Skewness and shock formation in laboratory-scale supersonic jet data

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Abstract: Spatial properties of noise statistics near unheated, laboratory-scale supersonic jets yield insights into source characteristics and near-field shock formation. Primary findings are (1) waveforms with positive pressure skewness radiate from the source with a directivity upstream of maximum overall level and (2) skewness of the time derivative of the pressure waveforms increases significantly with range, indicating formation of shocks during propagation. These results corroborate findings of a previous study involving full-scale engine data. Further, a comparison of ideally and over-expanded laboratory data show that while derivative skewness maps are similar, waveform skewness maps are substantially different for the two cases.

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1. Introduction

Various studies[1–5] have demonstrated far-field nonlinear propagation effects in the noise radiated by high-performance jet engines. Less understood, however, has been the behavior of the nonlinearity in the geometric near field. Some have thought the noise radiates from the shear layer as well-formed acoustic shocks, but a recent Letter showed otherwise using data from the F-35A Joint Strike Fighter.[6] The current Letter describes the results of a corroborative analysis using both ideally and over-expanded, unheated, supersonic jet data where fundamental parameters (e.g., temperature ratio and Mach number) are available. Furthermore, a comparison between the laboratory jet in ideal and nonideal expansion reveals fundamental differences in spatial radiation characteristics.

Nonlinear propagation in laboratory jets has been explored, from early measurements by Gallagher and McLaughlin[7] to more recent analyses.[8–11] However, authors have disagreed on the attribution of observed phenomena to nonlinearity. Recent work by Gee *et al.*[12,13] has examined unheated, Mach-2.0 data in the geometric near field using higher-order spectral analysis methods. Both bispectral analysis[12] and a quadspectrum indicator,[13] based on work by Morfey and Howell,[2] showed distinct evidence of quadratic phase coupling and nonlinear propagation. This Letter examines the same data set using a different analysis method—the skewness of the pressure waveform and of its time derivative—to compare with the analysis in Ref. 6.

Use of the skewness, the normalized third central moment of the probability density function, to characterize jet noise properties stems from the work by Ffowcs Williams *et al.*,[14] who linked the jet phenomenon of crackle to positive skewness values. However, skewness of the time derivative, which increases significantly as waveforms steepen and acoustic shocks form, was proposed as a useful measure by

McInerny.[15] Shepherd *et al.*[16] recently calculated the statistics in the preshock region for the canonical nonlinear example, a planar, initially sinusoidal wave. They found a large increase in derivative skewness near the shock formation distance. The F-35A analysis[6] utilized both the skewness of the pressure waveform and its time derivative to examine whether waveform asymmetry and shock formation are source and/or propagation phenomena. The authors concluded that skewed waveforms radiate from the shear layer but that the shock content forms during the course of propagation, which could have important implications regarding the perception of jet crackle.[17,18] The ideally and over-expanded laboratory data discussed in this Letter both strengthen and extend these prior conclusions.

2. Results and analysis

To summarize briefly the experiment described in Ref. 12, unheated jet data from a 3.49 cm diameter, convergent-divergent nozzle were collected with Type-1, 6.35 and 3.18 mm pressure microphones located between 10 and 75 nozzle diameters (D_j) from a reference position 4 D_j downstream of the nozzle exit. The microphones were mounted at nozzle centerline height on a boom that swept out a measurement arc between 80° and 150° (relative to the inlet and the reference position) in 5° increments. Data were acquired at a sampling rate of 192 kHz. Although only the ideally expanded Mach-2.0 data have been analyzed previously, waveform data were also acquired with the same nozzle in an over-expanded, Mach-1.8 condition.

The principal results in this Letter are displayed in Fig. 1 as maps of the overall sound pressure level (OASPL), waveform skewness ($\mathrm{Sk}\{p\}$), and derivative skewness ($\mathrm{Sk}\{\partial p/\partial t\}$). For convenience, the Mach-2.0 (ideally expanded) results are shown in the left column and the Mach-1.8 (over-expanded) results are displayed in the right column. The Mach-2.0 OASPL maximum at 75 D_j occurs at 145° or greater with a single relatively narrow lobe and a smooth roll-off toward the sideline. On the other hand, the OASPL for the Mach-1.8, over-expanded jet reaches its maximum at 150° or possibly greater angles, outside the measurement aperture. In addition, the Mach-1.8 jet radiates a secondary lobe at around 125° and has generally higher levels of radiation to the sideline.

The maps of $\mathrm{Sk}\{p\}$ for the two jet conditions are displayed in the middle of Fig. 1. For Mach 2.0, the pressure skewness peaks at a lesser angle than the OASPL—140° rather than 145°. Note further that there is a significant $\mathrm{Sk}\{p\}$ near the jet that increases slightly in the 40–60 D_j range to greater than 0.4 before decreasing. Finally, examination of the map suggests that $\mathrm{Sk}\{p\}$ originates 3–5 D_j downstream of the nozzle; this happens to correspond to the selected boom origin. The Mach-1.8 waveform skewness shares similarities with the Mach-2.0 case in that both have maxima 5° less than each of the OASPL lobes, and increases slightly in the 40–60 D_j range, but otherwise differs significantly. Specifically, the maximum skewness ($\mathrm{Sk}\{p\} \sim 0.37$) corresponds not with the maximum OASPL region but rather with the minor directivity lobe in the OASPL. Additionally, $\mathrm{Sk}\{p\}$ is more spatially uniform for the over-expanded case with a nominal baseline value of $\sim$0.2 to the sideline.

Displayed in the last row of Fig. 1 are maps of $\mathrm{Sk}\{\partial p/\partial t\}$ for the ideally expanded and over-expanded supersonic jets. For both jet conditions, the derivative skewness appears to originate approximately at the selected boom origin and monotonically increases with range along an observation angle. The derivative skewness is increasing over the same range where the waveform skewness has begun to decline. Unlike the maps for $\mathrm{Sk}\{p\}$, the maximum regions of $\mathrm{Sk}\{\partial p/\partial t\}$ appear to be aligned along the directions of maximum OASPL. This observation is qualified by the possibility that the true maximum OASPL for the Mach 1.8 case may be outside the measurement aperture. The region of significant growth in $\mathrm{Sk}\{\partial p/\partial t\}$ for both jets appears to be largely confined in a narrow region around the peak OASPL. However, there is growth at other angles; for example, at 125°, $\mathrm{Sk}\{\partial p/\partial t\}$ increases from 0.09 at 10 D_j to 0.54 at 75 D_j for the ideally expanded jet and from 0.14 to 0.75 for the

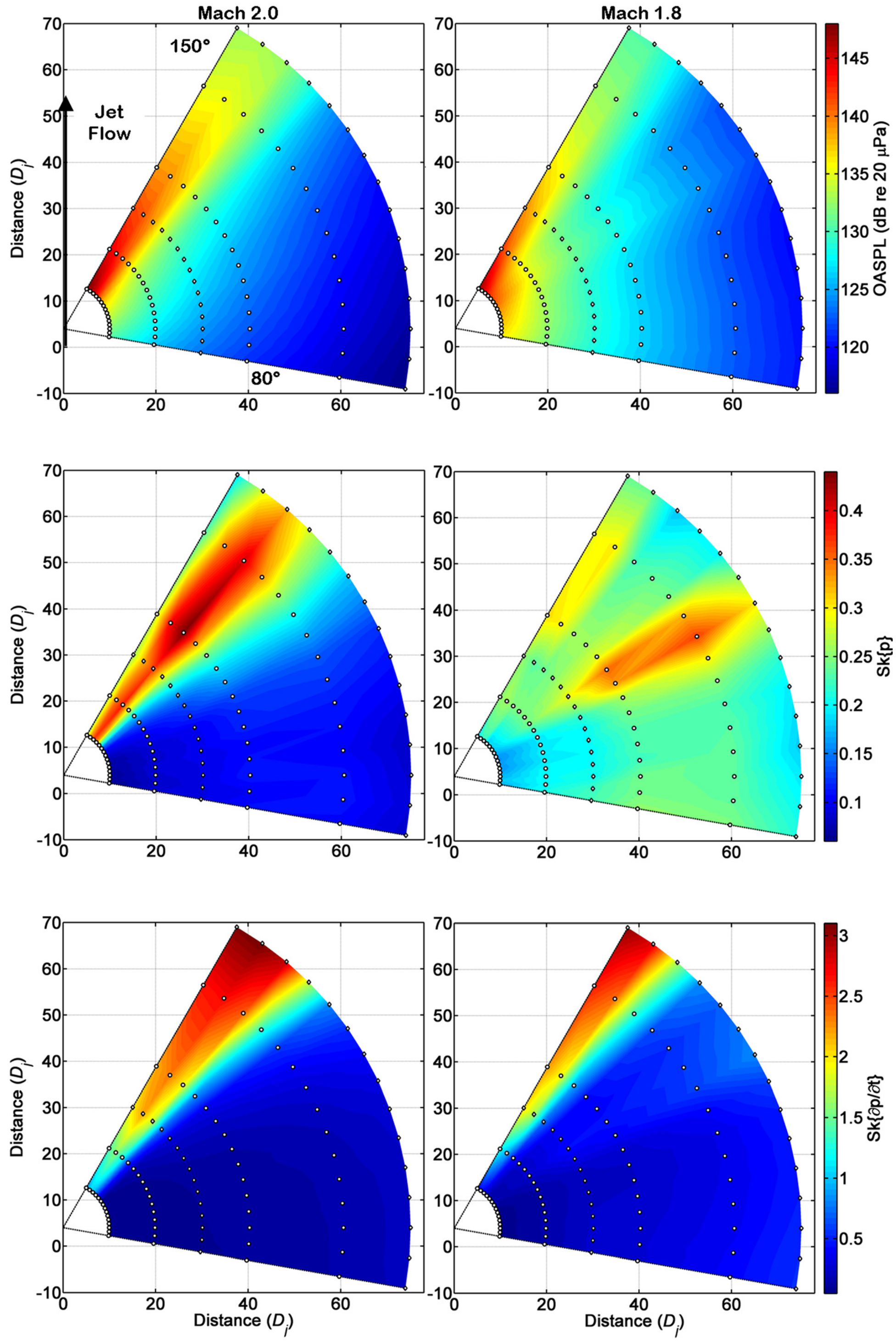

Fig. 1. (Color online) Interpolated maps of the overall sound pressure level (OASPL), waveform skewness, Sk{p}, and derivative skewness, Sk{$\partial p/\partial t$}, relative to the nozzle exit at (0,0). Measurement locations are marked in 5° increments between 80° and 150° and distances of 10–75 D_j. Mach-2.0 data (ideally expanded) are shown on the left and Mach-1.8 data (over-expanded) are shown on the right. Color scales are consistent between the two conditions to permit direct visual comparison.

over-expanded jet and the same range. This indicates nonlinear waveform steepening in directions other than the peak radiation angles.

To summarize thus far, the laboratory-scale, unheated jet noise radiates as skewed waveforms with the maximum derivative skewness tracking the principal OASPL lobe. However, the maximum waveform skewness occurs slightly upstream of the OASPL lobes, which can be also seen in the data of Krothapalli *et al.*[19] Because high-frequency levels are more predominant to the sideline of the jet,[5,20,21] this upstream directivity shift in skewness suggests that the jet's high-frequency noise radiation is correlated with $\mathrm{Sk}\{p\}$. This hypothesis is confirmed by an analysis of the Mach-2.0 waveform at 75 D_j and 145°, for which the OASPL is 133.2 dB re 20 μPa and $\mathrm{Sk}\{p\} = 0.28$. The waveform is divided into "low" and "high" frequency components and the OASPL and $\mathrm{Sk}\{p\}$ are calculated for each. The cutoff between the low- and high-pass filtered versions of the waveform was selected to be 6 kHz, above the peak-frequency region of the spectrum.[12] The filters were fourth-order, Butterworth-magnitude, zero phase-distortion filters. The low-pass filtered waveform has an OASPL of 130.2 dB, with $\mathrm{Sk}\{p\} = 0.04$. On the other hand, the high-pass filtered waveform has an OASPL of 127.7 dB re 20 μPa and $\mathrm{Sk}\{p\} = 0.65$. This confirms that despite the lesser total energy, skewness is predominately caused by the high-frequency waveform component. This analysis helps explain the directivity shift between the OASPL, dominated by lower frequencies, and waveform skewness, dominated by higher frequencies.

The main difference between the ideally and over-expanded jets is the variation of OASPL and $\mathrm{Sk}\{p\}$ over the measurement aperture: Mach 2.0 has only one peak region, whereas Mach 1.8 has two. For the over-expanded case, power spectral densities (PSDs) help explain the secondary OASPL peak at 125°. The PSDs at 125° and 150° and 75 D_j are displayed in Fig. 2. The 150° PSD shows a familiar haystack-like spectrum related to large turbulent structures in jet mixing noise.[21,22] The 125° PSD has a rounder shape and reveals a secondary high-frequency peak around 20 kHz. Though not shown, the angular variation in the PSD shapes for the Mach 1.8 jet match published curves of jet mixing and broadband shock-associated noise[20,23] in which the high-frequency secondary peak at similar angles is caused by shock-associated noise. (See 120° data in Fig. 3 of Ref. 20.) Thus the secondary lobe in OASPL results from a combination of significant mixing and broadband shock-associated noise in that direction. It is interesting that this combination results in the greatest skewness values (at 120°), whereas angles at which either the mixing noise or the broadband shock-associated noise dominates have lower $\mathrm{Sk}\{p\}$.

The skewness maps in Fig. 1 show a slight increase in $\mathrm{Sk}\{p\}$ between 40 and 60 D_j along many angles. This occurs both for the ideally and over-expanded jets in the vicinity of the maxima but does not occur in the same fashion for all angles, disqualifying individual microphones as being the cause. As waveform skewness could be correlated with high-frequency content, a decrease in skewness could be related to the absorption of high-frequency energy. On the other hand, the attenuation of propagating acoustic shocks most rapidly reduces the largest amplitude waveform peaks,[24]

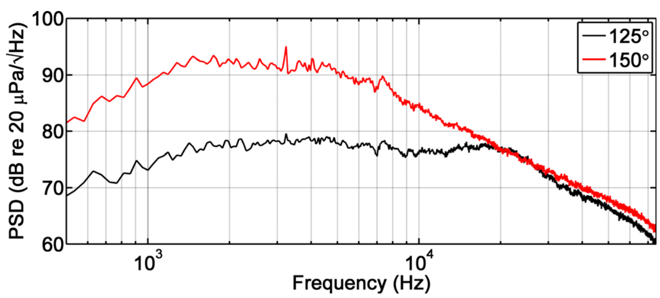

Fig. 2. (Color online) Power spectral densities (PSD) of the over-expanded Mach-1.8 jet at 75 D_j.

which would also reduce skewness. In any case, the reason(s) for the increase and subsequent decrease remains an open question that requires further investigation.

Although the properties of $\mathrm{Sk}\{p\}$ permit comparison with other experiments, it is $\mathrm{Sk}\{\partial p/\partial t\}$ that reveals the critical information regarding nonlinear propagation. Figure 1 displays a clear evolution of $\mathrm{Sk}\{\partial p/\partial t\}$ with range for both the ideally and over-expanded jets with the greatest change occurring in the direction of the maximum OASPL. To illustrate the significance of the maps, two short segments of the waveforms used to calculate the PSDs in Fig. 2 are displayed along with forward-difference derivative estimates in Fig. 3. Only the Mach-1.8 case is shown for brevity, but similar behavior is observed for Mach 2.0. The positive skewness of the waveform data at 125° and 75 D_j ($\mathrm{Sk}\{p\} = 0.26$) is further emphasized by differentiation ($\mathrm{Sk}\{\partial p/\partial t\} = 0.75$), but the contrast is marginal compared to the behavior at 150°. The 150° data clearly show weak shocks, such that only two samples describe the sharpest positive rises, corresponding to high-amplitude peaks in the derivative. The sampling frequency used to obtain the data is insufficient to capture the true shock rise times, a common difficulty with laboratory-scale measurements.[12] However, the combination of derivative skewness growth in Fig. 1 and the clear shock content in the 150° waveform in Fig. 3 shows evidence of waveform steepening and shock formation in the maximum OASPL direction.

3. Comparison with prior F-35A analysis

There are several similarities between the observations here and the findings of the F-35A analysis.[6] The basic phenomena observed are the same: (1) The supersonic jet source radiates skewed waveforms that are only slightly steepened near the shear layer and (2) the nonlinear evolution of the waveforms results in a rapid increase in the derivative skewness centered around the peak OASPL direction. Additionally, both skewness phenomena appear to originate relatively close to the nozzle. The relative upstream directivity of waveform skewness and its slight increase with range suggested in the F-35A analysis have been corroborated here. However, this study indicates that the skewness eventually decreases as the far field is approached. Quantitatively, both the ideally expanded case and the engine data have maximum pressure skewness values between 0.4 and 0.5 despite the fact that military jet engine flow is typically

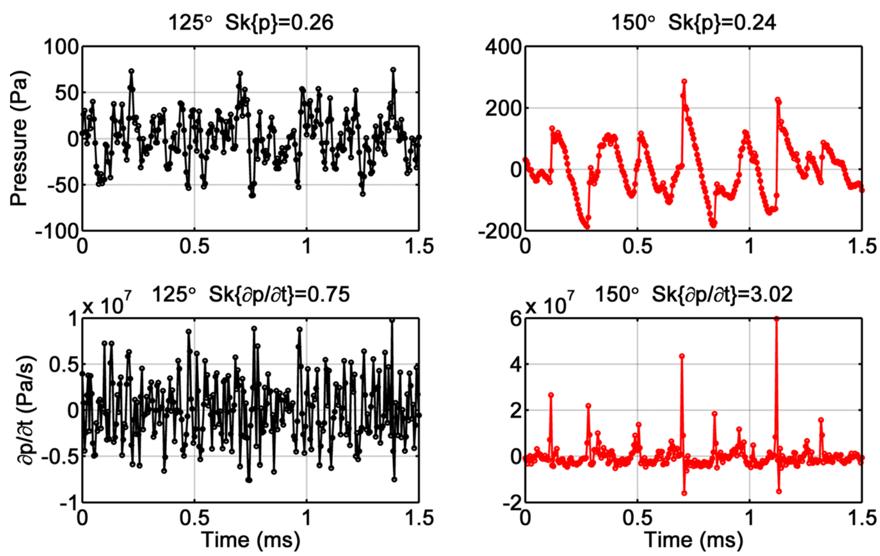

Fig. 3. (Color online) Waveform and time derivative segments for the Mach-1.8 over-expanded jet at 75 D_j at 125° and 150°. Skewness values for the entire waveforms are also provided.

over-expanded at take-off conditions. Nevertheless, the full-scale engine's baseline skewness of $\sim$0.2 in the forward direction is more similar to that of the over-expanded, laboratory-scale jet.

There are a few differences between skewness properties of the F-35A military engine and the laboratory-scale jet. First, the angular range over which significant waveform and derivative skewness occur is greater in the case of the full-scale engine. This is possibly due to the broader maximum OASPL region caused by jet heating.[20] Other possibilities include differences in Mach number and the influence of ground reflections in the F-35A data. In addition, there is no clearly separated secondary OSAPL or skewness lobe in the engine data as in the over-expanded laboratory data. Comparisons of the spatial distribution of the statistical properties of noise from full-scale, high-performance engines relative to that from laboratory-scale jets is a topic that merits further investigation.

A final important consideration is the quantitative assessment of the derivative skewness. Acoustic shocks are clearly seen in the laboratory data when $\mathrm{Sk}\{\partial p/\partial t\} \approx 3$. However, skewness values approaching nine were calculated in the shock-containing military engine noise. The difference has to do with sampling frequency and, thus, the maximum derivative values achievable. Data with a greater bandwidth allow for steeper shock content, which results in greater maximum derivative values and a corresponding greater skewness. Reexamination of the results of Shepherd *et al.*[16] for the evolution of derivative skewness for a nonlinearly propagating initial sinusoid while using different sampling rates has revealed sensitivity of the results to the relative data discretization near the shock formation distance. Further quantitative differences are still being explored.[18]

4. Concluding discussion

The principal findings in this Letter corroborate the previous full-scale engine study:[6] Skewed waveforms originating at the shear layer undergo significant waveform steepening and shock formation in the near field along the maximum radiation direction. Additionally, the skewness of the time derivative waveform is a convenient metric for examining nonlinear evolution, but quantitative differences due to varying relative sampling rates between experiments need to be further investigated.

The comparison between the unheated, ideally and over-expanded supersonic laboratory-scale jets has also resulted in observations not previously described in the literature. The maximum waveform skewness for the over-expanded case is correlated with a secondary radiation lobe in the overall level that seems to result from the combination of mixing and broadband shock-associated noise. In both the ideally and over-expanded cases, the maximum skewness regions occur upstream of the overall radiation lobes. Additional studies over a range of temperature ratios and Mach numbers for ideally and nonideally expanded jets are required to extend and further quantify these conclusions.

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References and links

[1] D. T. Blackstock, "Nonlinear propagation of jet noise," in *Proceedings of the Third Interagency Symposium on University Research in Transportation Noise* (University of Utah, Salt Lake City, UT, 1975), pp. 389–397.

[2] C. L. Morfey and G. P. Howell, "Nonlinear propagation of aircraft noise in the atmosphere," AIAA J. **19**, 986–992 (1981).

[3]K. L. Gee, V. W. Sparrow, M. M. James, J. M. Downing, C. M. Hobbs, T. B. Gabrielson, and A. A. Atchley, "The role of nonlinear effects in the propagation of noise from high-power jet aircraft," J. Acoust. Soc. Am. **123**, 4082–4093 (2008).
[4]B. Greska and A. Krothapalli, "On the far-field propagation of high-speed jet noise," *Proceedings of NCAD2008* (July 2008), Paper No. NCAD2008–73071.
[5]K. L. Gee, J. M. Downing, M. M. James, R. C. McKinley, R. L. McKinley, T. B. Neilsen, and A. T. Wall, "Nonlinear evolution of noise from a military jet aircraft during ground run-up," AIAA Paper No. 2012-2258 (2012).
[6]K. L. Gee, T. B. Neilsen, J. M. Downing, M. M. James, R. L. McKinley, R. C. McKinley, and A. T. Wall, "Near-field shock formation in noise propagation from a high-power jet aircraft," J. Acoust. Soc. Am. **133**, EL88–EL93 (2013).
[7]J. A. Gallagher and D. K. McLaughlin, "Experiments on the non-linear characteristics of noise propagation from low and moderate Reynolds number supersonic jets," AIAA Paper No. 81-2041.
[8]B. P. Petitjean and D. K. McLaughlin, "Experiments on the nonlinear propagation of noise from supersonic jets," AIAA Paper No. 2003-3127 (2003).
[9]B. P. Petitjean, K. Viswanathan, and D. K. McLaughlin, "Acoustic pressure waveforms measured in high speed jet noise experiencing nonlinear propagation," Int. J. Aeroacoust. **5**, 193–215 (2006).
[10]K. Viswanathan and M. J. Czech, "Role of jet temperature in correlating jet noise," AIAA J. **47**, 1090–1106 (2009).
[11]W. J. Baars, C. E. Tinney, and M. S. Wochner, "Nonlinear noise propagation from a fully expanded Mach 3 jet," AIAA Paper No. 2012-1177 (2012).
[12]K. L. Gee, A. A. Atchley, L. E. Falco, M. R. Shepherd, L. S. Ukeiley, B. J. Jansen, and J. M. Seiner, "Bicoherence analysis of model-scale jet noise," J. Acoust. Soc. Am. **128**, EL211–EL216 (2010).
[13]K. L. Gee, A. A. Atchley, L. E. Falco, and M. R. Shepherd, "Nonlinearity analysis of model-scale jet noise," in *Proceedings of 19th International Symposium on Nonlinear Acoustics* (May 2012), Paper No. 3pC11.
[14]J. E. Ffowcs Williams, J. Simson, and V. J. Virchis, " 'Crackle': An annoying component of jet noise," J. Fluid Mech. **71**, 251–271 (1975).
[15]S. A. McInerny, "Launch vehicle acoustics part 2. Statistics of the time domain data," J. Aircr. **33**, 518–523 (1996).
[16]M. R. Shepherd, K. L. Gee, and A. D. Hanford, "Evolution of statistics for a nonlinearly propagating sinusoid," J. Acoust. Soc. Am. **130**, EL8–EL13 (2011).
[17]K. L. Gee, V. W. Sparrow, A. A. Atchley, and T. B. Gabrielson, "On the perception of crackle in high-amplitude jet noise," AIAA J. **45**, 593–598 (2007).
[18]K. L. Gee, T. B. Neilsen, M. B. Muhlestein, A. T. Wall, J. M. Downing, and M. M. James, "On the evolution of crackle in jet noise from high-performance engines," AIAA Paper No. 2013-2190 (2013).
[19]A. Krothapalli, L. Venkatakrishnan, and L. Lourenco, "Crackle: A dominant component of supersonic jet mixing noise," AIAA Paper No. 2000-2024 (2000).
[20]C. K. W. Tam, "Supersonic jet noise," Annu. Rev. Fluid Mech. **27**, 17–43 (1995).
[21]T. B. Neilsen, K. L. Gee, A. T. Wall, and M. M. James, "Similarity spectra analysis of high-performance jet aircraft noise," J. Acoust. Soc. Am. **133**, 2116–2125 (2013).
[22]T. B. Neilsen, K. L. Gee, A. T. Wall, M. M. James, and A. A. Atchley, "Comparison of supersonic full-scale and laboratory-scale jet data and the similarity spectra for turbulent mixing noise," Proc. Mtgs. Acoust. **19**, 040071 (2013).
[23]P. J. Morris and S. A. E. Miller, "Prediction of broadband shock-associated noise using Reynolds-averaged Navier–Stokes computational fluid dynamics," AIAA J. **48**, 2931–2944 (2010).
[24]S. N. Gurbatov and O. V. Rudenko, "Statistical phenomena," in *Nonlinear Acoustics*, edited by M. F. Hamilton and D. T. Blackstock (Academic, San Diego, 1998), Chap. 13, pp. 386–387.