Coherence length as a figure of merit in multireference near-field acoustical holography[a)]

Alan T. Wall,[b)] Michael D. Gardner, Kent L. Gee, and Tracianne B. Neilsen
Department of Physics and Astronomy, Brigham Young University, Provo, Utah 84602
alantwall@gmail.com, gardnerfrance@gmail.com, kentgee@byu.edu, tbn@byu.edu

Abstract: Multireference partial field decomposition (PFD) can be used to generate coherent holograms for near-field acoustical holography measurements. PFD is most successful when the reference array completely senses all independent subsources, but meeting this requirement is not straightforward when the number of subsources and their locations are ambiguous (such as in aeroacoustic sources). A figure of merit based on spatial coherence lengths, called references per coherence length (RPL_C), is a useful metric to guide inter-reference spacing in the array design. For numerical, extended, arbitrarily coherent sources one reference per coherence length results in a sufficient reference array.

PACS numbers: 43.60.Fg, 43.60.Sx, 43.28.Lv [JL]
Date Received: May 24, 2012 **Date Accepted:** July 18, 2012

1. Introduction

Near-field acoustical holography (NAH)[1] requires a spatially coherent hologram to give a successful reconstruction of the sound field. This may be obtained from a simultaneous measurement of all field points. However, hologram measurements often consist of hundreds or thousands of measurement grid points, making the number of field microphones required impractical. In these cases, a scan-based measurement with a small, dense, field array, in combination with a fixed-location reference array, is used.

When a sound field is generated by a single, coherent source, only one reference microphone is required to tie the scans together. Hald[2] developed a multireference procedure called spatial transformation of sound fields (STSF), which is a partial field decomposition (PFD) method to accommodate sound fields of multiple independent (incoherent) subsources. In STSF, a singular value decomposition (SVD) of the cross-spectral matrix of reference signals can be used as the basis for a linear projection of the field measurement, which results in a set of mutually incoherent partial fields. The incoherent sum of these partial fields represents the total field. Each partial field may then be projected individually using NAH and summed to provide the total reconstruction. Because the partial fields in this method are based on an SVD, they do not directly represent independent physical subsources.

Additional PFD methods have been developed to generate partial fields that can be associated with physical subsources.[3,4] These are most successful when each reference is located as closely as possible to each individual subsource. Kim *et al.*[5] introduced a method that uses holographic projection to determine the optimal reference locations and then places a set of virtual references at those locations. However, in the STSF, virtual reference, and all PFD methods, it is important to understand that the physical reference array must completely sense all independent sources to begin with—the number of reference microphones must equal or exceed the number of subsources and each subsource must be sensed by at least one reference—if the total signal energy

[a)]Portions of this work were presented at the 157th meeting of the Acoustical Society of America in Portland, Oregon and at Inter-Noise 2009 in Ottawa, Canada.
[b)]Author to whom correspondence should be addressed.

at the hologram is to be represented in the decomposition.[5] In practice, when the subsources are localized or spatially distinct, this requirement is simple to meet.

What is to be done when independent subsources are not localized, such as in the case of an aeroacoustic source? An aeroacoustic source may be considered a continuum of partially coherent subsources where the number of subsources is ambiguous. Lee and Bolton,[6] Shah *et al.,*[7] Wall *et al.,*[8] and others performed scan-based NAH on jet-noise sources. In each of these experiments, the sufficiency of the reference array in completely sensing all subsources was verified after the measurement was taken, using the virtual coherence method.[9–11] However, no quantitative guidelines exist in current literature to predict reference-array sufficiency *a priori* or guide the design of a reference array when the number and locations of independent subsources are unknown. Although this study is directed toward an application in aeroacoustic measurements, no attempt is made here to model the sound field of an actual aeroacoustic source. Rather, the partial spatial coherence of an aeroacoustic source is represented by a simple array of partially coherent point monopoles.

The purpose of this paper is to present a simple, quantitative figure of merit that yields the inter-microphone spacing necessary, given an array aperture, for the deployment of a sufficient reference array near partially coherent complex sources. This guideline is based on a spatial analysis of the (ordinary) coherence in the region of the source, such as those performed by Wall *et al.,*[12] that can be performed with relatively few sensors. From such coherence measurements, an average spatial coherence length in the sound field is determined. The spatial density of microphones in the reference array dictates how many references, on average, are located within an average coherence length. This figure of merit is called "references per coherence length" (RPL_C).

2. References per coherence length

An analysis of the spatial coherence measured in the geometric near field of partially spatially coherent sources leads to the definition of RPL_C.[13] To obtain a spatial coherence measurement, a linear sensor array (collinear with the eventual location of the reference array) is placed near and along the length of an extended source. Coherence values,[14] $\gamma^2_{z_m,z}$, between a given sensor at a location z_m and all sensors along the array are calculated. From these data, the coherence length, L_C, is determined. Although the term "coherence length" is used in other studies, such as in optical holography applications[15] and underwater acoustics,[16] here it is defined as the spatial distance L_C along the array over which $\gamma^2_{z_m,z}$ drops from unity to some desired threshold. In this paper, a coherence threshold of $\gamma^2_{z_m,z} = 0.5$ is used. The value of L_C is assigned to the location z_m of the given sensor. This process is repeated for all N_R sensor locations, resulting in an array of coherence length values, $L_C(z_m)$. The mean of $L_C(z_m)$ over all locations yields the mean coherence length,

$$\langle L_C \rangle = \frac{1}{N_R} \Sigma_m L_C(z_m), \qquad (1)$$

which summarizes the spatial coherence of the sound field into a single quantity.

Note that $\gamma^2_{z_m,z}$ generally drops off in both directions of increasing and decreasing z away from z_m, so $L_C(z_m)$ may be calculated in either direction. If the source radiates symmetrically, then $\langle L_C \rangle$ is the same for either direction chosen. This may not be the case when the sound field is asymmetric.[12]

With $\langle L_C \rangle$ established, references per coherence length is defined as

$$\mathrm{RPL}_C = \langle L_C \rangle / \Delta z_R, \qquad (2)$$

where the distance Δz_R is the physical spacing between sensors in a reference array. The figure of merit RPL_C quantifies the spatial density of sensors in the reference array in terms of spatial field coherence. As spatial source coherence decreases, $\langle L_C \rangle$ decreases and the number of effective independent subsources increases. A greater number of reference sensors are required to completely sense these subsources, so the inter-reference spacing must be more dense (i.e., for a reference array of fixed aperture length, the sensor spacing Δz_R must decrease). The quantity RPL_C takes advantage of the fact that information about the number of independent subsources is contained in $\langle L_C \rangle$. One might expect that the value of RPL_C required for a sufficient reference array is invariant with source coherence because the necessary Δz_R decreases with decreasing $\langle L_C \rangle$. Such a relationship is demonstrated in this paper.

To assess the utility of RPL_C, a quantitative method of determining the sufficiency of a reference array is desired. Therefore the quantity "mean virtual coherence sum," $\langle \Sigma \tilde{\gamma}^2 \rangle$, is defined. This quantity is calculated after application of the virtual coherence method.[9] In a PFD based on the SVD of the reference cross-spectral matrix, the strengths of the individual partial fields are determined by the ordered, monotonically decreasing singular values, each of which corresponds to one of the normalized partial fields. The partial fields with the highest singular values correspond to dominant noise sources. Theoretically, when there are S independent subsources and low measurement noise, there will be S large source-related singular values. Then the first S partial fields can be propagated using NAH; the rest can be discarded.

However, in aeroacoustic sources, the number of subsources is ambiguous. This is reflected in a more gradual decrease of the ordered singular values with no clear distinction between source and noise-related singular values.[9,10] Virtual coherence, in conjunction with the STSF method, provides a way to estimate the number of significant partial fields.[9,17] The virtual coherence function,

$$\tilde{\gamma}_{i,j}^2 = \frac{|C_{v_i p_j}|^2}{C_{v_i v_i} C_{p_j p_j}}, \tag{3}$$

quantifies the amount of (normalized) coherent energy in the ith partial field at the jth measurement location. The quantity $C_{v_i p_j}$ is the cross-spectrum between the virtual reference signal v_i and the field signal p_j, $C_{v_i v_i}$ is the autospectrum of the virtual reference, and $C_{p_j p_j}$ is the autospectrum of the field signal. If the sum of the virtual coherence function over all the partial fields, $\sum_i \tilde{\gamma}_{i,j}^2$, approaches unity for each field location j, then the reference array has sufficiently sensed all of the subsources. If a subset of the first K partial fields returns a sum that approaches unity for each j, then these K partial fields fully contain the source information (i.e., $K \geq S$), and the remaining partial fields are discarded. Examples of $\sum_i \tilde{\gamma}_{i,j}^2$ plotted over all j are shown by Lee and Bolton (see Ref. 9, their Figs. 15 and 17).

It is useful to collapse the information in the spatial maps of maps of $\sum_i \tilde{\gamma}_{i,j}^2$ to a single number, which has not been done previously. Taking the average over field locations, the "mean virtual coherence sum" is obtained, i.e.,

$$\left\langle \sum \tilde{\gamma}^2 \right\rangle = \frac{1}{J} \sum_j \sum_i \tilde{\gamma}_{i,j}^2, \tag{4}$$

where J is the total number of field measurements. If $\langle \sum \tilde{\gamma}^2 \rangle$ approaches unity, then it is likely $\sum_i \tilde{\gamma}_{i,j}^2$ is nearly unity for all j. Hence, $\langle \sum \tilde{\gamma}^2 \rangle$ quantifies the sufficiency of a reference array in this paper. A calculation of $\langle \sum \tilde{\gamma}^2 \rangle$ as a function of RPL_C results in an RPL_C criterion for a sufficiently dense reference array regardless of frequency and spatial coherence.

3. Numerical experiment setup: A spatially noncompact source

To investigate the relationship between RPL_C and $\langle \sum \tilde{\gamma}^2 \rangle$, a numerical experiment was performed in which an approximately continuous, partially coherent source was

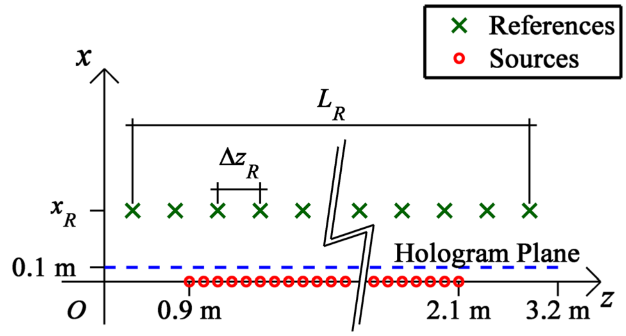

Fig. 1. (Color online) Schematic of relative locations of sources, references, and the hologram plane for the numerical experiment (not to scale).

generated one frequency at a time. See Fig. 1 for a schematic. An array of $N_Q = 1000$ point monopoles, marked by red circles, were spaced evenly along the z axis between $z = 0.9$ and 2.1 m with an inter-source separation of d such that $kd \ll 1$, k being the acoustic wavenumber. A "primary" signal vector, $\hat{s}_1$, was generated of complex numbers with unit magnitude and random phase. This signal defined the amplitude of the first monopole source, i.e., $\hat{q}_1 = \hat{s}_1$. Subsequent source amplitudes were defined by

$$\hat{q}_n = \frac{\hat{q}_{n-1} + b \times \hat{s}_n}{\|\hat{q}_{n-1} + b \times \hat{s}_n\|_2} \quad \text{for} \quad n = 2, \ldots, N_Q, \tag{5}$$

where $\hat{q}_{n-1}$ were the complex amplitudes of the previous source, $\hat{s}_n$ were newly generated random complex signals, and the coherence factor b dictated the portion of new, random signal energy that was added to the previous source signal. Division by the L2 norm, $\|\cdot\|_2$, of the total signal ensured that all monopole magnitudes were unity. The result was a line array of monopoles with normalized magnitude, comprising a partially coherent source. The spatial coherence properties of this source, and consequently the radiated field, depend on the coherence factor, b, and on frequency, f. For example, a b value of zero results in a perfectly coherent source, and as b approaches infinity the sources becomes completely incoherent.

A scan-based measurement of the sound field was simulated by propagating sound pressures from the monopoles to the measured hologram using the free-space Green's function. The measurement grid was 10 cm from the sources and consisted of a linear array of 11 receivers, placed at 43 locations, to generate a hologram of 11×43 points with equal 7.62 cm (3 in.) spacing. Its location is marked by the blue dashed line in Fig. 1. An infinite signal-to-noise ratio (SNR) was assumed in the measurement. The holographic projection of these data was not carried out in this work. Rather the hologram data serve as the field signals for which the reference array sufficiency is determined. When a reference array sufficiently senses all sources, then all energy in the field measurements is represented in the partial field decomposition.

The reference array, shown by the green "×" symbols in Fig. 1, was placed at a standoff distance of $x_R = 0.3$ m from the source with an aperture that spanned from $z = 0.3$ to 2.7 m (twice the length of the source) and with variable spacing, Δz_R, as illustrated in Fig. 1. With the fixed aperture length, L_R, Δz_R depended on the number of references, N_R.

The simulated scan-based measurement of the partially coherent source was repeated as f was varied from 0.1 to 10 kHz, b was varied from 0.01 to 10, and RPL_C was varied from 0.2 to 2. The analysis for each parameter set proceeded as follows. (1) From the L_C values found in the direction of increasing z, $\langle L_C \rangle$ was calculated. (2) Reference sensor spacing Δz_R was determined for a given RPL_C value using Eq. (2).

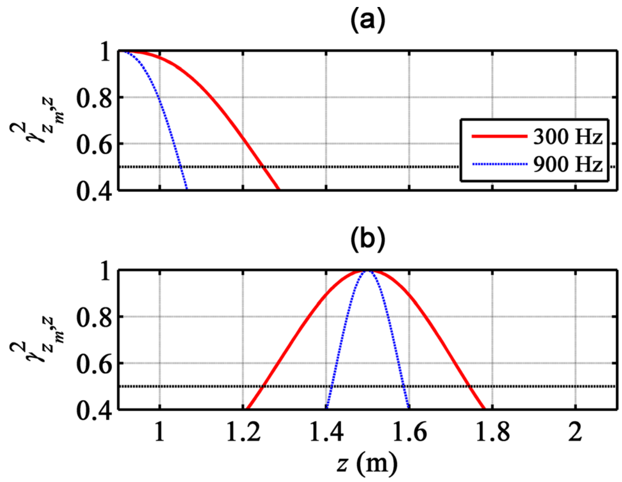

Fig. 2. (Color online) Near-field coherence values (along the reference array) for a source with a coherence factor $b = 0.05$ (a) calculated between $z_m = 0.9$ m and all other z values, and (b) calculated between $z_m = 1.5$ m and all other z values. Also shown are dashed horizontal lines denoting a coherence value of 0.5.

(3) A PFD of the sound field was performed using a reference array with spacing Δz_R. (4) The mean virtual coherence sum, $\langle\sum\tilde{\gamma}^2\rangle$, was calculated to determine reference-array sufficiency. Results of this experiment are reported in the following section.

4. Results

Figure 2 shows examples of coherence $\gamma^2_{z_m,z}$ as a function of z, referenced to two values of z_m, and for 300 and 900 Hz. The coherence factor was $b = 0.05$. Note the locations where $\gamma^2_{z_m,z} = 0.5$, which value was used to define L_C. The values of L_C tend to be smaller for large f and lower b. This trend also holds true for $\langle L_C\rangle$. It is also important to note that $\langle L_C\rangle$ depends somewhat on the reference-array aperture; calculated coherence lengths vary over location and can become large far from the source.

The values of $\langle\sum\tilde{\gamma}^2\rangle$ are plotted against N_R in Fig. 3(a) for two frequencies (300 and 900 Hz), and two b values: 0.05 for the highly coherent case and and 10 for the highly incoherent case. For the purpose of comparison, $\langle\sum\tilde{\gamma}^2\rangle \geq 0.99$ is considered to be "sufficient" in sensing all subsources. The minimum N_R at which this occurs is represented by $\tilde{N}_R$, the critical number of reference microphones. Figure 3(a) illustrates how $\tilde{N}_R$ depends on both b and f, although it is far more sensitive to f than to b in

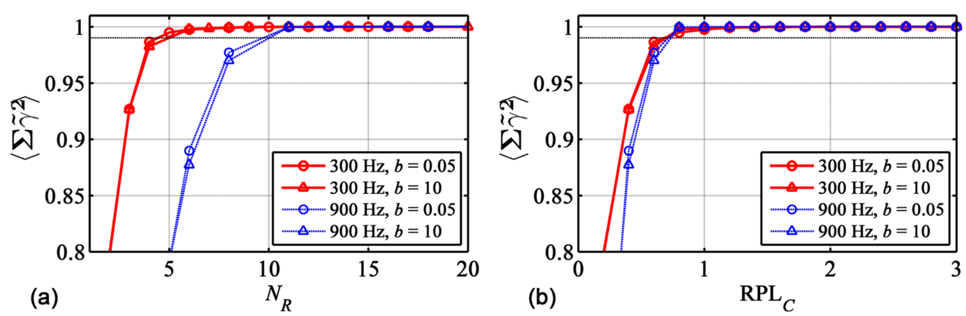

Fig. 3. (Color online) The mean virtual coherence sum, $\langle\sum\tilde{\gamma}^2\rangle$, (a) as a function of the number of references, N_R, for two frequencies and two different coherence factors, b, and (b) the mean virtual coherence sum $\langle\sum\tilde{\gamma}^2\rangle$ as a function of the new figure of merit, reference microphones per coherence length (RPL_C), for the same parameters.

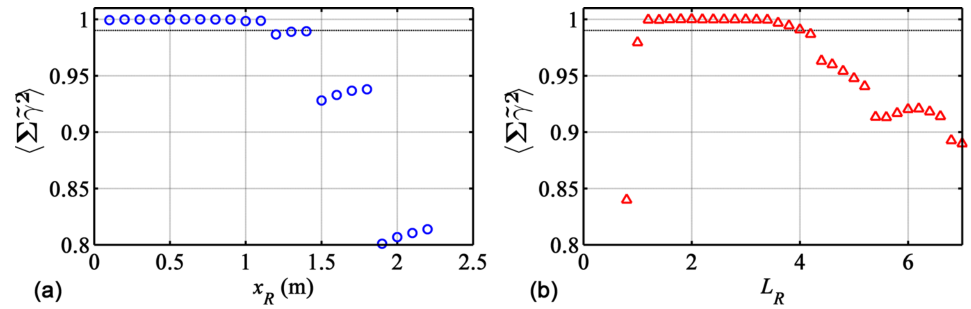

Fig. 4. (Color online) The mean virtual coherence sum, $\langle\sum\tilde{\gamma}^2\rangle$, for $f = 900$ Hz and $b = 0.05$ (a) as a function of standoff distance of the reference array from the source, x_R, with a fixed reference aperture $L_R = 2.4$ m, and (b) as a function of L_R with a fixed $x_R = 0.3$ m. Reference spacing Δz_R was always determined such that $\mathrm{RPL}_C = 1$.

this case. In general, it is difficult to predict the sufficiency of a reference array for an arbitrary source configuration based on an analysis of $\langle\sum\tilde{\gamma}^2\rangle$ as a function of N_R.

The figure of merit RPL_C provides an alternative way to view these data that shows a more consistent trend. The data in Fig. 3(a) are shown again in Fig. 3(b), but this time they are plotted against RPL_C. Note how the curves now more closely collapse, and $\langle\sum\tilde{\gamma}^2\rangle$ exceeds 0.99 above $\mathrm{RPL}_C \approx 1$. For all f and b tested in this work, with the definition of $\langle L_C\rangle$ given in Sec. 2, one reference microphone per coherence length ($\mathrm{RPL}_C = 1$) is considered sufficient to fully decompose the subsources.[13,18] Hence, RPL_C, as defined in Sec. 2, is a useful figure of merit when source coherence properties or the number of independent subsources is ambiguous. It should be noted that although the criterion $\mathrm{RPL}_C = 1$ may be intuitive, it is dictated by the definition of the coherence length, L_C, which in turn is dictated by the coherence threshold. For example, a coherence threshold of $\gamma^2_{z_m,z} = 0.8$ resulted in a criterion of $\mathrm{RPL}_C = 0.2$ for the same experiment.

It is important to understand the extent to which $\mathrm{RPL}_C = 1$ is a reasonable guideline. This guideline can become invalid when test parameters are varied to the extreme. As one example, decreasing b to smaller orders of magnitude than those shown here results in a highly coherent source, with long coherence lengths, and the calculation of RPL_C becomes unreliable. However, in this case, only one or two references are needed, eliminating the utility of such an analysis. For all practical realizations of source coherence and frequency, $\mathrm{RPL}_C = 1$ is quite robust.

To test the robustness of $\mathrm{RPL}_C = 1$, the experiment was repeated with $b = 0.05$, $f = 900$ Hz and with a varying reference array aperture and standoff distance. The criterion $\mathrm{RPL}_C = 1$ was used to determine Δz_R. First, $\langle\sum\tilde{\gamma}^2\rangle$ was calculated as a function of x_R for $L_R = 2.4$ m (twice the source length) and then as a function of L_R for $x_R = 0.3$ m. The results are shown in Fig. 4. Note that $\langle\sum\tilde{\gamma}^2\rangle > 0.99$ for a range of $x_R = 0.1$ to 1.1 m in Fig. 4(a) and for a range of $L_R = 1.2$ to 4.0 m in Fig. 4(b). This suggests that $\mathrm{RPL}_C = 1$ results in a sufficient reference array for a wide range of apertures and standoff distances. Extreme values of L_R and x_R can cause the $\mathrm{RPL}_C = 1$ guideline to break down, such as moving the reference array far from the source. However, this experiment shows that a reference-array design that satisfies $\mathrm{RPL}_C = 1$ based on a rudimentary knowledge of the source location and extent sufficiently senses all subsources.

5. Conclusion

A useful figure of merit for predicting a sufficient inter-microphone spacing of a reference array, deployed near a partially spatially coherent, extended source for the application of partial field decomposition (PFD) and near-field acoustical holography (NAH) methods, has been introduced. This quantity, "references per coherence length" (RPL_C) can be calculated from simple coherence measurements near the source. It has been shown that if microphones are placed such that there is one reference per coherence length, according to the definition of RPL_C given here, all independent subsources will be sufficiently sensed. This has been shown to be a robust guideline regardless of frequency, spatial source coherence, reference aperture, and standoff distance, over a broad range of these parameters.

There exist a myriad of additional source and measurement configurations to which such an RPL_C study can be applied to further ensure that $RPL_C = 1$ is a robust criterion. Future investigations could include the effects of amplitude-weighted sources and the directional radiation due to phase shading, and measurement noise. In addition, the validity of $RPL_C = 1$ to aeroacoustic or other physical, partially coherent sources needs to be investigated.

Acknowledgments

Special thanks to Scott D. Sommerfeldt and Jonathan D. Blotter for their insightful contributions.

References and links

[1] J. D. Maynard, E. G. Williams, and Y. Lee, "Nearfield acoustic holography. 1. Theory of generalized holography and the development of NAH," J. Acoust. Soc. Am. **78**, 1395–1413 (1985).

[2] K. B. Ginn and J. Hald, "STSF-practical instrumentation and applications," Bruel and Kjaer Tecnical Review, 1–27 (1989).

[3] H. S. Kwon and J. S. Bolton, "Partial field decomposition in nearfield acoustical holography by the use of singular value decomposition and partial coherence procedures," in *Proceedings of NOISE-CON* (1998), pp. 649–654.

[4] K.-U. Nam and Y.-H. Kim, "Visualization of multiple incoherent sources by the backward prediction of near-field acoustic holography," J. Acoust. Soc. Am. **109**, 1808–1816 (2001).

[5] Y. J. Kim, J. S. Bolton, and H. S. Kwon, "Partial sound field decomposition in multireference near-field acoustical holography by using optimally located virtual references," J. Acoust. Soc. Am. **115**, 1641–1652 (2004).

[6] M. Lee and J. S. Bolton, "Source characterization of a subsonic jet by using near-field acoustical holography," J. Acoust. Soc. Am. **121**, 967–977 (2007).

[7] P. N. Shah, H. Vold, and M. Yang, "Reconstruction of far-field noise using multireference acosutical holography measurements of high-speed jets," AIAA Paper 2011-2772, June 5–8, 2011.

[8] A. T. Wall, K. L. Gee, T. B. Neilsen, D. W. Krueger, M. M. James, S. D. Sommerfeldt, and J. D. Blotter, "Full-scale jet noise characterization using scan-based acoustical holography," AIAA Paper 2012-2081, June 4–6, 2012.

[9] M. Lee and J. S. Bolton, "Scan-based near-field acoustical holography and partial field decomposition in the presence of noise and source level variation," J. Acoust. Soc. Am. **119**, 382–393 (2006).

[10] A. T. Wall, K. L. Gee, M. D. Gardner, T. B. Neilsen, and M. M. James, "Near-field acoustical holography applied to high-performance jet aircraft noise," Proc. Mtg. Acoust. **9**, 040009 (2011).

[11] S. M. Price and R. J. Bernhard, "Virtual coherence: A digital signal processing technique for incoherent source idientification," in *Proceedings of 4th International Modal Analysis Conference* (1986), pp. 1256–1262.

[12] A. T. Wall, K. L. Gee, and T. B. Neilsen, "On near-field acoustical inverse measurements of partially coherent sources," Proc. Mtg. Acoust. **11**, 040007 (2012).

[13] M. Gardner, "Scan-based near-field acoustical holography on partially correlated sources," M.S. thesis Brigham Young Univ., Provo, UT, 2009.

[14] J. S. Bendat and A. G. Piersol, *Random Data: Analysis and Measurement Procedures*, 4th ed. (Wiley and Sons, Hoboken, NJ, 2010), pp. 134–135.

[15] G. K. Ackermann and J. Eichler, *Holography: A Practical Approach* (Wiley-VCH Verlag GmbH and Co. KGaA, Weinheim, 2007), pp. 75–78.

[16] J. M. Collis, T. F. Duda, J. F. Lynch, and H. A. DeFerrari, "Observed limiting cases of horizontal field coherence and array performance in a time-varying internal wavefield," J. Acoust. Soc. Am. **124**, EL97–EL103 (2008).

[17] J. Hald, "STSF—A unique technique for scan-based near-field acoustical holography without restriction on coherence," Technical Report No. 1, from Bruel & Kjaer, Naerum, Denmark, 1989.

[18] It should be noted that this result differs from that achieved by Gardner, who performed this analysis for a more specific source and set of reference configurations. This work builds upon those intial investigations.