Collective Excitations of a Bose-Einstein Condensate in a Magnetic Trap

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Collective excitations of a dilute Bose condensate have been observed. These excitations are analogous to phonons in superfluid helium. Bose condensates were created by evaporatively cooling magnetically trapped sodium atoms. Excitations were induced by a modulation of the trapping potential, and detected as shape oscillations in the freely expanding condensates. The frequencies of the lowest modes agreed well with theoretical predictions based on mean-field theory. Before the onset of Bose-Einstein condensation, we observed sound waves in a dense ultracold gas. [S0031-9007(96)00900-3]

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In 1941 Landau introduced the concept of elementary excitations to explain the properties of superfluid helium [1]. This phenomenological approach, based on quantum hydrodynamics, gave a quantitative description of the thermodynamic properties and transport processes in liquid helium. Landau rejected any relation to Bose-Einstein condensation (BEC). A microscopic derivation of the elementary excitation spectrum for a weakly interacting Bose gas was given by Bogoliubov in 1947 [2] and for He II by Feynman in 1955 [3], emphasizing the role of Bose statistics [3] and reconciling Landau's approach with London's explanation of superfluidity as being due to BEC [2,4].

The elementary excitations determine the spectrum of density fluctuations in a Bose liquid, and have been directly observed in He II by neutron scattering [5]. The low-frequency excitations are phonons, long-wavelength collective modes of the superfluid. So far, a satisfactory microscopic theory for an interacting bosonic system exists only for the dilute quantum gas. The recent realization of BEC in dilute atomic vapors [6–8] has opened the door to test this theory experimentally. In this paper we report on the observation of shape oscillations of a trapped Bose condensate, modes analogous to phonons in homogeneous systems [9].

The experimental setup for creating Bose condensates was the same as in our previous work [10]. Briefly, sodium atoms were optically cooled and trapped, and transferred into a magnetic trap where they were further cooled by rf-induced evaporation [11,12]. Every 30 s, condensates containing 5×10^6 sodium atoms in the $F = 1, m_F = -1$ ground state were produced. Evaporative cooling was extended well below the transition temperature to obtain a condensate without a discernible normal component. The condensate was confined in a cloverleaf magnetic trap which had cylindrical symmetry with trapping frequencies of 19 Hz axially and 250 Hz radially (see below). The trapping potential is determined by the axial curvature of the magnetic field $B'' = 125\ \mathrm{G\,cm^{-2}}$, the radial gradient $B' = 150\ \mathrm{G\,cm^{-1}}$, and the bias field $B_0 = 1.2$ G.

The condensate was excited by a time-dependent modulation of the trapping potential. First, we used a sudden step in the gradient B' to identify several collective modes of the condensate and to find their approximate frequencies. B' was decreased by 15% for a duration of 5 ms with a transition time of about 1 ms, and then returned to its original value. A variable time delay was introduced between the excitation and the observation of the cloud. In this way, we strobed the free time evolution of the system after the excitation. The cloud was observed by absorption imaging after a sudden switch off of the magnetic trap and 40 ms of ballistic expansion. No trap loss was observed during the interval over which the delay was varied. The images were similar to the series shown in Fig. 1. Four modes were identified from the measured center-of-mass positions and the widths of the condensate. The radial and axial center-of-mass oscillations (dipole modes) were excited because a change in B' displaced the center of the trap slightly due to asymmetries in the field-producing coils. A fast shape oscillation predominantly showed up as a sinusoidal modulation of the radial width while a slow sinusoidal shape oscillation was observed in the axial width. When a strong parametric drive (see below) was used to excite the slow shape oscillation, a weak oscillation of the radial width was also detected. Note that the widths were observed after ballistic expansion and reflect a convolution of the initial spatial and velocity

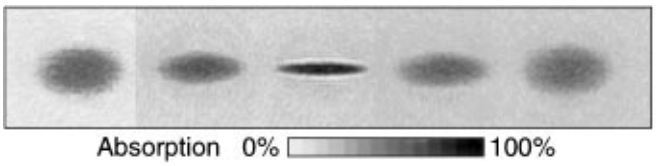

FIG. 1. Shape oscillation of a Bose-Einstein condensate. After excitation the condensate was allowed to freely oscillate in the trap for a variable time, ranging from 16 ms (left) to 48 ms (right). The absorption images were taken after a sudden switch-off of the trapping potential and 40 ms of ballistic expansion. The horizontal width of each cloud is 1.2 mm.

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distributions, further complicated by the acceleration due to the repulsive mean field.

From these "kicked" excitation experiments, we determined $\nu_\rho = 250(5)$ Hz [13] for the radial trapping frequency and $\nu = 510(15)$ Hz for the faster shape oscillation. The slower shape oscillation was studied further in the following manner. A better "mode selectivity" in the excitation was obtained by modulating B'' at 30 Hz sinusoidally for five full cycles. The amplitude of the parametric drive was varied between 0.5% and 6% of the dc field strength. Axial and radial widths of the cloud were then determined as a function of delay time. The axial widths were fitted with an exponentially decaying sine function. For the smallest drive an improved fit was obtained by fitting the aspect ratio of the cloud instead of the width itself (Fig. 2); this procedure eliminated the effect of fluctuations of the number N_0 of condensed atoms, which were about 20%. Note that in the hydrodynamic limit the frequencies of the normal modes are independent of N_0 (see below). The amplitudes of the axial width modulation Δw_z, after a time Δt of ballistic expansion, varied from 77.8 for the smallest drive to 527 μm for the largest drive. From $\Delta w_z/(2\Delta t)$ we obtained an upper bound for the kinetic energy of the collective mode, which ranged from 1.3 to 60 nK. The highest value is comparable to the typical mean-field energy per atom in our experiment [10]. It is, therefore, remarkable that the frequency of the collective excitation was found not to depend on the strength of the drive (Fig. 3). For the smallest amplitudes, the frequency was determined to be 30.0(2) Hz which will be compared below to a theoretical prediction for small oscillations. An accurate value of 19.28(11) Hz for the axial trapping frequency was obtained by using a five-cycle sinusoidal modulation of B' at 18 Hz. In this way, the simultaneous excitation of the radial dipole mode at 250 Hz was suppressed.

We first discuss the nature of the shape oscillations for a noninteracting gas and a weakly interacting condensate in the simplest case of an isotropic harmonic potential. For a noninteracting ideal gas (both in the normal and in the Bose-condensed states), all modes have frequencies which are integer multiples of the harmonic trapping frequency ν_0. In particular, the lowest quadrupole oscillation occurs at a frequency of $2\nu_0$. In a homogeneous weakly interacting Bose condensate of density n_0, the lowest frequency excitations are phonons propagating at the speed of sound $c = \sqrt{n_0\tilde{U}/m}$ at $T = 0$ [2,14]. $\tilde{U} = 4\pi\hbar^2 a/m$ characterizes the interactions of bosons with mass m and scattering length a. In a sample of size d, the lowest frequency ν of a phonon is obtained from the dispersion relation $\nu = c/\lambda$ with $\lambda = 2d$. In the Thomas-Fermi approximation the condensate wave function is nonvanishing over a size $d = (1/\pi\nu_0)\sqrt{2n_0\tilde{U}/m}$ [14,15] resulting in $\nu = (\pi/2^{3/2})\nu_0$. For an inhomogeneous condensate this result is only an estimate, but it correctly shows that the lowest excitation frequencies are proportional to ν_0. These frequencies should be independent of the number of atoms N_0 in the condensate since the dependences of the sound velocity and of the size of the condensate on N_0 exactly cancel.

The normal modes of the inhomogeneous interacting condensate are obtained by solving the corresponding wave equation which is the nonlinear Schrödinger equation, as recently discussed by several groups [5,14, 16–21]. Stringari presented the analytical solution for an isotropic harmonic potential in the Thomas-Fermi regime which gave $\sqrt{2}\,\nu_0$ as the frequency of the lowest normal mode, aside from the center-of-mass oscillation [16]. The normal modes of the condensate are classified by quantum numbers (n, l, m) where n is the radial quantum number and l, m denote quantum numbers for the total angular momentum and its axial projection,

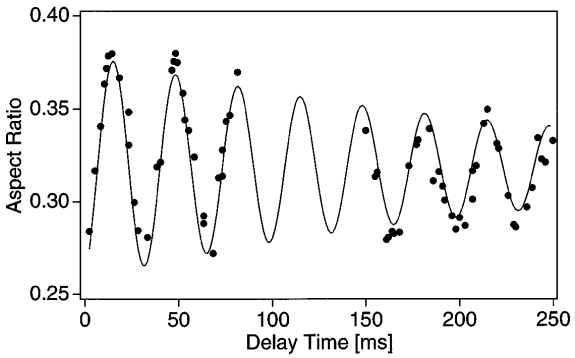

FIG. 2. Analysis of the "quadrupole" oscillation at 30 Hz. The aspect ratio of the expanding cloud is plotted against the free oscillation time between excitation and switch off of the trap. Damping of the harmonic oscillations was observed with a decay time of 250 ms. Note that the analagous mode in the non-interacting ideal gas occurs at $2\nu_z = 38.6$ Hz.

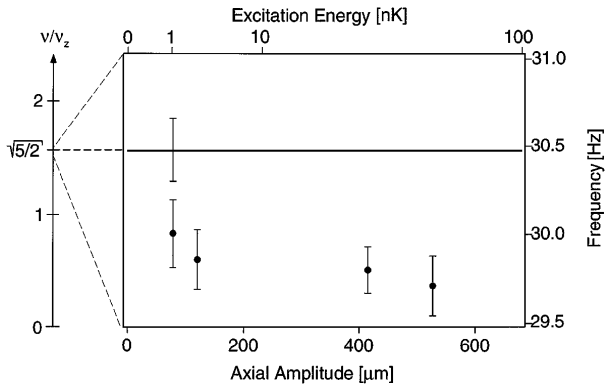

FIG. 3. Frequency of the collective excitation at 30 Hz as a function of driving amplitude. The abscissa shows the amplitude of the axial width modulation after 40 ms of ballistic expansion, which is used to estimate the excitation energy. The solid line is the theoretical prediction of Stringari [16] when combined with the measured axial trapping frequency.

respectively. For cylindrical symmetry (as in our trap), m is still a good quantum number, but l is not. Thus, the normal modes are superpositions of wave functions with the same m. Since our excitation scheme preserves the axial symmetry of the trapping potential, we expect to observe only $m = 0$ modes. (However, the excitation of the dipole modes shows that slight asymmetries in the trapping coils also excite $m = 1$ modes.) Stringari discussed an anisotropic harmonic potential with axial symmetry in the Thomas-Fermi limit [16]. He showed that the lowest $m = 0$ modes are coupled excitations of $(0, 2, 0)$ (which is a quadrupolar surface oscillation) and $(1, 0, 0)$ symmetries. For a cigar-shaped condensate with a large aspect ratio, these lowest modes were predicted at frequencies $\sqrt{5/2}\,\nu_z$ and $2\nu_\rho$, where ν_z and ν_ρ are the axial and radial trapping frequencies, respectively [16,22].

The observed ratio $\nu/\nu_z = 1.556(14)$ is in good agreement with the predicted ratio of $\sqrt{5/2} = 1.581$. It is the main result of the present paper and should be regarded as a critical quantitative test of the mean-field theory describing excited states of a Bose condensate. The fast collective excitation which was observed is probably the high frequency mode of the mixed $(0, 2, 0)$ and $(1, 0, 0)$ excitations for which a frequency of $2\nu_\rho$ was predicted [16], in good agreement with our measurement of $2.04(6)\nu_\rho$. Note that a noninteracting condensate also has a mode at $2\nu_\rho$.

The solutions of the linearized Gross-Pitaevskii equation are the normal modes of the condensate, also called the elementary excitations or quasiparticles of the macroscopic quantum system [5,19]. The collective excitations which we have observed, such as in Fig. 1, are large-scale density fluctuations which obey the hydrodynamic equation for superfluid flow at zero temperature [5,16]. They are a coherent excitation of many quasiparticles at the frequency of the normal mode. The lifetime of the quasiparticles can thus be determined from the damping of the shape oscillations.

Damping may be caused either by interactions between collective and thermal excitations, or by nonlinear interactions which couple the normal modes of the condensate [20]. For a nearly pure condensate ($T \approx 0$), the damping due to thermal excitations should be negligible. In our experiment, we observed a damping time of 250(40) ms for the collective excitation at 30 Hz (Fig. 2). So far there is no theoretical prediction for the damping of collective excitations of a trapped condensate.

Similar excitation experiments were performed on a thermal cloud at $T/T_c \approx 2$. For an ideal gas at low density one expects quadrupole oscillations at $2\nu_0$, the damping of which is described by a Q factor given by $Q = \pi\tau\nu \sim l/d$, where d is the size of the sample, l is the mean-free path between elastic collisions, and τ is the amplitude damping time. Our experiments were carried out at a density of 10^{14} cm^{-3} and an axial rms sample length of 300 μm. Using our recent determination of the cross section for elastic collisions of 6×10^{-12} cm^2 [11], we obtain 4 kHz for the elastic collision rate in the center of the cloud, and 12 μm for the mean-free path, much shorter than the size of the cloud. We are therefore well in the hydrodynamic regime, where the normal modes of the cloud are sound waves. One can estimate the frequency ν of the lowest mode from the speed of sound which is $c = \sqrt{5k_BT/3m}$ for a monatomic ideal gas, and the rms diameter d of the thermal cloud $d = (1/\pi\nu_0)\sqrt{k_BT/m}$. These give a lowest mode frequency of $\nu = c/2d \approx 2\nu_0$. This estimate agrees coincidentally with the frequency obtained in the collisionless regime. Damping in a classical gas is due to thermal conduction and shear viscosity, with each mechanism contributing almost equally for an ideal gas [23]. From the kinetic theory of gases we obtain the Q factor for a sound wave of wavelength λ as $Q = \pi\tau\nu \approx (1/2\pi)\lambda/l$. The damping therefore decreases with increasing density in the hydrodynamic regime, in contrast to the behavior at low density. For our experimental conditions this estimate gives an amplitude damping time of about 100 ms. Our measurements on a thermal cloud are in agreement with these predictions. We observed oscillations in the axial width at a frequency of 35(4) Hz $= 1.8(2)\nu_z$ and a damping time of about 80 ms, much longer than the collision time.

Theoretical discussions of collective excitations of a condensate have emphasized that the frequency shift of the normal modes compared to the uncondensed atoms is clear evidence for the order parameter associated with BEC [19]. We point out that the normal modes of dense clouds can show frequency shifts even above T_C, due to the hydrodynamic propagation of sound waves. It will be interesting to study the behavior of such shifts across the BEC phase transition. For the accurate comparison to theory we have normalized the frequencies of the condensate with the center-of-mass oscillation frequencies which are unshifted and identical to the single-particle trapping frequencies.

The results reported above were all obtained at small and medium drive amplitudes. At larger amplitude, we observed striations in the time-of-flight pictures of Bose condensates parallel to the radial direction. We conjecture that these interferencelike structures are the self-diffraction of an excited macroscopic matter wave and might reflect the nodal structure of a strongly driven condensate.

This paper is only the first step of a systematic study of the elementary excitations of a Bose condensate. The ultimate goal is a complete survey of the spectrum of collective excitations, including the lifetimes of the quasiparticles and the behavior at different temperatures and higher excitation energies. For excitation frequencies larger than the mean interaction energy (typically 2 kHz or 100 nK in our samples) one expects a transition from collective to single-particle behavior [5]. One limitation of the current experiment is that the

time-of-flight probing technique is intrinsically destructive. We have recently demonstrated dispersive imaging as a nondestructive technique to spatially resolve Bose condensates [24]. Combining this technique with a high-speed camera should enable us to observe collective modes of a Bose condensate *in situ* and in real time.

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[1] L. D. Landau, J. Phys. (Moscow) **5**, 71 (1941).
[2] C. N. Bogoliubov, J. Phys. (Moscow) **11**, 23 (1947).
[3] R. P. Feynman, in *Progress in Low Temperature Physics*, edited by C. J. Gorter (North-Holland, Amsterdam, 1955), Vol. 1, p. 17.
[4] F. London, Nature (London) **141**, 643 (1938).
[5] P. Nozieres and D. Pines, *The Theory of Quantum Liquids* (Addison-Wesley, Redwood City, CA, 1990), Vol. 2.
[6] M. H. Anderson *et al.*, Science **269**, 198 (1995).
[7] C. C. Bradley *et al.*, Phys. Rev. Lett. **75**, 1687 (1995). Note that some results of this paper have been corrected (R. Hulet, presented at the DAMOP meeting in Ann Arbor, MI, 1996).
[8] K. B. Davis *et al.*, Phys. Rev. Lett. **75**, 3969 (1995).
[9] First results of this study were presented at the DAMOP meeting in Ann Arbor, MI, 1996. At the same meeting the JILA group gave a report on unsubmitted related work. See D. S. Jin *et al.*, Phys. Rev. Lett. **77**, 420 (1996).
[10] M.-O. Mewes *et al.*, Phys. Rev. Lett. **77**, 416 (1996).
[11] K. B. Davis *et al.*, Phys. Rev. Lett. **74**, 5202 (1995).
[12] W. Ketterle and N. J. van Druten, in Advances in Atomic, Molecular and Optical Physics, edited by B. Bederson and H. Walther (to be published), Vol. 37, and references therein.
[13] All quoted experimental uncertainties in this paper correspond to 1 standard deviation in the fit.
[14] G. Baym and C. J. Pethick, Phys. Rev. Lett. **76**, 6 (1996).
[15] V. V. Goldman, I. F. Silvera, and A. J. Leggett, Phys. Rev. B **24**, 2870 (1981).
[16] S. Stringari, Report No. cond. mat. 9603126 (to be published).
[17] L. You *et al.* (to be published).
[18] A. L. Fetter, Phys. Rev. A **53**, 4245 (1996).
[19] M. Edwards *et al.* (to be published).
[20] P. A. Ruprecht *et al.*, Phys. Rev. A **51**, 4704 (1995).
[21] M. Marinescu and A. F. Starace (to be published).
[22] This prediction for $\nu_\rho/\nu_z \rightarrow \infty$ is good to 10^{-3} accuracy for $\nu_\rho/\nu_z = 13$.
[23] P. M. Morse and K. U. Ingard, *Theoretical Acoustics* (Princeton University Press, Princeton, NJ, 1986).
[24] M. R. Andrews *et al.*, Science **273**, 84 (1996).