respectively and  $\hat{\Phi}'_c \equiv \frac{d}{d(kr_c\theta)} \hat{\Phi}_c$ .

We now turn to the boundary action for  $q_s$ . After rescaling so that the kinetic term in  $S_{\text{UV}}$  is canonically normalized, the action is

$$S = \int \frac{d^4p}{(2\pi)^4} [q_s^\dagger \not{p} q_s + \dots] + \hat{\xi}_q q_s^\dagger \Sigma(p) q_s \quad (\text{C9})$$

where  $\hat{\xi}_q$  is a normalization that is independent of  $p$  and we have defined

$$\Sigma(p) = \frac{\not{p} f_R(p, 0)}{p f_L(p, 0)}. \quad (\text{C10})$$

This quantity  $\Sigma(p)$  determines the scaling dimension of the operator  $\mathcal{O}_\mathcal{Q}$  defined in Eq. (C2). This follows from the holographic principle which associates the generating function of a CFT

$$\mathcal{Z}[q_s] = \int \mathcal{D}\phi_{\text{CFT}} e^{iS_{\text{CFT}} + \int q_s \mathcal{O}_\mathcal{Q} + \text{H.c.}} \quad (\text{C11})$$

with the AdS partition function

$$\mathcal{Z}[q_s] = \int_{q_s} \mathcal{D}\mathcal{Q} e^{iS[\mathcal{Q}]} = e^{iS_{\text{eff}}[q_s]} \quad (\text{C12})$$

where the subscript on the integral means that the integration is to be performed subject to the condition that  $\mathcal{Q}$  takes on the value  $q_s$  on the UV boundary. In general  $S_{\text{eff}}[q_s]$  represents a nonlocal action in 4D for the source field  $q_s$ . In its original incarnation, the AdS/CFT correspondence applied to the scenario when the UV boundary, where the source field resides, corresponded to the boundary of AdS space. However, since different positions of the UV brane correspond simply to different choices of the cutoff scale, and are therefore related by RG transformations, the holographic principle can also be applied to the situation when the UV brane sits at an arbitrary location in the bulk. In general, it is also possible to promote  $q_s(x)$  to a dynamical field. The  $\mathcal{L}_{\text{UV}}$  term we can add to the UV brane in Eq. (C5) exactly captures this freedom.

Since the scaling dimension of an operator is associated with physics above the conformal symmetry breaking scale, we work in the limit of the IR brane being far away. This is done by taking the limit  $p \gg k \exp(-k\pi r_c)$ , where  $p$  represents the momentum scales being probed. It is likewise convenient to work in the limit that the UV brane is also far away, so that  $p \ll k$ . The reason is that the hard momentum cutoff associated with the presence of the UV brane constitutes an explicit violation of conformal symmetry by the regulator. When working at momenta well below the cutoff scale, spurious effects associated with the regulator are suppressed. In this limit, for instance, the correlator

$$\langle \bar{\mathcal{O}}_{\mathcal{Q}} \mathcal{O}_{\mathcal{Q}} \rangle \simeq \int \frac{d^4 p}{(2\pi)^4} e^{-ix \cdot p} \frac{\delta^2 S_{\text{eff}}}{\delta q_s^\dagger \delta q_s} \quad (\text{C13})$$

has dimension  $2\Delta_{\mathcal{Q}}$ . This allows us to relate

$$\lim_{\substack{kr_c \rightarrow \infty \\ p/k \rightarrow 0}} (\Sigma(p) + \text{counterterms}) \quad (\text{C14})$$

to  $\Delta_{\mathcal{Q}}$ . The counterterms are included because divergent terms in  $\Sigma(p)$  which are local, and hence analytic, are renormalized by local counterterms. This implies that the leading nonanalytic term in  $\Sigma(p)$  gives the dimension of  $\mathcal{O}_{\mathcal{Q}}$ ; specifically the leading nonanalytic term goes like [73]

$$\lim_{\substack{kr_c \rightarrow \infty \\ p/k \rightarrow 0}} (\Sigma(p) + \text{counterterms}) \propto p p^{2\Delta_{\mathcal{Q}}-5}. \quad (\text{C15})$$

Therefore once we compute  $\Sigma(p)$ , it will give us the scaling dimension of the dual operator.

The bulk RS metric possesses an isometry under shifts in the extra dimensional coordinate  $\theta$ , when combined with a rescaling of the 4D coordinates  $x^\mu$ . This isometry corresponds to the symmetry under scale invariance of the dual 4D theory. After the introduction of the stabilization mechanism, the isometry of the bulk 5D metric is no longer exact. In the dual description, the scale invariance of the 4D theory is now explicitly broken, and the scaling dimensions of operators are no longer strictly defined. However, the scenario we are interested in is one where the dilaton is light as a consequence of the fact that the operator that breaks the symmetry is close to marginal, and so the theory is approximately conformally invariant at all scales. In this limit, the scaling behavior of operators only changes very slowly as a function of the renormalization scale. Therefore we can continue to associate each operator with an approximate scaling dimension that changes very slowly with the renormalization scale. From the holographic perspective, the scaling dimension is dual to a function of the parameters of the 5D theory that changes with the extra dimensional coordinate  $\theta$ , but only very slowly.

Ultimately, we are interested in comparing with the dual picture [11], where the dilaton couplings are related to the scaling dimensions of operators evaluated near the scale where conformal symmetry is spontaneously broken. Therefore, we need to relate the scaling dimensions to 5D parameters evaluated near the IR brane. This is challenging because at  $\theta = \pi$  there is a phase transition where a boundary layer forms. Therefore we need to be careful in taking the limit approaching the IR brane, and we use the specific procedure described below.

We first modify the analysis above to compute the scaling dimension,  $\Delta_{\mathcal{Q}}(\theta_0)$ , in the neighborhood of an arbitrary point  $\theta = \theta_0$  in the bulk. We imagine that there is a UV brane at  $\theta = \theta_0$  such that

$$\mathcal{Q}_L(x, \theta)|_{\theta=\theta_0} = q_s(x|\theta_0), \Rightarrow \mathcal{L}_{\text{CFT}} \supset q_s(x|\theta_0) \mathcal{O}_{\mathcal{Q}}. \quad (\text{C16})$$

We can now follow the analysis above to get the equations analogous to Eqs. (C9) and (C10), namely

$$S = \int \frac{d^4 p}{(2\pi)^4} [q_s^\dagger(\theta_0) \not{p} q_s(\theta_0) + \dots] + \hat{\zeta}_q q_s^\dagger(\theta_0) \Sigma_{\theta_0}(p) q_s(\theta_0) \quad (\text{C17})$$

with the definition

$$\Sigma_{\theta_0}(p) = \frac{\not{p} f_R(p, \theta_0)}{p f_L(p, \theta_0)}. \quad (\text{C18})$$

which is then related to the scaling dimension  $\Delta_{\mathcal{Q}}(\theta_0)$  in the same way as before.

We now turn to calculating  $\Sigma(p)$  and its generalization  $\Sigma_{\theta_0}(p)$ . From Eq. (C8) we find

$$f_R = \frac{e^{-kr_c \theta}}{p r_c} \left[ \partial_\theta f_L - 2kr_c f_L + \frac{kr_c}{2} \frac{\alpha_q \hat{\Phi}'_c}{k^{3/2} + \alpha_q \hat{\Phi}_c} f_L + kr_c c_q \frac{k^{3/2} + \beta_q \hat{\Phi}_c}{k^{3/2} + \alpha_q \hat{\Phi}_c} f_L \right]. \quad (\text{C19})$$

Substituting this back into the companion relation for  $f_L$  we find

$$0 = \partial_\theta^2 f_L - kr_c (5 - C_1) \partial_\theta f_L + f_L (kr_c)^2 \times \left\{ \frac{p^2}{k^2} e^{2kr_c \theta} + 6 - c_q (1 + c_q) - \frac{5}{2} C_1 - \frac{1}{4} C_1^2 + C_2 - c_q C_3 \right\} \quad (\text{C20})$$

where

$$C_1 = \frac{\alpha_q \hat{\Phi}'_c}{k^{3/2} + \alpha_q \hat{\Phi}_c}, \quad (C21)$$

$$C_2 = \frac{1}{2} \frac{\alpha_q \hat{\Phi}_c''}{k^{3/2} + \alpha_q \hat{\Phi}_c} + \frac{c_q(\beta_q - \alpha_q) \hat{\Phi}'_c \hat{\Phi}_c'}{k^{3/2} (1 + \frac{\alpha_q}{k^{3/2}} \hat{\Phi}_c)^2}, \quad (C22)$$

$$C_3 = \frac{(\beta_q - \alpha_q) \hat{\Phi}_c}{k^{3/2} + \alpha_q \hat{\Phi}_c} \left[ 1 + 2c_q + \frac{c_q(\beta_q - \alpha_q) \hat{\Phi}_c}{k^{3/2} + \alpha_q \hat{\Phi}_c} \right]. \quad (C23)$$

The  $C_i$  above conveniently encapsulate the  $\hat{\Phi}_c$  dependence in the differential equation for  $f_L$ . We are interested in theories where the dilaton is light, which correspond to scenarios where  $\hat{\Phi}_c$  is a slowly varying function of  $\theta$ . In this limit we can solve the differential equation by making a WKB approximation, treating the  $C_i$  as constants independent of  $\theta$ . With this assumption the solution to Eq. (C20) takes the form

$$f_L(\theta) = e^{kr_c(5-C_1)\theta/2} \left[ A_1 J_{n(\theta)} \left( \frac{p}{k} e^{kr_c\theta} \right) + A_2 J_{-n(\theta)} \left( \frac{p}{k} e^{kr_c\theta} \right) \right] \quad (C24)$$

where

$$n(\theta) = \left| \left( c_q + \frac{1}{2} \right) \sqrt{1 + \frac{2C_1^2 - 4C_2 + 4c_q C_3}{(1 + 2c_q)^2}} \right|. \quad (C25)$$

We can check if this function solves the ODE Eq. (C20). In so doing we employ an identity, which can be most easily seen from the infinite series definition of the Bessel function:

$$\frac{d}{dx} J_{n(x)}(x) = J'_{n(x)}(x) + \frac{dn(x)}{dx} \frac{d}{dn} J_n(x) \quad (C26)$$

where  $J'_n$  is the usual derivative of the Bessel function with respect to its argument. The first term of the above is like those terms that appear when  $n$  has no dependence on  $x$  and will satisfy the ODE. The second term leads to terms that will not satisfy the ODE. These terms, however, are proportional to  $\frac{dn}{dx}$  which is in turn proportional to  $\hat{\Phi}'_c$  or higher order derivatives of  $\hat{\Phi}_c$  with no compensating large factors. Therefore, our solution for  $f_L$  is valid up to small corrections proportional to derivatives of the slowly varying function  $\hat{\Phi}_c$ .

Then, to be consistent, we must drop all  $\hat{\Phi}'_c$  terms in the  $C_i$ . In this limit  $C_1$  and  $C_2$  vanish, leaving us with the solution

$$f_L(\theta) = e^{\frac{5}{2}kr_c\theta} \left[ A_1 J_{n(\theta)} \left( \frac{p}{k} e^{kr_c\theta} \right) + A_2 J_{-n(\theta)} \left( \frac{p}{k} e^{kr_c\theta} \right) \right] \quad (C27)$$

where

$$n(\theta) = \left| \left( c_q + \frac{1}{2} \right) \sqrt{1 + \frac{4c_q C_3(\theta)}{(1 + 2c_q)^2}} \right| = \left| \frac{1}{2} + c_q + \frac{c_q(\beta_q - \alpha_q) \hat{\Phi}_c(\theta)}{k^{3/2} + \alpha_q \hat{\Phi}_c(\theta)} \right|. \quad (C28)$$

Now satisfied that Eq. (C27) solves the ODE we enforce the IR brane boundary condition  $f_L(\pi) = 0$ . This yields

$$f_L(\theta) = N_L e^{\frac{5}{2}kr_c\theta} \left[ J_{-n(\pi)} \left( \frac{p}{k} e^{kr_c\pi} \right) J_{n(\theta)} \left( \frac{p}{k} e^{kr_c\theta} \right) - J_{n(\pi)} \left( \frac{p}{k} e^{kr_c\pi} \right) J_{-n(\theta)} \left( \frac{p}{k} e^{kr_c\theta} \right) \right], \quad (C29)$$

with  $N_L$  a UV dependent normalization constant. We also find

$$f_R(\theta) = N_L e^{\frac{5}{2}kr_c\theta} \left[ J_{-n(\pi)} \left( \frac{p}{k} e^{kr_c\pi} \right) J_{n(\theta)-1} \left( \frac{p}{k} e^{kr_c\theta} \right) + J_{n(\theta)} \left( \frac{p}{k} e^{kr_c\pi} \right) J_{1-n(\theta)} \left( \frac{p}{k} e^{kr_c\theta} \right) \right]. \quad (C30)$$

With  $f_L$  and  $f_R$  in hand, for slowly varying  $\hat{\Phi}_c$ , we evaluate the two point correlator on the UV brane at  $\theta_0$ . Using (C18) we find

$$\Sigma_{\theta_0}(p) = \frac{\not{p} J_{-n(\pi)} \left( \frac{p}{k} e^{kr_c\pi} \right) J_{n(\theta_0)-1} \left( \frac{p}{k} e^{kr_c\theta_0} \right) + J_{n(\pi)} \left( \frac{p}{k} e^{kr_c\pi} \right) J_{1-n(\theta_0)} \left( \frac{p}{k} e^{kr_c\theta_0} \right)}{p J_{-n(\pi)} \left( \frac{p}{k} e^{kr_c\pi} \right) J_{n(\theta_0)} \left( \frac{p}{k} e^{kr_c\theta_0} \right) - J_{n(\pi)} \left( \frac{p}{k} e^{kr_c\pi} \right) J_{-n(\theta_0)} \left( \frac{p}{k} e^{kr_c\theta_0} \right)}. \quad (C31)$$



We wish to express the two point function as a power series in  $p/k$  to determine the scaling dimension of  $\mathcal{O}$ . To suppress effects associated with spontaneous conformal symmetry breaking we work in the limit that the IR brane is far away by choosing  $p$  such that  $\frac{p}{k} e^{kr_c \pi} \gg 1$ . In order to avoid spurious conformal symmetry violating effects associated with the regulator, we must also stay away from the UV brane by choosing  $p$  such that  $\frac{p}{k} e^{kr_c \theta_0} \ll 1$ . In this limit we can employ the small Bessel expansion for the terms with  $\theta_0$  in Eq. (C31). We also Wick rotate the momenta to tame the oscillations of the Bessel functions. Using the asymptotic expansions of the Bessel functions for both small and large argument we obtain a result of the form

$$\lim_{kr_c \rightarrow \infty} \Sigma_{\theta_0}(p) = \frac{p}{k} \left[ a_1(\theta_0) + a_2(\theta_0) \left( \frac{p}{k} \right)^2 + \dots + b_1(\theta_0) \left( \frac{p}{k} \right)^{|2n(\theta_0)-2|} + \dots \right] \quad (\text{C32})$$

where the  $a_i$  are the coefficients of analytic terms and the  $b_i$  those of the nonanalytic terms. The coefficients depend on the location of the extra dimension,  $\theta_0$ , but they are independent of  $p$ . Comparing the power of the  $b_1$  term to Eq. (C15) we immediately find that for  $n > 1$  ( $c_q > 1/2$  at leading order)

$$\Delta_{\mathcal{Q}}(\theta_0) = \frac{3}{2} + n(\theta_0). \quad (\text{C33})$$

A similar expression can be derived for  $n < 1$ .

We are interested in the scaling dimensions just above the conformal symmetry breaking scale, which corresponds to the region just outside the boundary layer near the IR brane. This corresponds to

$$\theta_0 = \pi - \frac{x}{kr_c}. \quad (\text{C34})$$

where  $x$  is a number of order a few. We must check that the approximations that led to Eq. (C33) continue to remain valid this close to the IR brane. In order for the asymptotic forms of the Bessel functions to be applicable,  $p$  must be chosen to simultaneously satisfy

$$\frac{p}{k} e^{kr_c \pi} \gg 1, \quad \frac{p}{k} e^{kr_c \pi} e^{-x} \ll 1. \quad (\text{C35})$$

These conditions can indeed be satisfied provided  $e^{-x} \ll 1$ , which corresponds to  $x$  of order a few.

We now evaluate  $n(\theta_0)$  in this limit. The  $\theta_0$  dependence of  $n(\theta_0)$  comes from the outer region GW solution Eq. (2.11)  $\hat{\Phi}_{\text{OR}}(\theta_0)$ . In the limit of large  $kr_c$  we find

$$\hat{\Phi}_{\text{OR}}(\theta_0) = \hat{\Phi}_{\text{OR}}(\pi) - x \hat{\Phi}'_{\text{OR}}(\pi) + \dots \quad (\text{C36})$$

Because we have been dropping all  $\hat{\Phi}'_{\text{OR}}$  to obtain the Bessel function solution (C27) we must also drop the second and higher terms in the expansion above. We are left with  $\hat{\Phi}_{\text{OR}}(\theta_0) = \hat{\Phi}_{\text{OR}}(\pi)$ . Therefore,

$$\begin{aligned} \Delta_{\mathcal{Q}}|_{\text{IR}} &= \frac{3}{2} + n(\pi) \\ &= \frac{3}{2} + \left| \frac{1}{2} + c_q + \frac{c_q(\beta_q - \alpha_q) \hat{\Phi}_{\text{OR}}(\pi)}{k^{3/2} + \alpha_q \hat{\Phi}_{\text{OR}}(\pi)} \right| \\ &\quad + \mathcal{O}(k^{-3/2} \hat{\Phi}'_{\text{OR}}). \end{aligned} \quad (\text{C37})$$

## APPENDIX D: NAIVE DIMENSIONAL ANALYSIS ESTIMATION OF PARAMETERS

In this appendix we estimate the sizes of the various parameters in the theory, using the methods of naive dimensional analysis (NDA) [74,75] as generalized to higher dimensions [76]. A more detailed explanation of some of these estimates may be found in Appendix C of [30]. The underlying philosophy behind NDA estimates is that in a strongly coupled theory, the radiative corrections to any process are expected to be comparable at every loop order. Since holography relates the interactions in the bulk and on the IR brane to the dynamics of a strongly coupled CFT, we expect that NDA will offer a guide to the sizes of the parameters in these regions. The dynamics on the UV brane, on the other hand, is associated with the interactions of states external to the CFT. Therefore, we do not expect that NDA will offer a useful guide to the sizes of parameters on this brane.

Following [76] we can write the  $D$ -dimensional Lagrangian of a strongly coupled theory as

$$\mathcal{L}_D \sim \frac{N \Lambda^D}{\ell_D} \tilde{\mathcal{L}}(\bar{\Phi}, \partial/\Lambda), \quad (\text{D1})$$

Here  $\bar{\Phi}$  represents the fields in the theory normalized so as to be dimensionless,  $\Lambda$  is the cutoff of the theory, and  $N$  is the number of states going around the loops. The loop factor, which comes from integrating over  $D$  dimensional phase space, is given in four dimensions by  $\ell_4 = 16\pi^2$ , while in five dimensions it is given by  $\ell_5 = 24\pi^3$ . All parameters in  $\tilde{\mathcal{L}}$  are dimensionless and taken to be  $\mathcal{O}(1)$ . Rescaling the fields so that kinetic terms are canonically normalized then gives all Lagrangian parameters in terms of the cutoff, the loop factor, and the number of states participating in the correcting loops.

We begin by analyzing the gravity Lagrangian,  $\mathcal{L} \sim 2M_5^3 \mathcal{R}$ . The above prescription allows us to relate the cutoff  $\Lambda_{\text{IR}}$  to the 5-dimensional Planck mass  $M_5$ ,

$$\Lambda_{\text{IR}} \sim \left( \frac{\ell_5}{N} \right)^{1/3} M_5. \quad (\text{D2})$$

We can also estimate the size of the bulk cosmological constant  $\Lambda_b$  that would be radiatively generated by the strong dynamics,

$$\Lambda_b \sim \frac{N\Lambda_{\text{IR}}^5}{\ell_5} \sim \left(\frac{\ell_5}{N}\right)^{2/3} M_5^5. \quad (\text{D3})$$

Einstein's equations then allow us to estimate the natural size of the curvature  $k$ , in units of the cutoff. From Eq. (2.3) we obtain

$$k = \sqrt{\frac{-\Lambda_b}{24M_5^3}} \sim \frac{\Lambda_{\text{IR}}}{\sqrt{24}}, \quad (\text{D4})$$

from which we can express

$$\left(\frac{k}{M_5}\right)^3 \sim 24^{-3/2} \frac{\ell_5}{N} \sim \frac{6}{N}. \quad (\text{D5})$$

### 1. GW scalar potential

Next we analyze the bulk potential for the GW field. The potential in Eq. (2.8) is parametrized by  $\epsilon = m^2/4k^2$  and  $\xi = \eta v/8\sqrt{k}$ . The bulk mass of the GW scalar is estimated to be simply  $m^2 \sim \Lambda_{\text{IR}}^2$ , but we need the bulk mass to be small in order for the size of the extra dimension to be stabilized at a large value [29]. Therefore, we take the NDA estimate to be an upper bound:

$$\epsilon \lesssim 6. \quad (\text{D6})$$

The bulk cubic is estimated as

$$\eta \sim \sqrt{\frac{\ell_5 \Lambda_{\text{IR}}}{N}}. \quad (\text{D7})$$

If we use NDA on the UV brane, we can estimate  $v \sim 0.4$ , but the dynamics of the UV brane is weakly coupled, so  $v$  is expected to be smaller than its NDA value. Furthermore, we need  $v$  to be small in order to have an approximately conformal dual because it corresponds to explicit breaking of the CFT. Putting it all together, we get an NDA upper bound of

$$\xi \lesssim \frac{3}{\sqrt{N}}. \quad (\text{D8})$$

Finally, we can now estimate the size of the VEV  $\hat{\Phi}$  on the IR brane by looking at the IR brane potential parameter  $\alpha$ . Using the NDA prescription on the 4D brane we find that

$$\frac{\hat{\Phi}(\pi)}{k^{3/2}} \simeq \frac{\alpha}{4} \sim \frac{\sqrt{N}\ell_5}{8\ell_4} \left(\frac{\Lambda_{\text{IR}}}{k}\right)^{5/2} \sim 1.1\sqrt{N}. \quad (\text{D9})$$

### 2. Couplings of SM fields

In order to estimate the size of the gauge couplings in Eq. (3.1), we work in a convention where the gauge field is treated on the same footing as a spacetime derivative so that the gauge covariant derivative is  $D_\mu = \partial_\mu - iA_\mu$ . This allows us to generalize Eq. (D1) to  $\mathcal{L}_D \sim \frac{N\Lambda_D}{\ell_D} \tilde{\mathcal{L}}(\tilde{\Phi}, \partial/\Lambda, A/\Lambda)$ , where  $\tilde{\Phi}$  represents the non-gauge fields and  $A$  the gauge fields. We can then estimate the size of the visible brane gauge coupling in terms of the 4D loop factor,

$$g_{\text{IR}} \sim \frac{4\pi}{\sqrt{N}}. \quad (\text{D10})$$

The bulk gauge coupling is dimensionful, and we can estimate the following useful combination

$$g_5^2 k \sim \sqrt{24} \frac{\pi^3}{N}. \quad (\text{D11})$$

The IR coupling  $g_{\text{IR}}$  and bulk gauge coupling  $g_5$  are expected to be of order their NDA values, because they are associated with the strong dynamics. The UV coupling  $g_{\text{UV}}$ , on the other hand, is associated with physics external to the strong dynamics, so it can naturally be smaller than its NDA value.

We now estimate the couplings of the GW field to SM-like fields. We begin with the coupling to gauge bosons as in Eq. (3.17) which is given schematically by

$$\frac{\Phi}{k^{3/2}} \left\{ \frac{\beta_{\text{UV}}}{4g_{\text{UV}}^2} \delta(\theta) + \frac{\beta}{4g_5^2} + \frac{\beta_{\text{IR}}}{4g_{\text{IR}}^2} \delta(\theta - \pi) \right\} F^2. \quad (\text{D12})$$

Because the gauge coupling is already scaled out of the definition of  $\beta$ , all one needs to do to estimate its size is rescale  $\Phi$  so it is canonically normalized using the prescription of Eq. (D1). Therefore we find that

$$\beta \sim \sqrt{\frac{\ell_5}{N}} \left(\frac{k}{\Lambda_{\text{IR}}}\right)^{3/2} \sim \frac{2.5}{\sqrt{N}}. \quad (\text{D13})$$

To estimate the IR brane coupling to bulk fields  $\beta_{\text{IR}}$ , we note that  $F^2$  is normalized as a 5D operator, while in the NDA prescription the brane operator is multiplied by the 4D loop factor. Therefore, we find that

$$\beta_{\text{IR}} \sim \frac{\ell_5^{3/2}}{\ell_4 \sqrt{N}} \left(\frac{k}{\Lambda_{\text{IR}}}\right)^{5/2} \sim \frac{2.4}{\sqrt{N}}, \quad (\text{D14})$$

which is numerically similar to the estimate for the bulk coupling  $\beta$ . The size of  $\beta_{UV}$  is not correlated with the NDA estimate.

The GW scalar can also couple to the Higgs kinetic term on the IR brane as in Eq. (4.3),

$$\beta_W \frac{\Phi}{k^{3/2}} \delta(\theta - \pi) (\mathcal{D}^\mu H)^\dagger (\mathcal{D}_\mu H). \quad (\text{D15})$$

An estimate of the NDA size of  $\beta_W$  yields a result similar to that of the gauge kinetic term on the IR brane,  $\sqrt{N}\beta_W \sim \sqrt{N}\beta_{IR} \sim 2.4$ . The coupling of the GW field to the fermions is given in Eq. (5.9),

$$\frac{\Phi}{k^{3/2}} \left[ \left( \alpha_q \frac{i}{2} e_a^M \bar{Q} \Gamma^a \overleftrightarrow{\partial}_M Q - \beta_q k c_q \bar{Q} Q \right) + \delta(\theta - \pi) \alpha_y \left( \frac{Y}{k} \bar{Q} H U + \text{H.c.} \right) \right]. \quad (\text{D16})$$

We have only shown  $Q$ , but the generalization to other fermions is clear. We find that  $\sqrt{N}\alpha_y \sim \sqrt{N}\beta_v \sim 2.4$ , and that  $\sqrt{N}\alpha_q \sim \sqrt{N}\beta \sim 2.5$ . Here we are assuming that the

coupling to the GW scalar does not break the SM flavor symmetries.

Finally we come to  $\beta_q$ . Before we can determine this, we must first estimate the size of the dimensionless coefficient  $c_q$  that parametrizes the bulk mass term. This is given by

$$c_{q,u} \sim \frac{\Lambda_{IR}}{k} \sim 4.9. \quad (\text{D17})$$

If the mass term  $c_q$  was of order its NDA size, then we would have  $\sqrt{N}\beta_q \sim \sqrt{N}\alpha_q \sim 2.5$ . However, in order to generate a realistic spectrum of fermion masses, it is necessary to take values of  $c_q$  close to 1/2, significantly below its NDA value. It follows that the estimate of the coupling to the GW scalar is modified to

$$\beta_q \sim \frac{2.5}{\sqrt{N}} \frac{c_q^{\text{NDA}}}{c_q}, \quad (\text{D18})$$

where  $c_q^{\text{NDA}}$  is given in Eq. (D17).

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- [1] M. A. Luty and T. Okui, Conformal technicolor, *J. High Energy Phys.* **09** (2006) 070.
- [2] B. Holdom, Technicolor, *Phys. Lett. B* **150**, 301 (1985).
- [3] T. W. Appelquist, D. Karabali, and L. Wijewardhana, Chiral Hierarchies and the Flavor Changing Neutral Current Problem in Technicolor, *Phys. Rev. Lett.* **57**, 957 (1986).
- [4] K. Yamawaki, M. Bando, and K.-i. Matumoto, Scale Invariant Technicolor Model and a Technidilaton, *Phys. Rev. Lett.* **56**, 1335 (1986).
- [5] T. Appelquist and L. Wijewardhana, Chiral hierarchies and chiral perturbations in technicolor, *Phys. Rev. D* **35**, 774 (1987).
- [6] A. Salam and J. Strathdee, Nonlinear realizations. 2. Conformal symmetry, *Phys. Rev.* **184**, 1760 (1969).
- [7] C. Isham, A. Salam, and J. Strathdee, Spontaneous breakdown of conformal symmetry, *Phys. Lett. B* **31**, 300 (1970).
- [8] B. Zumino, in *1970 Brandeis Summer School*, edited by S. Deser (MIT Press, Cambridge, MA, 1970).
- [9] J. R. Ellis, Aspects of conformal symmetry and chirality, *Nucl. Phys.* **B22**, 478 (1970).
- [10] J. R. Ellis, Phenomenological actions for spontaneously-broken conformal symmetry, *Nucl. Phys.* **B26**, 536 (1971).
- [11] Z. Chacko and R. K. Mishra, Effective theory of a light dilaton, *Phys. Rev. D* **87**, 115006 (2013).
- [12] B. Bellazzini, C. Csaki, J. Hubisz, J. Serra, and J. Terning, A Higgslike dilaton, *Eur. Phys. J. C* **73**, 2333 (2013).
- [13] R. Rattazzi, in Proceedings of Planck 2010, <https://indico.cern.ch/event/75810/contribution/163/attachments/1050757/1498158/Rattazzi.pdf>; A. Pomarol, in Proceedings of Xmas10; F. Coradeschi, P. Lodone, D. Pappadopulo, R. Rattazzi, and L. Vitale, A naturally light dilaton, *J. High Energy Phys.* **11** (2013) 057.
- [14] T. Appelquist and Y. Bai, A light dilaton in walking gauge theories, *Phys. Rev. D* **82**, 071701 (2010).
- [15] D. Elander, C. Nunez, and M. Piai, A light scalar from walking solutions in gauge-string duality, *Phys. Lett. B* **686**, 64 (2010).
- [16] D. Elander and M. Piai, On the glueball spectrum of walking backgrounds from wrapped-D5 gravity duals, *Nucl. Phys.* **B871**, 164(2013).
- [17] N. Evans and K. Tuominen, Holographic modelling of a light technidilaton, *Phys. Rev. D* **87**, 086003 (2013).
- [18] D. Elander, Light scalar from deformations of the Klebanov-Strassler background, *Phys. Rev. D* **91**, 126012 (2015).
- [19] W. D. Goldberger, B. Grinstein, and W. Skiba, Distinguishing the Higgs boson from the dilaton at the Large Hadron Collider, *Phys. Rev. Lett.* **100**, 111802 (2008).
- [20] J. Fan, W. D. Goldberger, A. Ross, and W. Skiba, Standard model couplings and collider signatures of a light scalar, *Phys. Rev. D* **79**, 035017 (2009).
- [21] L. Vecchi, Phenomenology of a light scalar: The dilaton, *Phys. Rev. D* **82**, 076009 (2010).
- [22] J. M. Maldacena, The large N limit of superconformal field theories and supergravity, *Adv. Theor. Math. Phys.* **2**, 231 (1998).
- [23] S. Gubser, I. R. Klebanov, and A. M. Polyakov, Gauge theory correlators from noncritical string theory, *Phys. Lett. B* **428**, 105 (1998).

- [24] E. Witten, Anti-de Sitter space and holography, *Adv. Theor. Math. Phys.* **2**, 253 (1998).
- [25] I. R. Klebanov and E. Witten, AdS/CFT correspondence and symmetry breaking, *Nucl. Phys.* **B556**, 89 (1999).
- [26] N. Arkani-Hamed, M. Porrati, and L. Randall, Holography and phenomenology, *J. High Energy Phys.* **08** (2001) 017.
- [27] R. Rattazzi and A. Zaffaroni, Comments on the holographic picture of the Randall-Sundrum model, *J. High Energy Phys.* **04** (2001) 021.
- [28] L. Randall and R. Sundrum, A large mass hierarchy from a small extra dimension, *Phys. Rev. Lett.* **83**, 3370 (1999).
- [29] W. D. Goldberger and M. B. Wise, Modulus stabilization with bulk fields, *Phys. Rev. Lett.* **83**, 4922 (1999).
- [30] Z. Chacko, R. K. Mishra, and D. Stolarski, Dynamics of a stabilized radion and duality, *J. High Energy Phys.* **09** (2013) 121.
- [31] B. Bellazzini, C. Csaki, J. Hubisz, J. Serra, and J. Terning, A naturally light dilaton and a small cosmological constant, *Eur. Phys. J. C* **74**, 2790 (2014).
- [32] T. Konstandin, G. Nardini, and M. Quiros, Gravitational backreaction effects on the holographic phase transition, *Phys. Rev. D* **82**, 083513 (2010).
- [33] Y. Eshel, S. J. Lee, G. Perez, and Y. Soreq, Shining flavor and radion phenomenology in warped extra dimension, *J. High Energy Phys.* **10** (2011) 015.
- [34] E. Megias and O. Pujolas, Naturally light dilatons from nearly marginal deformations, *J. High Energy Phys.* **08** (2014) 081.
- [35] P. Cox and T. Gherghetta, A soft-wall dilaton, *J. High Energy Phys.* **02** (2015) 006.
- [36] C. Csaki, M. Graesser, L. Randall, and J. Terning, Cosmology of brane models with radion stabilization, *Phys. Rev. D* **62**, 045015 (2000).
- [37] W. D. Goldberger and M. B. Wise, Phenomenology of a stabilized modulus, *Phys. Lett. B* **475**, 275 (2000).
- [38] G. F. Giudice, R. Rattazzi, and J. D. Wells, Gravitational mixing, *Nucl. Phys.* **B595**, 250 (2001).
- [39] C. Csaki, M. L. Graesser, and G. D. Kribs, Radion dynamics and electroweak physics, *Phys. Rev. D* **63**, 065002 (2001).
- [40] T. G. Rizzo, Radion couplings to bulk fields in the Randall-Sundrum model, *J. High Energy Phys.* **06** (2002) 056.
- [41] C. Csaki, J. Hubisz, and S. J. Lee, Radion phenomenology in realistic warped space models, *Phys. Rev. D* **76**, 125015 (2007).
- [42] N. Arkani-Hamed and M. Schmaltz, Hierarchies without symmetries from extra dimensions, *Phys. Rev. D* **61**, 033005 (2000).
- [43] Y. Grossman and M. Neubert, Neutrino masses and mixings in nonfactorizable geometry, *Phys. Lett. B* **474**, 361 (2000).
- [44] T. Gherghetta and A. Pomarol, Bulk fields and supersymmetry in a slice of AdS, *Nucl. Phys.* **B586**, 141 (2000).
- [45] K. Agashe, G. Perez, and A. Soni, Flavor structure of warped extra dimension models, *Phys. Rev. D* **71**, 016002 (2005).
- [46] D. B. Kaplan, Flavor at SSC energies: A new mechanism for dynamically generated fermion masses, *Nucl. Phys.* **B365**, 259 (1991).
- [47] K. Cheung and T.-C. Yuan, Could the Excess Seen at 124-126 GeV Be due to the Randall-Sundrum Radion?, *Phys. Rev. Lett.* **108**, 141602 (2012).
- [48] S. Matsuzaki and K. Yamawaki, Techni-dilaton at 125 GeV, *Phys. Rev. D* **85**, 095020 (2012).
- [49] B. Grzadkowski, J. F. Gunion, and M. Toharia, Higgs-Radion interpretation of the LHC data?, *Phys. Lett. B* **712**, 70 (2012).
- [50] S. Matsuzaki and K. Yamawaki, Discovering 125 GeV techni-dilaton at LHC, *Phys. Rev. D* **86**, 035025 (2012).
- [51] S. Matsuzaki and K. Yamawaki, Is 125 GeV techni-dilaton found at LHC?, *Phys. Lett. B* **719**, 378 (2013).
- [52] D. Elander and M. Piai, The decay constant of the holographic techni-dilaton and the 125 GeV boson, *Nucl. Phys.* **B867**, 779 (2013).
- [53] S. Matsuzaki and K. Yamawaki, Holographic techni-dilaton at 125 GeV, *Phys. Rev. D* **86**, 115004 (2012).
- [54] Z. Chacko, R. Franceschini, and R. K. Mishra, Resonance at 125 GeV: Higgs or Dilaton/Radion?, *J. High Energy Phys.* **04** (2013) 015.
- [55] G.-C. Cho, D. Nomura, and Y. Ohno, Constraints on radion in a warped extra dimension model from Higgs boson searches at the LHC, *Mod. Phys. Lett. A* **28**, 1350148 (2013).
- [56] N. Desai, U. Maitra, and B. Mukhopadhyaya, An updated analysis of radion-Higgs mixing in the light of LHC data, *J. High Energy Phys.* **10** (2013) 093.
- [57] J. Cao, Y. He, P. Wu, M. Zhang, and J. Zhu, Higgs phenomenology in the minimal dilaton model after Run I of the LHC, *J. High Energy Phys.* **01** (2014) 150.
- [58] D.-W. Jung and P. Ko, Higgs-dilaton (radion) system confronting the LHC Higgs data, *Phys. Lett. B* **732**, 364 (2014).
- [59] E. Boos, S. Keizerov, E. Rahmetov, and K. Svirina, Higgs boson-radion similarity in production processes involving off-shell fermions, *Phys. Rev. D* **90**, 095026 (2014).
- [60] S. Bhattacharya, M. Frank, K. Huitu, U. Maitra, B. Mukhopadhyaya, and S. K. Rai, Probing the light radion through diphotons at the Large Hadron Collider, *Phys. Rev. D* **91**, 016008 (2015).
- [61] G.-C. Cho and Y. Ohno, Production and decay of radion in Randall-Sundrum model at a photon collider, *Mod. Phys. Lett. A* **29**, 1450136 (2014).
- [62] Y. Bai, M. Carena, and J. Lykken, Dilaton-assisted dark matter, *Phys. Rev. Lett.* **103**, 261803 (2009).
- [63] K. Agashe, K. Blum, S. J. Lee, and G. Perez, Astrophysical implications of a visible dark matter sector from a custodially warped GUT, *Phys. Rev. D* **81**, 075012 (2010).
- [64] K. Blum, M. Cliche, C. Csaki, and S. J. Lee, WIMP dark matter through the dilaton portal, *J. High Energy Phys.* **03** (2015) 099.
- [65] A. Efrati, E. Kuflik, S. Nussinov, Y. Soreq, and T. Volansky, Constraining the Higgs-dilaton with LHC and dark matter searches, *Phys. Rev. D* **91**, 055034 (2015).
- [66] G. Servant, Baryogenesis from Strong CP Violation and the QCD Axion, *Phys. Rev. Lett.* **113**, 171803 (2014).
- [67] P. Breitenlohner and D. Z. Freedman, Stability in gauged extended supergravity, *Ann. Phys. (N.Y.)* **144**, 249 (1982).



- [68] W. D. Goldberger and I. Z. Rothstein, Quantum stabilization of compactified AdS(5), *Phys. Lett. B* **491**, 339 (2000).
- [69] J. Garriga and A. Pomarol, A Stable hierarchy from Casimir forces and the holographic interpretation, *Phys. Lett. B* **560**, 91 (2003).
- [70] C. Csaki, J. Erlich, and J. Terning, The effective Lagrangian in the Randall-Sundrum model and electroweak physics, *Phys. Rev. D* **66**, 064021 (2002).
- [71] J. Hewett, F. Petriello, and T. Rizzo, Precision measurements and fermion geography in the Randall-Sundrum model revisited, *J. High Energy Phys.* 09 (2002) 030.
- [72] A. Azatov, M. Toharia, and L. Zhu, Radion mediated flavor changing neutral currents, *Phys. Rev. D* **80**, 031701 (2009).
- [73] R. Contino and A. Pomarol, Holography for fermions, *J. High Energy Phys.* 11 (2004) 058.
- [74] A. Manohar and H. Georgi, Chiral quarks and the non-relativistic quark model, *Nucl. Phys.* **B234**, 189 (1984).
- [75] H. Georgi and L. Randall, Flavor conserving  $CP$  violation in invisible axion models, *Nucl. Phys.* **B276**, 241 (1986).
- [76] Z. Chacko, M. A. Luty, and E. Ponton, Massive higher dimensional gauge fields as messengers of supersymmetry breaking, *J. High Energy Phys.* 07 (2000) 036.