Surface Excitations of a Bose-Einstein Condensate

R. Onofrio, D. S. Durfee, C. Raman, M. Köhl, C. E. Kuklewicz, and W. Ketterle

Department of Physics and Research Laboratory of Electronics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139

(Received 24 August 1999)

Surface modes in a Bose-Einstein condensate of sodium atoms have been studied. We observed excitations of standing and rotating quadrupolar and hexadecapolar modes. The modes were excited with high spatial and temporal resolution using the optical dipole force of a rapidly scanning laser beam. This novel technique is very flexible and should be useful for the study of rotating Bose-Einstein condensates and vortices.

PACS numbers: 03.75.Fi, 32.80.Lg, 67.40.Db, 67.57.Jj

Elementary excitations play a crucial role in the understanding of many-body quantum systems. Landau derived the properties of superfluid liquid helium from the spectrum of collective excitations [1]. After the observation of Bose-Einstein condensation in dilute alkali gases [2], considerable theoretical and experimental efforts focused on collective excitations. This has already led to advances in our understanding of the weakly interacting Bose gas [3]. In most studies, collective modes were excited by modulating the parameters of the magnetic trapping potential [4,5]. This method of exciting collective modes is limited to spatial perturbations that reflect the geometry of the trapping coils. Such a limitation is particularly severe for the widely used dc magnetic traps, where only modes with cylindrical symmetry have been excited [6].

Studies of high multipolarity modes are important for a number of reasons. First, high multipolarity modes are the closest counterpart to the surface excitations in mesoscopic liquid helium droplets. These surface modes are considered crucial to understand finite size effects in superfluids, but are difficult to achieve experimentally [7]. Second, for higher angular momentum the surface modes change their character from collective to single particle type [3]. This crossover could be crucial for the existence of a critical rotational velocity for vortex formation [8,9]. Also, because at low temperatures the thermal atoms are localized around the Thomas-Fermi radius, surface modes should be very sensitive to finite temperature effects [8].

In this Letter we report on the observation of surface excitations of a Bose-Einstein condensate confined in a dc magnetic trap. The excitations were induced by the optical dipole force of a focused red-detuned laser beam which was controlled by a two-axis acousto-optic deflector. With these tools, local and controllable deformations of the magnetic trapping potential with both arbitrary spatial symmetry and timing can be achieved. This opens the way to selectively excite modes with higher multipolarity and complex spatial patterns.

Elementary excitations in a dilute Bose condensate are usually described by the hydrodynamic equations derived from the Bogoliubov theory [10], which closely resemble the equations describing superfluids at zero temperature [11],

$$m\,\frac{\partial}{\partial t}\,\mathbf{v} + \nabla\!\left(\frac{1}{2}\,m v^2 + V_{\mathrm{ext}}(\mathbf{r}) - \mu + \frac{4\pi\hbar^2 a}{m}\,\rho\right) = 0\,. \tag{1}$$

Here $\rho(\mathbf{r},t)$ and $\mathbf{v}(\mathbf{r},t)$ are the condensate density and velocity, respectively (linked by a continuity equation), m the atomic mass, a the s-wave scattering length, μ the chemical potential, and V_{ext} the external trapping potential. For an isotropic harmonic oscillator potential $V_{\mathrm{ext}} = m\omega_0^2 r^2/2$ the solution for the density perturbation $\delta\rho$ can be expressed as

$$\delta\rho(\mathbf{r}) = P_\ell^{(2n)}(r/R)\,r^\ell Y_{\ell m}(\theta,\phi)\,, \tag{2}$$

where $P_\ell^{(2n)}(r/R)$ are polynomials of degree $2n$ (R being the Thomas-Fermi radius $R = \sqrt{2\mu/m\omega_0^2}$, $Y_{\ell m}(\theta,\phi)$ are the spherical harmonics, and ℓ, m are the total angular momentum of the excitation and its z component, respectively. The dispersion law for the frequency of the normal modes is expressed in terms of the trapping frequency $\nu_0 = \omega_0/2\pi$ as [12]

$$\nu(n,\ell) = (2n^2 + 2n\ell + 3n + \ell)^{1/2}\nu_0\,, \tag{3}$$

which should be compared to the prediction for an ideal Bose gas in a harmonic trap, $\nu_{\mathrm{HO}} = (2n + \ell)\nu_0$. The effect of interactions in determining the transition from a collective to a single-particle regime is particularly evident for the excitations whose radial dependence of the density perturbation has no nodes ($n = 0$). These modes are referred to as *surface excitations* since the density perturbation, while vanishing at the origin, is peaked at the surface of the condensate. In thin films of superfluid liquid ^{4}He and ^{3}He, their study has led to the observation of third sound [13]. In a semiclassical picture these excitations can be considered the mesoscopic counterpart to tidal waves at the macroscopic level [14,15].

The experimental results were obtained using a newly developed apparatus for studying Bose-Einstein condensates of sodium atoms. A Zeeman slower with magnetic

 0031-9007/00/84(5)/810(4)$15.00

field reversal [16] delivers 10^{11} slow atoms s^{-1} which are collected in a magneto-optical trap. A loading time of 3 s allowed us to obtain 10^{10}–10^{11} atoms in a dark spontaneous-force optical trap [17] at $\simeq$1 mK. After 5 ms of polarization gradient cooling, atoms in the $F = 1, m_F = -1$ ground state at $\simeq$50–100 μK are loaded into a magnetic trap. The latter realizes a Ioffe-Pritchard configuration modified with four Ioffe bars and two strongly elongated pinch coils, symmetrically located around a quartz glass cell. This novel design combines excellent optical access with tight confinement. The typical values for the axial curvature and radial gradient of the magnetic field at the trap center are 202 and 330 G/cm, among the largest ever obtained in such magnetic traps. The resulting trapping frequencies are $\nu_r = 547$ Hz and $\nu_z = 26$ Hz for the radial and the axial directions, respectively. The background gas-limited lifetime of the atoms in the magnetic trap at $\simeq 10^{-11}$ Torr is around 1 min. After evaporative cooling with an rf sweep lasting 20 s, around $(5–10) \times 10^6$ atoms are left in a condensate with a chemical potential of 200 nK and a negligible thermal component (condensate fraction $\geq$ 90%). A decompression to the final radial and axial trapping frequencies of 90.1 $\pm$ 0.5 Hz and 18 Hz lowered the density to 2×10^{14} cm^{-3} where three-body recombination losses were less prominent. The radial trapping frequency was measured by exciting the condensate motion with a short modulation of the bias magnetic field and looking at the free center of mass oscillation in the magnetic trap.

Surface modes were excited by perturbing the magnetic trapping potential with light from a Nd:YAG laser (emitting at 1064 nm) traveling parallel to the axis of the trap and focused near the center of the magnetic trap. Because of the low intensity of the laser beam and the large detuning from the sodium resonance, heating from spontaneous scattering was negligible. The laser beam was red detuned from the sodium resonance and therefore gave rise to an attractive dipole potential [18]. The 1 mm Rayleigh range of the beam waist is considerably longer than the 220 μm axial extent of the condensate. Therefore, the laser only created radial inhomogeneities in the trapping potential, leaving the axial motion almost undisturbed.

The spatial and temporal control of the beam was achieved with two crossed acousto-optic deflectors. Using the two-axis deflector arbitrary laser patterns could be scanned in a plane transverse to the propagation of the laser beam. The maximum size of these patterns is 100 beam widths in both directions. The scan rate was chosen to be 10 kHz, which is much larger than the trapping frequencies. Thus, the atoms experienced a time-averaged potential that is superimposed upon the magnetic trap potential as depicted in Fig. 1a. For these experiments a beam width of 15 μm and a total power of 80 μW were used to generate a potential depth corresponding to 20% of the chemical potential.

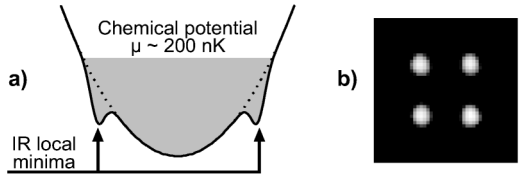

FIG. 1. Scheme for exciting collective modes by optically deforming the magnetic trap potential. (a) The combined potential of the magnetic trap and two red-detuned beams. (b) An image of the four-point IR pattern taken with a CCD camera. The laser beams were arranged on the corners of a 40 μm $\times$ 40 μm square centered on a condensate with a Thomas-Fermi radius of $\simeq$25 μm.

For an anisotropic axially symmetric trapping potential, only the z component of the angular momentum is conserved and the eigenfunctions are more complicated than in the isotropic case. However, surface modes of the form as in Eq. (2) with $m = \pm\ell$ are still solutions with a frequency [12],

$$\nu(m = \pm\ell) = \sqrt{\ell}\,\nu_r\,. \tag{4}$$

Quadrupolar standing waves were studied by exciting a superposition of $\ell = 2, m = 2$ and $\ell = 2, m = -2$ modes with a pattern of two points located on opposite sides of the condensate. The light intensity was modulated in phase at the expected quadrupole frequency $\nu_2 = \sqrt{2}\,\nu_r$. After five cycles the IR light was turned off, leaving the condensate free to oscillate in the magnetic trap. The condensate was then released from the magnetic trap and after 20 ms of ballistic expansion it was probed by resonant absorption imaging along the axis of the trap. In Fig. 2a, images are shown for different phases of the oscillation. The aspect ratio of the condensate oscillates at a frequency of 130.5 $\pm$ 2.5 Hz with a damping time of about 0.5 s. A similar damping time was observed for the lowest $m = 0$ mode of an almost pure condensate [19].

A rotating wave $\ell = 2, m = 2$ was excited with two IR spots of constant intensity rotating around the axis at half the measured quadrupole frequency. This excitation

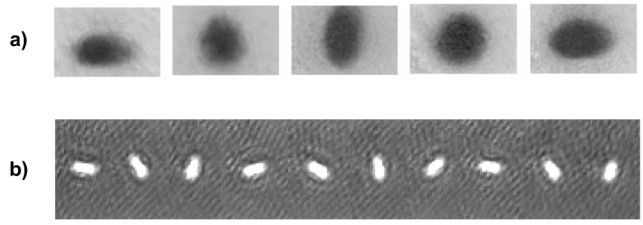

FIG. 2. Observation of standing and rotating surface waves. (a) Shows absorption images of a standing $\ell = 2, m = \pm 2$ quadrupole mode. They were taken after free oscillation times of 3.5, 5.25, 7, 8.75, and 10.5 ms (from left to right) in the magnetic trap and 20 ms of ballistic expansion. In (b) multiple phase-contrast images of a clockwise rotating $\ell = 2, m = 2$ quadrupole surface excitation are shown, each frame being 2 ms apart. The field of view of each image in (a) is 720 μm $\times$ 480 μm.

scheme was highly frequency selective. When the rotation frequency deviated by 10% from the resonance, no excitation of this mode was observed, consistent with the narrow bandwidth of the mode. In Fig. 2b we show a set of 10 pictures of the rotating mode taken with nondestructive phase-contrast imaging [20] in the magnetic trap.

The higher lying $\ell = 4$ surface mode (superposition of $m = \pm 4$) was driven with a four-point pattern that was intensity modulated at the expected frequency $\nu_4 = 2\nu_r$ (Fig. 1b). In Fig. 3a time of flight absorption images are shown for variable hold times in the magnetic trap after stopping the drive. These can be compared to the time evolution for a pure $\ell = 4$ surface mode whose contours are shown in Fig. 3b. The excitation was characterized by a multipole analysis of the absorption images. By introducing polar coordinates in the radial plane with respect to the center of mass of the condensate we determined the first radial moment $r(\theta)$ of the column density distribution $n(r, \theta)$:

$$r(\theta) = \int r^2 n(r, \theta)\, dr \Big/ \int r n(r, \theta)\, dr\,. \qquad (5)$$

The integration region was limited to a region between two contours having relative absorption with typical values of 40% and 70%, avoiding the noisier areas of larger and smaller absorption. The largest coefficient in the Fourier analysis of $r(\theta)$ was the $\ell = 4$ coefficient (in the cosine quadrature reflecting the excitation pattern). The time evolution of this coefficient was obtained by repeating the analysis for various hold times (Fig. 3c). We observed an exponentially decaying oscillation at $\nu_4 = 177 \pm 5$ Hz with a damping time of $\tau_4 = 9.5 \pm 2.2$ ms. The agreement between the measured frequencies and the hydrodynamic predictions is very good (see Table I).

The hexadecapolar mode was damped much faster than the quadrupole mode. Most likely, the observed decay is caused by Landau damping [21,22]. It is strongest in regions where the densities of the quasiparticle, the thermal cloud, and the condensate are high. Surface excitations are localized in a region where the condensate density drops to zero, and the density of the thermal cloud is maximized due to the mean-field repulsion by the condensate. It is therefore not obvious, and worth further study, whether higher surface modes are more weakly or strongly damped.

For higher ℓ, the crossover from the hydrodynamic regime to the single particle picture could be explored. This is expected to occur for $\ell \geq \ell_{\rm crit} = 2^{1/3}(R/a_{\rm HO})^{4/3} \simeq 24$ [8] for our trap parameters, where $a_{\rm HO} = [\hbar/2\pi m(\nu_r^2\nu_z)^{1/3}]^{1/2}$ is the harmonic oscillator length. The excitation of such modes would require smaller beam waists. However, attempts to do this resulted either in a very weak excitation or, when the power was increased, the condensate became strongly distorted and/or localized around the laser focus leading to high densities and to large recombination losses. We plan to use blue-detuned light in the future, which will make it easier to create stronger perturbations without loss mechanisms.

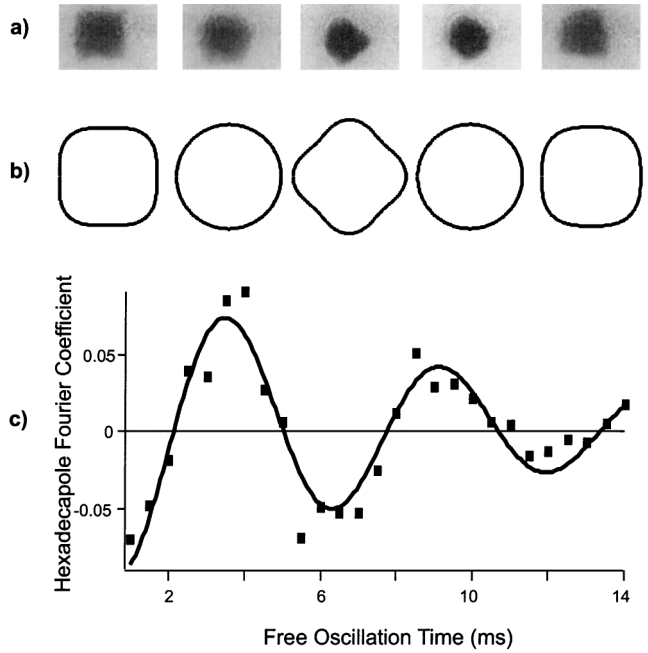

FIG. 3. Observation of the hexadecapolar mode of a Bose-Einstein condensate. (a) Absorption images of a condensate for various hold times (1, 2, 3.5, 4.5, and 6.5 ms from left to right) in the magnetic trap after the excitation. The shape of a pure $\ell = 4$ oscillation is schematically depicted in (b) for one cycle. (c) shows the $\ell = 4$ Fourier coefficient as a function of the free oscillation time in the magnetic trap. The field of view of each image is 720 μm $\times$ 480 μm.

Our method of generating time-averaged optical potentials can also be used to create purely optical traps in a variety of geometries. By increasing the laser intensity and shutting off the magnetic trap we were able to transfer the condensate into multiple optical dipole traps, as shown in Fig. 4. They can be used for interference of multiple condensates and studies of coherence and decoherence.

Another interesting possibility is the study of condensates in rotating potentials where vortices should be stable [23]. Our first attempts showed a very short trapping time, probably caused by heating due to micromotion. It should be possible to overcome this limitation by increasing the scan frequency beyond the current maximum value of 100 kHz.

In conclusion, we have developed a technique to excite surface modes in a Bose-Einstein condensate by inducing

TABLE I. Comparison between observed and predicted frequencies for the quadrupole and the hexadecapole surface excitations, normalized to the radial trapping frequency (dipole mode) ν_1.

ℓ	ν_ℓ (Hz)	ν_ℓ/ν_1 (Expt.)	ν_ℓ/ν_1 (Theor.)
1	90.1 $\pm$ 0.5	...	...
2	130.5 $\pm$ 2.5	1.45 $\pm$ 0.04	$\sqrt{2}$
4	177 $\pm$ 5	1.96 $\pm$ 0.06	2

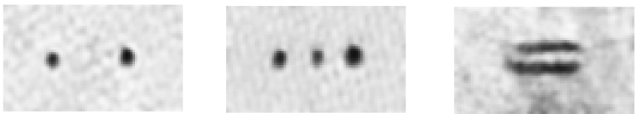

FIG. 4. Bose-Einstein condensates in time-averaged potential optical traps. Patterns with two (a) and three (b) points and a double sheet (c) are shown. The absorption images are taken along the axis of the laser beam, with resonant probe light. The frame size is 200 μm $\times$ 120 μm.

deformations of the trap potential with a rapidly scanning red-detuned laser beam. With this technique we could excite both standing and rotating modes. The measured frequencies for quadrupole and hexadecapole modes are in agreement with the predictions of the hydrodynamic theory for collective excitations of dilute Bose gases. This flexible technique should be useful for the investigation of the interplay between collective excitations and the physics of rotating Bose-Einstein condensates [24].

We thank J. Gore, Z. Hadzibabic, and J. Vogels for experimental assistance and useful discussions. This work was supported by the ONR, NSF, JSEP (ARO), NASA, and the David and Lucile Packard Foundation. M. K. also acknowledges support from Studien-stiftung des Deutschen Volkes.

[1] L. D. Landau, J. Phys. USSR **5**, 71 (1941).
[2] M. H. Anderson *et al.*, Science **269**, 198 (1995); K. B. Davis *et al.*, Phys. Rev. Lett. **75**, 3969 (1995); C. C. Bradley *et al.*, Phys. Rev. Lett. **78**, 985 (1997).
[3] F. Dalfovo, S. Giorgini, L. P. Pitaevskii, and S. Stringari, Rev. Mod. Phys. **71**, 463 (1999).
[4] D. S. Jin, J. R. Ensher, M. R. Matthews, C. E. Wieman, and E. A. Cornell, Phys. Rev. Lett. **77**, 420 (1996).
[5] M.-O. Mewes, M. R. Andrews, N. J. van Druten, D. M. Kurn, D. S. Durfee, C. G. Townsend, and W. Ketterle, Phys. Rev. Lett. **77**, 988 (1996).
[6] The time-dependent potential of the TOP-trap allowed the excitation of $m = 2$ modes [4].
[7] S. A. Chin and E. Krotscheck, Phys. Rev. Lett. **74**, 1143 (1995); D. L. Whitaker, C. Kim, C. L. Vincente, M. A. Weilert, H. J. Maris, and G. M. Seidel, J. Low Temp. Phys. **113**, 491 (1998).
[8] F. Dalfovo, S. Giorgini, M. Guilleumas, L. P. Pitaevskii, and S. Stringari, Phys. Rev. A **56**, 3840 (1997).
[9] E. C. Lundh, C. J. Pethick, and H. Smith, Phys. Rev. A **55**, 2126 (1997).
[10] N. N. Bogoliubov, J. Phys. USSR **11**, 23 (1947).
[11] P. Nozières and D. Pines, *The Theory of Quantum Liquids* (Addison-Wesley, Reading, MA, 1990), Vol. II.
[12] S. Stringari, Phys. Rev. Lett. **77**, 2360 (1996).
[13] C. W. F. Everitt, K. R. Atkins, and A. Denenstein, Phys. Rev. **136**, A1494 (1964); A. M. R. Schechter, R. W. Simmonds, R. E. Packard, and J. C. Davis, Nature (London) **396**, 555 (1998).
[14] I. Newton, *Mathematical Principles of Natural Philosophy* (University of California Press, Berkeley, 1947), p. 435 [English translation by F. Cajori].
[15] U. Al Khawaja, C. J. Pethick, and H. Smith, Phys. Rev. A **60**, 1507 (1999).
[16] A. Witte, Th. Kisters, F. Riehle, and J. Helmcke, J. Opt. Soc. Am. B **9**, 1030 (1992).
[17] W. Ketterle, K. B. Davis, M. A. Joffe, A. Martin, and D. E. Pritchard, Phys. Rev. Lett. **70**, 2253 (1993).
[18] S. Chu, J. E. Bjorkholm, A. Ashkin, and A. Cable, Phys. Rev. Lett. **57**, 314 (1986).
[19] D. M. Stamper-Kurn, H.-J. Miesner, S. Inouye, M. R. Andrews, and W. Ketterle, Phys. Rev. Lett. **81**, 500 (1998).
[20] M. Andrews, M.-O. Mewes, N. J. van Druten, D. S. Durfee, D. M. Kurn, and W. Ketterle, Science **273**, 84 (1996).
[21] L. P. Pitaevskii and S. Stringari, Phys. Lett. A **235**, 398 (1997).
[22] P. O. Fedichev, G. V. Shlyapnikov, and J. T. M. Walraven, Phys. Rev. Lett. **80**, 2269 (1998).
[23] B. M. Caradoc-Davies, R. J. Ballagh, and K. Burnett, Phys. Rev. Lett. **83**, 895 (1999).
[24] D. Guéry-Odelin and S. Stringari, e-print cond-mat/9907293.