Active noise control of an exhaust-mounted, two-fan array

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Fan arrays are used to cool various types of electronic equipment. In addition to adding multiple noise sources, using two nominally identical fans with closely spaced blade passage frequencies (BPF) can create an annoying beating effect. In this work, a two-fan, exhaust-mounted array has been considered for active control of the radiation at the BPF and second harmonic of each fan. Two control configurations were theoretically and experimentally compared. The first control configuration consisted of one controller using six control sources and six error sensors in a fully coupled control system designed to control the noise from both fans simultaneously. The second configuration used two independent controllers with three control sources and three error sensors, i.e., one noise controller per fan. Each noise control configuration was theoretically modeled to examine minimum radiated power and appropriate near-field error sensor locations. Experimental results suggest that independent controllers perform better than a single controller at lower computational cost. It is also demonstrated that active reduction of radiation at both BPFs is sufficient to significantly reduce the audible acoustic beating between the two fans.

Primary subject classification: 38.2; Secondary subject classification: 11.4

1 INTRODUCTION

Arrays of axial fans are often used in applications such as servers and cooling trays. Compared to a single fan, multiple fans improve cooling but can also result in increased noise levels. Huang[1] showed the tonal noise at the blade passage frequency (BPF) and harmonics in axial cooling fans comes from periodic aerodynamic interactions between the fan blades and the stator vanes. Broadband noise from cooling fans comes from various sources, including the vortex shedding at the blade trailing edges and from turbulence in the flow[2,3].

When using two or more cooling fans, manufacturing differences cause identical model fans to generally rotate at slightly different speeds, even when the same voltage is applied. When the BPFs are sufficiently close together and of similar amplitude, they can create audible beating, whose frequency is the difference between the two closely spaced BPFs. Means of eliminating beating have been explored by various researchers, including synchronizing the speed of one fan to match the speed of the other[4] and using one fan as the control source to actively control the noise of the other fan[5].

The global active control of the tonal noise content from a single axial cooling fan has been studied using various control configurations[6]. Control source configurations have ranged from a single control source to multiple control sources surrounding the fan[7–12]. Notably, Gee and Sommerfeldt[7,8] achieved global sound field reductions with a multichannel system with error microphones located as suggested by a theoretical model intended to minimize radiated sound power[13]. Because it directly relates to the methods and results of this paper, the method is summarized.

Gee and Sommerfeldt[7,8] modeled the fan and four symmetrically placed control sources as baffled coplanar monopoles. Following the approach by Nelson et al.[13], they determined the complex control source strengths that minimized the sound power. Using these source strengths, optimal near-field error microphone locations were found by locating regions of maximum sound pressure attenuation[14]. With the microphones in

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these theoretically determined positions, greater global sound reductions were obtained on average[7]. Monson et al.[11] and Shafer et al.[12] also investigated the approach of Gee and Sommerfeldt and helped to verify the robustness of the technique.

The purpose of this paper is to discuss the application of the prior modeling technique by Gee and Sommerfeldt to an exhaust-mounted, two-fan array and other implementation considerations. This includes a modified control source arrangement, because the proximity of the two fans precludes using the four symmetric source arrangement of Gee and Sommerfeldt. In addition, the presence of additional "virtual" sources must be considered in that the second fan opening constitutes an additional radiation path for the first fan and vice versa. Finally, the controller design is investigated in order to determine if independent control systems can be appropriately used for each, or if one coupled controller should be used to control the radiation from both fans.

2 THEORETICAL MODELING

The overall theoretical modeling approach follows that of Gee and Sommerfeldt[7,8] and is not repeated. However, details specific to its implementation for the two-fan array are considered. Following Gee and Sommerfeldt[7,8], all primary and control sources considered are modeled as baffled monopoles. For the primary sources, the two fans have slightly different BPFs, which allows them to be treated as independent, uncoupled sources and the system is modeled at each BPF and their respective second harmonics. However, the secondary radiation path for each fan still results in a two-source system centered at the locations of the two fans for each of the four frequencies. The relative complex source strength of the "virtual" primary source is considered subsequently.

Another consideration is that of control source arrangement to be used in the theoretical model. As mentioned, Gee and Sommerfeldt[7,8] placed four equally-spaced control sources surrounding the fan with one source at each corner of a square surrounding the fan. In this study, however, the sources A and B were surrounded by six control sources as shown in Fig. 1. Source A is representative of the direct acoustic path of the fan from the front side of the fan, e.g., the physical "primary" source. Source B is a flanking source representative of the acoustic path from the back side of the fan through the box and out the other fan hole.

Two control source configurations were investigated to determine the sound power reduction possible from these closely spaced primary sources. The first control configuration used control sources 1–3. If Source B is neglected, this is actually the "3 x 3 adjacent" control configuration considered by Gee and Sommerfeldt[8]. The second control implementation uses all six control sources shown in Fig. 1. Because the BPFs of the two physical fans used in the experiment were determined to be 712 and 714 Hz, a frequency of 712 Hz was used in this theoretical analysis.

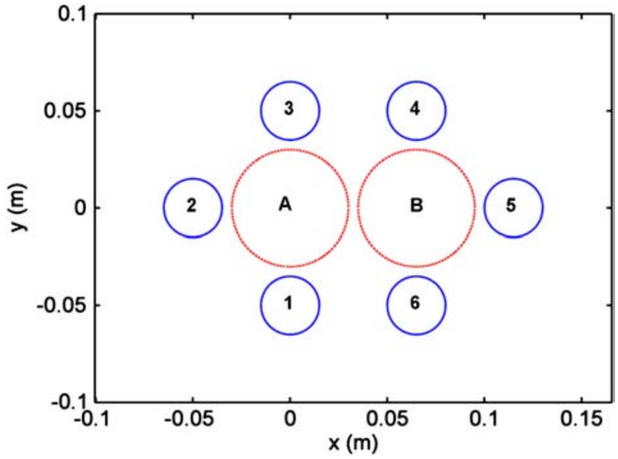

Fig. 1—The location of the primary and control sources with the primary sources being red dashed circles and the control sources blue solid circles.

The final consideration is the phase and magnitude of primary source B relative to A to use in the theoretical analysis, as this will influence the sound power radiated by the primary source system. To determine the potential impact of this flanking source B, the sound power attenuation of various primary source arrays was calculated for both the three and six-control-source arrangements. In Table 1, the sound power level attenuation (in dB) is given for various relative magnitudes and phases of sources B and A, and for three and six control sources.

The results in Table 1 indicate that the six-control source configuration is robust enough to be relatively unaffected by different possible primary source magnitudes and phases. The exception, which results in over 33 dB attenuation but is unlikely to occur in practice, is where sources A and B comprise a true dipole. For the other configurations, the theoretical attenuation ranges between 27 and 28 dB across all the primary source arrays tested. For the three-source configuration in Table 1, the attenuation is much more sensitive to the relative strength of source B. In the absence of this flanking source, the maximum attenuation is 24 dB, but can be as little as 7 dB in the dipole primary source configuration.

Ideal microphone locations were determined for the three and six-control source arrangements in Fig. 1, in the same manner as Gee and Sommerfeldt[7,8]. To

Table 1—Sound power attenuation of a two primary source array using a three-and six-control-source configuration. Results are shown for various values of the magnitude and phase of primary source B (flanking path) relative to primary source A.

		Attenuation (dB)	
Relative Magnitude	Relative Phase	3 Sources	6 Sources
1	0	16	27
1	90	14	27
1	180	7	34
0.5	0	18	27
0.5	90	18	27
0.5	180	17	28
0.25	0	20	27
0.25	90	21	27
0.25	180	26	27
0	0	24	27

review, the control source strengths that minimize the total radiated power were found. With these source strengths, a map of the near-field pressure attenuation was created. The places of maximum attenuation of the primary field correspond to optimal error microphone locations. An example of the near-field attenuation map for the three-control-source configuration and the special case that |B| = 0 is shown in Fig. 2. In this case, the strength of source 2 (see Fig. 1) approaches zero as the distance between sources becomes small. Note that ideal microphone locations are represented by the near-field pressure nulls. For the three-control source configuration each fan had an independent controller. Thus, for the second fan in the array the same plot (mirrored about the vertical axis) would be used for the second fan at the frequency of interest. The relative near-field pressure map for the six-control-source configuration is shown in Fig. 3, again for the case of having only one primary source.

The results in Figs. 2 and 3 are for a single fan, but the experiment involves two fans operating at slightly different frequencies and involves the sharing of error microphones for the six-channel coupled controller configuration. Therefore, the error microphones need to be in optimal locations for each fan. A similar pressure map can be made for the second fan in the array; note that the change in near-field null characteristics for 712 and 714 Hz is negligible. Shown in Fig. 4 are the mirroring and superposition of Fig. 3 and the resulting overlapping pressure map with ideal "composite"

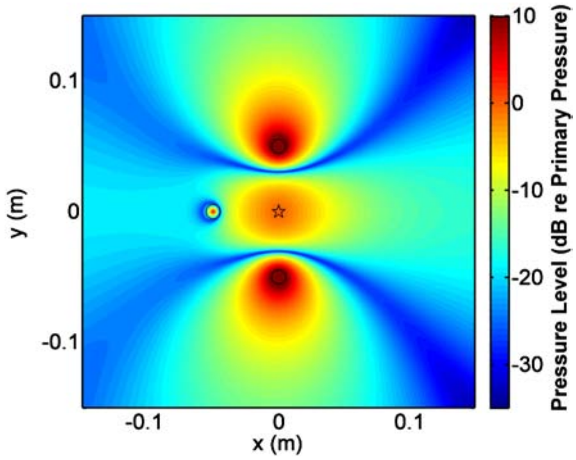

Fig. 2—The near-field pressure of the controlled field relative to the primary field, in dB, for a three-control-source and one primary source configurations. The nulls in the near field are the ideal locations for the error sensors, the circles show the locations of the control sources and the star is the primary source location.

microphone locations. Because of the symmetry of the control arrangement, these ideal microphone locations are limited to a small region in between the primary and secondary sources. Note that this is for the case that the flanking source for each of the two fans is negligible. The near-field pressure field for the second

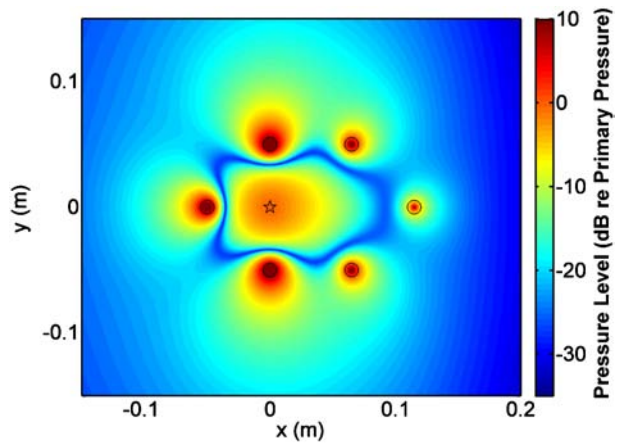

Fig. 3—The near-field pressure of the controlled field relative to the primary field, in dB, for a six control source and one primary source configurations. The nulls in the near field are the ideal locations for the error sensors, the circles show the locations of the control sources and the star is the primary source location.

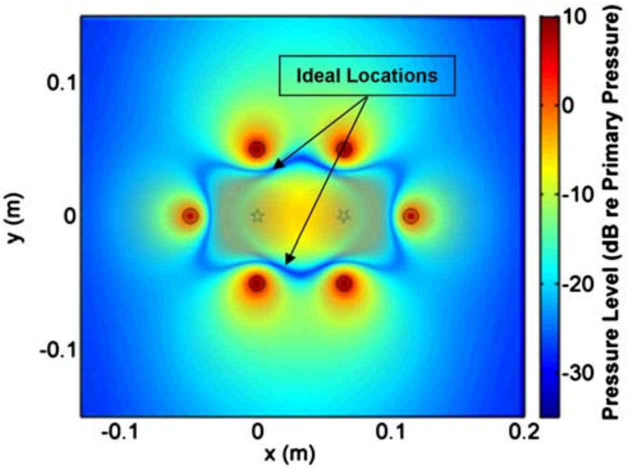

Fig. 4—The overlapped near-field pressure of the controlled field relative to the primary field, in dB, for a six control source and one primary source configurations for each fan in a two-fan array. The cross section of the nulls in the near field are the ideal locations for the error sensors, the circles show the locations of the control sources and the stars the locations of primary sources.

harmonic behaves similar to the BPF with slight changes in the relative pressure field with increasing frequency comparable to what Gee and Sommerfeldt[7,8] found. Optimal microphone locations for both frequencies can be found in the regions closer to the control sources.

In the experimental set up used to test the active noise control on the fan array, the magnitude of the primary source B relative to source A was unknown; therefore, the ideal microphone placements were also unknown. An estimate of primary source B relative to source A was determined for a fan in the experimental array. This was used to guide further theoretical modeling of ideal error microphone locations for the independent and coupled control configurations. To estimate the relative primary source strengths, a baffle was placed in between the two fans on the exhaust to isolate the two primary sources. A microphone was placed on each side of the baffle just outside each fan opening. With one fan in operation, the transfer function between the two microphones was used to estimate the magnitude difference at the BPF between the two acoustic paths. The magnitude difference was found to be 7 dB, which gives a magnitude ratio of 0.44.

Examination of Table 1 suggests that the six-control-source coupled noise control configuration could theoretically achieve as much as 10 dB more attenuation than two three-control-source configurations. Using the magnitude data and assuming no phase difference, the error sensor placement can be approximated by creating new pressure maps. The near-field controlled pressure field relative to the primary field for the three-control-source configuration is shown in Fig. 5 with a second primary source B with a relative magnitude of 0.44 and between the two sources. Note that there are several locations that nearly overlap with those in Fig. 2, suggesting that robust microphone locations may be found regardless of the relative importance of the flanking path source.

Similar to Fig. 4, the overlapped pressure attenuation maps were created for the two-fan, two-flanking source case with the flanking-to-direct source strength ratio of 0.44. The results in Figs. 5 and 6 were used to guide the placement of error microphones for the control of the two-fan array. The maximum theoretical sound power attenuation expected for the three-control-source configuration is about 16–17 dB, and for the six-control-source configuration is about 27 dB at a BPF of 712 Hz.

3 EXPERIMENTAL SETUP

An exhaust-mounted two-fan array was constructed with two identical 60 mm axial cooling fans. The fan array was in a computer-sized box with dimensions of 46 x 42 x 22 cm. The two fans were spaced 6.5 cm apart, center to center, which is 0.135 of an acoustic

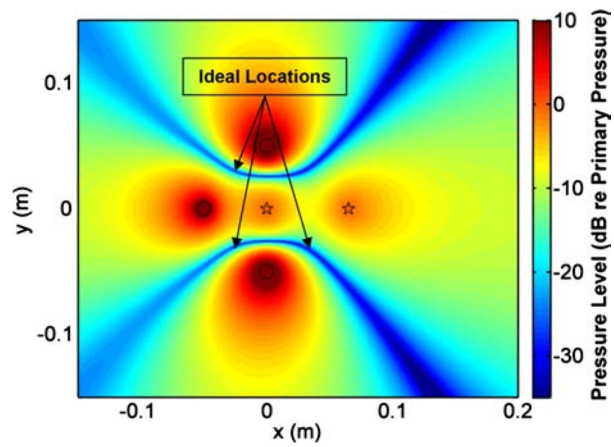

Fig. 5—The overlapped near-field pressure of the controlled field relative to the primary field, in dB, for a three-control-source and two-primary source configurations. The nulls in the near field are the ideal locations for the error sensors, the circles show the locations of the control sources and the stars are the primary source locations.

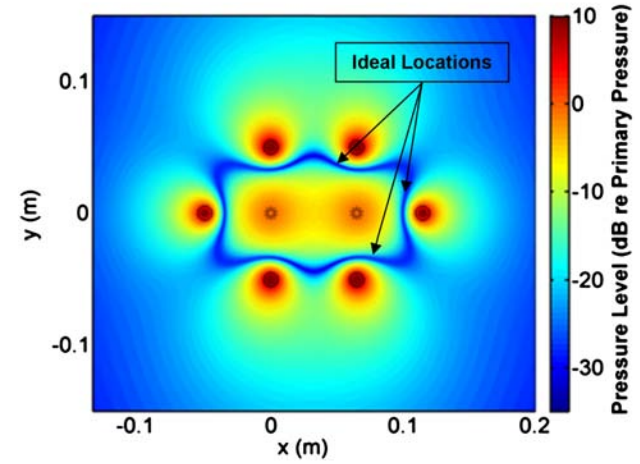

Fig. 6—The overlapped near-field pressure of the controlled field relative to the primary field, in dB, for a six control source and two primary sources configurations. The cross section of the nulls in the near field are the ideal locations for the error sensors, the circles show the locations of the control sources and the stars the locations of primary sources.

wavelength of a 712 Hz wave. The control sources were 2.5 cm HiVi loudspeakers and were placed according to Fig. 1, with the complete set up shown in Fig. 7. A feedfoward active control system was used to target the tonal noise from the fans, using the filtered-x algorithm. Because a feedforward controller requires a reference signal that is correlated to the noise, and the fans used did not have a built-in tachometer, the reference signal was taken from lowpass-filtering the output of an infrared emitter/detector pair mounted on either side of the fan blade. This method has been used in a number of previous studies[7,8,11,12].

The control was implemented using a DSP board with a Texas Instruments 320c6713 DSP. For the experimentation, the DSP was set at a sampling frequency of 4000 Hz. The anti-aliasing filters were implemented using an 8-pole Krohn-Hite 3384 set to lowpass filter the signal at 1800 Hz. The six-control-source and three-control-source configurations were experimentally tested. The six-control-source configuration was used to control the noise from both fans simultaneously, which required summation of the reference signals from each fan's emitter/detector. The control system utilized 128 control filter coefficients with 20 secondary path model coefficients for each secondary path modeled. For the two independent three-source configurations, each of the controllers had its corresponding reference signal with periodicity equal to that of the BPF and used 20 control filter coefficients and 20 secondary path model coefficients. The error microphones were 6 mm Hosiden electret microphones. The error sensors were placed on the theoretical pressure minimum locations found previously in the pressure nulls in Figs. 5 and 6. In this study, zero phase difference between the two primary sources was considered in modeling the configuration to place the error microphones.

Similar to previous studies[7,8,11,12], the fan array was placed in a fully anechoic chamber to measure the sound power reduction at the tonal frequencies. The noise spectrum of the experimental fan array was taken using a GRAS 12.7 mm Type I microphone with a 0.6 Hz frequency resolution to determine the beat frequency.

The performance of the control system was determined using a hemispherical sound power measurement. The measurement was obtained using thirteen Type 1 12.7 mm microphones located every 15° along a semicircular arc with a radius of 1.7 m. The fan enclosure was placed under the center microphone of the semicircular array, and the measurement array was rotated about the center microphone at 15° increments to create a full hemispherical scan. At each microphone position, the autospectrum with and without control was calculated using 15 averages and a frequency resolution of 6.1 Hz. The sound power was calculated from the autospectra with an area-weighting method used previously by Monson et al.[11]. The measurement was repeated five times with and without control running and the results averaged to find the sound power reduction for both control configurations.

4 RESULTS AND DISCUSSION

Results from the fan array radiation and its control are discussed in this section. First, the characteristic noise

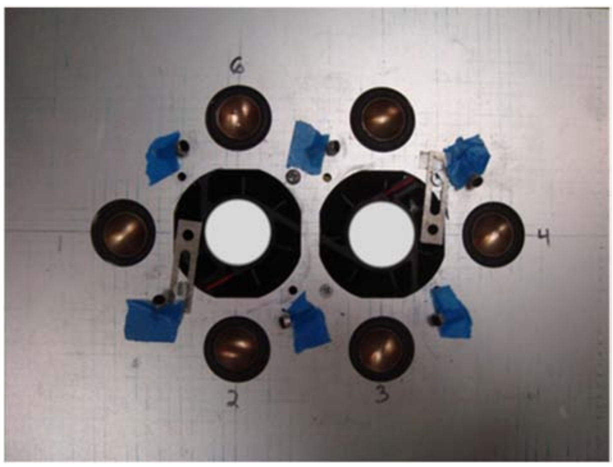

Fig. 7—The control setup used for the active control of a two-fan array.

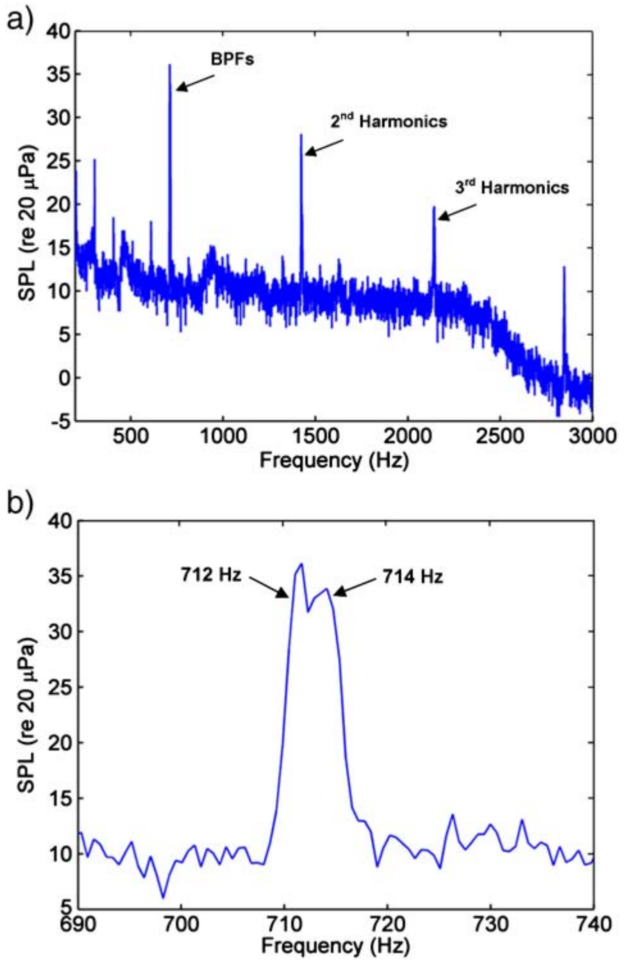

Fig. 8—(a) Noise spectrum of a two-fan array measured 1.7 m on axis from the center of the array. (b) Noise spectrum of a two-fan array, showing the two BPFs approximately 2 Hz apart.

spectrum of this two-fan array is shown in Fig. 8(a). This figure shows the frequencies that were targeted by the control system, consisting of the BPFs and second harmonics at 712, 714, 1424 and 1428 Hz, respectively. Highlighting the beating of the fan, Fig. 9 shows a zoom spectrum of the BPFs. The BPF peaks are 2 Hz apart, creating an audible beat frequency.

Shown next are representative sound power spectra calculated from the field microphones. These are shown for one trial of the three and six-channel control configurations in Fig. 9. The figures show that both control systems were able to control the BPFs and the second harmonics of both fans in the two-fan array. The total attenuation at the BPFs and the second harmonics was determined for each trial by integrating across a 12 Hz band centered on the frequency of interest and is displayed in Table 2. This table shows that the two independent noise controller approaches were actually superior to the single controller at both the BPFs and second harmonics by an average of 2 and 1 dB, respectively.

Two aspects of the fully coupled noise control system degraded the performance compared to the two independent noise controllers. First, the microphone placement had to be ideal for each fan, meaning that it limited the ideal microphone locations for the fully coupled system compared to the independent noise controller approach. Additionally, when finding the optimal locations, zero phase difference between the two primary sources was used. Looking at Table 1, phase difference at 0.5 relative magnitude makes very little difference in the potential reductions (17–18 dB using two independent noise controllers versus 27–28 for a single noise controller). However, optimal microphone locations shift with phase and since the error

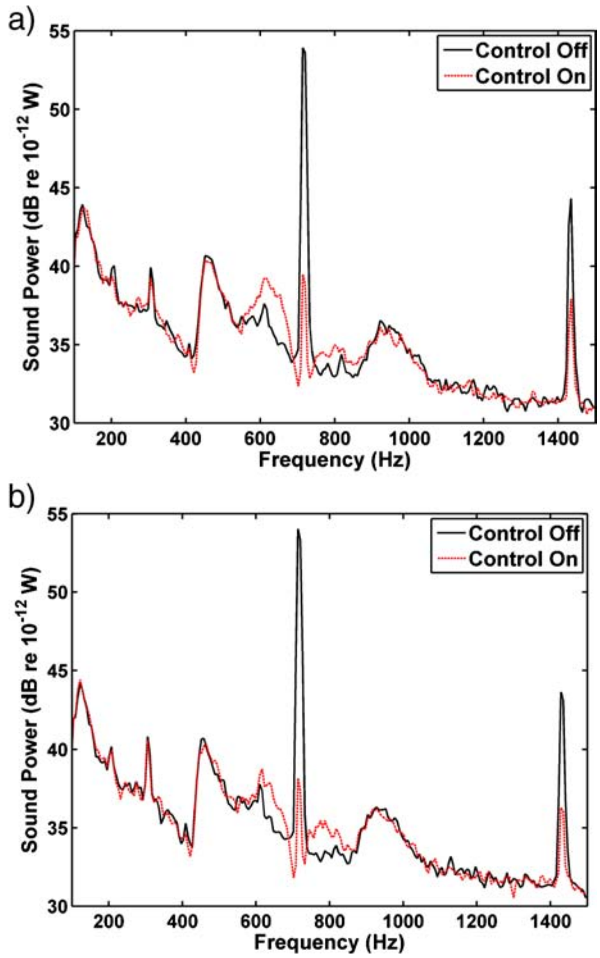

Fig. 9—Comparison of the estimated hemispherical sound power with control on and off using (a) One controller with six control sources to control the noise from both fans and (b) Two independent noise controllers (one for each fan) consisting of three control sources.

Table 2—12 Hz narrow band sound power reduction (in dB) of the BPFs and 2nd harmonics of a two fan array using one control filter and two independent filters.

	1 Control Filter		2 Control filters	
runs	BPFs	2x BPFs	BPFs	2x BPFs
1	14	6	16	7
2	14	6	16	7
3	14	6	15	8
4	14	5	16	7
5	14	6	16	8
Average	14	6	16	7

microphones for the single controller need to be optimal for both fans, there was a larger impact on this configuration than for two individual noise controllers. For this study, the tonal noise from the fan array was able to be controlled modeling with zero phase difference between the fan and virtual sources. Further examination of the relative phase between the two primary sources could be a subject for future studies to try and improve the performance.

The second aspect that appeared to limit the fully coupled system was processing related. The controller required an increased number of control taps to achieve the same results as the independent controllers. The greater number of taps was required to represent and control the independent, closely spaced frequencies of the two fans. This consideration is eliminated when two independent controllers were used. In addition, increased computational requirements of the coupled controller due to all the system paths could be an important limitation in some applications. Although the sound power reductions for the two controllers are similar, it is also worthwhile to examine the spatial radiation patterns provided by the hemispherical measurement. The spatial sound pressure level with and without control for a single trial with a 12-Hz–integration band for the BPFs and second harmonics and the single coupled controller setup are shown in Fig. 10. Similar plots for the two independent controllers are shown in Fig 11. The results indicate that although the single controller produces slightly more uniform radiation patterns, the maximum attenuation is not as great as achieved with the two independent controllers. This result is not unexpected; the fully coupled system surrounds the primary source with control sources, which would lead to attenuation in all directions. In contrast, the independent controller approach had speakers on one side of the primary sources which limits the attenuation in certain directions.

While the uniformity suffers a little with the independent controllers, the experimental results of this study suggest that using two independent control systems is a suitable approach for the active control of noise from closely spaced cooling fans. For example, individual controllers require fewer computations than a single coupled controller. In the case of the systems examined here, we can use the number of finite impulse response filters used in the filtered-x algorithm as a figure of merit. The one six control source by six error microphone control system used six control filters for the actuators in addition to the 36 (6 x 6) filters for the secondary path model, giving a total of 42 filters. The two control loops each used three control filters and nine (3 x 3) secondary path model filters, for a total of 24 filters. Using the independent controllers required 18 fewer filters than the fully coupled system, greatly reducing the computation time. As mentioned earlier, in

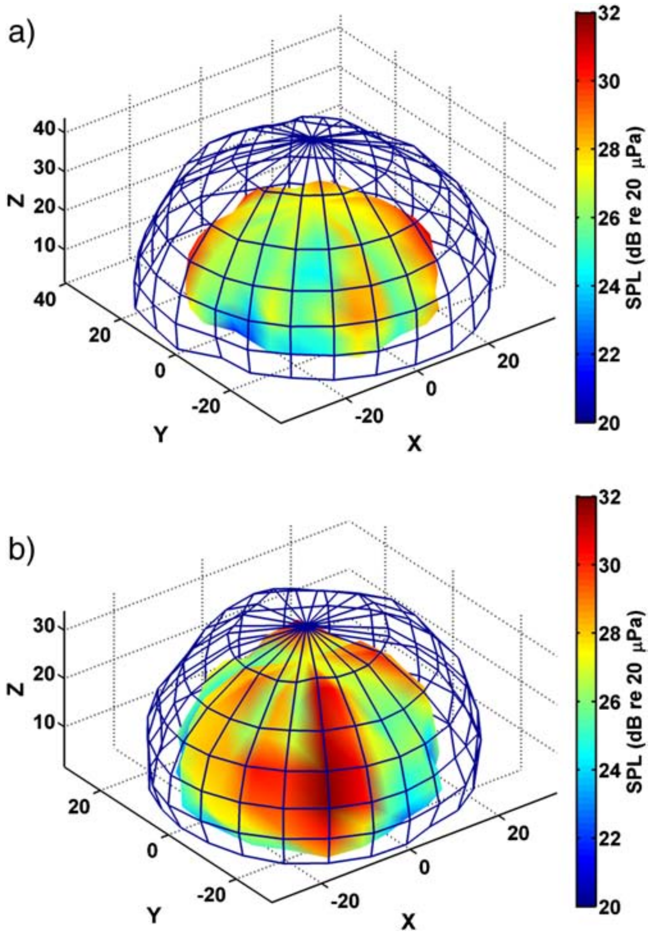

Fig. 10—Directivity plot of a two-fan array at the BPFs using one control filter with the control on (color) and control off (mesh), showing that the tones are reduced in all directions for the (a) BPFs of the two fans and (b) The second harmonics of the two fans.

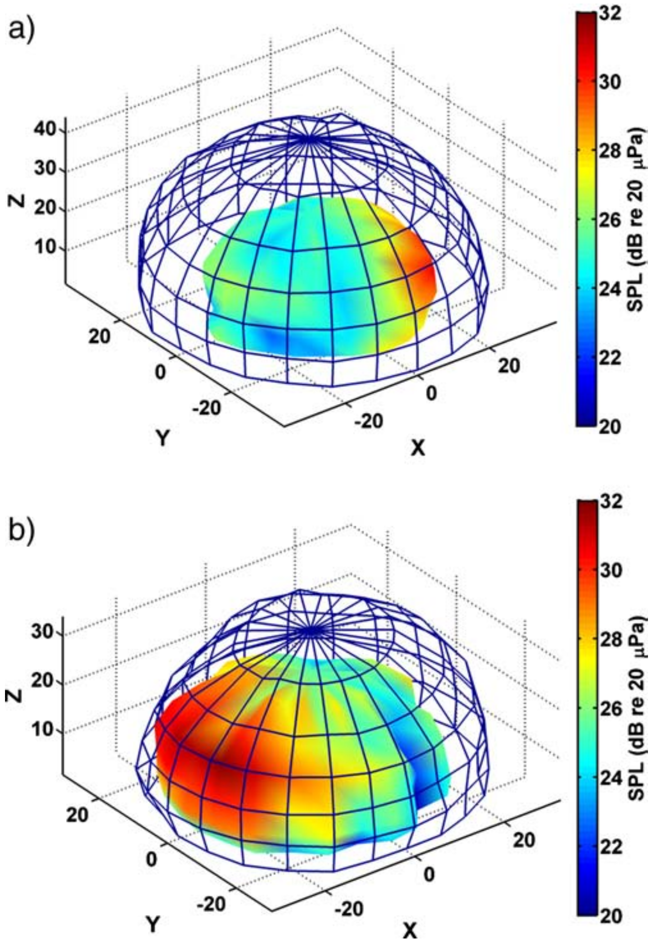

Fig. 11—Directivity plot of a two-fan array at the BPFs using two independent control filters, one for each fan in the array, with the control on (color) and control off (mesh), showing that the tones are reduced in all directions for the (a) BPFs of the two fans and (b) The second harmonics of the two fans.

order to achieve comparable experimental results, the number of control taps had to be increased for the single fully coupled controller compared to using independent controllers. Another benefit to smaller systems is that microphones do not have to be shared between control systems. Microphones used for two different noise sources have to be placed in optimal locations for both noise sources causing limited optimal placement positions that may lead to degraded performance. Lastly, the independent controller can be modularized and used for larger fan arrays.

Overall, for both control configurations, the amplitude of the acoustic beating was reduced significantly. Applying the active noise control to the fan array lowers the magnitudes of the two BPFs to the point that the beating was almost completely nonexistent. To demonstrate, Fig. 12 shows a bandpass-filtered (between 661.5 and 771.8 Hz) version of the time signal of a far-field microphone 1.7 m from the fan array. The figure shows before and after the controller is turned on, demonstrating the reduced amplitude of the beat frequency because of the control system. Figure 12 also shows that the beat frequency appears to fluctuate around 2 Hz because of the slight variation in rotation speeds of the two fans. This result shows that the active control system can be used to reduce the amplitude of the components that produce the beating, thereby making the fan array less annoying.

5 CONCLUSIONS

The active noise control of a two fan exhaust mounted array has been studied. The tonal noise from the fan array was controlled using two different control configurations. First, a fully coupled six-control source system to control the noise from both fans was implemented, and second, two independent noise controllers, one for each fan were implemented. Each fan in the array operated at different frequencies, so they were modeled individually. A single fan has two acoustic paths: a direct radiation path and an acoustic path through the box and out the other fan opening. Therefore, each fan was modeled as a two source array. Since the effect of the second radiation path was unknown, the sound power attenuation of a two source array with various relative source strengths was theoretically determined. When the second acoustic path was significant, the six-control-source configuration outperformed the three-control-source configuration

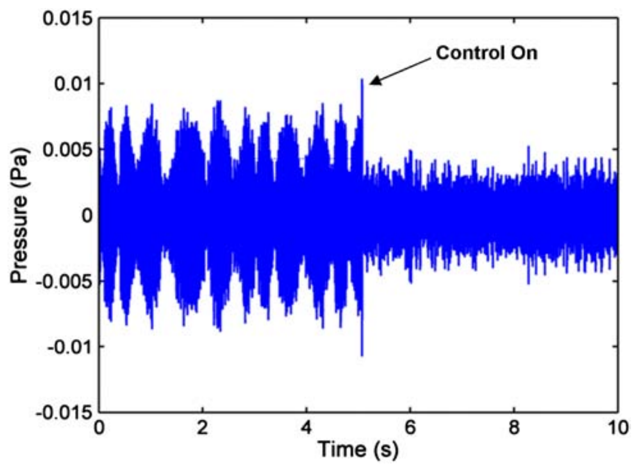

Fig. 12—Bandpass filtered time signal of a far-field microphone with control off and then turned on. The noticeable beating at ~2 Hz drops out of the time signal after control is turned on.

by 8–27 dB. However, as the relative magnitude of the second primary source dropped, the difference in performance of the two control systems was reduced until there was only 3 dB separating them. This suggests that if the second acoustic path is significant, a fully coupled control source configuration that surrounds the two-fan array should be used. However, this was not required in this study.

Experimental results showed that both control approaches were able to control the BPF and the second harmonics of each fan with significant sound power reduction. This extends the single-fan global control approaches developed in the past to a multi-fan problem. Additionally, the active noise control of the two-fan array reduced the components that contributed to an audible beat frequency, thereby making it less annoying. In regard to the individual noise control approaches, the two independent controllers actually achieved slightly more attenuation than the single fully-coupled controller. The narrowband sound power attenuation for two independent controllers was 16 dB for the BPFs and 7 dB for the second harmonics, as compared to 14 and 6 dB for the larger fully coupled system. This result suggests that using independent controllers was the better approach for this two-fan array. Larger fan arrays amplify the problems associated with a fully coupled noise control system, making the independent controller approach more attractive for these types of implementations. Additionally, using smaller independent controllers means a reduction in computation time over a fully coupled system. Furthermore, the larger fully coupled system had limited optimal microphone placement locations for both fans. This tends to reduce the performance of the control system, while this issue is not apparent with individual noise controllers. The results obtained to date suggest that these conclusions should be scalable for different sizes of fan arrays.

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