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Multi-objective optimization of acoustic black hole vibration absorbers

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Abstract: Structures with power law tapers exhibit the acoustic black hole (ABH) effect and can be used for vibration reduction. However, the design of ABHs for vibration reduction requires consideration of the underlying theory and its regions of validity. To address the competing nature of the best ABH design for vibration reduction and the underlying theoretical assumptions, a multi-objective approach is used to find the lowest frequency where both criteria are sufficiently met. The Pareto optimality curve is estimated for a range of ABH design parameters. The optimal set could then be used to implement an ABH vibration absorber.

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1. Introduction

Recent studies have shown the benefits of using the acoustic black hole (ABH) effect for vibration and radiated noise reduction.[1–4] By introducing a power-law taper in a beam, the wavespeed will approach zero as the beam thickness decreases and a wave will never reflect back into the beam once it enters the taper. Higher-power tapers have been shown to have a lower total reflection coefficient and therefore better performance. In practice, the taper can never be thin enough to approach the theoretical zero wavespeed. However, good vibration reduction has still been achieved with finite-length tapers by including damping material in the taper region. The combination of a power-law taper with a viscoelastic damping layer or internal material damping is the most efficient and practical implementation of an ABH for vibration reduction.

Feurtado *et al.* recently showed that although greater attenuation is predicted for higher-power thickness tapers, the underlying smoothness assumption that predicts the increased performance becomes more violated as the taper power increases.[5] The normalized wavenumber variation (NWV) is shown to be an effective metric for addressing the smoothness violation that occurs along the taper. Further work investigating the boundary-taper reflection found that $\mathrm{NWV} < 0.4$ is a good rule of thumb for designing ABHs without introducing excessive violation of the smoothness criterion.[6] By violating the smoothness requirement, reflection can occur due to the thickness changing too abruptly and the entire purpose of the taper becomes compromised.[7] The desire for lower frequency damping performance of the ABH then leads to a conflict of design options: increase the power of the taper to reduce reflections but decrease the taper power to prevent additional reflections from rapid thickness changes.

Such competing objectives can be naturally handled by modern multi-objective optimization algorithms. This paper presents the multi-objective optimization of a power-law taper that considers both the performance of the ABH for vibration reduction as well as the requirement to maintain adequate smoothness. The lowest frequency for a beam where both conditions is met was located by estimating the non-dominated set (i.e., Pareto optimality front) using an evolutionary search framework. The knee region of this set indicates the best design possibilities for low frequency performance of the ABH.

2. Evolutionary search

Since evolutionary algorithms are typically more effective than gradient methods for problems with discontinuous, multi-modal, or noisy objective functions, a real-valued, multi-objective evolutionary search algorithm was used to estimate the Pareto optimality front.[8] The search framework utilizes an auto-adaptive, multi-operator recombination step to adaptively learn and use the most efficient recombination operator(s) for the problem being solved.[9] enabling the algorithm to outperform other modern evolutionary algorithms on a number of multi-objective test functions.

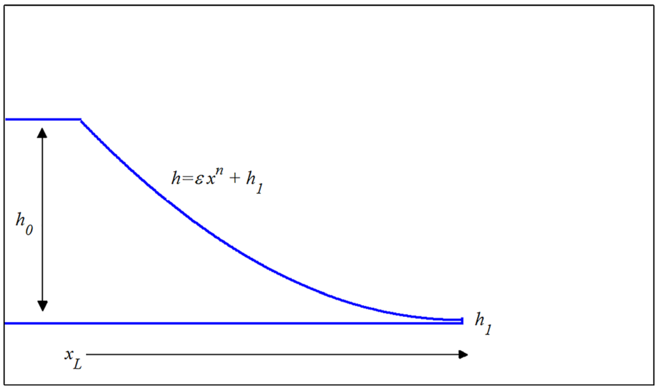

Fig. 1. (Color online) A beam with a power law taper behaves as an acoustic black hole (ABH). The initial thickness (h_0), end thickness (h_1), power of the taper (n), and length of the taper (x_L) define the performance of the ABH and are used as design variables during optimization.

The design variables to be varied in the optimality search are the initial thickness (h_0), end thickness (h_1), power of the taper (n), and taper length (x_L). These design variables are shown graphically in Fig. 1. The search was limited to be within the upper and lower limit of each design variable as listed in Table 1. The design variables were then scaled to be between 0 and 10 to prevent bias caused by different search ranges of each design variable. An initial population size of 100 was used and epsilon, a parameter to determine dominance of candidate solutions, was set to 0.01.

The goal is to minimize the frequency above which the total reflection coefficient (R) of the ABH is small and the NWV is also small. The total reflection coefficient, a metric describing the overall performance of the ABH, was calculated by integrating the imaginary portion of the wavenumber over the taper length.[1,2] A value of $R < 0.4$ was used to show effective performance of the ABH. For the second objective, Feurtado and Conlon established the minimal NWV value of 0.4 to ensure that the ABH theory has not been violated.[6] Thick beam theory was used to compute the complex bending wavenumber with aluminum material properties and a constant damping loss factor of 0.1. The damping loss factor of 0.1 was used to simulate the damping achieved by a viscoelastic damping layer. R was then computed using trapezoidal integration of the wavenumber and NWV was computed with a second-order finite difference approximation of the change in wavenumber over the frequency range of 0–15 kHz with 5 Hz spacing. Additional details regarding the computation of NWV and R can be found in Ref. 5.

3. Results

Optimization was executed for 30 000 function evaluations and the estimated Pareto optimality front is shown is Fig. 2. The knee in the curve shows when both objectives are equally weighted. This occurs at a frequency of 360 Hz indicating the lowest frequency where both the total reflection coefficient and the normalized wavenumber variation are less than 0.4. At the extreme ends of the curve, optimality in one objective function is found at the expense of an increased value of the other objective.

The four design variables are each a solution of the non-dominated set and are shown in Figs. 3 and 4. The solutions that consider mostly NWV are shown on the left while the solutions that consider mostly R are shown on the right. The knee region is designated by the red box. The initial height is found to be constant at it is lowest possible value 0.5 cm, while the final height stays at its lower bound (0.001 cm) except at the far left. The taper length is constant at its upper limit while the taper power increases gradually from left to right. The taper power at the knee is 2.75. This indicates that almost all of the competition between R and NWV relates to the power

Table 1. The upper and lower limits of the design variables.

Design variable	Lower limit	Upper limit
Initial thickness	0.5 cm	5 cm
End thickness	1.0×10^{-3} cm	1.0×10^{-2} cm
Taper power	2	10
Taper length	1 cm	20 cm

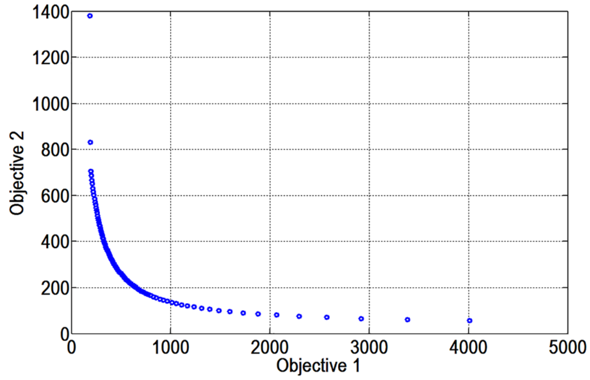

Fig. 2. (Color online) The Pareto optimality front showing the competing nature of the two objectives. Objective 1 is the lowest frequency where NWV < 0.4 while objective 2 is the lowest frequency where $R < 0.4$. The knee in the curve occurs at 360 Hz.

of the taper. However, the other variables still contribute to the overall prediction of the ABH.

4. Discussion

As previously mentioned, the optimization results show that the taper power contains the competing information between total reflection coefficient and normalized wavenumber variation. By estimating the Pareto optimality curve, a taper can now be designed to emphasize reduction in wavespeed with less consideration of NWV or alternatively to consider a more conservative design that falls entirely within the valid theoretical realm. In other words, the optimal taper can be tailored to the situation.

Another interesting note from the optimization results is that the initial and final beam height should be small. A small initial thickness creates smaller wavelengths at the entry into the taper such that the bending wavelength equals the taper length at a lower frequency. However, the bounds set on these design variables during optimization, listed in Table 1, keep the initial/final thickness from becoming too small/large such that the design is unreasonable.

The optimization also shows that having the longest taper is optimal. This result has been previously shown anecdotally.[1] As the wave travels along the taper and the wavespeed slows, the wavenumber increases. The higher wavenumbers have shorter wavelengths and will have more cycles to dissipate energy.

The optimization results also show that ABH vibration reduction can be achieved at frequencies lower than expected by the wavelength smaller than the taper length limit ($\lambda_b < x_L$) defined by Conlon *et al.*[4] For the aluminum taper length of 20 cm, this occurs at 243 Hz. However, these results are achieved only when NWV is significantly violated (see objective 2 of Fig. 2). While the $\lambda_b < x_L$ criterion (or equivalently $ka > 2\pi$, where a is the taper length) is an important factor to consider, considering the taper length alone does not capture all relevant physics (i.e., the value of ka is the same for a high-power taper and a lower power taper).[6] Thus, to fully consider all aspects of ABH design, multi-objective optimization must be performed.

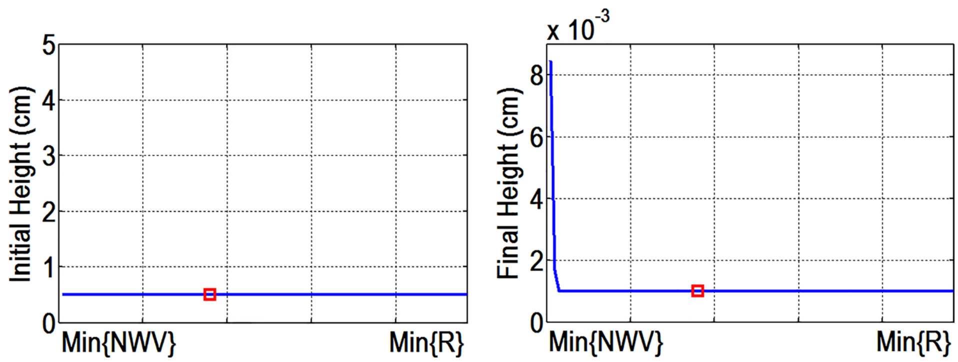

Fig. 3. (Color online) The initial and final heights for each point in the Pareto optimality front. The red box indicates the location where the knee exists in the curve.

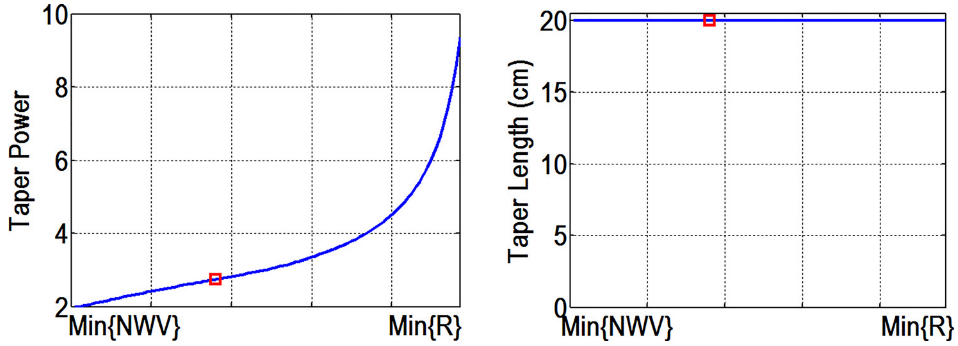

Fig. 4. (Color online) The taper power and length for each point in the Pareto optimality front. The box indicates the location where the knee exists in the curve.

5. Conclusion

Multi-objective optimization has been performed to address the conflicting design criteria for ABHs. The frequency of good performance of the ABH for vibration reduction as well as the requirement to maintain adequate smoothness were simultaneously reduced and the Pareto optimality curve was estimated using an evolutionary search framework. The knee region of this set indicates the best design possibilities for low frequency performance of the ABH. Overall, given the design space available for ABHs and the limited design evaluations that have been published, optimization shows significant promise for designing ABH vibration absorbers.

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