

## Propagation of Sound in a Bose-Einstein Condensate

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Sound propagation has been studied in a magnetically trapped dilute Bose-Einstein condensate. Localized excitations were induced by suddenly modifying the trapping potential using the optical dipole force of a focused laser beam. The resulting propagation of sound was observed using a novel technique, rapid sequencing of nondestructive phase-contrast images. The speed of sound was determined as a function of density and found to be consistent with Bogoliubov theory. This method may generally be used to observe high-lying modes and perhaps second sound. [S0031-9007(97)03665-X]

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The study of quantum liquids has revealed a wealth of physics such as superfluidity, second sound, and quantized vortices. A microscopic picture of these macroscopic quantum phenomena was developed based on elementary excitations and quantum hydrodynamics [1,2]. For a long time such studies were limited to  $^4\text{He}$  and  $^3\text{He}$ . The realization of Bose-Einstein condensation in atomic vapors [3–5], however, has provided a new class of macroscopic quantum fluids which are dilute gases. An important issue, which applies both to quantum liquids and quantum gases, is the characterization of the system by its collective excitations. Several experimental [6,7] and theoretical [8,9] papers have studied oscillations in Bose-Einstein condensed gases of trapped atoms. So far, the resonance frequencies and the damping of only the few lowest modes have been investigated. Due to the small size of the trapped clouds, the discrete nature of the spectrum is dominant, in contrast to the continuous spectrum of quantum liquids, which is phononlike at low frequencies. The correspondence between the excitation spectrum of a quantum liquid [2], which consists of zeroth, first, and second sound, and the collective modes of a trapped Bose-Einstein condensate have not yet been delineated.

In this Letter we study zeroth sound in Bose-Einstein condensed samples. After exciting density perturbations much smaller than the size of the condensate, we directly observed the propagation of sound waves, analogous to studies in superfluids, and determined the speed of sound. In this study, we demonstrated a method for locally exciting a condensate using a focused off-resonant laser beam, and a new technique of observing dynamical processes using rapid sequencing of nondestructive phase-contrast images.

Bose-Einstein condensed gases at temperature  $T = 0$  are described by the nonlinear Schrödinger equation, which, in the limit of a large number of atoms, yields hydrodynamic equations [1,9]. Density perturbations propagate according to a wave equation for zeroth sound,

$$\partial_t^2 \delta n(r) = \nabla \cdot [c^2(r) \nabla \delta n(r)], \quad (1)$$

where  $\delta n(r)$  is the deviation from the equilibrium density  $n(r)$ . The local speed of sound  $c(r)$  is given by an expression first derived by Bogoliubov [10] and Lee, Huang, and Yang [11]

$$c(r) = \sqrt{n(r)\tilde{U}/m}, \quad (2)$$

where  $\tilde{U} = 4\pi\hbar^2 a/m$  characterizes the (repulsive) interaction of bosons with mass  $m$  and scattering length  $a$ . In the Thomas-Fermi approximation for a harmonic oscillator potential with frequency  $\nu_0$ , the wave function of a condensate is nonvanishing over a size [12]

$$d = \sqrt{2n_0\tilde{U}/m} / \pi\nu_0 = (\sqrt{2}/\pi)c_0/\nu_0, \quad (3)$$

where  $c_0$  is the speed of sound at peak density  $n_0$ . The frequencies of the lowest collective excitations are proportional to  $c_0/d$ , and thus, from Eq. (3), do not depend on the speed of sound, being simply proportional to the trapping frequencies. Therefore, previous experiments on collective excitations [6,7] did not yield direct information on the speed of sound.

The experimental setup for creating Bose-Einstein condensates was similar to our previous work [7,13,14]. Briefly, sodium atoms were optically cooled and trapped, and transferred into a magnetic trap where they were further cooled by rf-induced evaporation [15]. Condensates typically containing  $5 \times 10^6$  sodium atoms in the  $F = 1, m_F = -1$  ground state were produced every 30 s. In most cases we studied condensates with no discernible thermal component by extending the evaporative cooling well below the transition temperature. The condensate was confined in a cloverleaf magnetic trap, with the trapping potential determined by the axial curvature of the magnetic field of up to  $B'' = 125 \text{ G cm}^{-2}$ , the radial gradient  $B' = 120 \text{ G cm}^{-1}$ , and the bias field as low as  $B_0 = 1.5 \text{ G}$ . The atom clouds were cigar-shaped, with the long axis horizontal.

The condensate was directly observed by nondestructive phase-contrast imaging [16]. This technique is an extension of our previous work on dispersive imaging [17] and

greatly improved the signal-to-noise ratio. Images were formed by photons scattered coherently in the forward direction. The phase modulation in the probe beam caused by the condensate was transformed into an intensity modulation at the camera by retarding the transmitted probe beam by a quarter-wave. This was done using a glass plate with a small raised spot ( $500\ \mu\text{m}$  diameter) at its center, placed in the Fourier plane of the imaging system, where the transmitted probe beam is focused. Since the probe light was detuned far off-resonance (1.7 GHz to the red), absorption was very small [17], and the small-angle forward scattering imparted negligible recoil energies to the condensed atoms. Thus our imaging was nondestructive, allowing multiple images of the same condensate. Rapid images were taken by dividing a CCD camera chip into eleven strips, and shifting the accumulated charge after each exposure from the illuminated region of the chip into a covered "storage" region. After eleven exposures taken with a repetition rate of up to 1 kHz, the full chip was read out slowly with low noise.

Localized density perturbations were generated by using the repulsive optical dipole force of a focused blue-detuned far-off-resonant laser beam. Heating from spontaneous emission was negligible due to the far detuning of the argon ion laser line (514 nm) relative to the sodium resonance (589 nm). This laser beam was focused into the center of the trap, and could be switched on and off in less than a millisecond. The  $1/e^2$  half-widths were about  $12\ \mu\text{m}$  and  $100\ \mu\text{m}$ , which created a light shift of  $\sim 70\ \text{nK}$  per 1 mW of laser power. Localized increases in density (positive perturbations) were created by suddenly switching on the argon ion laser beam after the condensate had formed [see Fig. 1(a)]. The repulsive optical dipole force expelled atoms from the center of the

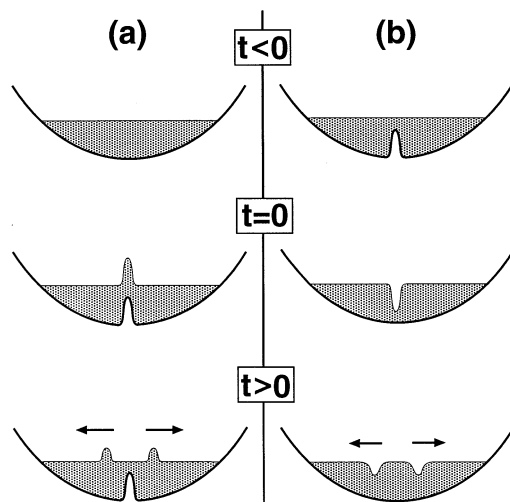


FIG. 1. Excitations of wave packets in a Bose-Einstein condensate. A condensate is confined in the potential of a magnetic trap. At time  $t = 0$ , a focused, blue-detuned laser beam is suddenly switched on (a) or off (b) and, by the optical dipole force, creates, respectively, two positive or negative perturbations in density which propagate at the speed of sound.

condensate, creating two density peaks which propagated symmetrically outward. Alternatively, we formed a condensate by evaporative cooling in the presence of the argon ion laser light, and then switched the laser off [Fig. 1(b)]. This created localized depletions of density (negative perturbations) which also propagated outward.

Figures 2 and 3 show the propagation of density perturbations observed by sequential phase-contrast imaging of a single condensate. The position of the density maxima varied linearly with time, and the speed of propagation was easily extracted. By changing the power of the argon ion laser, the relative amplitude of the density perturbation was varied between 20 and 100%. Within the accuracy of the measurement, the speed of sound was independent of amplitudes and was the same for positive and negative perturbations.

The density dependence of the speed of sound was studied using adiabatically expanded condensates. The weakest trap was formed when the field curvature was reduced to  $20\ \text{G}/\text{cm}^2$  and the bias field increased to 4 G. The critical temperatures in the strongest and weakest traps

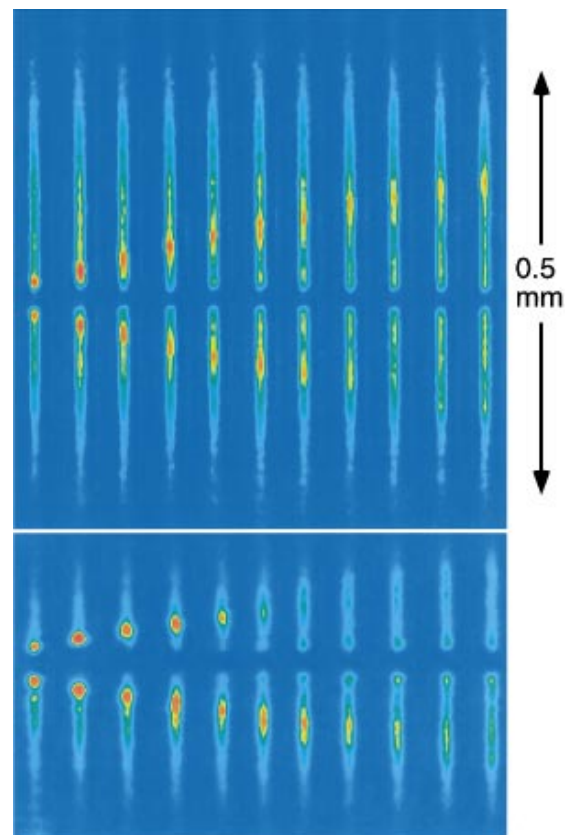


FIG. 2(color). Observation of sound propagation in a condensate by nondestructive rapid phase-contrast imaging. An image was taken every 1.3 ms, beginning 1 ms after switching on the argon ion laser. A power of 7 mW was used, just splitting the condensate into two separated parts. Two pulses traveled outward with the speed of sound. The condensate in the upper sequence was  $450\ \mu\text{m}$  long. The lower sequence was taken at lower radial confinement and thus lower peak density. As a result, the pulse propagation was slower.

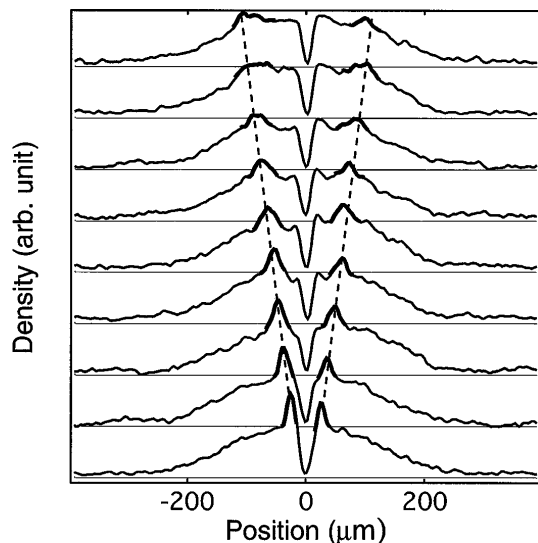


FIG. 3. Vertical profiles through the images in the upper sequence of Fig. 2.

were estimated to be 1.6 and 0.4  $\mu\text{K}$ , respectively. The peak density  $n_0$  was determined using Eqs. (2) and (3):

$$n_0 = (\pi m^2 / 8 \hbar^2 a) d^2 \nu_0^2. \quad (4)$$

Here  $d$  is the measured axial extension of the condensate, and  $\nu_0$  is the trap frequency, which was measured in a single shot by exciting a dipole oscillation of the condensate and using rapid image sequences. The scattering length  $a$  was recently determined spectroscopically to be 2.75 nm [18]. Condensates up to 450  $\mu\text{m}$  in length were observed (Fig. 2). Alternatively, the density was obtained from the observed phase shift of the probe light and the directly observed size of the condensate. Both measurements agreed to within 20% as long as the condensate was wider than the imaging resolution.

We observed one-dimensional, axial propagation of sound near the center of the cloud, where the axial density varies slowly. However, the local speed of sound varies over the radial cross section of the cloud. One can obtain a one-dimensional wave equation by eliminating the radial degrees of freedom. This is done by assuming a Thomas-Fermi solution for the radial wave function,  $\psi(r)$ , which adiabatically follows axial density variations, and performing a radial integral of the energy functional. This gives a one-dimensional speed of sound as determined by Eq. (2) for the density at  $r = 0$ . As is shown in Fig. 4, our experimental results agree well with the theoretical prediction, which has no adjustable parameters. The small discrepancy at higher densities is larger than our statistical error and might indicate that the assumptions of adiabaticity of the radial motion and the neglect of damping and dispersion are not strictly valid.

Indeed, in addition to propagation at a constant speed, we observed spreading of the pulse (Fig. 3). The pulses dispersed typically after traveling half their way to the ends of the condensate, preventing the observation of

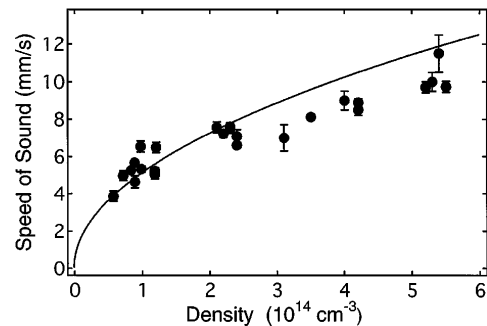


FIG. 4. Speed of sound versus condensate peak density. The solid line is the speed of sound [Eq. (2)] using the maximum cloud density [Eq. (4)] with no adjustable parameter. The error bars show only the statistical error.

a reflection of the pulse or of a decrease in the speed of propagation in the lower-density outer regions of the condensate.

In the language of collective excitations, a localized perturbation is a coherent superposition of many modes [1]. In a homogenous Bose-Einstein condensate, the dispersion relation is linear (phononlike) up to wave vectors comparable to the inverse of the healing length  $\xi = (8\pi an)^{-1/2}$ , which is approximately 0.2  $\mu\text{m}$  at the peak density of the condensate. Since the extent of the initial perturbation was larger than the healing length, we should have excited predominantly phononlike quasiparticles. The dispersion of the wave packet can be due either to damping or to dephasing of the modes, and we expect the inhomogeneous density distribution of the condensate to play an essential role. After a long time one expects the mode with the smallest damping to dominate. Indeed, in the case of a negative density perturbation, after the propagating pulses died out during the first 25 ms, we observed the lowest collective excitation (the 30 Hz quadrupolelike mode) with a damping time of  $300 \pm 120$  ms, consistent with our earlier measurement [7].

We also studied sound propagation at various temperatures. By varying the final frequency of the rf sweep during evaporative cooling, we prepared samples with condensate fractions varying from 10% to larger than 60% [19]. The speed of sound decreased with temperature in accordance with Eq. (2) where  $n(r)$  accounts for the smaller condensate density. Thus, within the accuracy of measurement we observed no effect of the normal component on the speed of zeroth sound. The dispersion of the pulses was similar over the whole temperature range studied. Further experimental work is necessary to study finite temperature effects in detail.

Above the phase transition we saw no clear evidence of sound wave propagation in the normal cloud. At an estimated density of  $10^{14} \text{ cm}^{-3}$  and for an elastic scattering cross section of  $1.9 \times 10^{-12} \text{ cm}^2$  [18], the mean free path between collisions is  $l \approx 35 \mu\text{m}$ . Thus the argon ion laser-induced density modulations of spatial extent  $\lambda < l$ . In contrast to the “hydrodynamic” regime

( $\lambda \gg l$ ), such a density perturbation expands ballistically and the amplitude decreases rapidly, which is consistent with our observation.

Additional information on the pulse propagation can be obtained by using the time-of-flight technique. After creating two Bose-Einstein condensates by evaporative cooling in a double-well potential (see [14] for details), we switched off the argon ion laser beam a variable time (up to 40 ms) before switching off the magnetic trap, and let the cloud expand ballistically. Absorption images taken after 40 ms expansion showed “interference”-like structures with coarse (100  $\mu\text{m}$ ) and fine (15  $\mu\text{m}$ ) striations [20]. The latter were similar to those observed in the interference of two Bose-Einstein condensates [14], whereas the coarse structures are probably related to collective excitations caused by propagating wave packets.

Focused laser beams can excite collective modes which cannot be excited by simply varying the magnetic trapping potential. Short-wavelength excitations can be studied by using tightly focused beams. To excite a specific high-frequency mode, one could periodically modify the laser power, or even focus several laser beams near the antinodes of the mode.

Another intriguing prospect is the excitation of second sound. Second sound is a collective excitation for which the density of the condensate and the normal component oscillate out of phase; this corresponds to a temperature wave. By focusing near-resonant light into the condensate, one could locally heat the sample by spontaneous scattering and excite such a wave. In our quasi-one-dimensional geometry, the signature of second sound could be a density minimum in the condensate accompanied by a density maximum in the surrounding normal cloud, both propagating together along the axial direction. When we replaced the argon ion laser beam by a beam of near-resonant light, we locally “evaporated” the condensate, but could not discern a localized normal fraction. This was probably due to the ballistic spreading discussed above—the excitation is rapidly damped. Furthermore, the near-resonant light also caused trap loss due to optical pumping to nontrapped states.

In conclusion, we have demonstrated a new method to excite collective excitations in a Bose-Einstein condensed cloud and have determined the speed of sound as a function of condensate density. This method allows for comparison with sound propagation in He II and is promising for the study of higher-lying excitations and perhaps for studying second sound. Furthermore, the method of rapid sequencing of phase-contrast images was used to determine the speed of sound and the frequency and damping of collective modes. This technique allows for single shot measurements of dynamical processes

and has two main advantages over previous techniques. The measuring time is greatly reduced, and studies of dynamical properties are possible that might be otherwise obscured by experimental or statistical fluctuations.

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