On the contributions of David T. Blackstock to understanding nonlinear propagation of jet noise

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On the contributions of David T. Blackstock to understanding nonlinear propagation of jet noise

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This paper describes the contributions of David T. Blackstock to the understanding of nonlinear propagation of jet noise. Although he investigated this problem with students in the 1970s, their findings directly formed the foundation for my own doctoral studies begun in 2002, partly caused by a relatively lack of high-amplitude jet noise propagation research in the intervening years. Moreover, exchanges with David during my doctoral program and after led to improved physical insights, additional research directions, and meaningful interactions.

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1. INTRODUCTION

This article stems from my talk at the memorial session held for David T. Blackstock at the 181st Meeting of the Acoustical Society of America (ASA). Its purpose is three-fold. First, it documents David's contributions to understanding and modeling nonlinearities in high-amplitude jet noise propagation. Second, it describes how those contributions laid the foundation for other research, including my own. Third, the article describes my personal interactions with David over many years that typify his concern for, and interest in, others. I believe it important that these be part of the archival record, as they help illustrate David's character and why he is the namesake of the ASA's student mentoring award.

2. BLACKSTOCK AND JET NOISE

To say David's early career contributed significantly to nonlinear acoustic theory is an understatement. A Lagrangian description[1] of hydrodynamics in lossy fluids and model for finite-amplitude piston motion[2] preceded several foundational papers on nonlinear propagation of planar and nonplanar waves in lossless and lossy fluids.[3-7] Part of David's early work was experimental; e.g., plane-wave tube measurements[8] were conducted with periodic waves in which the combined effects of nonlinearity and tube boundary layer dispersion were described.

While David was fast becoming one of the foremost experts on nonlinear acoustics, the introduction of high-power jet aircraft and the study of their noise was resulting in reports of anomalously low atmospheric absorption at high frequencies. Nonlinear propagation, with its accompanying transfer of spectral energy to high frequencies, was believed to be a potential cause. This belief was strengthened by Pernet and Payne's[9] 1971 analysis in which propagation of band-limited noise in a tube was extrapolated to a wideband, spherically spreading scenario representative of jet noise.

Like Pernet and Payne, nonlinear propagation of jet aircraft noise motivated David to begin studies of finite-amplitude noise. David's doctoral student at the University of Texas at Austin, Mike Pestorius, began to study nonlinear propagation of noise in a long tube, also like Pernet and Payne. Unlike Pernet and Payne, however, a relatively wideband noise (500 Hz – 3500 Hz) waveform was used as an input to the "Pestorius algorithm" shown in Figure 1. The algorithm,[10] based on the generalized Burgers equation (GBE), incorporated tube boundary-layer absorption and dispersion, waveform steepening via the Earnshaw solution, and weak shock theory to account for shock formation and coalescence in a stepwise fashion. Rigorous mathematical understanding, careful experiments, and novel numerical modeling led to a new understanding of nonlinear noise propagation and an unprecedented ability to predict it, as shown in Figure 2.[11,12] An increase in high-frequency energy was attributed to shock formation, whereas efficient low-frequency energy generation was attributed to shock coalescence and an associated reduction in zero crossings.

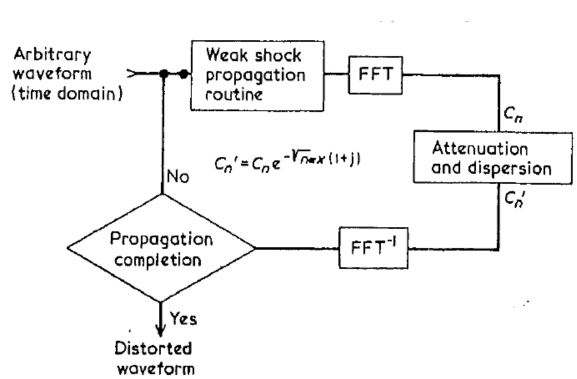

Figure 1. Schematic of the "Pestorius" algorithm, with waveform steepening and weak-shock theory implemented in the time domain and (frequency-dependent) absorption and dispersion applied in the frequency domain.

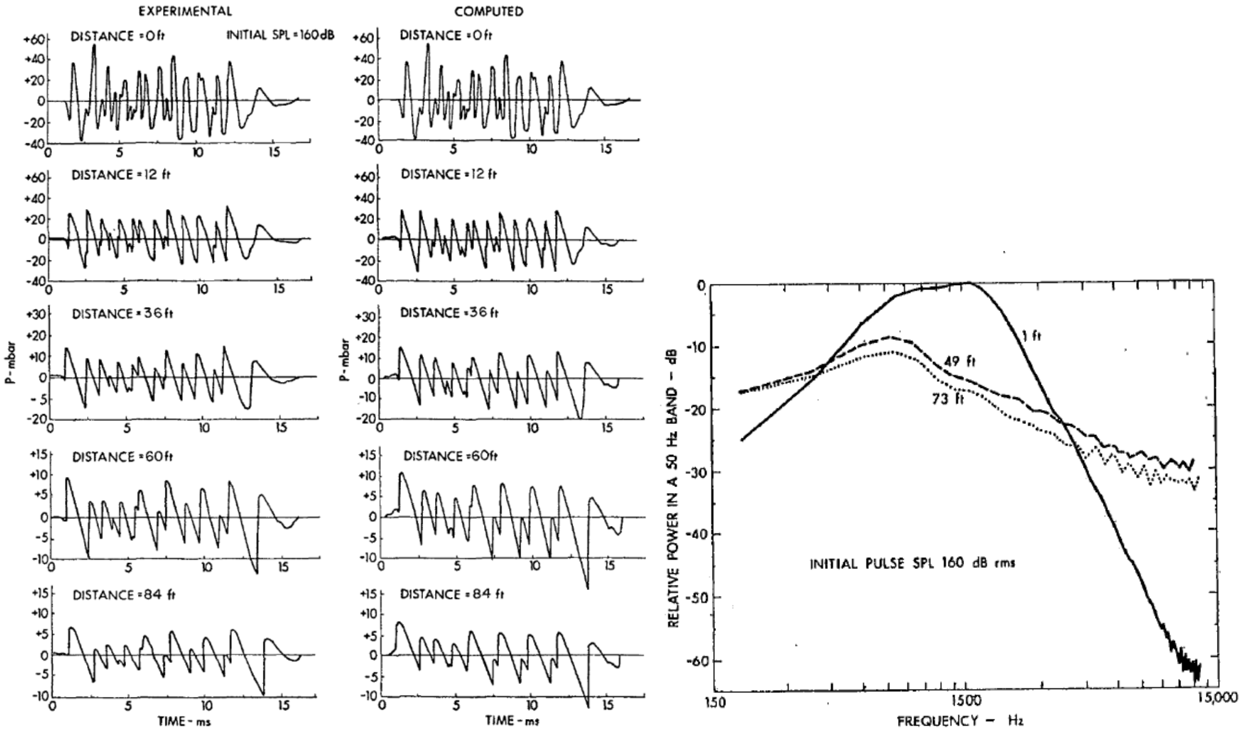

Figure 2. Left: Measured (experimental) and numerically predicted (computed) noise waveforms at different distances in an anechoically-terminated plane wave tube. Right: Predicted spectra at different distances, showing increase in energy at low and high frequencies, and a loss of energy from the peak-frequency region.[11]

The success of the initial work by Pestorius and Blackstock significantly advanced understanding of finite-amplitude noise propagation, but its dissemination in the open, reviewed literature is limited. (Although Pestorius and Williams[13] published a Journal of the Acoustical Society of America (JASA) paper on weak-shock theory limits, Mike Pestorius had military commitments and David had significant, concurrent work with finite-amplitude source[14] and parametric sound generation.[15]) Nonetheless, the subsequent research trail in this area is relatively easy to track because of sponsor reports, conference proceedings, and ASA meeting abstracts. First, Pestorius *et al*.[16,17] extended the plane-wave study of noise to the problem of cylindrical and spherical noise propagation, resulting in the conclusion that random sawtooth waves have a high-frequency rolloff of 6 dB/octave and that initial spectral phase had little impact on predicted spectra. The abstract for Ref. 17, a technical report version of Ref. 16, also contained the following statement: *"Finally, distortion computations were made using a particular example of actual jet noise, noise of not very high intensity from the British-French Concorde. In this particular case nonlinear effects did not prove to be very important.*"

Although a bit of an aside, the above quote is significant in the context of other contemporaneous finite-amplitude jet noise research. Ffowcs Williams[18] had been investigating the origin of "positive spikes" in the noise signature of the Concorde's Rolls-Royce Olympus 593 engines and had, apparently, reached out to David to conduct the simulation with a measured waveform. The results of the Pestorius *et al*.[17] simulation, which showed little evidence of nonlinear propagation, were discussed by Ffowcs Williams *et al*.[19] and caused them to conclude that jet "crackle" – a raspy, staccato-like noise characteristic that the authors associated with the pressure distribution skewness – must be a source phenomenon. Jet crackle has since been a much-debated topic within the jet noise community – with competing and complementary descriptions of origins and characteristics – and a detailed discussion goes beyond the scope of the paper. However, it has now been shown that crackle is the perception of shocks embedded in the broadband noise and that pressure skewness has little to do with crackliness.[20] Skewed pressure waveforms, however, do appear to be a source phenomenon related to high-power jet exhausts.[21] Recent work on crackle is linked back to the Pestorius *et al*. simulations through Ffowcs Williams *et al*.'s reasoning.

The Concorde simulation notwithstanding, nonlinear jet noise propagation research persisted. Blackstock[22] obtained a T-38 waveform and numerically propagated it according to spherical spreading, waveform steepening, and weak-shock theory. Although the algorithm did not include atmospheric absorption or dispersion, his overall argument was similar to that made by Ffowcs Williams *et al*.: *"If appreciable distortion, e.g., the formation of numerous shocks, does not become evident until the noise has traveled a great distance, nonlinear effects are probably not important. The reason is that ordinary absorption, which was ignored in the computation, can in practice be counted on to damp out the wave before much distortion accrues. If, on the other hand, the predicted waveform exhibits many shocks after a relatively short propagation distance, we assume that nonlinear propagation distortion probably competes favorably with absorption (and other effects). It would then be a mistake to ignore nonlinear distortion."*

The results of the Blackstock[22] simulation are shown in Figure 3, with the waveform on the left and predicted spectra on the right. Clearly, shock formation is predicted, with evidence of the high-frequency 6 dB/octave rolloff (which is the f^{-2}, shock-related slope described by Gurbatov and Rudenko[23] and others). Although David listed numerous caveats to his results, he concluded that geometric spreading alone was insufficient to prevent the formation of significant shocks and that the results seem to indicate that nonlinear distortion is important to intense jet noise propagation.

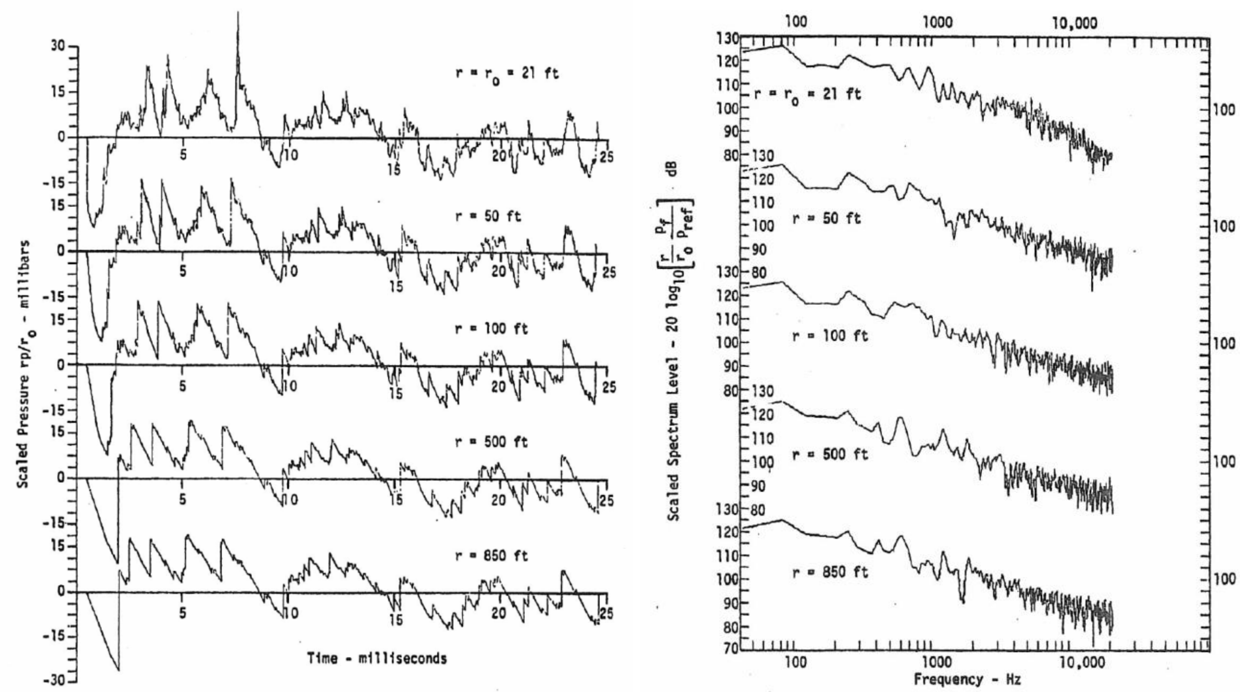

Figure 3. Numerical propagation of a T-38 waveform measured at a distance of 21 ft.

From this lone investigation involving actual jet noise numerical propagation, David's research turned to outdoor experiments with loudspeaker arrays able to produce finite-amplitude signals. Theobald, Webster, and Blackstock [24-27] made measurements of acoustic propagation from vertically-fired loudspeaker arrays with sinusoidal source signals and measurements made along an 85-m tower (see Figure 4). Webster, Alexander, and Blackstock [28,29] extended the measurements to noise, with overall source levels ranging from 121 to 145 dB at 1 m. For the higher source levels, they concluded that nonlinear propagation distortion caused generation of high-frequency noise over the entire propagation path; at no point was small-signal behavior observed. They further saw that, unlike the Pestorius's plane-wave tube, little low-frequency noise was generated. Finally, repeated in Figure 5 is one of the primary outcomes of the research program. When frequency/amplitude-scaled and compared against a KC-135A noise spectrum, the loudspeaker spectrum was appreciably lower. To quote from Webster *et al*.[29], *"[T]he level of the KC-135A noise is roughly 10 dB higher in the mid- and high-frequency regions than our noise. By demonstration, our noise was definitely affected by nonlinear propagation distortion. The implication is that even stronger nonlinear effects were at work during the propagation of the KC-135A noise. Moreover, although the KC-135A is a very noisy aircraft, many other current aircraft produce noise whose spectrum levels are higher than our scaled spectra. One therefore concludes that nonlinear effects are probably common in jet noise."*

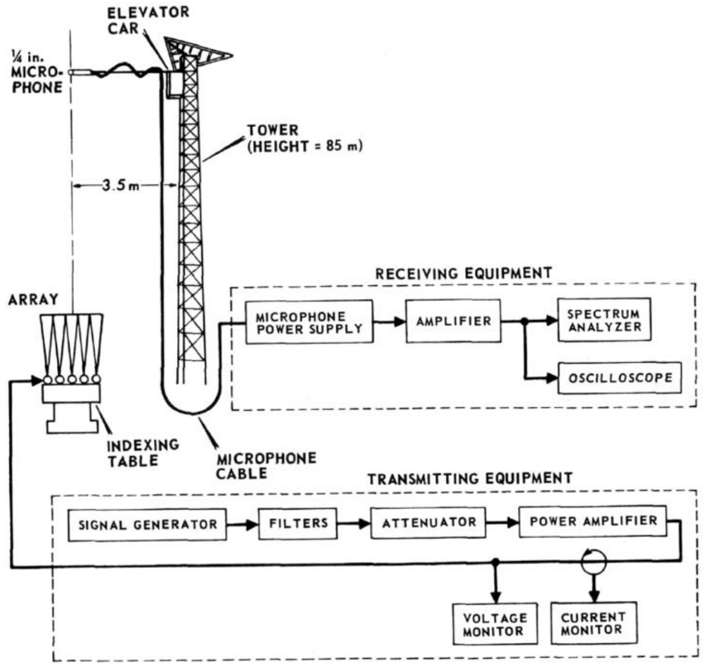

Figure 4. Schematic of the Theobald (sinusoids) and Webster (noise) experimental setup for outdoor finite-amplitude acoustic propagation.[29]

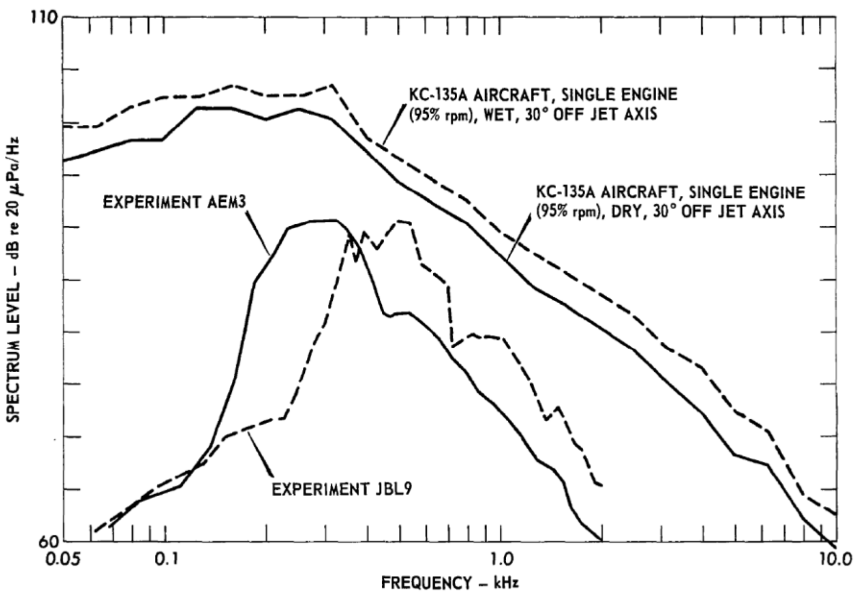

Figure 5. The Webster* et al*. experiments amplitude/frequency-scaled against KC-135A aircraft spectra.[28]

David's research program into nonlinear propagation of noise concludes here, although Webster and Blackstock published some other important studies regarding nonlinear acoustic theory.[30-32] The reasons are unclear, but a report[33] summarizing David's research activities and publications from 1975-1984 reflects a rather sudden end to continuous noise research in favor of N-wave, suppression of sound by sound, noncollinear wave interaction, and finite-amplitude sound beam studies. Some of these areas of investigation ultimately resulted in David's impactful sonic boom and biomedical research programs. It is also unclear why David abandoned further use of jet noise waveforms for propagation measurements and simulations after his 1975 conference paper,[22] in favor of controlled sources of lower amplitudes and narrower bandwidths. However, David's prior research had been based on developing physical insights and models with as simple as scenarios as possible. The conceptual jump from a plane-wave tube to measurements made on a supersonic jet engine exhaust – representing an extended, turbulent aeroacoustic source – and I believe David saw gaps that needed to be filled in with sequential steps for a more complete understanding. Although he never again had the direct opportunity to work on the nonlinear jet noise problem, he did return briefly to the problem of nonlinear noise modeling years later with a publication with Menounou.[34]

3. POST-BLACKSTOCK NONLINEAR NOISE INVESTIGATIONS

Although David moved on to other research, the work by his students and him influenced other investigations, particularly in the early 1980s. One example was the work of McDaniel *et al*. [35] at Penn State in their investigation of free-field, high-intensity noise. In Japan were noise propagation studies in pipes by Watanabe and Urabe[36] and Sakagami *et al*.[37] More prominently, there were two efforts in the UK to develop prediction algorithms directly from the noise power spectrum in the pursuit of modeling simplicity. One was Crighton and Bashforth[38]; David Crighton was a Cambridge mathematician specializing in fluid mechanics and wave propagation theory. He and Bashforth developed a truncated series solution for the weakly nonlinear evolution of the noise. Another was Geoffrey Howell, a doctoral student of well-known aeroacoustician Chris Morfey at Southampton. Howell and Morfey[3940] developed a different nonlinear spectral evolution solution based on a quasi-normal hypothesis to relate higher-order joint spectra and moments to the autospectrum. Their most well-known paper[41] has been made famous not for the success of the propagation algorithm but for the identification of a third-order quadspectrum, the "Morfey-Howell indicator" or "Q/S" within the GBE that has been the subject of several recent papers in nonlinear acoustics theory[42,43] and high-amplitude jet aeroacoustics.[44-47]

After the early 1980s investigations, ties back to David's work in nonlinear noise propagation nearly goes cold for two decades, because of a dearth of research in high-amplitude jet noise. One exception is Sally Anne McInerny's studies of rocket noise. Her 1996 paper[48] references the work of Webster and Blackstock and a desire to propagate rocket noise waveforms using the Pestorius algorithm. (Although this never happened, it motivated at least one example of this.[49]) In a more detailed investigation of rocket noise nonlinearity, McInerny and Ölçmen[50] use some of Blackstock and colleagues' subsequent research involving atmospheric sonic boom propagation to explain far-field rise times of shocks in rocket launch noise. Beyond these, I should note that the monograph, *Nonlinear Acoustics*, edited by Mark Hamilton and David, includes a chapter by Gurbatov and Rudenko[23] on statistical phenomena. Other connections to David's research on nonlinear noise propagation will certainly be found therein.

4. MY DOCTORAL RESEARCH

A. RESUSCITATION OF AN OLD PROBLEM

In August 2002, I began doctoral work under the direction of Dr. Victor Sparrow at Penn State. Vic was part of a Strategic Environmental Research and Development Program grant[51] with Wyle Laboratories, along with aeroacousticians Phil Morris and Dennis McLaughlin of Penn State and Sally Anne McInerny. Separately, Anthony Atchley and Tom Gabrielson of Penn State had a grant from the Office of Naval Research to study high-amplitude jet noise propagation. I was surrounded by an unprecedented confluence of activity on high-amplitude jet noise, with many opportunities to learn.

The objective of my research was to implement a nonlinear propagation methodology for modern tactical aircraft. I spent the first several months of my program studying and implementing the methods of Crighton and Bashforth[38] and Morfey and Howell[41] and reading many of the references in Gurbatov and Rudenko.[23] After trying to implement the methods for F/A-18E noise spectra,[52] I concluded that the assumptions made in both methods were limiting to the point of being nonphysical. I presented[53] this conclusion at David's tribute session held in Austin at the Fall 2003 ASA meeting. I outlined that we planned to return to the original line of research begun by David and his students: to numerically propagate the waveform with a GBE-based algorithm that incorporates the necessary physics.

I will deviate from the technical narrative with a personal anecdote. I should note that I was the only student and the only contributed talk in the tribute session. It was my second ASA talk ever, and I was extremely nervous to be presenting in a session along with all of these people whose papers I had read: Mark Hamilton, Robin Cleveland, Allan Pierce, Chris Morfey, and others. I was the concluding speaker (as the lone contributed talk) and after it was over, the audience gave David a standing ovation. The moment the ovation ended, he made a beeline to the front to talk to me. It was startling. He told me that he never understood how spectrum-only methods were sufficient and that he thought I was pursuing the right approach in directly modeling the waveform steepening. Those few minutes he spent on me, when he should have been the center of attention, were significant.

With this confidence boost by a legend – whose work with Pestorius happened before I was born and with Webster before I started Kindergarten – my research progressed rapidly. I implemented a modification of the Pestorius algorithm that had been developed by David's student, Mark Anderson,[54] for N-wave propagation and then further adapted it for continuous noise propagation. In reverse order of David's program, we first applied the algorithm to sinusoid and noise propagation from a large horn-coupled pneumatic source known affectionately known as the Mother of All Speakers.[55] Then came the opportunity to make propagation measurements on a tied-down, full-scale tactical aircraft: the F-22. (This was the first of many career field tests for me.) The results[56,57] showed clearly what David and students had supposed: that nonlinear effects were common in high-power jet noise. Observed waveform steepening and shock formation, and the success of the nonlinear modeling approach at different engine powers and propagation angles showed that nonlinearity is an integral part of tactical jet noise radiation.

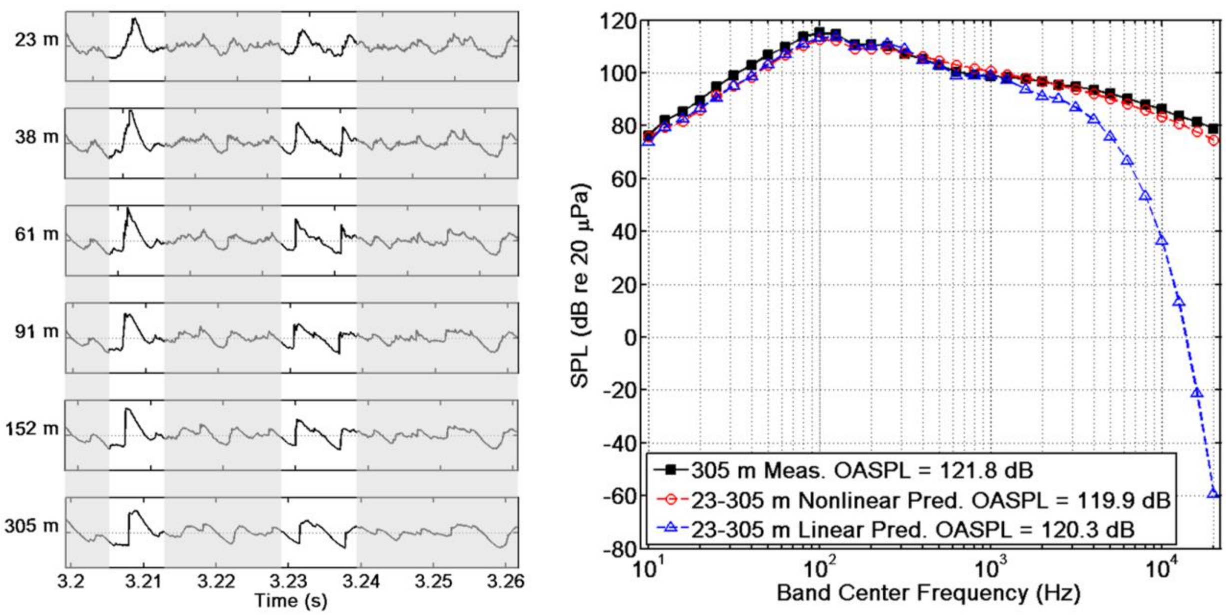

Figure 6. Observed shock formation and measured, linearly predicted, and nonlinearly predicted one-third octave spectra.[58]

Others became interested in nonlinear jet noise work about the same time and my following the 1970s research thread of David brought new awareness of his and others' work. From concurrent laboratory-scale work at Penn State by Petitjean *et al*. [59,60] to alternate prediction methods[61-63] and predicted impact on noise metrics,[64] there was a flurry of work regarding nonlinear jet noise propagation. More recently, some research in nonlinear jet noise propagation has been at the University of Texas at Austin, where David spent most of his career.[46,65,66]

B. ONCE NONLINEAR, ALWAYS NONLINEAR

As I neared completion of my doctoral work, I had one more fundamental question to answer: when does the noise stop being nonlinear? This question was crucial for my work to be included in tactical aircraft noise modeling programs. As I numerically propagated simulated and measured jet noise waveforms long distances using my algorithm, I found that there was no distance at which the additional high-frequency nonlinearity generated dropped to zero. In fact, the spatial rate of change in high-frequency sound level due to nonlinearity appeared to converge to a constant number of decibels per meter for a given frequency. Puzzled by this result and uncertain if I had an error, I showed it to Vic Sparrow, who quipped (not for the first time), "Once nonlinear, always nonlinear." I asked who coined that phrase, and he thought it had come from David. Seeking greater understanding, I searched David's papers and the *Nonlinear Acoustics* text and was unable to find it anywhere. So, again with some trepidation, I wrote to David. (I am extremely grateful to have found in my files printed copies of the email exchange.) A portion of my first email is shown in Figure 7. His response, which arrived the same day, is shown in Figure 8. I was astounded. Not only had he responded, but he had done so promptly and thoroughly – citing references and LaTeX-formatted equations with incredible detail. For me, this was the gold standard of email responses and is the reason I am including them for the historical record.

02-24-05

Dr. Blackstock,

I've heard the statement "once nonlinear, always nonlinear" attributed to you several times and I was wondering if a) you said/wrote it, and b) what was the context?

As you probably recall, I'm working on the nonlinear propagation of jet noise using an Anderson-type code I wrote, and as a final point to the thesis, I was going to discuss at what distance the propagation becomes essentially linear. Anyway, what I've been finding is that the difference between the linearly and nonlinearly predicted power spectra continues to grow as a function of range, especially at high frequencies (10-20 kHz), out to where I start running into numerical noise floor issues. It looks to me that the rate of change between the nonlinear and linear power spectra approaches a constant (in dB/m), which surprises me because I would have expected it to continue to diminish as overall amplitudes decay and the time scale distortion afforded by the Earnshaw solution becomes negligible. I can't decide if it's a numerical issue with the algorithm or if the result is believable, meaning "once nonlinear, always nonlinear."

Figure 7. A portion of an email message I sent to David Blackstock on 24 February 2005.

A couple of months passed and I was writing my Ph.D. thesis and a paper[67] on this topic of "once nonlinear, always nonlinear" in the context of jet noise for the 2005 International Symposium on Nonlinear Acoustics (ISNA). I was still wrestling with parts of David's explanation and took him up on his offer for a deeper physical explanation (see Figure 8). So, I wrote to him again. His response, which again arrived the very same day, is shown in Figure 9. Particularly in his second response, David brilliantly connected my jet noise work to nonlinear parametric sound generation theory. I had my answer to "once nonlinear, always nonlinear." I will also admit I was rather enthusiastic to read that my questions were motivating David to write a paper for ISNA (see the conclusion of the email at the end of Figure 9). In fact, at the conference, Mark Hamilton thanked me for "getting David to write a paper." How little I knew.

The week before ISNA, I was at my cubicle when the phone in the Penn State acoustics graduate student area rang. The feeling was surreal when the student who answered announced that it was David Blackstock on the phone for me. David was calling to inquire if he could use my email to help introduce his ISNA paper.[68] What I failed to understand was that he was going to use the text in Figure 7 nearly verbatim in his paper. David then introduced the paper the same way at the conference. It was exciting to have my work called out as correct by David, but I will also admit it was a bit embarrassing; I felt silly that I had not properly understood the physical principles of "old-age decay" described in the literature decades before. Thankfully, everyone laughed when I chimed in from the back of the session room when it was over that, "You can never be too careful about what you put in an email!" In all seriousness, though, this email exchange with David Blackstock has been one of the highlights of my career. I was a student – not-quite-random because of the 2003 Austin ASA meeting and having had occasion to talk briefly at other meetings – and a world-renowned physical acoustician had treated me to an in-depth technical exchange, and then validated my work as he used that interaction to teach others. I learned about mentorship from this exchange, and I am grateful.

From: David Blackstock [dtb@mail.utexas.edu]
Sent: Thursday, February 24, 2005 3:20 PM
To: Kent Gee
Cc: vws1@psu.edu; hamilton@mail.utexas.edu
Subject: Re: "Once nonlinear, always nonlinear"

02-24-05
Kent,

I think the description "once nonlinear, always nonlinear" may have originated with David Pernet, an English scientist who did some early work on finite-amplitude noise propagation. I assume you have NASA CR 2992, by Don A. Webster and me, August 1978. References 3 (1969), 4 (1971), 5 (1971), and 8 (1975) are works by Pernet and Payne. It may be that the phrase "once nonlinear, always nonlinear" can be found in one of those references. However, I suspect that the term came out more informally. It's been many years since a conversation I had with him where recall his using the term. I don't remember the details, but we must have been discussing the curious phenomenon that some properties associated with finite-amplitude propagation seem to persist forever, despite the apparent demise of the wave to what ought to be small-signal status. Both of us agreed that the phenomenon is real, and I think that he came up with the very succinct way of putting it: "Once nonlinear, always nonlinear." He had encountered the phenomenon in his studies of intense noise propagation. I don't remember the particulars, but it must have been something about the decay of the higher harmonic bands. Once they are generated, they never seem to reach a stage (distance) at which they decay the way small signals of the same frequency would be expected to decay.

I had noticed the same thing years earlier in my studies of the distortion of a sinusoidal (at the source) wave. Take a look at my March 1964 paper, "Thermoviscous attenuation of plane, periodic, finite-amplitude sound waves," JASA **36**, 534-542. Equation 31 shows that in old age the fundamental decays as exp(-\alpha x), where \alpha is the attenuation coefficient at the fundamental frequency. Nothing surprising about this, since a small-signal wave of this frequency would have exactly this sort of decay. However, look at the second harmonic: it decays as exp(-2\alpha x), whereas a small signal of that frequency should decay as exp(-4\alpha x) (for a thermoviscous fluid the absorption coefficient varies as f^2). Similarly, all the other harmonics (n) decay as exp(-n\alpha x) rather than exp(-n^2\alpha x), the latter being the way a small-signal wave "ought" to decay. At the time I presented my work, there was considerable skepticism because "old age" was supposed to be the stage at which the wave behaves as a small signal. Alas, I didn't coin Pernet's expression at the time. Incidentally, in Appendix A of NASA CR 2992, the same "peculiar" results for the absorption coefficient are shown to occur for spherical waves; see Equations A-22. However, an even more interesting (mind-boggling) result is that the geometrical spreading for the nth harmonic is r^{-n}, NOT r^{-1}! At the top of p. 103, note the conclusion: "In other words, the higher harmonic sound never establishes a farfield where traditional small-signal laws take over." (That would have been a good place to use Pernet's words.) See also conclusion 3, p. 94, in Chapter 8 "Summary."

I found a physical reason that the harmonic tones (or harmonic bands of noise) never reach small-signal behavior. The explanation was given in paper N6 of the 1978 ASA Meeting in Honolulu. The program of that meeting gives the abstract, but unfortunately the physical explanation is not part of the text. If you are interested, that will have to be the subject of another email message.

The bottom line is that I'm not surprised at your computational results. Of course the practical aspect is that at far enough distance the noise has to compete with ambient sound (like your numerical noise floor), and so the fact that the noise is still not behaving as a small signal doesn't matter.

---David Blackstock---

Figure 8. Response from David Blackstock to my email in Figure 7.

06-07-05
Kent,

I think the easiest physical explanation is in terms of virtual sources of the distortion components. The virtual-source description of growth of distortion components was developed by Westervelt to explain why the parametric array works. If you are not familiar with the virtual-source idea, reading about it may help you understand the description below.

To keep it simple, consider plane waves in a tube containing an absorbing fluid, let the source emit a single-frequency (f) sound wave, and focus attention on the second-harmonic distortion component p_2 (the most prominent distortion component). Second-harmonic sound is generated, not back at the source x=0, but throughout the medium by the propagating fundamental as it interacts nonlinearly with itself. The second-harmonic sound thus seems to come from a continuous array of sources of frequency 2f, from x=0 out to x, the location of the receiver. These are the virtual sources. Their phasing is such that 2f sound produced by a virtual source and point x_1 is in phase with all the 2f signals coming from all earlier virtual sources. That's why the second-harmonic sound grows with propagation distance. In a lossless fluid, for example, the second-harmonic sound grows linearly with x, at least until it gets big enough to lose some of its energy to higher-order distortion components.

Although the virtual-source description was first used by Westervelt to explain generation of difference-frequency sound in the parametric array, the description is equally applicable to second harmonic generated by the propagation of a wave launched by a single-frequency (physical) source.

Now consider the effect of absorption. The strength of a virtual source at x_1 is proportional to the square of the local fundamental amplitude there. That amplitude is p_1(x_1) = p_10 e^{-\alpha_1 x_1}, where p_10 is the amplitude of the fundamental back at the (physical) source, and alpha_1 is the absorption coefficient at frequency f. The second-harmonic sound produced at x_1 is proportional to the square of the fundamental, i.e.,

p_2 (x_1) \propto [p_10 e^{-\alpha_1 x_1}]^2 = [p_10]^2 e^{-2\alpha_1 x_1}.

Once the second-harmonic signal is generated at x_1, however, it travels to the receiver at x > x_1 as a second harmonic, i.e., its attenuation over the distance x-x_1 is e^{-\alpha_2 (x-x_1)}. In other words, the second-harmonic signal reaching the receiver has two different absorption coefficients:

(a) 2\alpha_1 for travel from x=0 to x=x_1
(b) \alpha_2 for travel from x_1 to x, the position of the receiver.

The total second harmonic signal at the receiver is, of course, the sum of what arrives there from all the virtual sources. Note that \alpha_2 is normally greater than 2\alpha_1, e.g., for a thermoviscous fluid \alpha_2 = 4\alpha_1. Therefore, 2f signals from virtual sources close to the physical source are much weaker when they arrive at the receiver than 2f signals coming from virtual sources close to the receiver. The result is that the second-harmonic sound at the receiver is dominated by the virtual sources close to it. The higher absorption coefficient \alpha_2 has little effect because it operates for only a very short distance. Asymptotically, therefore, the second-harmonic sound at the receiver x is proportional to [p_10]^2 e^{-2\alpha_1 x} because, for sources very near the receiver, x_1 is approximately the same as x.

Now extend this description to spherical waves. The virtual source strength for sources very near the receiver is practically proportional to

[p_10/r]^2 e^{-2\alpha_1 r}.

Despite the fact that the second harmonic from virtual sources near the physical source travels most of the way under simple 1/r spreading, the much higher absorption suffered, e^{-\alpha_2 x}, is the factor that renders this second-harmonic sound so weak as to be negligible.

In summary, the received second harmonic is dominated by virtual sources near the receiver, whether for plane waves or for spreading waves.

The result is different for difference frequency sound (f_2 - f_1, where f_2 and f_1 are the two primary frequencies) generated by the parametric array. In this case the virtual sources near the physical source are dominant because their effective absorption coefficient is \alpha_2 - \alpha_1. The effective absorption coefficient for virtual sources near the receiver is so high, \alpha_2 + \alpha_1, that their contributions are nil.

Let me know if you want to discuss any of this or have any questions. The argument above will be the heart of the paper I'll give at the ISNA. So if my argument is not clear, I'll have to work to make it clearer.

---David Blackstock---

Figure 9. Email response by David Blackstock to a follow-up question by the author on 07 June 2005.

5. POSTDOCTORAL INTERACTIONS

I am also grateful my interactions with David did not stop after I graduated and became a faculty member. More than once, I helped him match professional ASA members with Brigham Young University (BYU) students while he was coordinating the Education in Acoustics "Students Meet Members for Lunch" program. But, David also frequently attended my talks, and my students' talks on nonlinear propagation, jet noise, or both. He always had something positive to say, and suggestions for possible next steps. Looking back, I believe David's interest in our work was in part because we were rekindling the research torch he probably thought had been extinguished decades before. From David's investigations, I can easily trace a thread through dozens of BYU student-authored papers on nonlinear theory, nonlinearity in jet and rocket noise, and even weak-shock decay of muzzle blasts and explosions. In one recent paper,[69] the understanding of parametric sound generation and old-age asymptotic decay first provided to me by David was merged with our pursuit of understanding the physics of the Morfey-Howell nonlinearity indicator, which we were then able to connect directly back to the far-field jet noise propagation modeling results[67] presented at the 2005 ISNA.

The ongoing interactions with David resulted in a unique, collaborative research opportunity. Students and I had noted a curious phenomenon about nonlinear propagation from an open-ended pipe: the measured pressure waveforms appeared to take on the derivative of a sawtooth waveform, with large positive spikes. The same occurred for nonlinear noise transmission, but with random timing and derivative-like amplitudes. When occasion permitted, I asked David about it and learned that there are probably few problems in nonlinear acoustics that David had not at least thought about. He thought for a moment and said, "I think we worked on something similar once." He then asked if we had made measurements off-axis. I replied that we had not and he responded that he thought diffraction was playing a role and that the spikes would be less pronounced off-axis. (He was right.) A little while later, David sent me an email with an ASA meeting abstract for a talk a student had given about work they had done with Wayne Wright. David had, in fact, done similar work to ours – in 1979.[70] David then sent an unpublished manuscript (see Figure 10) that contained details on the diffraction theory. The discussions led to a joint publication with David and Wayne at the 2013 ICA/ASA meeting in Montreal, where we showed that nonlinear propagation outside the pipe was also occurring.[71] In true form, David complimented the undergraduate student on his work and presentation.

Radiation of sawtooth waves from the open end of a pipe[a]

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The radiation of sawtooth waves from the open end of a circular pipe is considered. Time domain piston radiation theory is used to compute the waveform for all regions of the field. For example, the waveform on the axis in the farfield is predicted to be a series of impulses, one for each of the shocks in the sawtooth at the mouth of the pipe. Experiments done with a 3.04-m long, 0.0497-m id. pipe generally confirm the predictions. The sawtooth wave was achieved by generating intense sinusoidal waves (8-kHz tone bursts, SPL138 dB re

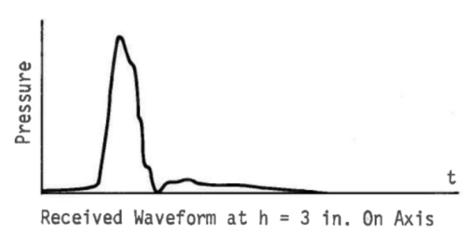

Figure 10. Excerpt from an unpublished manuscript by Blackstock and Wright, along with one of their figures showing the "derivative"-like on-axis phenomenon.

6. ONE FINAL ANECDOTE

This paper has accomplished its purposes, to describe a) David's contributions to understanding nonlinear propagation of noise and jet noise, in particular; b) how it impacted later work, including my own, and c) some of David's personal attributes as a mentor, shown to me and others. However, because of its remarkable nature, I would like to end with one final anecdote that somehow ties together the effects of high-amplitude jet noise with the importance of personal relationships in science.

In October 2010, David had agreed to come to BYU to give a colloquium in the Department of Physics and Astronomy. He was to speak on the parametric array. A couple of weeks before his colloquium, the BYU Acoustics Research Group had held the very first of an ongoing public acoustics outreach show called "Sounds to Astound." An elderly gentleman by the name of Ronald Hansen had attended the show and stayed afterward to visit. When he learned of my work in jet noise, he told me of how he used to develop hearing protectors for the military, at Wright-Patterson Air Force Base (WPAFB) in Dayton, Ohio. Knowing that David's start in acoustics was at WPAFB before he went on to earn a Ph.D., and knowing something about David's work there with hearing protectors, I asked, "Did you know a David Blackstock?" Ron's eyes lit up and he responded, "I remember David! He left to go get a Ph.D. Whatever happened to him?" I was able to arrange for Ron and David to meet during David's time at BYU, two former colleagues meeting each other for the first time after a half-century. Although I do not know what they talked about in their half-hour visit, it was while preparing for my ASA talk that preceded the paper that I learned at least part of the significance of this connection.

In 1956, David gave his first talk[72] at an ASA meeting, beginning a decades-long relationship with the Society. His topic? "Evaluation of Ear Protective Devices," in which he discussed an effort to evaluate a certain plug with 20 test subjects. His coauthor? Ronald G. Hansen, with whom David authored two additional technical reports.[73,74]

For most of his career, David Blackstock's research involved describing physical interactions of nonlinear acoustic waves, sometimes with unanticipated results. One of these areas was in the finite-amplitude propagation of random noise, laying the foundation for additional work into understanding nonlinear jet noise phenomena. Although this interaction with Ron Hansen dealt with the interpersonal rather than the physical, it feels much the same: a seemingly random encounter, an unanticipated interaction, and an underlying deeper meaning made clear by careful study of the observables. It seems a fitting conclusion for this tribute to the man, scholar, and mentor extraordinaire, David T. Blackstock.

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I sincerely thank Mark Hamilton for reviewing a draft of this manuscript and for organizing David Blackstock's 2003 tribute session and his 2021 memorial session and for allowing me to speak at both. Without that 2003 opportunity, I am unsure what might have, or have not, happened. Without the 2021 opportunity, this paper would not have happened. I also thank Vic Sparrow for his exceptional mentorship, giving me so many opportunities during my doctoral program to interact with David and others.

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