Bicoherence analysis of model-scale jet noise

Kent L. Gee[a)]
Department of Physics and Astronomy, Brigham Young University, Provo, Utah 84602
kentgee@byu.edu

Anthony A. Atchley, Lauren E. Falco, and Micah R. Shepherd
Graduate Program in Acoustics, The Pennsylvania State University, University Park, Pennsylvania 16802
atchley@engr.psu.edu, laurenefalco@yahoo.com, mrs30@psu.edu

Lawrence S. Ukeiley
Department of Mechanical and Aerospace Engineering, University of Florida, Shalimar, Florida 32579
ukeiley@ufl.edu

Bernard J. Jansen and John M. Seiner
National Center for Physical Acoustics, University of Mississippi, Oxford, Mississippi 38677
bjansen@olemiss.edu, jseiner@olemiss.edu

Abstract: Bicoherence analysis has been used to characterize nonlinear effects in the propagation of noise from a model-scale, Mach-2.0, unheated jet. Nonlinear propagation effects are predominantly limited to regions near the peak directivity angle for this jet source and propagation range. The analysis also examines the practice of identifying nonlinear propagation by comparing spectra measured at two different distances and assuming far-field, linear propagation between them. This spectral comparison method can lead to erroneous conclusions regarding the role of nonlinearity when the observations are made in the geometric near field of an extended, directional radiator, such as a jet.

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1. Introduction

After early studies a few decades ago,[1–3] appreciable progress has been made recently in understanding the role that nonlinear effects play in high-amplitude jet noise propagation. Gee *et al.*[4] analyzed F/A-18E data for evidence of nonlinear propagation effects; this study has been followed up by further full-scale engine tests that showed evidence of nonlinear propagation along the peak directivity direction.[5,6] Laboratory experiments[7] performed on model-scale jets have shown a modest nonlinear transfer of spectral energy to high frequencies [<10 dB relative to far-field, linear predictions out to a scaled distance of 289 jet nozzle diameters (D_j)] and that the range of angles over which nonlinear effects are present increases[8] as the jet's convective Mach number becomes progressively greater than one. However, the strongest evidence of the relevance of nonlinearity in high-amplitude jet noise propagation has come from work that involved field measurements of the noise radiated by the F-22 Raptor.[9,10] In these studies, a nonlinear propagation model predicted significant waveform steepening and a spectral energy transfer to high frequencies that agreed closely with measured data. With one aircraft engine at afterburner power, nonlinear propagation out to 305 m results in levels at 20 kHz that are approximately 100 dB greater than predicted by linear propagation. Nonlinear effects were found to be present along the full 55° measurement span to the side and aft of the aircraft, for even intermediate engine conditions.

[a)] Author to whom correspondence should be addressed.

Most studies have examined jet noise data for nonlinear effects by comparing measurements made at two distances along the same radial line and then comparing the measured spectrum with a spectrum obtained by assuming far-field, free-field linear propagation (i.e., by applying spherical spreading and atmospheric absorption between the two measurement distances). This approach requires a) placement of both microphones in the geometric far field of the jet, and b) a sufficient measurement bandwidth and propagation range in order to observe the differences between linearly predicted and measured spectra. (The more limited the bandwidth, the greater the propagation range required.) These two requirements make model-scale laboratory measurements challenging. First, the size of most anechoic jet facilities makes it difficult to place two microphones in the geometric far field over a large nozzle-scaled propagation range. Second, the microphones used for these measurements are of the same scale as those used in full-scale engine studies. In other words, the relative measurement bandwidth cannot currently be scaled to be the same as for a full-scale test. As an example, if we were to scale the F-22 geometric conditions and analysis bandwidth[9,10] to those of the model-scale experiment described in this Letter, we would need an anechoic chamber that allowed over 17 m of far-field propagation and microphones with a response exceeding 2 MHz.

If the far-field or bandwidth/propagation range requirement is not met, erroneous conclusions regarding the nonlinearity of the propagation when using the spectral comparison method can be reached. Consequently, other analysis techniques are desirable. This Letter describes the use of bispectral analysis to analyze noise data collected largely in the geometric near field of a Mach-2.0 unheated jet. Using the bicoherence, a normalized form of the bispectral density, we demonstrate that nonlinear effects are predominantly limited to near the peak directivity angle for this jet noise source and propagation range. We further use the bicoherence and the far-field spectral comparison method to show a) how incorrect conclusions about the significance of nonlinear effects can be reached when comparisons are made in the near field, and b) that the onset of the geometric far field is being approached by 60 D_j. In summary, the bicoherence provides a way to essentially separate nonlinear and (linear) geometric near-field effects; this separation cannot be done using power spectral comparisons alone.

2. Bicoherence

Bispectral analysis is a form of higher-order spectral analysis that has been used to examine data for quadratic nonlinearities in a variety of applications from astronomy to economics. In acoustics, quadratic nonlinearities reveal themselves in a propagating pressure waveform through harmonic and sum and difference-frequency generation, which cause the energy present at different frequencies to become phase coupled. This process is referred to as quadratic phase coupling (QPC). A convenient normalization of the bispectral density, $S_{ppp}(f_1,f_2)=\lim_{T\to\infty}(1/T)\langle P(f_1)P(f_2)P^*(f_1+f_2)\rangle$, is the bicoherence. The bicoherence may be defined[11] as

$$b^2(f_1,f_2)=\frac{|S_{ppp}(f_1,f_2)|^2}{Z(f_1,f_2)S_{pp}(f_1+f_2)}, \tag{1}$$

where $S_{pp}(f)=\lim_{T\to\infty}(1/T)\langle P(f)P^*(f)\rangle$, $Z(f_1,f_2)=\lim_{T\to\infty}(1/T)\langle|P(f_1)P^*(f_2)|^2\rangle$, $P(f)$ is the Fourier transform of the pressure waveform, and * denotes the complex conjugation operator.

Although the bicoherence provides a measure of the degree to which QPC exists among spectral components in a signal, $b^2(f_1,f_2)$ currently has a quantitative interpretation only if the signal is periodic. For a periodic signal with spectral components at f_1, f_2, and f_1+f_2, Kim and Powers[11] contend that $b^2(f_1,f_2)$ is the fraction of power at f_1+f_2 that is present due to QPC between f_1 and f_2. If the power spectral component at f_1+f_2 exists solely because of a nonlinear interaction between f_1 and f_2, then $b^2(f_1,f_2)\to 1$. However, for a nonlinear random noise signal, multiple frequency pairs may interact nonlinearly to yield a single component of $S_{pp}(f)$. Consequently, there is a cascading of sum and difference-frequency generation that makes quantitative analysis of the bicoherence difficult. An additional limitation of the bicoherence is that it does not independently indicate energy transfer upward or downward in the spectrum, only that

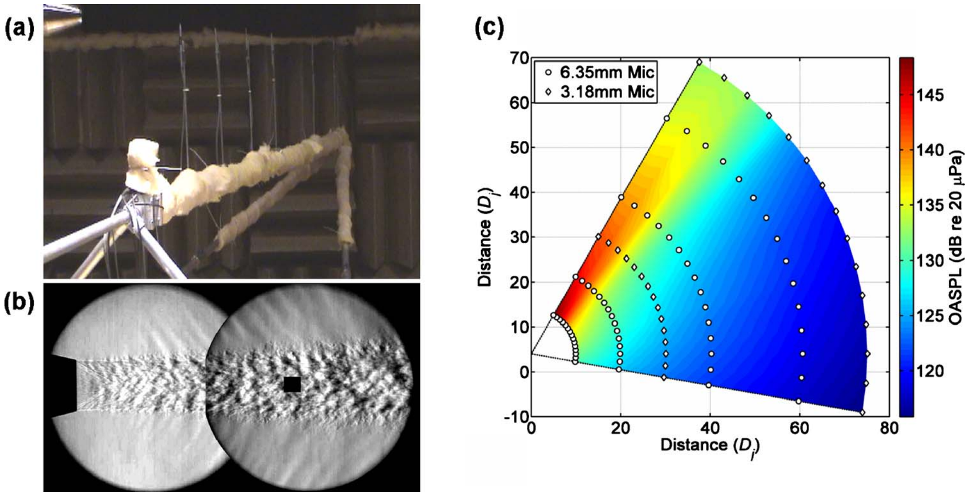

Fig. 1. (Color online) (a) Measurement setup with the nozzle and the boom-mounted microphones. (b) Overlapped instantaneous Schlieren images showing the measurement origin (black square) relative to the high-density-gradient Mach waves originating near the nozzle. (c) Overall sound pressure level (OASPL) map with measurement locations and 80°–150° (relative to nozzle inlet) aperture shown.

QPC exists between f_1, f_2, and f_1+f_2. However, one of the principal benefits of bispectral analysis is that it does not depend explicitly on second-order wave equation assumptions and so can be used in both the acoustic and geometric near and far fields for the purposes of determining relative QPC.

3. Experiment

Acoustic pressure data were collected on an unheated jet produced by a 3.49 cm, Mach-2.0 convergent-divergent nozzle operated on design at the National Center for Physical Acoustics anechoic jet noise facility whose dimensions yield a maximum scaled propagation distance of 80 D_j. The frequency-to-Strouhal number scaling for this experiment is 6.74E-5 Hz^{-1}. During the experiments, the ambient pressure, temperature, and relative humidity were monitored inside the anechoic chamber and were found to be nearly constant at 1.0 atm, 25.5 °C, and 50%, respectively. Bruel and Kjaer 6.35 mm 4938 microphones (at 10, 20, 40, and 60 D_j) and 3.18 mm 4138 microphones (at 30 and 75 D_j) were mounted at nozzle height on a stepper-controlled microphone boom [see Fig. 1(a)]. For each boom location, 2^{20} samples of time waveform data were acquired with a 24-bit Motu 896 recorder at 192 ksamples/s, but the maximum analysis frequency was limited to 75 kHz because of the 6.35 mm microphones' frequency response.[12]

The boom's axis was located 4 D_j downstream of the nozzle exit plane. This location is within the visible Mach-wave radiation region of the jet [see the Schlieren visualization in Fig. 1(b)] but is upstream of the dominant apparent low-frequency noise source region estimated by far-field elliptical mirror measurements made on a Mach-1.9 unheated jet.[13] Figure 1(c) depicts microphone locations produced by boom rotation between 80° and 150° (relative to the nozzle inlet) in 5° increments and an overall sound pressure level (OASPL) map. The OASPL map shows that although the peak directivity angle beyond 40 D_j is 145°, the 10 and 20 D_j measurements suggest a dominant overall noise source region that is slightly farther downstream than 4 D_j. Note, however, that the jet noise source location and spatial extent varies appreciably as a function of frequency, which illustrates a difficulty with attempting to align propagation and observation angles in order to compare spectra for nonlinear propagation along a radial line in the geometric near field.

This difficulty is shown explicitly in Fig. 2, which shows both the measured PSD at all six measurement positions along 145° [Fig. 2(a)] and the far-field, linear (i.e., spherical spread-

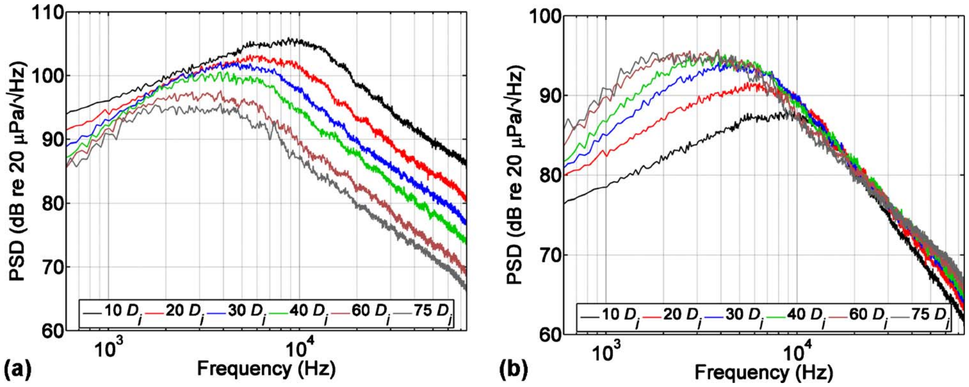

Fig. 2. (Color online) (a) Measured power spectral densities (PSD) along 145° for the Mach-2.0 unheated jet. (b) Extrapolation of the measured PSDs along 145° to 75 D_j by applying spherical spreading and atmospheric absorption.

ing and atmospheric absorption) propagation extrapolation out to 75 D_j from the other positions [Fig. 2(b)]. If the source were a compact source aligned at the measurement origin undergoing linear atmospheric propagation, all six spectra would collapse. Although this collapse occurs from about 15 to 30 kHz, there are differences above and below, which indicate a problem with the initial compact, linear source assumption. This means that geometric near-field and/or nonlinear propagation-related effects exist. The measured f^{-2} slope at high frequencies for all distances is already evidence that acoustic shocks are occurring, but bispectral analysis is a potentially useful tool to distinguish between nonlinear and near-field behavior.

4. Bicoherence results

Two sets of results for the bicoherence as defined in Eq. (1) are now shown for the Mach-2.0 unheated jet. To calculate a digital estimate of Eq. (1), a block size of 512 samples and a Hamming window with 50% overlap were used. This was done according to recommended practices for digital bispectrum estimation[14] and, based on Elgar and Guza's[15] work, resulted in a 99% confidence threshold for significant bicoherence of 0.05. In Fig. 3, $b^2(f_1,f_2)$ is displayed along 60 D_j for three angles, 120°, 135°, and 150°. Note that because $b^2(f_1,f_2)$ is symmetric about the line $f_1=f_2$, only the unique portion ($f_1 \leq f_2$) is shown. At 120°, the results show only traces of QPC and primarily above 15 kHz. For 135° and 150°, however, most frequencies are above the significant threshold of 0.05. Examination of Fig. 1(c) shows that the 10-dB increase in OASPL over these three angles is correlated with the relative degree of QPC.

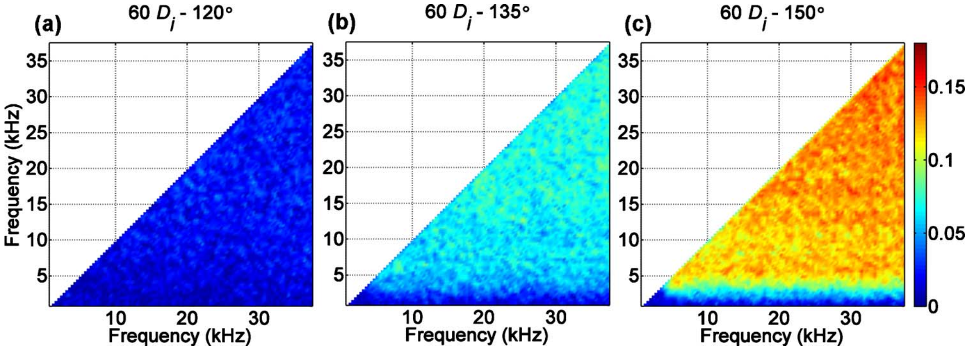

Fig. 3. (Color online) Bicoherence along 60 D_j for the Mach-2.0 unheated jet at (a) 120°, (b) 135°, and (c) 150°.

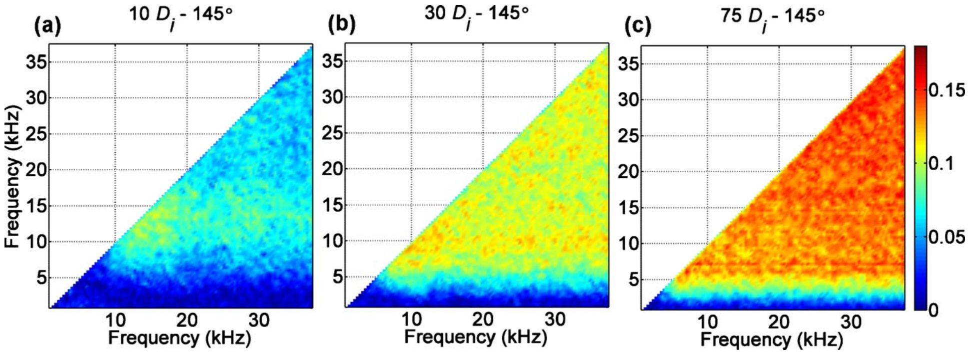

Fig. 4. (Color online) Bicoherence along 145° for the Mach-2.0 unheated jet at (a) 10 D_j, (b) 30 D_j, and (c) 75 D_j.

In Fig. 4, $b^2(f_1,f_2)$ is displayed along 145° for three distances, 10, 30, and 75 D_j. We note again that this is a measurement angle relative to our chosen origin, rather than a true propagation angle. However, calculation of $b^2(f_1,f_2)$ for all angles revealed that the average bicoherence was greatest along 145° for all distances, despite the fact that the 145° OASPL was not always the largest. Consequently, we have chosen to observe the evolution of $b^2(f_1,f_2)$ along our measurement angle. In Fig. 4(a), the 10 D_j result reveals that the greatest QPC is not at the highest frequencies as was the case with Figs. 3(b) and 3(c), but in a frequency region around 10–15 kHz, which appears to be interacting with itself and higher frequencies as evidenced by the horizontal band of greater bicoherence. Given that ~ 10 kHz is roughly the peak-frequency region of the PSD [see Fig. 2(a)], this behavior is qualitatively consistent with sine-wave or narrowband noise nonlinear propagation in that the initial frequency component or band couples with itself to produce harmonics.

At 30 and 75 D_j [Figs. 4(b) and 4(c), respectively], there is a distinct change in $b^2(f_1,f_2)$ in that it is nearly uniform above 5 kHz in Fig. 4(b) and appears to increase as a function of frequency in Fig. 4(c). This suggests that the fraction of energy present in the spectrum at high frequencies (i.e., f_1+f_2) is increasingly quadratically-phased-coupled with lower frequencies (i.e., f_1 and f_2) as the waveform propagates. Considering the high-frequency PSD evolution in Fig. 2, an increase in QPC with a simultaneous decrease in level suggests an atmospheric filtering of random-phase high-frequency energy that is replaced by shock-related, phased-coupled energy through ongoing nonlinear steepening.

In Fig. 4, there is a low-frequency band where $b^2(f_1,f_2)\approx 0$ for all distances. Examination of the PSDs in Fig. 2(a) shows that this band corresponds to frequencies below the peak-frequency region. This same behavior is also evident in Figs. 3(b) and 3(c). This result indicates that the dominant nonlinear transfer of energy is from the peak frequencies upward in the spectrum and not downward. Thus, in reexamining the discrepancies between assumed far-field, linear propagation and measured spectra in Fig. 2(b), the high-frequency differences above the peak-frequency region are primarily caused by nonlinear propagation and that the low-frequency differences are caused by locating microphones in the geometric near field of an extended, directional radiator. There is no evidence of appreciable nonlinear energy transfer downward in the spectrum over this propagation range, which could be erroneously concluded if Fig. 2 were examined in isolation. With the information yielded by the bicoherence analysis, the improved collapse of the 60 and 75 D_j spectra at low frequencies in Fig. 2(b) indicates that the geometric far field for an unheated Mach-2.0 jet is nearly reached by 60 D_j for an assumed source origin of 4 D_j.

5. Conclusion

The bicoherence has been used to examine nonlinear effects in near-field noise measurements of an unheated supersonic jet. The results have shown that this analysis technique is useful in determining nonlinear interactions that cannot be gauged by power spectral comparisons alone.

However, as this study is only a preliminary investigation into the use of bispectral analysis to characterize nonlinear propagation effects for jet noise, significant work remains. For example, a more complete understanding of the theory of bispectrum evolution for nonlinear propagation could be gained by using inputs with well-established theoretical models for nonlinear waveform and power spectral evolution.

Acknowledgment

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