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JASA ARTICLE

A coherence-based phase and amplitude gradient estimator method for calculating active acoustic intensity

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ABSTRACT:

The phase and amplitude gradient estimator (PAGE) method [Thomas, Christensen, and Gee, J. Acoust. Soc. Am. **137**, 3366–3376 (2015)] has been developed as an alternative to the traditional p-p method for calculating energy-based acoustic measures such as active acoustic intensity. While this method shows many marked improvements over the traditional method, such as a wider valid frequency bandwidth for broadband sources, contaminating noise can lead to inaccurate results. Contaminating noise degrades performance for both the traditional and PAGE methods and causes probe microphone pairs to exhibit low coherence. When coherence is low, better estimates of the pressure magnitude and gradient can be obtained by using a coherence-based approach, which yields a more accurate intensity estimate. This coherence-based approach to the PAGE method, known as the CPAGE method, employs two main coherence-based adjustments. The pressure magnitude adjustment mitigates the negative impact of uncorrelated contaminating noise and improves intensity magnitude calculation. The phase gradient adjustment uses coherence as a weighting to calculate the phase gradient for the probe and improves primarily the calculation of intensity direction. Though requiring a greater computation time than the PAGE method, the CPAGE method is shown to improve intensity calculations, both in magnitude and direction. *© 2022 Acoustical Society of America.*

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I. INTRODUCTION

Coherence is an important measure in many signal processing applications, including wind noise,[1] beamforming,[2] underwater acoustics,[3] and intensity calculations.[4] It is a frequency-space measure that gives the correlation of signals received by a pair of microphones, and ranges between values of zero and one. Low coherence is often indicative of extraneous noise, multiple sound sources, or nonlinear effects.[5] As such, it can give insight into the nature of a sound field.

Coherence is sometimes also called spatial coherence, as it measures the spectral similarity between simultaneous sound signals measured at two spatial locations.[6] It is defined by using the auto- and cross-spectra for two microphones μ and ν as

$$\gamma^2_{\mu\nu}(\omega) = \frac{|G_{\mu\nu}(\omega)|^2}{G_{\mu\mu}(\omega)G_{\nu\nu}(\omega)}, \tag{1}$$

where ω is the angular frequency, $G_{\mu\mu}$ and $G_{\nu\nu}$ are auto-spectra, and $G_{\mu\nu}$ is the cross-spectrum. Although $G_{\mu\nu}$ can be complex, the auto-spectra are purely real, and so the coherence will always be real-valued and between 0 and 1.[7]

For a probe consisting of n microphones, there will be $n_p = n(n-1)/2$ microphone pairs, and hence n_p coherence spectra. These coherence spectra are useful because, as the coherence is a measure of the similarity of signals measured by the microphones, they can be used to account for the effects of contaminating noise and improve intensity calculations. Coherence is particularly useful for removing low-frequency noise caused by wind, as it is a non-acoustic source that can exhibit low coherence.[8–10]

Active acoustic intensity, which is a frequency- and spatially dependent vector measuring the propagation of sound energy, is useful for a number of applications, including source characterization and localization.[11] To calculate the intensity, both the pressure and particle velocity are necessary. Some methods exist to measure particle velocity directly, though many methods instead use a multi-microphone probe to measure pressure at different spatial locations and then obtain a value for the particle velocity using the calculated pressure gradient. Different methods have different ways to obtain the pressure gradient, which can result in different calculated intensity vectors.

A commonly used method for acoustic intensity known as the p-p or traditional method calculates the pressure gradient by separating the complex pressure in frequency space into real and imaginary parts. Using the five-microphone orthogonal probe pictured in Fig. 1, where the coordinate system is defined in the caption, the two-dimensional intensity is given by

$$\boldsymbol{I}(\omega) = \frac{\mathrm{Im}\{G_{12}(\omega)\} - \mathrm{Im}\{G_{13}(\omega)\}}{4a\rho_0\omega}\hat{x} + \frac{\mathrm{Im}\{G_{15}(\omega)\} - \mathrm{Im}\{G_{14}(\omega)\}}{4a\rho_0\omega}\hat{y}, \tag{2}$$

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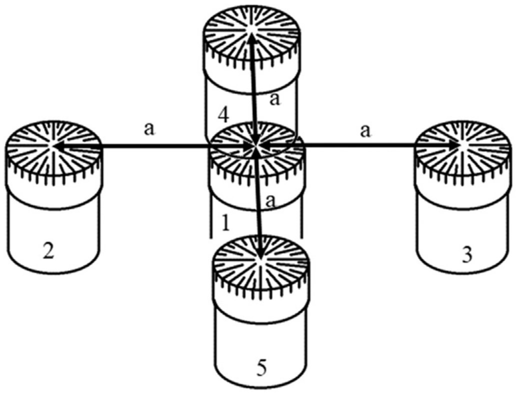

FIG. 1. A five-microphone orthogonal probe used for two-dimensional intensity measurements. For the coordinate system defined, microphone 1 is at the origin, while $\hat{x}$ points in the direction from microphone 1 to microphone 3, and $\hat{y}$ points in the direction from microphone 1 to microphone 4. The microphones in numerical order are then positioned at locations $(0,0)$, $(-a,0)$, $(a,0)$, $(0,a)$, $(0,-a)$.

where ρ_0 is the air density and a is the microphone spacing. The traditional method is fairly robust to uncorrelated contaminating noise, as it uses cross-spectral values to calculate the intensity, and cross-spectral values are not impacted by uncorrelated noise with a high enough signal-to-noise ratio or SNR. For correlated contaminating noise—such as noise emitted by additional sources—results are more complicated. However, this method can only be used for frequencies below the spatial Nyquist frequency, defined as when the microphone spacing is equal to one-half of an acoustic wavelength, i.e., the sound speed divided by $2a$. Even below this frequency, the intensity magnitude estimate rolls off,[12] and so the traditional method is only used for frequencies well below the spatial Nyquist frequency[13]—level bias errors due to processing are about 1 dB at half the spatial Nyquist frequency.[7]

Another method, known as the phase and amplitude gradient estimator (PAGE) method,[12,21] separates the complex pressure into the magnitude and phase instead of real and imaginary parts. By using phase unwrapping,[14,22] this method can yield reliable intensity estimates well above the spatial Nyquist frequency for radiating sources. The general equation for the PAGE method intensity estimate is

$$\boldsymbol{I}(\omega) = \frac{1}{\rho_0 \omega} P^2(\omega) \nabla \phi(\omega), \tag{3}$$

where P is the pressure magnitude, and $\nabla\phi$ is the phase gradient. Using the five-microphone orthogonal probe shown in Fig. 1, the PAGE method calculated intensity is

$$\boldsymbol{I}(\omega) = \frac{-G_{11}(\omega)\arg\{G_{23}(\omega)\}}{4a\rho_0\omega}\hat{x} + \frac{-G_{11}(\omega)\arg\{G_{54}(\omega)\}}{4a\rho_0\omega}\hat{y}, \tag{4}$$

where G_{11} is the auto-spectrum of the center microphone. Note that while the argument of a transfer function is typically used in such equations, the argument of the corresponding cross-spectrum is equivalent, since the transfer function is simply the complex cross-spectrum divided by the product of real-valued auto-spectra. The auto-spectra are affected by noise, and because of this, though the PAGE method can be used for higher frequencies, significant contaminating noise can reduce the usefulness of the PAGE method.

The calculation bias errors for both of these pressure-gradient-based intensity methods have been studied,[15] including the bias for different probe configurations and orientations relative to the source, as well as in the presence of contaminating noise. Bias errors for the PAGE method are essentially caused by a combination of two separate mechanisms: errors due to pressure magnitude calculation and errors due to phase gradient calculation. Because errors are the result of two mechanisms, two main adjustments can be made. Both adjustments make use of the coherence measured by microphone pairs. These adjustments can be implemented into the PAGE method calculation; the resulting approach is called the coherence-based phase and amplitude gradient estimator (CPAGE) method. The pressure magnitude adjustment is discussed in Sec. II, and the phase gradient adjustment is discussed in Sec. III. Experimental validation for these adjustments is given in Sec. IV.

II. PRESSURE MAGNITUDE ADJUSTMENT

Using the PAGE method, one type of error in intensity calculations caused by contaminating noise is encountered when obtaining the pressure magnitude estimate. Because auto-spectral values are used, as detailed in Eq. (4), the pressure of any contaminating noise is included in the pressure measurements and therefore in the intensity calculation. The pressure magnitude is squared to obtain the auto-spectrum, and so even relatively small errors can have a large impact on intensity calculations. In previous research,[7] a magnitude adjustment was found that can reduce the pressure magnitude bias errors of the PAGE method. This adjustment employs an improved estimate of the pressure magnitude.

Probe geometry determines how the acoustic pressure is obtained. There are two qualitatively different types of probes: those with a microphone located at the geometric center of the probe, and those without. When there is a microphone at the probe center, the magnitude of the pressure measured by this microphone is taken to be the acoustic pressure magnitude, and no calculation is needed,

$$P^2(\omega) = \left|\tilde{p}_c(\omega)\right|^2, \tag{5}$$

where the subscript c indicates that this is the probe's center microphone. For probes without a center microphone, the pressure magnitude is calculated by averaging the pressure magnitudes obtained at all of the microphones,

$$P^2(\omega) = \left(\frac{1}{n}\sum_{i=1}^{n}\left|\tilde{p}_i(\omega)\right|\right)^2. \tag{6}$$

The PAGE method uses the pressure magnitude obtained from Eq. (5) or (6) directly, while the CPAGE method makes an adjustment to the pressure magnitude.

The bias errors of both the PAGE and traditional methods in the presence of contaminating noise have been found previously.[7] Because the traditional method incorporates cross-spectra, it is unaffected by uncorrelated noise in some cases, such as when the contaminating noise is plane wave like.[7] For the PAGE method, however, uncorrelated noise simply adds to the overall pressure magnitude, increasing the magnitude of the auto-spectra and therefore the calculated intensity magnitude. However, by using the coherence values of the probe microphone pairs, the pressure magnitude of the uncorrelated noise relative to that of the signal can be calculated. The coherence can therefore be used to account for the additional pressure caused by the contaminating noise. Thus, the resulting pressure estimate will more accurately estimate the pressure magnitude of the coherent sound, rather than the combined pressure of the source and contaminating noise. For example, when measuring wind noise, which is non-acoustic in nature, additional pressures are measured at microphone locations which can be incoherent with one another,[1] and the additional pressure can be accounted for with the CPAGE method.

As a basic example, consider a case where all microphones in a probe measure the combined pressure from uncorrelated plane wave-like noise and a plane wave signal that are of equal amplitude, i.e., the SNR is 1, which is equivalent to 0 dB. The PAGE-calculated pressure is double what would be calculated without the contaminating noise, resulting in a +3 dB bias caused by the contaminating noise.[7] The coherence gives the amount of contamination measured between each microphone pair, so if the coherence values are the same for each microphone pair, the exact amount of contamination caused by uncorrelated noise—and hence the SNR—can be found. Accounting for the additional pressure caused by the contaminating noise would make the calculation bias error go to zero.

Unfortunately, in practice, coherence values are not identical for all probe microphone pairs, and so obtaining the SNR for the probe is non-trivial. When microphone pairs have different coherence values, an effective SNR—or an effective coherence for the probe—must be calculated. This is not as simple as averaging all coherence values, however. As an example, consider the five-microphone orthogonal probe in Fig. 1, for a case where all microphones record the desired the signal, while a single microphone also measures additional pressure from uncorrelated noise (e.g., electrical noise). For this probe, there are ten distinct microphone pairs, and with only one microphone picking up a contaminated signal, the coherence of four of these microphone pairs is reduced. Averaging the coherence across all microphone pairs gives a skewed result of the effective probe coherence or overall SNR.

There are multiple possible solutions to the problem of estimating the SNR; the CPAGE method uses one that is easy to implement and gives a conservative pressure magnitude adjustment. The maximum coherence value across all microphone pairs at a particular frequency is used as the effective probe coherence. Using the maximum coherence value ensures that only noise measured by all microphones is removed and therefore will not cause an over-adjustment. Using the pressure magnitude P—which is obtained from the composite signal of the source and noise together, as given in Eqs. (5) and (6)—the adjusted pressure magnitude utilized by the CPAGE method is

$$P^2_{\mathrm{CPAGE}}(\omega) = \left(\max_{\mu,\nu}\left\{\sqrt{\gamma^2_{\mu\nu}(\omega)}\right\}P(\omega)\right)^2 = \max_{\mu,\nu}\left\{\gamma^2_{\mu\nu}(\omega)\right\}P^2(\omega). \tag{7}$$

Note that any adjustment leads to a reduction rather than an augmentation of the pressure magnitude, since the coherence is always between zero and one. This adjustment is reasonable because contaminating noise always serves to increase, rather than decrease, the total pressure magnitude. The multiplicative factor $\sqrt{\gamma^2_{\mu\nu}}$ was found in previous research where the bias errors of the PAGE method were quantified,[7] and causes the calculation bias errors of the CPAGE method for a plane wave source and uncorrelated noise to go to zero, while causing a reduction in bias errors for other source and noise combinations.

The adjustment in Eq. (7) is most effective when all microphones record uncorrelated contaminating noise. All microphone pairs must exhibit a decrease in coherence at the same frequency in order for any adjustment to be made. When microphone pairs demonstrate vastly different coherence values—whether due to correlated contaminating noise or different levels of uncorrelated contaminating noise—an adjustment to the phase gradient can yield higher benefit.

III. PHASE GRADIENT ADJUSTMENT

The first adjustment utilized by the CPAGE method improves the intensity magnitude calculation; the phase gradient adjustment can have some impact on intensity magnitude, though primarily is of use in improving the intensity direction. Using the PAGE method, the intensity calculation is given in Eq. (3); calculating the intensity direction relies upon estimating the phase gradient, $\nabla\phi$. This frequency-dependent phase gradient is calculated by using the phase of the transfer functions between probe microphone pairs, then performing a least squares fit to obtain a phase gradient for the probe,

$$\nabla\phi = \left(\mathrm{X}^{\mathrm{T}}\mathrm{X}\right)^{-1}\mathrm{X}^{\mathrm{T}}\Delta\phi, \tag{8}$$

where X and $\Delta\phi$ are defined below.

The matrix X is of size $n_p \times d$, where n_p is the number of probe microphone pairs [$n_p = n(n-1)/2$ for a probe consisting of n microphones] and d is the probe dimensionality. The matrix X is composed of the physical distance between probe microphone positions,

$$X = \begin{bmatrix} x_2 - x_1 & \cdots & x_n - x_1 & x_3 - x_2 & \cdots & x_n - x_{n-1} \\ y_2 - y_1 & \cdots & y_n - y_1 & y_3 - y_2 & \cdots & y_n - y_{n-1} \\ z_2 - z_1 & \cdots & z_n - z_1 & z_3 - z_2 & \cdots & z_n - z_{n-1} \end{bmatrix}^T, \quad (9)$$

where x, y, and z are orthogonal coordinates in three-dimensional space, and the subscripts give the probe microphone numbers. For one- or two-dimensional intensity probes, only the first one or two columns, respectively, of X are used, or equivalently are nonzero. Because the physical positions of the probe are not frequency dependent, the pseudoinverse $(X^TX)^{-1}X^T$ utilized by the PAGE method is the same for all frequencies and must only be computed once.

The vector $\Delta\phi$—not to be confused with $\nabla\phi$, the phase gradient—is of length n_p, and gives the measured phase differences between microphone pairs. Phase differences are given by the argument of the transfer function (or cross-spectrum), and are frequency-dependent,

$$\Delta\phi(f) = [\arg\{G_{12}(f)\} \quad \cdots \quad \arg\{G_{1n}(f)\} \quad \arg\{G_{23}(f)\} \quad \cdots \quad \arg\{G_{n-1,n}(f)\}]^T. \quad (10)$$

Because measured phase differences lie in a 2π radian interval, this requires the use of phase unwrapping to get accurate total phase differences above the spatial Nyquist frequency.[12] The vector $\Delta\phi$, therefore, must contain unwrapped transfer function phase differences to be useful above the spatial Nyquist frequency.

The phase gradient adjustment implemented by the CPAGE method is conceptually simple: instead of using a least squares method, a weighted least squares method is used. The weighted least squares method—as the name implies—allows for data points (phase difference values, here) in the least squares fitting algorithm to be given different weights or importance values. The weights used for the CPAGE method are the square roots of the coherence values between microphone pairs, $\sqrt{\gamma^2_{\mu\nu}}$, the same values used in the pressure magnitude adjustment. Note that the weights used are frequency dependent. These weights are combined into a diagonal matrix of size $n_p \times n_p$,

$$W(f) = \mathrm{diag}\left[\sqrt{\gamma^2_{12}(f)} \quad \cdots \sqrt{\gamma^2_{1n}(f)} \quad \sqrt{\gamma^2_{23}(f)} \quad \cdots \quad \sqrt{\gamma^2_{n,n-1}(f)}\right], \quad (11)$$

where W is frequency dependent. The frequency-dependent phase gradient obtained by the CPAGE method using a weighted least squares algorithm is then

$$\nabla(f) = \left(X^TW(f)X\right)^{-1}X^TW(f)\Delta\phi(f). \quad (12)$$

Equation (12) uses the weighted pseudoinverse $(X^TWX)^{-1}X^TW$. Unlike the unweighted pseudoinverse $(X^TX)^{-1}X^T$, this pseudoinverse varies with frequency since the weighting matrix itself is frequency dependent. A possible disadvantage to this is that a pseudoinverse must be calculated for each frequency individually, which increases overall computation time, although the increase is hardly noticeable in practice. By using this approach, the phase gradient adjustment allows the CPAGE method to improve intensity calculation, most especially intensity direction, when contaminating noise is present.

IV. EXPERIMENTAL VERIFICATION

The two coherence-based adjustments to the PAGE method explained are uniquely suited for different situations. The magnitude adjustment—Eq. (7)—is most applicable when all probe microphone signals include uncorrelated noise, while the phase gradient adjustment—Eq. (12)—is most applicable when only some of the microphones are contaminated by noise at a particular frequency, or when the contaminating noise is self-correlated, which can cause an uneven coherence distribution in the data. The CPAGE method uses both the pressure magnitude and phase gradient adjustments simultaneously, though—depending on the situation—one adjustment can have a much larger impact than the other. Two different experiments are used to show the effects of each adjustment individually. Experimental results show how the CPAGE method calculation differs from the PAGE method calculation when microphone signals contain contaminating noise.

A. Pressure magnitude adjustment experiment

For the first experiment, measurements were taken in a large anechoic chamber. The five-microphone orthogonal probe for two-dimensional intensity calculation—pictured in Fig. 1—was used, where the probe radius was $a = 0.25$ m. This relatively large radius for a compact probe was used to reduce probe scattering at high frequencies. The sound source used was a loudspeaker emitting broadband noise and is pictured in Fig. 2. The calculation bias errors for the CPAGE and PAGE method are compared. Rather than using an analytical intensity for comparison, which would induce assumptions on the loudspeaker fidelity, the results of the PAGE method in the absence of noise are used as the benchmark value. This choice allows bias errors to show the differences in how both methods handle contaminating noise, compared to the noiseless case. A bias error of zero means that the contaminating noise has no effect on intensity calculation. The magnitude and direction bias errors are explicitly defined, respectively, as

$$L_{\epsilon,\boldsymbol{I}} = 10\log_{10}\left(\frac{|\boldsymbol{I}_{\text{noise}}|}{|\boldsymbol{I}|}\right)\text{dB}, \quad (13)$$

$$\theta_{\epsilon,\boldsymbol{I}} = \theta_{\text{noise}} - \theta, \quad (14)$$

where $|\boldsymbol{I}|$ and θ are the magnitude and direction, respectively, of the intensity as calculated by the PAGE method in

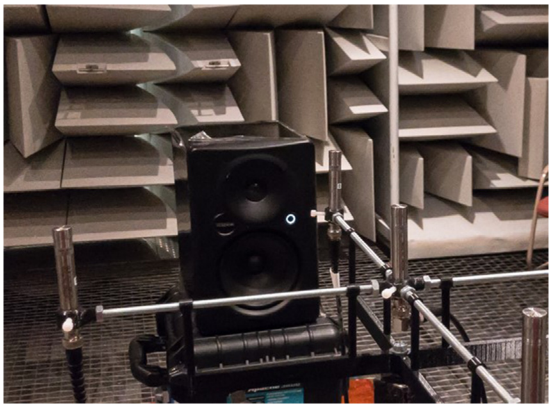

FIG. 2. (Color online) Experimental setup. The five-microphone orthogonal probe has a radius of a = 0.25 m. The loudspeaker was used as the sound source.

the absence of contaminating noise, and $|\boldsymbol{I}_{\text{noise}}|$ and θ_{noise} are the calculated intensity magnitude and direction of the method of choice when contaminating noise is present. Bias errors for the traditional p-p method are not shown as the spatial Nyquist frequency for this probe is 686 Hz, and so for frequencies above a few hundred Hertz the traditional method intensity calculations are unreliable and contain large bias errors.[13]

Measurements were first taken of the source alone—the results of the PAGE method for these measurements are the benchmark values. The signals acquired were then contaminated with computer-generated broadband white noise, with an independent contamination signal for each microphone. Because the contaminating signals were of approximately equal amplitude for each microphone, the resulting coherence values for all microphone pairs were also of similar amplitude, though not identical. The contaminated signals had much larger pressure magnitudes than the original signals (SNR ≈ -6 dB). This meant that the PAGE method calculated a much larger intensity magnitude for the contaminated case, resulting in a large level bias error due to noise.

The CPAGE method, on the other hand, can account for the uncorrelated contaminating noise, resulting in much smaller level bias errors than the PAGE method. Because intensity direction calculation uses arguments of cross-spectra, which are largely unaffected by uncorrelated contaminating noise, the calculated intensity direction for both methods approximate the noiseless case, resulting in only small angular bias errors for either method.

The results for this experiment are seen in Fig. 3. Sound pressure levels obtained from auto-spectral values for two of the probe microphones (numbered 1 and 2 in Fig. 1) are shown in Fig. 3(a). The solid lines give the sound pressure levels of the source alone, while the dashed lines are the results obtained from the contaminated signals. The coherence of the contaminated signals for the four microphone pairs which include the center microphone (number 1) are shown in Fig. 3(b).

The level bias errors for the PAGE and CPAGE method due to the contaminating noise are shown in Fig. 3(c). Because the contaminating noise is much louder than the source, the PAGE method shows bias errors of about 6 dB at all frequencies: calculated levels are a result of the noise, rather than indicative of the source. As expected, the CPAGE method can correctly account for a significant portion the uncorrelated contaminating noise. At all frequencies, this results in a smaller magnitude bias for the CPAGE method. The PAGE method calculates a larger magnitude because of the contaminating noise; while variation in the CPAGE method magnitude follows the same trend across frequency, it obtains much lower bias errors. As expected, the angular bias errors for both methods are nearly zero for all frequencies, as shown in Fig. 3(d).

B. Phase gradient adjustment experiment

To test the effectiveness of the coherence-based phase gradient adjustment—as well as the effectiveness of the CPAGE method in a real-world application—far-field measurements were taken at the static firing of an FSB-1 Artemis rocket flight support booster tested by NASA and Northrop Grumman in Promontory, Utah (see Fig. 4).[16,23,24] Measurements were taken approximately 2.5 km from the booster using a four-microphone equilateral triangular probe with a radius of 5 m, where the experiment is further described in Ref. 17. This large radius is typically used for only very low frequencies—the traditional method is limited to infrasonic frequencies, as the spatial Nyquist frequency for this probe setup is 34.3 Hz. For the PAGE method and the CPAGE method, accurate phase unwrapping is of crucial importance to calculate intensity above 34.3 Hz, and to reach 20 kHz unwrapping must be performed 583 times.[14]

With the 5-m microphone spacing, coherence between microphone pairs at higher frequencies can be vastly reduced not just by additional noise sources, but also because random fluctuations in the air can cause coherence loss for widely separated microphones.[18] The low coherence complicates intensity direction calculation,[19,20] and so this is a good test for the CPAGE intensity direction calculation method. Note also that while the apparent sound source location for jet noise sources can change across frequency,[17] at this distance the angular direction to the apparent source—and, hence, the ideal angular direction of the intensity—changes by only about 2°.

Calculated intensity magnitude results are seen in Fig. 5(a). Above around 300 Hz the CPAGE method calculates a lower intensity magnitude, as it is effectively removing at least some uncorrelated noise. This mismatch between amplitudes in the methods is caused by the drop in coherence, which is possibly due to atmospheric effects, which would require further investigation (along with the increase in coherence between 1 and 10 kHz). Further investigation would be needed to validate these levels, especially when the measured level is exceeded by the acoustic noise floor or when the microphone auto-spectral magnitudes are

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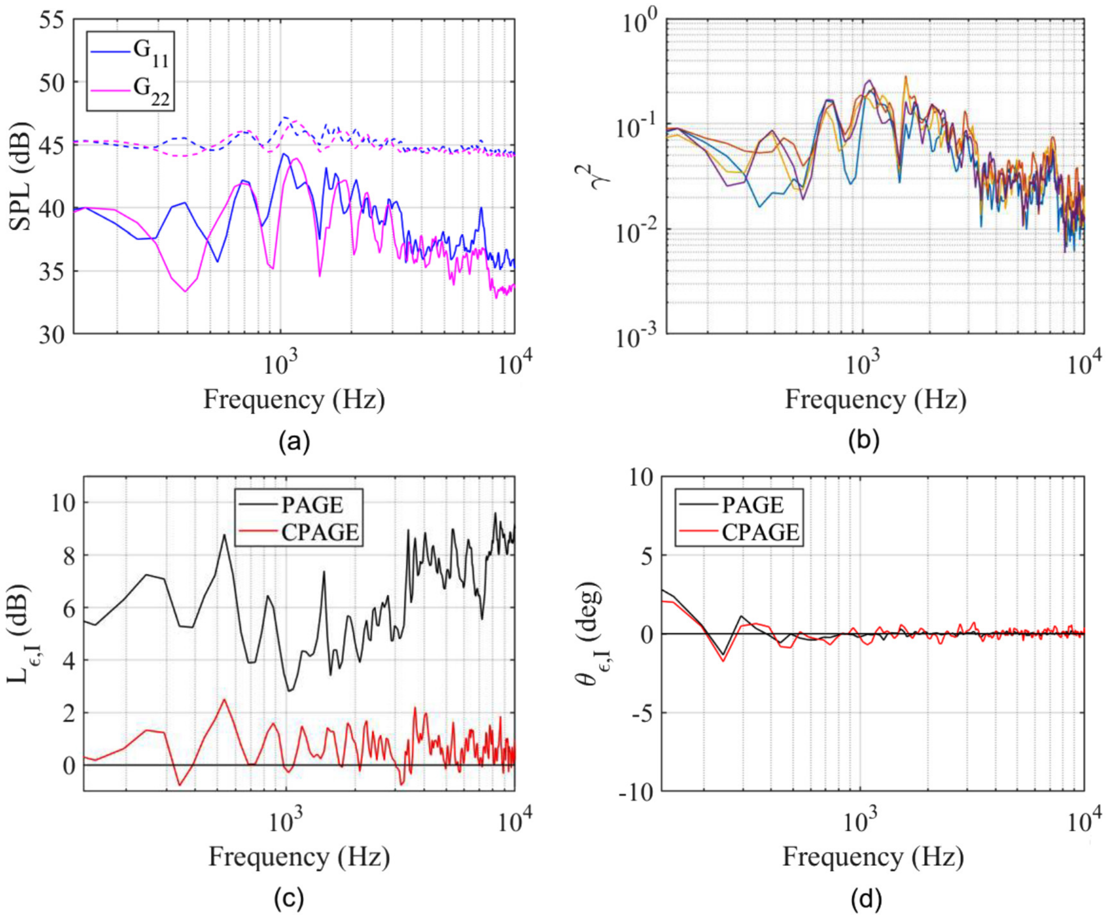

FIG. 3. (Color online) Bias errors for the PAGE and CPAGE methods—in relation to the noiseless PAGE intensity results—caused by adding uncorrelated broadband white noise to the signals recorded by all probe microphones independently. Sound pressure levels obtained from microphone auto-spectral values are shown in (a), where the solid lines are for the source alone, and the dashed lines are for the contaminated signal. The coherence of the contaminated signals for microphone pairs which include the center microphone are shown in (b). The CPAGE method is seen to have reduced magnitude bias errors (c), while the angular bias is mostly unchanged from that of the PAGE method (d).

different. The calculated intensity direction for both methods is shown in Fig. 5(b). For the PAGE method, the intensity direction begins to drift above about 300 Hz, while the CPAGE intensity direction remains consistent to within a few degrees across all frequencies. The drift in the PAGE method is caused by the decrease in signal coherence, which is shown in Fig. 5(c). The CPAGE method can accurately use the coherence-weighted method to obtain a more accurate phase gradient, and therefore a more accurate intensity direction, despite the very low intensity magnitude at high frequencies.

These experimental results highlight the effectiveness of the phase gradient adjustment in the CPAGE method. The results show not only a more accurate intensity direction calculation, but also an increase in the viable frequency bandwidth by using the CPAGE method, as the PAGE method intensity direction begins to drift and does not recover. While the PAGE method can increase the viable

FIG. 4. (Color online) Experimental setup for measuring a static rocket booster firing, with a booster in the far distance (a). The firing of the booster is also shown (b).

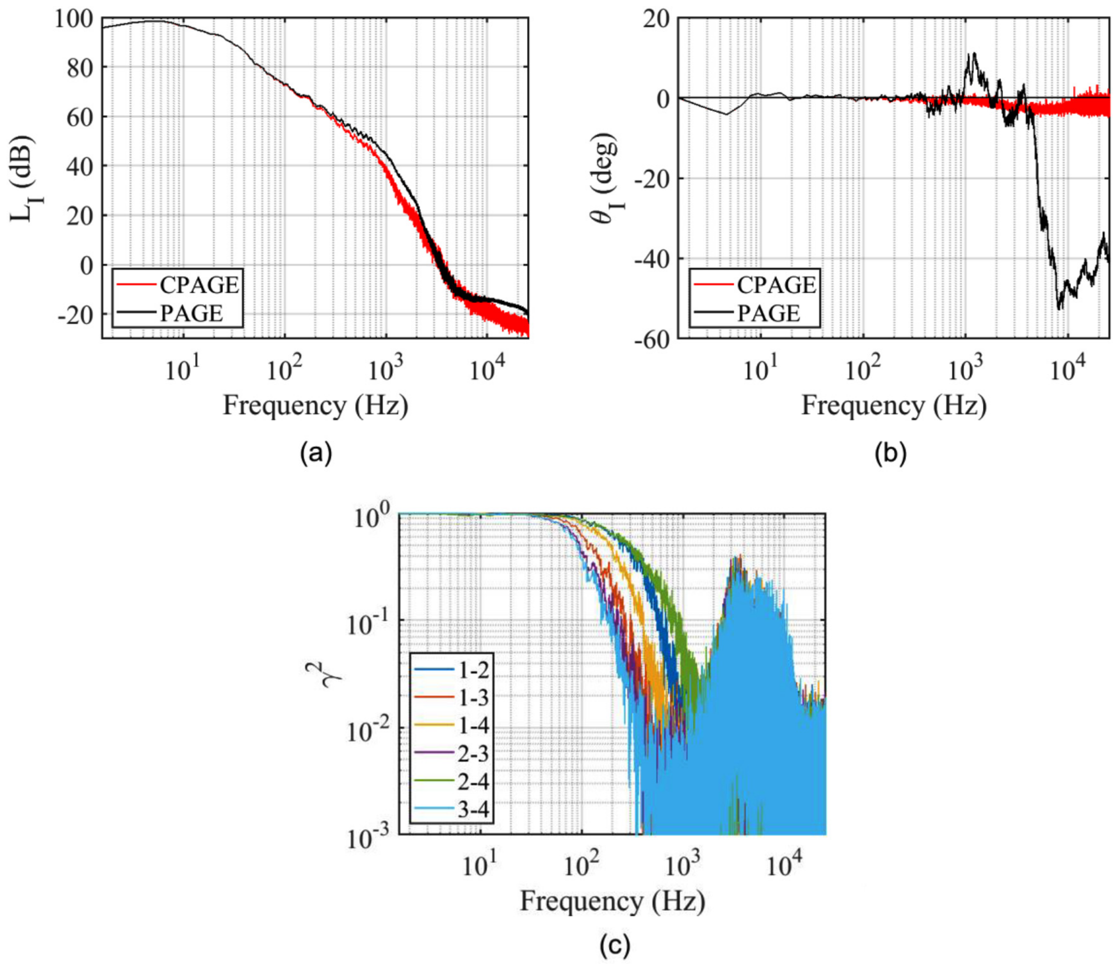

FIG. 5. (Color online) Results of rocket booster measurements, showing the calculated intensity magnitude (a), intensity direction with 0° being the correct direction of intensity propagation (b), and coherence between microphone pairs (c). The CPAGE method is seen to improve PAGE method errors in intensity direction at higher frequencies, despite poor coherence and low intensity magnitude.

frequency range when compared to the traditional method, as the traditional method can only be used below the spatial Nyquist frequency, the CPAGE method here appears to further increase the viable frequency range over the PAGE method's effectiveness, though the specifics of the apparent bandwidth extension seen in this instance remains a subject for future verification.

V. CONCLUSION

By using coherence in its calculations, the CPAGE method reduces the intensity calculation bias errors of the PAGE method and can also increase the viable frequency range for intensity calculations. Two main adjustments are used. A coherence-based magnitude adjustment can account for uncorrelated noise measured by all microphones and improves intensity magnitude calculation. A coherence-weighted pressure gradient calculation can account for different levels of noise measured by microphones and improves intensity direction calculation. These adjustments together make the CPAGE method better able to calculate intensity vectors in the presence of contaminating noise.

Though only active acoustic intensity results are investigated herein, the calculation of other energy-based acoustic measures—such as reactive intensity, potential energy density, kinetic energy density, and specific acoustic impedance—could also be improved with the CPAGE method. Coherence can effectively measure the signal contamination, and as such can account for contaminating noise recorded while measuring source properties.

The CPAGE method is limited in many of the same ways as the PAGE method. It relies upon correct phase unwrapping and broadband signals to be accurate above the spatial Nyquist frequency and is affected by probe scattering. Though the CPAGE method is not ideal for every situation, it broadens the applications of the PAGE method because it is more robust in handling contaminating noise.

The CPAGE method has been shown to be effective with large microphone separation distances, which can show very low coherence at high frequencies. Not only are calculated intensities more accurate, the CPAGE method can also increase the valid frequency range. These results strongly support the usefulness of the CPAGE method in calculating active acoustic intensity in any situation where contaminating noise is present, as it is in most real-world applications.

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APPENDIX

The CPAGE method uses two main adjustments to calculate intensity. However, there is another possible adjustment which is useful in very special cases. The adjustment proposed here should rarely be necessary in practice, though is worth considering for completeness. This adjustment is useful when the pressure measured by a probe's center microphone at a particular frequency is considered to be of dubious validity. This can result from poor experimental setup, where—for whatever reason—the center microphone records mostly noise instead of actual signal at a particular frequency. If this is known in practice then the experimental setup should clearly be changed, but if further data acquisition is infeasible, this additional adjustment can prove useful.

The CPAGE method only uses this adjustment at frequencies where the coherence of the center microphone with all other microphones is less than the coherence of all other microphone pairs,

$$\max_{\mu} \left\{ \gamma_{c\mu}^2(\omega) \right\} < \min_{\mu,\nu \neq c} \left\{ \gamma_{\mu\nu}^2(\omega) \right\}, \tag{A1}$$

where the subscript c signifies the center microphone, and μ and ν label the other probe microphones. When the condition in Eq. (A1) is met, then instead of using the pressure measured at the center microphone, the weighted average pressure of the other microphones should be used. In other words, at these frequencies, the CPAGE method treats the probe as if there is no center microphone since the measured center pressure is considered to be more erroneous than the measured pressure of all other probe microphones. Intensity probes are generally more accurate when a probe has a center microphone, so this adjustment is very rarely necessary. Because the CPAGE method calculates a pseudoinverse at every frequency to obtain the pressure gradient, the condition in Eq. (A1) can be checked—and the adjustment implemented—without causing a significant increase in computation time.

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