



# Experimental sound power from curved plates using the radiation resistance matrix and a scanning vibrometer

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## ABSTRACT

*Vibration-based sound power (VBSP) methods based on elemental radiators and measurements from a scanning vibrometer have been shown to be accurate for flat plates and cylinders. In this paper, the VBSP method is extended to account for simple curved structures, with a constant radius of curvature. Data are also presented that suggest the VBSP method is more accurate than the ISO 3741 standard for measuring sound power when significant background noise is present. Experimental results from ISO 3741 and the VBSP methods are compared for three simple curved plate structures with different radii of curvature. The results show good agreement for all three structures over a wide frequency range. The experimental results also indicate that the VBSP method is more accurate in the low frequency range where the curved plates radiated relatively little and significant background noise was present.*

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## 1. INTRODUCTION

Sound power is the standard metric for measuring and comparing noise radiation from a source. There are many methods for measuring sound power. For example, the International Organization for Standardization (ISO) has published ten standards and two technical specifications for measuring sound power with different levels of precision. There is no precision grade standardized method based on vibration measurements; the two technical specifications that are vibration measurement based are only capable of engineering or survey grade results. Theory was developed in the early 1990s for a vibration-based sound power measurement method which could potentially provide precision grade results; this method calculates sound power from a combination of measured surface velocities and the acoustic radiation resistance.<sup>1</sup> Recent work has been performed to validate a vibration-based method for measuring sound power from simple flat plates and cylinders and these promising results show potential for further development.<sup>2,3</sup> This paper details progress in developing the Vibration-Based Sound Power (VBSP) method for measuring sound power from a vibrating surface by experimentally validating the VBSP method for simple curved plates.

Acoustic radiation modes are a useful basis for describing acoustic radiation from a structure. They describe the acoustic field, with modes that are orthogonal with respect to sound radiation, and allow the surrounding acoustical field to be determined based on the vibrations of a structure. In contrast, structural vibration modes describe the displacement of the structural surface and must satisfy structural equations of motion and boundary conditions. Acoustic radiation modes can be obtained from the radiation resistance matrix,  $\mathbf{R}$ . The eigenvectors of the radiation resistance matrix are the acoustic radiation modes, and the eigenvalues are proportional to the radiation efficiency of their respective acoustic radiation modes. The radiation resistance matrix relates the normal surface velocities from discrete sections of the radiating structure to the radiated sound power through the equation

$$P(\omega) = \mathbf{v}_e^H(\omega)\mathbf{R}(\omega)\mathbf{v}_e(\omega) \quad (1)$$

where  $\mathbf{v}_e$  is a vector of the velocity of each elementary radiator,  $(\cdot)^H$  is the Hermitian transpose, and  $\omega$  is the frequency of interest. The form of the radiation resistance matrix is specific to the general shape of the structure of interest but does not depend on the boundary conditions associated with the structure. Previous work has established the form(s) of the radiation resistance matrix for flat plates and for cylinders.<sup>2,3</sup> For example, the form of the radiation resistance matrix for flat plates is

$$\mathbf{R}(\omega) = \frac{\omega^2 \rho_0 A_e^2}{4\pi c} \begin{bmatrix} 1 & \frac{\sin(kd_{12})}{kd_{12}} & \dots & \frac{\sin(kd_{1N})}{kd_{1N}} \\ \frac{\sin(kd_{21})}{kd_{21}} & 1 & \dots & \vdots \\ \vdots & \dots & \ddots & \vdots \\ \frac{\sin(kd_{N1})}{kd_{N1}} & \dots & \dots & 1 \end{bmatrix} \quad (2)$$

where  $\omega$  is the angular frequency,  $\rho_0$  is the density of the surrounding fluid,  $A_e$  is the area of a single discrete element,  $c$  is the speed of sound in the fluid,  $k$  is the acoustic wavenumber, and  $d_{ij}$  is the distance from the  $i^{th}$  to the  $j^{th}$  element.

The VBSP method is based on using complex-valued (frequency domain) surface velocity measurements with the radiation resistance matrix to compute sound power. This paper will present the method employed to obtain these surface velocity measurements for simple curved structures to enable sound power comparison between the VBSP and ISO 3741 methods. Experimental validation results of the VBSP method for curved plates will then be presented that quantify the differences between the experimental results of these two methods.

## 2. METHODS

Sound power measurements of three curved plates were performed using the VBSP and ISO 3741 methods to validate the VBSP approach for computing sound power from curved plates. Each of the three curved plates measured has a significantly different constant radius of curvature to provide evidence of consistent results for simple curved plates. The experimental results presented in this section are compared to sound power measurements obtained using ISO 3741 in a large reverberation chamber, the results being reported in one-third octave bands.

### 2.1. Experimental setup and measurement of three curved plates

The three curved plates used in obtaining the experimental results presented in this paper are of similar design, build, and materials, with the only significant difference between each being a different constant radius of curvature. As the setup for each of these curved plates was identical, the setup will be detailed in general and then results from each curved plate will be presented in turn.

The curved plates are identified as Tight Radius (TR) curved plate, Medium Radius (MR) curved plate, and Wide Radius (WR) curved plate. Each curved plate was composed of a thin aluminum rectangular plate curved in an approximately constant radius of curvature that was held in a heavy steel and aluminum frame (see Figure 1). The thin plate was clamped on the straight edges and approximately simply supported on the curved edges, with solid metal caps to prevent any radiation from the back of the plate. Table 1 summarizes the dimensions for each curved plate used in these experimental results.



Figure 1: Setup of the TR curved plate in a reverberation chamber with the wall of the reverberation chamber acting as a baffle. The MR and WR curved plates were setup in the same manner shown.

Table 1: Summary of curved plate dimensions used for experimental testing

Object	Height	Width	Thickness	Radius of curvature
<b>TR Curved Plate</b>	30 cm	29 cm	1.59 mm	15.5 cm
<b>MR Curved Plate</b>	30 cm	36 cm	1.59 mm	30 cm
<b>WR Curved Plate</b>	30 cm	40 cm	1.59 mm	51 cm

The setup of each curved plate approximates an infinitely baffled panel and was such that both the VBSP and ISO 3741 measurements could each be taken with the same setup. Each curved plate (one at a time) was mounted on one wall in a reverberation chamber with dimensions of approximately 5 m x 6 m x 7 m, as shown in Figure 1. To help seal the edges of the curved plate to the wall, black Gaff tape was applied around all four sides of the frame. Note that the additional blue masking tape shown was for the sole purpose of reducing reflections that interfere with the operation of the SLDV system. To excite the curved plate, a piezoelectric transducer was adhered to the backside of the upper right quadrant (when facing the front of the plate) of the curved plate.

Measurements were taken both with the SLDV system and as per ISO 3741. The SLDV system utilized for these measurements was a Polytec PSV-500-3D Scanning Vibrometer, featuring three laser heads that provide velocity data in three dimensions. Due to the curvature of the plates, the surface of each was scanned in two or three separate sections and then the sections were stitched together to provide a complete response of each curved plate (see Figure 2 and Figure 3).

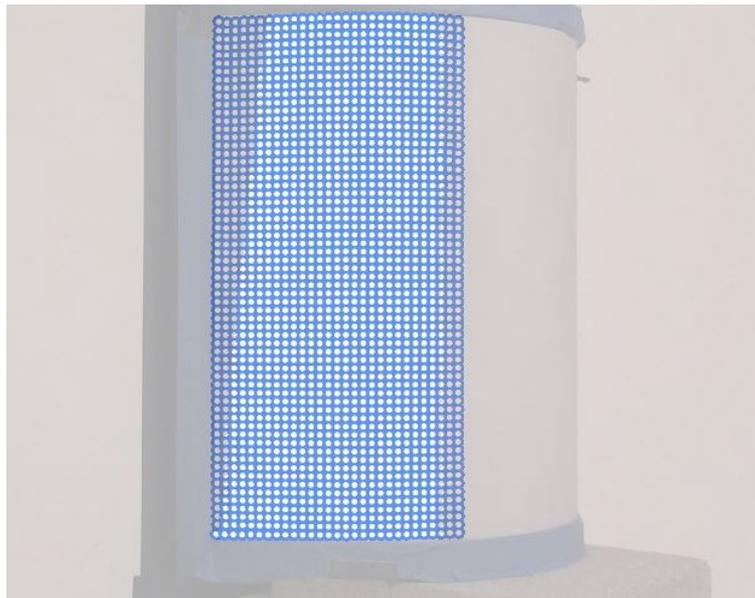


Figure 2: Example of one scan section taken over the surface of the TR curved plate to measure complex surface velocities, as seen in the SLDV software with scan points highlighted.

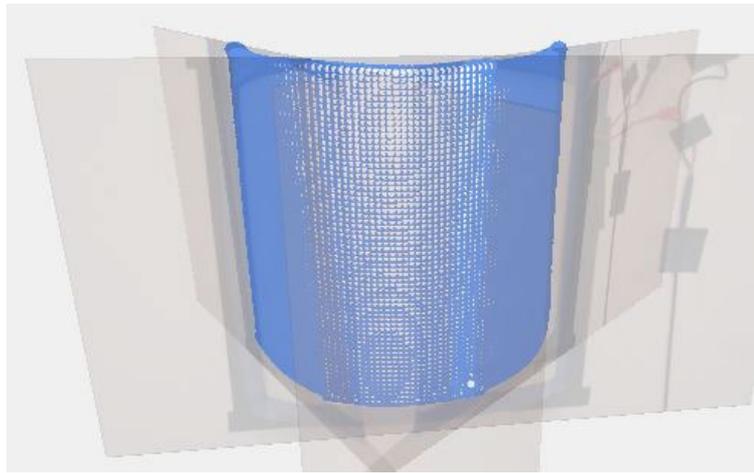


Figure 3: Example of stitched sections taken over the surface of the TR curved plate to measure the full complex surface velocity response, as seen in the SLDV software with scan points highlighted.

Calibrating the SLDV system to the surface of interest was performed to enable all three lasers to move precisely together to each specific scan location on the surface and provide three-dimensional (3D) data. To enable the 3D features of the SLDV device, known calibration points were marked on each surface of interest, based on a global coordinate system. This was accomplished by utilizing a 3D coordinate measurement arm system to measure and place precise marks on the surface of each plate, recording the corresponding coordinate location in 3D space. A separate SLDV calibration was performed for each section scanned on each curved plate.

After calibration, a scan grid was created to fit the scan section of interest with discrete scan locations where surface velocity data was to be measured. The density of the scan grid sections, or spatial sampling, was held constant for all scan sections of a curved plate but varied somewhat between each curved plate. All results presented here were obtained with a spatial sampling sufficient to resolve the curved plate velocity response up to 10 kHz. The piezoelectric transducer was excited with a pseudo random signal from 0 to 12.8 kHz and the SLDV was used to measure surface velocity data in three dimensions for each scan point of each section in turn. A full response for each curved plate was then obtained by stitching the scan sections together. Surface geometry data of the scan grid(s) were also collected in the form of an .STL file.

In preparation to make ISO 3741 sound power measurements, six microphones were placed inside the reverberation chamber as per the standard. After measuring the response of the curved plate with the SLDV, the SLDV system was completely shut down but left in the reverberation chamber for the ISO 3741 sound power measurement. Sound power was computed according to the VBSP (with the SLDV data) and ISO 3741 (with the sound pressure data from the microphone array) methods, respectively, with both sets of data reported and compared in one-third octave bands.

## 2.2. Surface normal velocity calculation

One critical element to the VBSP method is that the surface velocity normal to the vibrating surface must be known at each discrete scan point. Previous work on the VBSP method has accomplished this for simple flat plates by simply placing a single laser head normal to the surface and measuring the surface normal velocity directly.<sup>2</sup> This can also be done for a simple curved surface (constant radius of curvature in one direction) by setting a single laser head normal to a thin section of the surface at a time and performing many such narrow scans and rotating the structure or moving the laser head to capture each part of the full response. These narrow scan sections can then be stitched together to form a complete response.<sup>3</sup>

A 3D SLDV system, with three laser heads, has the advantage of scanning larger sections of a surface of interest and providing surface velocity data in three orthogonal directions. These results, magnitude and direction, are based on a global coordinate system set when calibrating the SLDV to a specific section of the surface of interest (see Figure 4). These surface velocity results must then be converted from 3D components of velocity to surface normal velocity, with respect to the orientation

of the surface at each discrete scan point. This was accomplished using the geometry data provided by the SLDV in the form of an .STL file. Data processing was used to smooth the geometry data as it contains significant noise. From the smoothed geometry data, surface normal vectors were then computed for each discrete scan point (see Figure 5). The normal vectors are matched with their corresponding discrete scan point location and 3D velocity vectors to compute the magnitude of the surface normal velocity by applying the dot product, defined as

$$V_n = \vec{V} \cdot \vec{N} \quad (3)$$

where  $V_n$  is the magnitude of the surface velocity in the surface normal direction,  $\vec{V}$  is the 3D velocity vector, and  $\vec{N}$  is the surface unit normal vector. This process was applied at each discrete scan point location for every frequency of interest. The surface normal velocity magnitude was then used to apply the VBSP method and compute sound power. This method for computing the surface normal velocity at each discrete scan point allows for the SLDV to scan larger sections of a surface at a time, thus making the process more efficient by requiring fewer laser calibrations for a given surface and fewer scan sections to stitch.

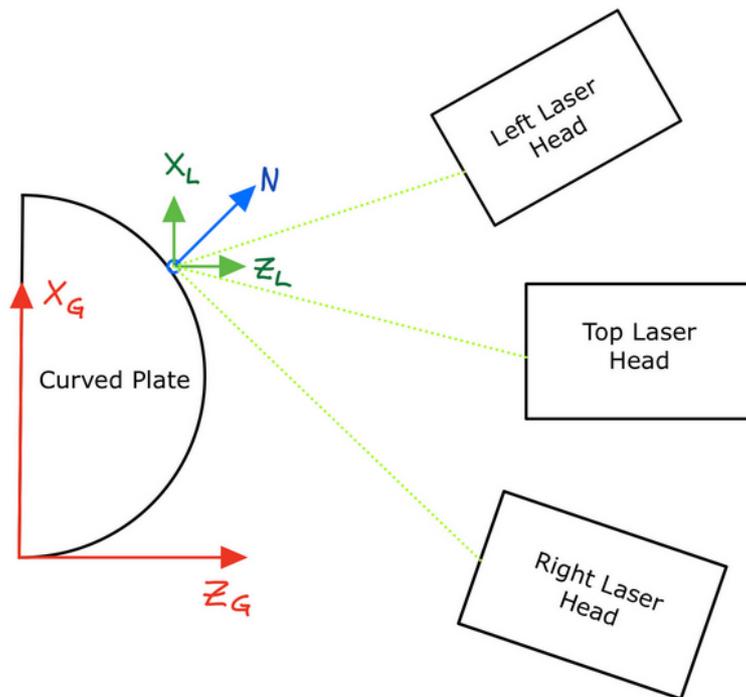


Figure 4: Schematic drawing (top view) of the SLDV three-laser system scanning an arbitrary scan point (indicated by a blue circle) on the surface of a curved plate. Note that the  $X_G$  and  $Z_G$  refer to the global coordinate system that the SLDV is calibrated to,  $X_L$  and  $Z_L$  refer to the coordinate location of the scan point as measured by the SLDV, and  $N$  is the surface normal vector at the scan point location. The SLDV system directly measures the three components of velocity at the scan point in the  $X_G$ ,  $Y_G$  (not shown but normal to the page), and  $Z_G$  directions. Data processing takes these components of velocity at each scan point and converts them to the velocity magnitude in the surface normal direction ( $N$ ).

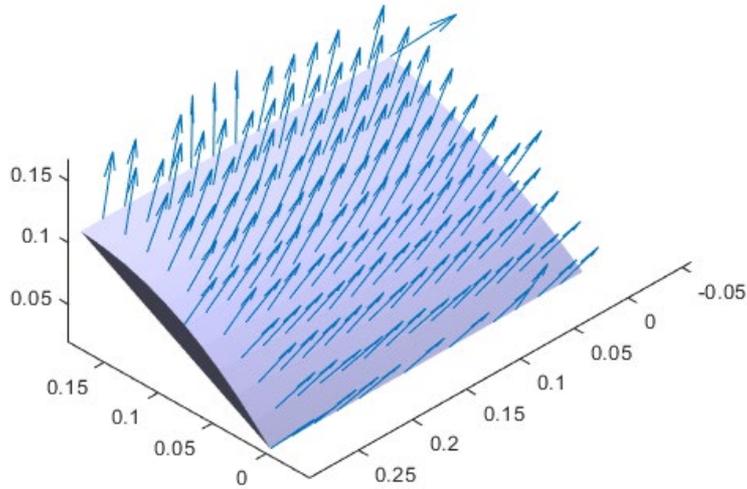


Figure 5: Example plot of unit normal vectors that are computed and placed on the surface of a curved plate. These vectors are obtained by processing geometry data from the SLDV.

### 3. SOUND POWER RESULTS OF THREE CURVED PLATES

#### 3.1. MR curved plate results

Figure 6 shows a comparison between the VBSP method and the ISO 3741 sound power results for the MR curved plate. The background noise of the reverberation chamber is also shown by the dashed black line plot. These results are also summarized in Table 2, which shows the difference between the methods at each one-third octave band frequency value. The results for the MR curved plate show good agreement between the VBSP and ISO 3741 methods from the one-third octave bands with center band frequencies between 315 Hz and 10 kHz.

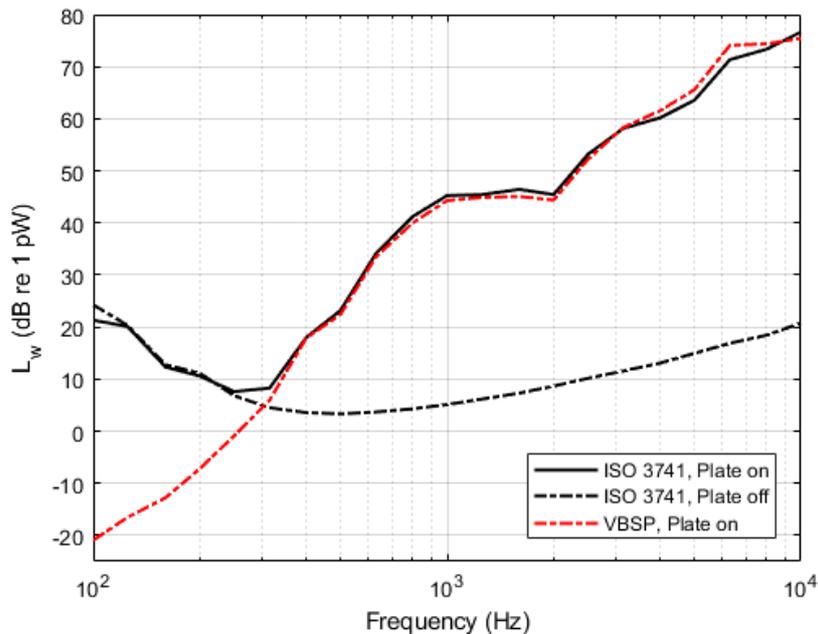


Figure 6: Results of the sound power measurements of the MR curved plate using the VBSP method compared to the ISO 3741 standard results. Background noise results from the reverberation chamber are also included (Plate off). Note that below 315 Hz the sound power level closely follows the background noise in the chamber. This indicates that the sound power measured in this frequency range is not from the curved plate. The VBSP results are specific to the curved plate and show a continuous decrease in sound power from 315 Hz down to 100 Hz.

Below 315 Hz, there is not good agreement between the two methods, but there is good agreement in this low frequency regime between the ISO 3741 sound power measurement of the MR curved plate and the background noise in the reverberation chamber (shown as ISO 3741, Plate off in Fig. 10). This suggests that the ISO 3741 method is measuring the background noise level at these frequencies because the sound power from the curved plate is well below the noise floor of the reverberation chamber when the frequency is below 315 Hz. According to ISO 3741, if the noise floor is within 10 dB of the measured sound power the results represent an upper bound on sound power. The ISO 3741 method relies on microphones to measure sound power and the method is unable to distinguish between noise energy from the curved plate and that from background noise in the reverberation chamber. It is therefore likely that the VBSP method may be a priori more accurate than established standard methods when the noise source of interest radiates below the environment noise floor.

For the frequency spectrum of good agreement between the VBSP and ISO 3741 methods (315 Hz to 10 kHz), the maximum difference is 2.8 dB at the 6.3 kHz one-third octave band (see Table 2), and the mean difference was 0.2 dB with a standard deviation of 1.4 dB. For the full frequency spectrum (100 Hz to 10 kHz) the total sound power is 79.7 dB re  $10^{-12}$  W using the VBSP method and 79.4 dB re  $10^{-12}$  W using the ISO 3741 method resulting in a total difference of 0.3 dB.

Table 2: Results of the sound power measurements of the MR curved plate using the VBSP method and ISO 3741, and the difference between the two.

		<b>Sound Power (dB)</b>		
		<b>ISO 3741</b>	<b>VBSP</b>	<b>Difference</b>
<b>Third octave band by center band frequency (Hz)</b>	100	21.3	-20.9	42.3
	125	20.1	-16.5	36.6
	160	12.3	-12.8	25.1
	200	10.6	-7.2	17.8
	250	7.6	-0.9	8.5
	315	8.3	6.0	2.3
	400	18.0	17.9	0.1
	500	23.2	22.4	0.8
	630	34.2	33.5	0.6
	800	41.2	40.0	1.2
	1000	45.3	44.3	1.0
	1250	45.4	44.9	0.5
	1600	46.4	45.1	1.4
	2000	45.4	44.4	1.1
	2500	53.2	52.2	1.0
	3150	58.2	58.3	-0.2
	4000	60.2	61.6	-1.4
	5000	63.6	65.6	-2.0
	6300	71.4	74.1	-2.8
	8000	73.4	74.4	-1.0
10000	76.7	75.4	1.2	
<b>Total</b>		79.4	79.7	0.3

### 3.2. TR curved plate results

Figure 7 shows a comparison between the VBSP method and the ISO 3741 sound power results for the TR curved plate. These results are also summarized in Table 3, which shows the difference between the methods at each one-third octave band frequency value. The results for the TR curved plate show good agreement between the VBSP and ISO 3741 methods from the one-third octave center bands with center band frequencies between 315 Hz and 10 kHz. In this frequency regime, the maximum difference was 1.8 dB at center band frequency 1.25 kHz (see Table 3), and the mean difference was 0.1 dB with a standard deviation of 0.8 dB. For the full frequency spectrum (100 Hz to 10 kHz) the total sound power level for the TR curved plate is 70.7 dB re  $10^{-12}$  W for the VBSP method and 70.2 dB re  $10^{-12}$  W for the ISO 3741 method for a total difference of 0.5 dB. At frequencies lower than 315 Hz, the noise floor again introduces significant error in the ISO 3741 method, with the VBSP method likely giving more accurate results.

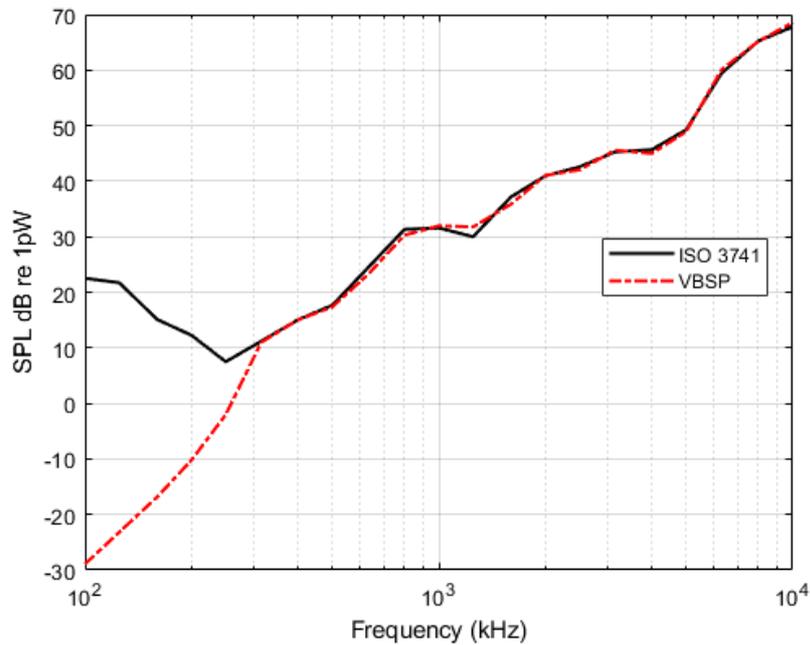


Figure 7: Results of the sound power measurements of the TR curved plate using the VBSP method compared to the ISO 3741 standard results.

Table 3: Results of the sound power measurements of the TR curved plate using the VBSP method and ISO 3741, and the difference between the two.

		Sound Power (dB)		
		ISO 3741	VBSP	Difference
Third octave band by center band frequency (Hz)	100	22.5	-28.9	51.5
	125	21.7	-23.2	44.9
	160	15.1	-16.8	31.9
	200	12.3	-10.2	22.5
	250	7.5	-2.0	9.5
	315	11.2	11.0	0.2
	400	15.1	15.0	0.0
	500	17.7	17.4	0.3
	630	24.4	23.2	1.2
	800	31.3	30.3	1.0
	1000	31.6	32.0	-0.4
	1250	30.0	31.8	-1.8
	1600	37.2	35.9	1.3
	2000	41.0	41.0	-0.1
	2500	42.6	42.1	0.5
	3150	45.3	45.6	-0.3
	4000	45.7	45.0	0.7
5000	49.3	48.9	0.3	
6300	59.5	60.1	-0.6	
8000	65.2	65.2	0.1	
10000	67.8	68.6	-0.8	
<b>Total</b>		70.2	70.7	0.5

### 3.3. WR curved plate results

Figure 8 shows a comparison between the VBSP method and the ISO 3741 sound power results for the WR curved plate. These results are also summarized in Table 3, which shows the difference between the methods at each one-third octave band frequency value. The results show good agreement from the one-third octave bands with center band frequencies between 250 Hz and 10 kHz. In this frequency regime, the maximum difference was 3.3 dB at center band frequency 400 Hz (see Table 4), and the mean difference between the two methods was 0.1 dB with a standard deviation of 2.1 dB. For the full frequency spectrum (100 Hz to 10 kHz), the total sound power was 81.0 dB re  $10^{-12}$  W using the VBSP method and 80.0 dB re  $10^{-12}$  W using the ISO 3741 method, resulting in a total difference of 1.0 dB. Similar to the above results for the MR and TR curved plates, the noise floor again introduces significant error in the ISO 3741 results below 250 Hz, with the VBSP method results likely being more accurate in this low frequency regime (see Figure 6).

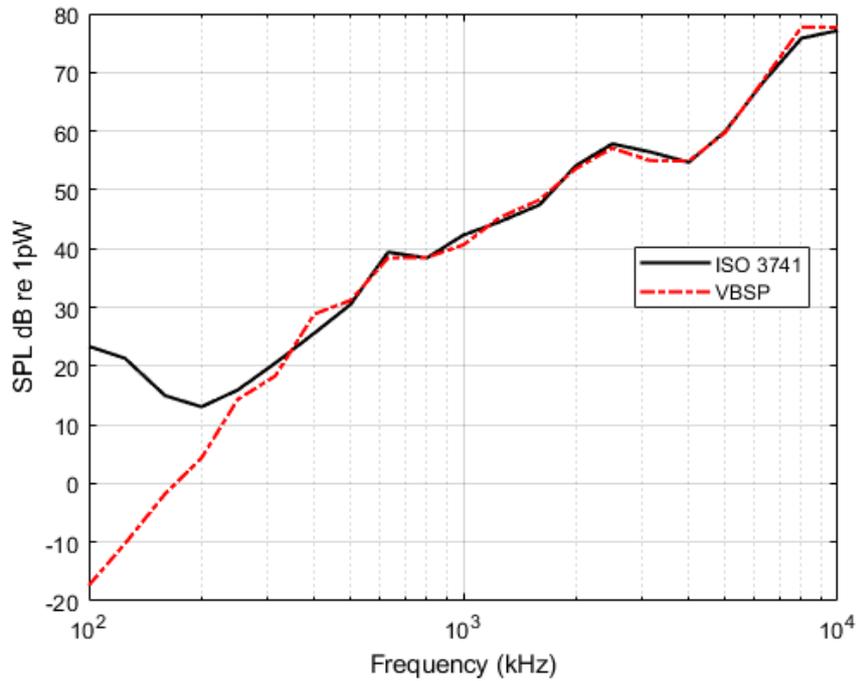


Figure 8: Results of the sound power measurements of the WR curved plate using the VBSP method (SLDV) compared to the ISO 3741 standard results.

Table 4: Results of the sound power measurements of the WR curved plate using the VBSP method and ISO 3741, and the difference between the two.

		Sound Power (dB)		
		ISO 3741	VBSP	Difference
<b>Third octave band by center band frequency (Hz)</b>	100	23.3	-17.4	40.7
	125	21.3	-10.2	31.5
	160	14.9	-1.7	16.7
	200	13.0	4.3	8.7
	250	15.9	14.3	1.6
	315	20.5	18.3	2.2
	400	25.6	28.8	-3.3
	500	30.5	31.2	-0.7
	630	39.4	38.4	1.0
	800	38.4	38.5	-0.1
	1000	42.3	40.6	1.7
	1250	44.5	45.3	-0.7
	1600	47.5	48.3	-0.9
	2000	54.1	53.6	0.5
	2500	57.8	57.1	0.7
	3150	56.5	55.0	1.5
	4000	54.7	54.9	-0.2
	5000	59.9	59.7	0.2
	6300	68.2	68.5	-0.3
	8000	75.8	77.7	-1.9
10000	77.1	77.7	-0.6	
<b>Total</b>		80.0	81.0	1.0

## 4. CONCLUSIONS

A brief review of the concept of acoustic radiation modes was presented as the basis for the VBSP method, a developing method that could provide a precision grade vibration-based sound power measurement method. Recent work on the VBSP method has validated its use for simple flat plates and cylinders. The results of these efforts show promise that the VBSP may be effectively developed further.

Following a brief review of the developing VBSP method, the experimental setup for applying the VBSP method to simple curved plates is presented. As part of this extension of the VBSP method, the work for incorporating a 3D SLDV device to provide a more efficient method of scanning a surface for velocity measurements is presented. The VBSP method relies on obtaining accurate complex surface normal velocity measurements of the radiating surface of interest. This was done more efficiently by measuring 3D components of velocity of a larger section of the surface than previously done by combining the velocity data with surface 3D geometry data to compute the surface normal velocity magnitude. This step significantly improves the practical application of the VBSP method by reducing the total number of individual scans required to measure the complete response of a surface of interest.

Experimental surface velocity measurements were obtained using a 3D SLDV to scan three curved plates with significantly different radii of curvature. The VBSP method was used to compute sound power from these measurements while the ISO 3741 method was applied to compute sound power for comparison. These experimental results show good agreement from the 315 Hz to 10 kHz one-third octave band spectrum for all three curved plates measured. The mean sound power difference between the two methods was 0.2 dB with a standard deviation of 1.4 dB for the MR curved plate, 0.1 dB with a standard deviation of 0.8 dB for the TR curved plate, and 0.1 dB with a standard deviation of 2.1 dB for the WR curved plate. Of particular note is that the background noise of the reverberation chamber closely agrees with the ISO 3741 result below 315 Hz while the VBSP result is significantly lower. This indicates that the VBSP method is more accurate than the ISO 3741 method when the environment noise is greater than the actual source of interest, thus the VBSP method is capable of ignoring environmental noise.

The results of the experimental work presented in this paper have shown validation for extending the VBSP method to accurately measure sound power from simple curved structures and have continued to establish the potential usefulness of the method. It has also shown that the VBSP method may be effectively improved by developing the method of measuring surface velocity by using a 3D SLDV such that a surface may be more efficiently scanned with fewer individual scan sections.

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