Quantum-electrodynamic treatment of photoemission by a single-electron wave packet

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A quantum-field-theory description of photoemission by a laser-driven single-electron wave packet is presented. We show that, when the incident light is represented with multimode coherent states then, to all orders of perturbation theory, the relative phases of the electron's constituent momenta have no influence on the amount of scattered light. These results are extended using the Furry picture, where the (unidirectional) arbitrary incident light pulse is treated nonperturbatively with Volkov functions. This analysis increases the scope of our prior results in [Phys. Rev. A **84**, 053831 (2011)], which demonstrate that the spatial size of the electron wave packet does not influence photoemission.

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I. INTRODUCTION

Recent advances in high-intensity laser physics have renewed interest in fundamental processes of quantum electrodynamics. Physicists can now probe new regimes of the theory, testing its framework as well as our intuition for its complexity. References [1,2] provide a summary of high-intensity processes of QED. In this paper, we examine the process of photoemission from a laser-driven electron wave packet, as an extension of our previous work in Ref. [3].

A brief summary of the question addressed in our previous paper is as follows: A classical description of radiation from a specified charge current density superposes the contributions from different spatial regions by use of the retarded Green function

$$A^{\mu}(x) = \frac{4\pi}{c} \int d^4x' G_{\mathrm{ret}}(x - x') J^{\mu}(x') \tag{1}$$

in the Lorenz Gauge [4]. In the classical case, radiation from different regions of the current density $J^{\mu}(x_1)$ and $J^{\mu}(x_2)$ can interfere at spacetime point x if $(x_1 - x)^2 = (x_2 - x)^2 = 0$. It is natural to wonder if this interference arises in the quantum mechanical problem where the source is a single (laser-driven) electron wave packet. If such interference exists in the quantum problem, one might expect its effects to become salient when the size of the electron wave packet spans a wavelength (or more) of the stimulating light.

Intuitively, one might want to replace the classical current density in Eq. (1) by the quantum probability current density (multiplied by the charge of the electron). The precedent for this line of thinking goes back to some of the founders of quantum mechanics, including Schrödinger [5], Gordon [6], and Klein [7]. By closely paralleling the classical-field solution, this model predicts interference akin to that obtained from a classical charge current [8–10].

In our previous paper, we emphasized that, in order for a first-quantized theory to match the lowest-order QED amplitude, one must perturb the laser-electron system with a fixed single-mode photon potential, representing a particular scattered photon. The single-electron transition amplitude is then reinterpreted as a probability amplitude for both the scattered electron and the scattered photon. From a classical-field perspective, this approach seems to suggest the repugnant idea that the scattered radiation is plane-wave in nature. Insights from QED, however, indicate that this reflects the incoherence of the electron's emitted radiation, rather than the particular form of the emitted field. Hence, this model predicts that emission from different spatial regions of the wave packet do not interfere. This conclusion supports those suggested by the fully quantized model of [11] and the numerical simulations of [12].

This "informed" semiclassical analysis only artificially connects to the framework and language of the fully quantized radiation theory. Moreover, it does not consider the possibility of multiphoton emissions and is restricted to the lowest order of perturbation theory. Interesting physics sometimes emerge beyond lowest-order calculations. The high-intensity case was considered only for an infinite-duration plane-wave field. This prevents one from conceptually specifying the spatial size of the electron wave packet *during* the interaction. In the analysis given here, we remove these shortcomings using the techniques of QED scattering theory.

Section II gives an overview of photon coherent states and provides the framework we use for counting scattered photons. We choose the stimulating light field to be a multimode coherent state representing an arbitrary unidirectional pulse. This choice allows for the conceptual possibility of phase-mismatching across the electron packet, as the induced oscillations of the probability current would carry the (reasonably) well-defined phase of the coherent state [13]. We choose the initial electron state to be an arbitrary superposition of momentum eigenstates. In Sec. III, we show how the disentangled initial state evolves into the final state via the scattering operator. To all orders of perturbation theory, the average number of scattered photons (detected in a different direction from that of the incident beam) does not depend on the relative phases of the momentum amplitudes of the initial electron state. The inevitable conclusion is that the size of the electron wave packet does not affect the detection of scattered photons.

In Sec. IV, we reexamine our results to lowest order in the Furry picture, which enables consideration of arbitrarily intense incident light. It is well known [2] that QED perturbation theory fails for ultra-intense light fields. In such cases, the stimulating light pulse must be treated nonperturbatively through quantization via Dirac Volkov states. The Furry picture treats the quantized light field as a perturbation against the backdrop of a classical high-intensity light field. We show that this method of quantization does not affect the validity of our conclusions. Our nonperturbative approach accommodates an *arbitrary* unidirectional light pulse. In Sec. V, we expand the analysis to all orders of perturbation theory and for all numbers of emitted photons in the Furry picture.

II. COUNTING SCATTERED PHOTONS

We begin by identifying two regions of k space that are of interest, depicted schematically in Fig. 1. We define the region V_{k_z} to contain photon momentum vectors comprising the incident light field, which propagates only along the $\hat{\mathbf{z}}$ direction. We define the region $V_{\mathbf{k}'}$ to contain photon momentum vectors that may be intercepted by a detector aligned off axis (blind to the incident light). The latter region need not be limited to a single ray emanating from the origin, as real photon detectors may subtend a nonvanishing solid angle. The regions V_{k_z} and $V_{\mathbf{k}'}$ should not be confused with the (position-space) quantization volume V.

Without loss of generality, we suppress spin and polarization indices. We also use scaled units such that $\hbar$ and c vanish from the equations. In calculating the amount of detected radiation, we are interested in the object

$$\langle N_{V_{\mathbf{k}'}}\rangle = \langle\Psi(t)|\sum_{V_{\mathbf{k}'}} a^{\dagger}_{\mathbf{k}'}a_{\mathbf{k}'}|\Psi(t)\rangle. \tag{2}$$

This quantity represents the expected number of photons scattered into the region $V_{\mathbf{k}'}$. The use of QED scattering theory will require the eventual limit that $t \to \infty$.

We write (2) in terms of traditional scattering amplitudes. In the space of states that includes a single electron and an arbitrary number of photons, we can resolve the identity as follows:

$$\begin{aligned}\mathbb{1} = \sum_{\mathbf{p}'}|\mathbf{p}'\rangle\langle\mathbf{p}'|\otimes\sum_{\{n_{\mathbf{k}}\}}|\{n_{\mathbf{k}}\}\rangle\langle\{n_{\mathbf{k}}\}| = \sum_{\mathbf{p}'}|\mathbf{p}'\rangle\langle\mathbf{p}'|\otimes\sum_{\{n_{k_z}\}}\Bigg(&\big|0_{\mathbf{k}''};\{n_{k_z}\}\big\rangle\big\langle 0_{\mathbf{k}''};\{n_{k_z}\}\big| + \sum_{\mathbf{k}''}\big|\mathbf{k}'';\{n_{k_z}\}\big\rangle\big\langle\mathbf{k}'';\{n_{k_z}\}\big|\\ &+\sum_{\mathbf{k}''}\sum_{\mathbf{k}'''}\big|\mathbf{k}'',\mathbf{k}''';\{n_{k_z}\}\big\rangle\big\langle\mathbf{k}'',\mathbf{k}''';\{n_{k_z}\}\big| + \cdots\Bigg),\end{aligned} \tag{3}$$

where $\{n_{k_z}\}$ represents a configuration of photons in modes $k_z \in V_{k_z}$, and it is understood that $\{\mathbf{k}'',\mathbf{k}''',\ldots\} \notin V_{k_z}$. This mixture of notations for modes in and out of V_{k_z} will prove useful in the scattering analysis, as it explicitly distinguishes newly scattered photons from those that were already present in the incident pulse. If we insert this identity between the creation and annihilation operators of (2) and note that $\mathbf{k}' \notin V_{k_z}$, we find that the detected photon number may be written as

$$\langle N_{V_{\mathbf{k}'}}\rangle = \sum_{\mathbf{p}'}\sum_{V_{\mathbf{k}'}}\sum_{\{n_{k_z}\}}\left[\big|\big\langle\mathbf{p}';\mathbf{k}';\{n_{k_z}\}\big|\Psi(t)\big\rangle\big|^2 + 2\big|\big\langle\mathbf{p}';2_{\mathbf{k}'};\{n_{k_z}\}\big|\Psi(t)\big\rangle\big|^2 + \sum_{\mathbf{k}''\neq\mathbf{k}'}\big|\big\langle\mathbf{p}';\mathbf{k}',\mathbf{k}'';\{n_{k_z}\}\big|\Psi(t)\big\rangle\big|^2 + \cdots\right]. \tag{4}$$

We see explicitly that the state $|\Psi(t)\rangle$ is projected onto a single basis vector *before* squaring and summing over the states of that basis. This is in agreement with the probability interpretation of quantum mechanics [14], where (4) is a weighted sum of the probabilities of scattering photons into the k-space region $V_{\mathbf{k}'}$.

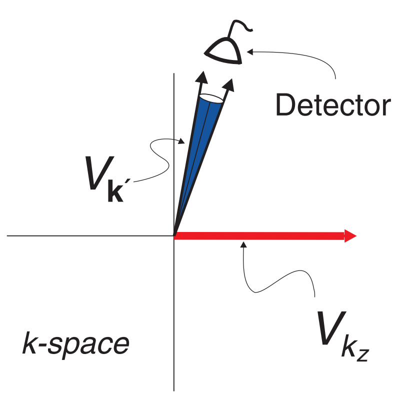

FIG. 1. (Color online) Depiction of k-space regions for the incident pulse (V_{k_z}) and photon detector ($V_{\mathbf{k}'}$).

We represent the incident laser field with a coherent state [15]. It is well known that coherent states, which satisfy

$$a_{\mathbf{k}_1}|\{\alpha_{\mathbf{k}}\}\rangle = \alpha_{\mathbf{k}_1}|\{\alpha_{\mathbf{k}}\}\rangle, \tag{5}$$

are the closest quantum analog of classical oscillating fields. Whereas the expectation value of the electric field operator vanishes for photon number states, coherent states can be constructed in such a way to match the field expectation value to any real-valued and well-behaved classical-field function. Such multimode coherent states are eigenvectors of the Maxwell field annihilation operator $A^{\nu(+)}(x)$, such that

$$\begin{aligned}A^{\nu(+)}(x)|\{\alpha_{\mathbf{k}}\}\rangle &= \sum_{\mathbf{k}_1}\frac{1}{\sqrt{2\omega_{k_1}V}}\alpha_{\mathbf{k}_1}\epsilon^{\nu}_{\mathbf{k}_1}e^{-ik_1\cdot x}|\{\alpha_{\mathbf{k}}\}\rangle\\ &= a^{\nu}(x)|\{\alpha_{\mathbf{k}}\}\rangle,\end{aligned} \tag{6}$$

where $a^{\nu}(x) \equiv \sum_{\mathbf{k}}\frac{1}{\sqrt{2\omega_k V}}\alpha_{\mathbf{k}}\epsilon^{\nu}_{\mathbf{k}}e^{-ik\cdot x}$ is the eigenvalue corresponding to the coherent state $|\{\alpha_{\mathbf{k}}\}\rangle$. We employ the Minkowski metric characterized by $(+1,-1,-1,-1)$. We see that $\{\alpha_{\mathbf{k}}\}$ is the set of Fourier coefficients of the corresponding classical field $a^{\nu}(x)$. There is, of course, some "quantum flesh on the classical bones" [16] that matches the uncertainty of the vacuum field.

III. SCATTERING OF COHERENT LIGHT STATES

We are now prepared to compute the average number of photons scattered to a detector that is aligned off axis to the incident photon beam. Let the initial state of the system be represented by the disentangled state

$$|\Psi_{\text{in}}\rangle = \left(\sum_{\mathbf{p}} \beta_{\mathbf{p}}|\mathbf{p}\rangle\right) \otimes \left|\{\alpha_{k_z}\}\right\rangle = \sum_{\mathbf{p}} \beta_{\mathbf{p}}\left|\mathbf{p};\{\alpha_{k_z}\}\right\rangle, \quad (7)$$

where $\{\alpha_{k_z}\}$ are chosen to represent a unidirectional light pulse. Note that only modes $k_z \in V_{k_z}$ are initially occupied in the light field. The coefficients $\beta_{\mathbf{p}}$ can be chosen to construct an arbitrary (potentially large) free-electron wave packet. We time-evolve this state in the interaction picture using the scattering operator S. The Dyson expansion is

$$S = \sum_{n=0}^{\infty} \frac{(-i)^n}{n!} \int d^4x_1 \cdots d^4x_n T[\mathcal{H}_{\text{int}}(x_1)\cdots\mathcal{H}_{\text{int}}(x_n)], \quad (8)$$

where

$$\mathcal{H}_{\text{int}}(x) = e : \bar{\psi}(x)\gamma_\mu\psi(x)A^\mu(x) : \quad (9)$$

is the normally ordered interaction Hamiltonian density, T is the time-ordering operator, and $\psi(x)$ and $A^\mu(x)$ are the standard free-field operators for electrons or positrons and photons, respectively [17].

As shown in Eq. (4), we must compute and then square amplitudes of the form $\langle \mathbf{p}';\mathbf{k}',\mathbf{k}'',\ldots;\{n_{k_z}\}|S|\Psi_{\text{in}}\rangle$, where primed wave vectors represent photons scattered outside of V_{k_z}. We emphasize that the parameters defining the bra are fixed before squaring. To properly characterize the Feynman diagrams that contribute to these amplitudes, we must examine the general framework (not the fine details) of the relevant Wick expansion of (8). Wick's theorem rewrites the time-ordered operator products in Eq. (8) as sums of normally ordered operator products. We find (after some algebra) that

$$\begin{aligned}
&\left\langle \mathbf{p}';\mathbf{k}',\mathbf{k}'',\ldots;\{n_{k_z}\}\right|S|\Psi_{\text{in}}\rangle \\
&= \sum_{n=2}^{\infty}\sum_{\mathbf{p}} \beta_{\mathbf{p}} \frac{(-ie)^n}{n!}\int d^4x_1\cdots d^4x_n \\
&\quad\times \sum_{\xi} C_\xi S_F\left(x_{\xi_1},x_{\xi_2}\right)\cdots S_F\left(x_{\xi_{n-1}},x_{\xi_n}\right) \\
&\quad\times \sum_{0\leqslant l\leqslant n-2}\sum_{\zeta} D\left(x_{\zeta_1},x_{\zeta_2}\right)\cdots D\left(x_{\zeta_{l-1}},x_{\zeta_l}\right) \\
&\quad\times \left\langle \mathbf{p}';\mathbf{k}',\mathbf{k}'',\ldots;\{n_{k_z}\}\right|\bar{\psi}^{(-)}\left(x_{\xi_n}\right) : A\left(x_{\zeta_{l+1}}\right)\cdots \\
&\quad\times A\left(x_{\zeta_n}\right) : \psi^{(+)}\left(x_{\xi_1}\right)\left|\mathbf{p};\{\alpha_{k_z}\}\right\rangle, \quad (10)
\end{aligned}$$

where ξ represents a particular set of $n-1$ contractions of fermion operators, ζ represents a set of contractions of an *even* number l of photon operators, and C_ξ contains all gamma matrices and any constants that arise from fermion contraction ξ. All polarization, spin, and spinor and gamma matrix indices have been suppressed. The functions $S_F(x,x')$ and $D(x,x')$ represent fermion and photon propagators, respectively. The photon propagators introduce radiative corrections, which, among other terms, require renormalization for explicit calculation. This does not affect our analysis. We note that (10) is valid only as an asymptotic series in n [18].

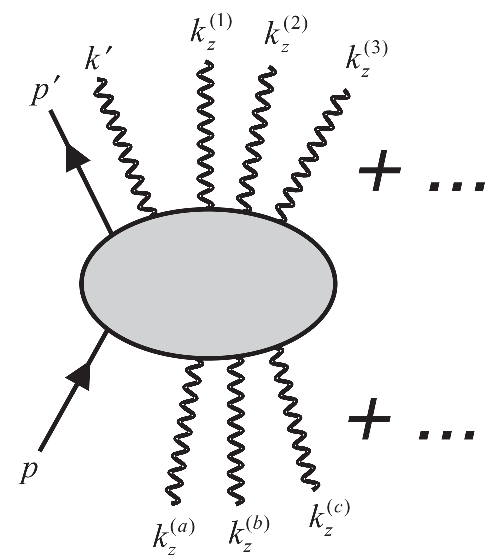

FIG. 2. Generic Feynman diagram showing possible external lines. Time runs upward.

We will not compute any terms of (10) explicitly, although a few comments are in order. Since $\mathbf{k}'$, $\mathbf{k}''$, etc. do not belong to V_{k_z}, there must be a creation operator $A^{(-)}(x_i)$ for every primed photon to "create" that state from the initial one (or else the amplitude would vanish from orthogonality between the bra and the ket). One can also show that all matter operators $\bar{\psi}$ and ψ must be contracted except for the two that annihilate and create the incoming and outgoing electron states; hence, there are $n-1$ fermion contractions. It can be shown kinematically that $\{\mathbf{k}',\mathbf{k}'',\ldots\} \notin V_{k_z}$ implies that $\mathbf{p}' \neq \mathbf{p}$ in nonvanishing diagrams. These arguments indicate that certain types of intuitively plausible Feynman diagrams vanish trivially. Figure 2 shows a generic nonvanishing Feynman diagram. The external lines referring to primed quantities are fixed before squaring, as demonstrated by (4).

For every field that is not contracted, there is an external particle line [19]. All $A^{(+)}(x)$ operators appear to the right, owing to normal ordering. Acting on the coherent state, they repeatedly pull out the eigenvalue

$$a^\nu(x) = \sum_{k_z\in V_{k_z}} \frac{1}{\sqrt{2\omega_{k_z}V}}\alpha_{k_z}\epsilon^\nu_{k_z}e^{-ik_z\cdot x}, \quad (11)$$

without changing the state. We note that each operator $A^{(+)}(x_i)$ produces a different sum $a^{\nu_i}(x_i)$ with its own summation index $k_z^{(i)}$. This feature will be important to our analysis. All $A^{(-)}$ operators appear on the left. Some of them produce the scattered photons $\mathbf{k}'$, $\mathbf{k}''$, etc., while the remainder produce photons that are forward scattered into V_{k_z}. In the usual manner, they contribute complex exponentials of the form

$$\frac{1}{\sqrt{2\omega_{k'}V}}e^{ik'\cdot x}, \quad \frac{1}{\sqrt{2\omega_{k''}V}}e^{ik''\cdot x}, \ldots \quad (12)$$

for photons scattered outside of V_{k_z}, and

$$\sum_{k_z \in V_{k_z}} g(\{n_{k_z}\}) \frac{1}{\sqrt{2\omega_{k_z} V}} e^{ik_z \cdot x} \tag{13}$$

for photons forward-scattered into V_{k_z}. The items (11)–(13) designate the external photon lines of Feynman diagrams. In typical low-order calculations, the external lines are determined uniquely by the initial state (ket) and the projection (bra). That is clearly not the case when considering coherent states, especially for high-order terms in the expansion. The electron may, in principle, absorb an arbitrary number of photons from V_{k_z} [dictated by the number of $A^{(+)}(x)$ operators in the product] or forward scatter as many photons as are allowed by the final projection onto $\langle\{n_{k_z}\}|$. In this important respect (owing to indeterminacy of photon number), our calculation differs from the packet-packet considerations of [20,21], who only considered narrow momentum distributions of *number* states. However, the kinematic principles have some similarities.

The integrations over $d^4x_1 \cdots d^4x_n$ produce delta functions that enforce energy-momentum conservation at every vertex. These delta functions allow for the evaluation of many of the momentum-space integrals that compose the electron and photon propagators in Eq. (10). When the smoke clears, there remains [for each summed term of (10)] a single four-dimensional delta function that enforces energy-momentum conservation of the external lines. (Three of the delta functions are of the Kronecker variety if we quantize in volume V, although this does not change the arguments that follow.) These kinematic constraints are well known and constitute one of the Feynman rules for evaluation of transition amplitudes [22]. Ignoring numerical factors, the complex exponentials in the previous paragraph indicate that (10) must include delta functions of the form

$$\begin{aligned}\delta^{(4)}\big(&p' + k' + k'' + \cdots + k_z^{(1)} \\ &+ k_z^{(2)} + \cdots - k_z^{(a)} - k_z^{(b)} - \cdots - p\big),\end{aligned} \tag{14}$$

where, as in Fig. 2, numerical superscripts indicate forward-emitted photons and letter superscripts indicate photons absorbed from the incident light. It appears, at first glance, that the square of the amplitude (10) might include cross terms between different electron momenta as well as different photon momenta, because a single four-delta cannot collapse the many sums in Eq. (10).

A careful examination of the kinematic constraints enforced by (14) demonstrates that the scattering does not depend on the relative phases of the momenta that compose the initial electron wave packet. We remind the reader that, in the amplitude (10), the momenta of all primed external lines (belonging to the bra) are fixed before the amplitude is squared. If the incident light pulse is unidirectional, then the kinematic constraints make the scattering amplitude (10) zero except when

$$\begin{gathered} p'_x + k'_x + k''_x + \cdots = p_x, \quad p'_y + k'_y + k''_y + \cdots = p_y, \\ p'_z + k'_z + k''_z + \cdots + k_z^{(1)} + k_z^{(2)} + \cdots = k_z^{(a)} + k_z^{(b)} + \cdots + p_z, \\ E_{\mathbf{p}'} + k' + k'' + \cdots + |k_z^{(1)}| + |k_z^{(2)}| + \cdots = |k_z^{(a)}| + |k_z^{(b)}| + \cdots + E_{\mathbf{p}}. \end{gathered} \tag{15}$$

The x and y constraints collapse two dimensions out of the sum over $\mathbf{p}$. Since the incident pulse is unidirectional, we have $k_z = |k_z|$ for all $k_z \in V_{k_z}$. Then both of the bottom two constraints contain the identical quantity $k_z^{(a)} + k_z^{(b)} + \cdots - k_z^{(1)} - k_z^{(2)} - \cdots$, which can be substituted between them. This results in

$$p'_z + k'_z + k''_z + \cdots = E_{\mathbf{p}'} + k' + k'' + \cdots - E_{\mathbf{p}} + p_z. \tag{16}$$

This constraint must be the same for *every* nonzero contribution to (10) (to all orders of perturbation theory), because the substitution of momenta from V_{k_z} can always be made for a unidirectional pulse. This final constraint, along with the simpler ones in the x and y directions, entirely determines the value of $\mathbf{p} = \bar{\mathbf{p}}$ for which the amplitude (10) is nonzero. Thus, kinematic constraints collapse the sum over $\mathbf{p}$, and the amplitude squared of (10) depends on $\beta_{\mathbf{p}}$ only via

$$\left|\langle \mathbf{p}'; \mathbf{k}', \mathbf{k}'', \ldots; \{n_{k_z}\}|S|\Psi_{\text{in}}\rangle\right|^2 \propto |\beta_{\bar{\mathbf{p}}}|^2. \tag{17}$$

That is, the relative phases of $\beta_{\mathbf{p}}$ have no influence on the scattered radiation.

The relative phases of $\beta_{\mathbf{p}}$ play a key role in determining the spatial size of an electron wave packet. A simple change of these phases such as

$$\beta_{\mathbf{p}} \rightarrow \beta_{\mathbf{p}} e^{-iE_{\mathbf{p}}T} \tag{18}$$

accounts for the natural quantum spreading experienced by a free particle during a time interval of duration T. This spreading can drastically change the spatial scale of a wave packet from being almost pointlike (relative to the wavelength of the stimulating field) to spanning many wavelengths. We have shown that such transformations have no effect on the scattered radiation; that is, size does not matter. This result holds for *all* emission configurations and to *all* orders of perturbation theory, instead of only to lowest order as done in our previous work [3].

Once the sum over $\mathbf{p}$ is collapsed, there remains only a single delta function. This delta function determines the precise value that

$$\Delta k_z \equiv k_z^{(a)} + k_z^{(b)} + \cdots - k_z^{(1)} - k_z^{(2)} - \cdots \tag{19}$$

must take for the amplitude to be nonzero. This suggests that absorption and reemission of multiple photons into V_{k_z} can effectively be treated kinematically as the absorption or emission of an single unidirectional photon of momentum Δk_z. Evidently the remaining delta function does not collapse all of the remaining sums over V_{k_z}. This indicates that the relative phases of α_{k_z} do matter. This result is unsurprising, however. The relative phases of α_{k_z} can determine the incident light's state of chirp, for example. Rearrangement of those phases can change the temporal profile of the pulse from short to long without changing the spectral content. This can drastically affect the instantaneous intensity experienced by the electron, thereby altering nonlinear radiative transitions [23,24].

If the incident light is not unidirectional, our analysis breaks down, and the photoemission indeed depends on the relative phases of $\beta_{\mathbf{p}}$. On physical grounds, we would expect this to be the case. A spatial translation such as

$$\beta_{\mathbf{p}} \to \beta_{\mathbf{p}} e^{-i\mathbf{p}\cdot\mathbf{r}_0} \tag{20}$$

could shift a wave packet into or out of the path of a focused laser [3]. Our choice of a unidirectional field is the only way to guarantee that the entire electron wave packet (large, small, or spatially translated) experiences the same incident light pulse.

IV. LASER-DRESSED PHOTOEMISSION

It is well known [2] that QED perturbation theory fails for ultra-intense fields. In such cases, the above arguments do not apply, and we must treat the incident light nonperturbatively. In the Furry picture of QED [25], we account for the intense (classical) light field A^{μ}_{ext} by requiring the matter field operator to satisfy

$$(i\gamma \cdot \partial - e\gamma \cdot A_{\text{ext}} - m)\psi_L = 0. \tag{21}$$

If A^{μ}_{ext} depends on x only via $\eta \equiv n \cdot x = x^0 - \hat{\mathbf{n}} \cdot \mathbf{x}$, then we may expand the quantized field operator $\psi_L(x)$ in the basis of Volkov functions $\{\psi^{v\pm}_{\mathbf{p}r}(x)\}$ instead of plane waves. Appendix gives explicit expressions for the laser-dressed matter field operator and its Volkov basis.

Scattering calculations proceed in much the same way as in regular perturbative QED. The interaction Hamiltonian (9) changes only in substituting the dressed operator ψ_L for the free-field operator ψ, as shown in Eq. (A4). We expand the scattering operator in a Dyson series, use Wick's theorem to produce sums of normally ordered operators (multiplied by propagators), and evaluate Furry-Feynman diagrams. The chief difference is that fermion lines must be calculated using Volkov functions instead of free-particle plane waves.

With the proper tools in hand, we now address the radiation of light from a laser-dressed electron wave packet. As in previous sections, we suppress spin and polarization indices. The initial electron state is given as a superposition of Volkov states individually denoted by $|\mathbf{p}\rangle$ (whereas in previous sections this ket denoted a free-particle state). Since the incident laser field is accounted for in the dressing of the Dirac field operator, the initial quantum state contains no photons [26]:

$$|\Psi_{\text{in}}\rangle = \left(\sum_{\mathbf{p}} \beta_{\mathbf{p}} |\mathbf{p}\rangle\right) \otimes |0_{\mathbf{k}'}\rangle = \sum_{\mathbf{p}} \beta_{\mathbf{p}} |\mathbf{p}; 0_{\mathbf{k}}\rangle. \tag{22}$$

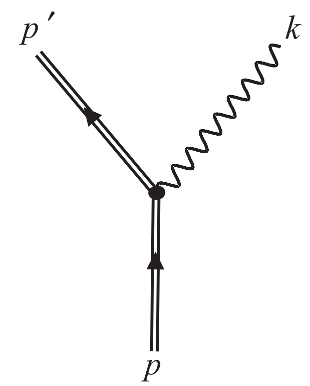

FIG. 3. Furry-Feynman diagram for photoemission from a laser-dressed electron. The double lines denote field-dressed electrons.

The lowest-order matrix element represents the emission of a single photon:

$$\begin{aligned}\langle \mathbf{k}'; \mathbf{p}'|S^{(1)}|\Psi_{\text{in}}\rangle &= \sum_{\mathbf{p}} \beta_{\mathbf{p}} \langle \mathbf{k}'; \mathbf{p}'|S^{(1)}|\mathbf{p}; 0_{\mathbf{k}'}\rangle \\ &= -ie \sum_{\mathbf{p}} \beta_{\mathbf{p}} \int d^4x \bar{\psi}^{v+}_{\mathbf{p}'}(x) \gamma_\mu \psi^{v+}_{\mathbf{p}}(x) \\ &\quad \times \frac{1}{\sqrt{2\omega_{k'} V}} \epsilon^{\mu}_{\mathbf{k}'} e^{ik' \cdot x}. \end{aligned} \tag{23}$$

The corresponding Furry-Feynman diagram is shown in Fig. 3. We note that, to lowest order, this matrix element is squared (with fixed $\mathbf{k}'$ and $\mathbf{p}'$) when computing the expectation of emitted photons in Eq. (4).

Most calculations make the assumption that $A^{\mu}_{\text{ext}}(x)$ is a single-mode plane-wave field. In contrast, we consider a (unidirectional) light *pulse* with arbitrary spectral content. This feature has the conceptual advantage of limiting the interaction time so that the particle does not have an infinite time interval during which it can spread. Hence, the spatial size of the wave packet during the interaction is well defined by (22). Treatments of arbitrary unidirectional pulses appear to be relatively new in the literature [23,27,28]. The approach introduced here makes overall energy-momentum conservation more transparent. The results also generalize naturally to arbitrary numbers of emitted photons and to all orders of perturbation theory, as outlined in the next section.

As usual, we let the intense light pulse propagate in the $\hat{\mathbf{z}}$ direction ($k_z > 0$ for all $k_z \in V_{k_z}$):

$$A^{\mu}_{\text{ext}}(x) = \sum_{k_z} A_{k_z} \epsilon^{\mu}_{k_z} \cos\left[k_z(t - z) + \phi_{k_z}\right], \tag{24}$$

where $\epsilon^{\mu}_{k_z}$ represents some nonscalar polarization orthogonal to $\hat{\mathbf{z}}$, and $A_{k_z} > 0$. Defining as before $\eta \equiv n \cdot x = t - z$, we have the unit propagation vector $n = (1,0,0,1)$.

We anticipate the appearance of kinematic delta functions in Eq. (23) that will collapse the sum over $\mathbf{p}$. To investigate this structure, we expand the Volkov functions as a series of complex exponentials. Ignoring the constant phase factor produced by the lower limit of integration in Eq. (A5), we find that the exponent becomes

$$\frac{1}{p\cdot n}\int^{\eta}\left(ep\cdot A_{\text{ext}}(\eta')-\frac{e^2}{2}A_{\text{ext}}^2(\eta')\right)=\frac{1}{p\cdot n}\left(e\sum_{k_z}A_{k_z}p\cdot\epsilon_{k_z}\int^{\eta}\cos\left(k_z\eta'+\phi_{k_z}\right)d\eta'\right.$$
$$\left.-\frac{e^2}{2}\sum_{k_z}\sum_{\tilde{k}_z}A_{k_z}A_{\tilde{k}_z}\epsilon_{k_z}\cdot\epsilon_{\tilde{k}_z}\int^{\eta}\cos\left(k_z\eta'+\phi_{k_z}\right)\cos\left(\tilde{k}_z\eta'+\phi_{\tilde{k}_z}\right)d\eta'\right). \qquad (25)$$

Evaluating these indefinite integrals yields

$$\frac{1}{p\cdot n}\left(e\sum_{k_z}\frac{A_{k_z}}{k_z}p\cdot\epsilon_{k_z}\sin\left(k_z\eta+\phi_{k_z}\right)+\frac{e^2}{8}\sum_{k_z}\frac{A_{k_z}^2}{k_z}\left[2\left(k_z\eta+\phi_{k_z}\right)+\sin 2\left(k_z\eta+\phi_{k_z}\right)\right]\right.$$
$$\left.-\frac{e^2}{4}\sum_{k_z}\sum_{\tilde{k}_z\neq k_z}A_{k_z}A_{\tilde{k}_z}\epsilon_{k_z}\cdot\epsilon_{\tilde{k}_z}\left\{\frac{\sin\left[(k_z-\tilde{k}_z)\eta+\phi_{k_z}-\phi_{\tilde{k}_z}\right]}{k_z-\tilde{k}_z}+\frac{\sin\left[(k_z+\tilde{k}_z)\eta+\phi_{k_z}+\phi_{\tilde{k}_z}\right]}{k_z+\tilde{k}_z}\right\}\right). \qquad (26)$$

Because $\eta\equiv n\cdot x$, the middle term $\frac{e^2}{4p\cdot n}\sum_{k_z}A_{k_z}^2\eta$ can be absorbed into the $p\cdot x$ term in the exponent of (A5) to produce $q\cdot x$, where we define the dressed momentum four-vector:

$$q^{\nu}\equiv p^{\nu}+\frac{e^2}{4p\cdot n}n^{\nu}\sum_{k_z}A_{k_z}^2. \qquad (27)$$

This is a natural generalization of single-mode [29,30] and double-mode [24,31] dressed momenta, but it is not necessarily a definitive expression for dressed momentum in all contexts. However, its suits our purposes here.

It can be algebraically shown that the integrand of (23) is proportional to

$$\bar{u}_{\mathbf{p}'}\left[1+\frac{e}{2p\cdot n}\gamma\cdot A_{\text{ext}}(\eta)\gamma\cdot n\right]\gamma\cdot\epsilon_{\mathbf{k}'}\left[1+\frac{e}{2p\cdot n}\gamma\cdot n\gamma\cdot A_{\text{ext}}(\eta)\right]u_{\mathbf{p}}e^{i(q'+k'-q)\cdot x}g_1(\eta)g_2(\eta)g_3(\eta)g_4(\eta), \qquad (28)$$

where

$$g_1(\eta)\equiv\exp\left[-i\sum_{k_z}\frac{e\alpha A_{k_z}}{k_z}\sin\left(k_z\eta+\phi_{k_z}\right)\right],\quad g_2(\eta)\equiv\exp\left[-i\sum_{k_z}\frac{\beta e^2A_{k_z}^2}{8k_z}\sin 2\left(k_z\eta+\phi_{k_z}\right)\right],$$
$$g_3(\eta)\equiv\exp\left\{i\sum_{k_z}\sum_{\tilde{k}_z\neq k_z}\frac{e^2\beta A_{k_z}A_{\tilde{k}_z}\epsilon_{k_z}\cdot\epsilon_{\tilde{k}_z}}{4(k_z-\tilde{k}_z)}\sin\left[(k_z-\tilde{k}_z)\eta+\phi_{k_z}-\phi_{\tilde{k}_z}\right]\right\}, \qquad (29)$$
$$g_4(\eta)\equiv\exp\left\{i\sum_{k_z}\sum_{\tilde{k}_z\neq k_z}\frac{e^2\beta A_{k_z}A_{\tilde{k}_z}\epsilon_{k_z}\cdot\epsilon_{\tilde{k}_z}}{4(k_z+\tilde{k}_z)}\sin\left((k_z+\tilde{k}_z)\eta+\phi_{k_z}+\phi_{\tilde{k}_z}\right)\right\},$$

and

$$\alpha\equiv\frac{p\cdot\epsilon_{k_z}}{p\cdot n}-\frac{p'\cdot\epsilon_{k_z}}{p'\cdot n},\quad \beta\equiv\frac{1}{p\cdot n}-\frac{1}{p'\cdot n}. \qquad (30)$$

We may expand the $g_i(\eta)$ further using the generating function of Bessel functions [32]

$$e^{iz\sin\theta}=\sum_{m=-\infty}^{\infty}J_m(z)e^{im\theta}, \qquad (31)$$

where the $J_m(z)$ are standard Bessel functions. We find that

$$g_1(\eta)=\prod_{k_z}\left[\sum_{\ell}J_{\ell}\left(\frac{e\alpha A_{k_z}}{k_z}\right)e^{-i\ell(k_z\eta+\phi_{k_z})}\right],\quad g_2(\eta)=\prod_{k_z}\left[\sum_{m}J_m\left(\frac{e^2\beta A_{k_z}^2}{8k_z}\right)e^{-i2m(k_z\eta+\phi_{k_z})}\right],$$
$$g_3(\eta)=\prod_{k_z}\prod_{\tilde{k}_z\neq k_z}\left[\sum_{r}J_r\left(\frac{e^2\beta A_{k_z}A_{\tilde{k}_z}\epsilon_{k_z}\cdot\epsilon_{\tilde{k}_z}}{4(k_z-\tilde{k}_z)}\right)e^{ir[(k_z-\tilde{k}_z)\eta+\phi_{k_z}-\phi_{\tilde{k}_z}]}\right], \qquad (32)$$
$$g_4(\eta)=\prod_{k_z}\prod_{\tilde{k}_z\neq k_z}\left[\sum_{s}J_s\left(\frac{e^2\beta A_{k_z}A_{\tilde{k}_z}\epsilon_{k_z}\cdot\epsilon_{\tilde{k}_z}}{4(k_z+\tilde{k}_z)}\right)e^{is[(k_z+\tilde{k}_z)\eta+\phi_{k_z}+\phi_{\tilde{k}_z}]}\right].$$

To more easily distinguish between product expansions, we use a different summation index letter for each product expansion $g_i(\eta)$. We remark that $A^{\mu}_{\text{ext}}(\eta)$, as defined in Eq. (24), is also a sum of complex exponentials. Hence the entire integrand, as a function of x, is equivalent to products of sums of complex exponentials. We are now prepared to compute the integral over d^4x in Eq. (23).

The integrals over x and y are straightforward because the integrand depends on those variables only through

$$e^{i(q'+k'-q)\cdot x}. \tag{33}$$

This indicates that (23) is proportional to

$$\begin{aligned}&\delta(q'_x + k'_x - q_x)\delta(q'_y + k'_y - q_y)\\ &\quad = \delta(p'_x + k'_x - p_x)\delta(p'_y + k'_y - p_y),\end{aligned} \tag{34}$$

since the incident field only dresses the momentum in the direction of its propagation. These delta functions uniquely determine p_x and p_y in Eq. (23) in terms of p'_x, p'_y, k'_x, and k'_y—quantities that are fixed before the square is performed. That is, the sums over p_x and p_y collapse.

The integrals over z and t in Eq. (23) require more care, since $g_i(\eta)$ and $A_{\text{ext}}(\eta)$ also depend on these variables of integration. At first glance, it might appear that the sum over p_z in Eq. (23) will not fully collapse. However, integrating the sums of exponentials in Eqs. (28) and (32) produces pairs of delta functions that are just right to fully collapse the sum over p_z, the reason being that $g_i(\eta)$ and $A_{\text{ext}}(\eta)$ depend on z and t only via exponentials of $\eta = t - z$. The important point is that the arguments of individual delta-function pairs *share* $\{k_z\}$ *dependence that can be substituted between them.* When this is done, one of the delta functions becomes identical for all pairs and can be factored out to collapse the sum over p_z.

To make this explicit, consider a generic exponential term of the integrand. We expand the products for each $g_i(\eta)$, enumerating k_z for $g_1(\eta)$ and $g_2(\eta)$, and enumerating pairs $(k_z,\tilde{k}_z)$ for $g_3(\eta)$ and $g_4(\eta)$. Before integration, the integrand contains terms of the form

$$e^{i(q'+k'-q)\cdot x}e^{-i(\ell_1 k_{z1}+\ell_2 k_{z2}+\cdots)\eta}e^{-i2(m_1 k_{z1}+m_2 k_{z2}+\cdots)\eta}e^{i[r_1(k_{z1}-k'_{z1})+r_2(k_{z2}-k'_{z2})+\cdots]\eta}e^{i[s_1(k_{z1}+k'_{z1})+s_2(k_{z2}+k'_{z2})+\cdots]\eta}. \tag{35}$$

If we define [33]

$$\begin{aligned}\Delta k_{z\{\ell_i,m_i,r_i,s_i\}} \equiv\ & (\ell_1 k_{z1} + \ell_2 k_{z2} + \cdots) + 2(m_1 k_{z1} + m_2 k_{z2} + \cdots) - r_1(k_{z1} - k'_{z1})\\ & - r_2(k_{z2} - k'_{z2}) - \cdots - s_1(k_{z1} + k'_{z1}) - s_2(k_{z2} + k'_{z2}) - \cdots,\end{aligned} \tag{36}$$

we find that (35) may be written compactly as

$$e^{i(q'+k'-q)\cdot x}e^{-i\Delta k_{z\{\ell_i,m_i,r_i,s_i\}}\eta}. \tag{37}$$

When integrated over z and t, the resulting delta functions are

$$\begin{aligned}&\delta\big(q'_0 + k' - q_0 - \Delta k_{z\{\ell_i,m_i,r_i,s_i\}}\big)\\ &\quad\times\delta\big(q'_z + k'_z - q_z - \Delta k_{z\{\ell_i,m_i,r_i,s_i\}}\big).\end{aligned} \tag{38}$$

As mentioned, we can solve for $\Delta k_{z\{\ell_i,m_i,r_i,s_i\}}$ in the argument of one of the delta functions and substitute that into the other delta function. One of the delta functions becomes

$$\delta(q'_0 - q'_z + k' - k'_z - q_0 + q_z). \tag{39}$$

The definition of dressed momentum q^{ν} in Eq. (27) indicates that (39) is equivalent to

$$\delta(E_{\mathbf{p}'} - p'_z + k' - k'_z - E_{\mathbf{p}} + p_z), \tag{40}$$

which is independent of the sums over $\{\ell_i,m_i,r_i,s_i\}$. Thus, p_z is uniquely determined from parameters that are fixed, and the sum over $\mathbf{p}$ in Eq. (23) is collapsed before squaring. This indicates that $|\beta_{\bar{\mathbf{p}}}|^2$ then neatly factors out of the detection probability. We conclude, as before, that the size of the electron wave packet does not matter. Notice that the delta functions enforce a constraint that agrees with the general result (16) obtained in the previous section by use of coherent states. We note that the constraints (34) and (40) can also be derived using lightcone coordinates, as in Ref. [27]. In fact, their approach is more suitable for calculating scattering spectra, particularly because it avoids the cumbersome expansions of Bessel functions. However, the approach we have taken parallels the low-intensity analysis in the previous section, as a net exchange of unidirectional laser photons (and the dressing of electron momenta) can be partially substituted out of energy-momentum conservation. Moreover, it also lends itself well to higher orders of perturbation theory and the emission of multiple photons, as will be discussed in the next section.

This exercise also confirms the previous result that the relative phases of momenta in the incident light, here denoted by $\{\phi_{k_z}\}$, do matter, as products of sums of these phases are different for every term. We argued in Sec. III that this is expected and does not affect our conclusion that radiation scattering is independent of the electron wave-packet size.

V. COMPLETE EXPANSION FOR LASER-DRESSED PHOTOEMISSION

The conclusions of the previous section generalize to higher orders of perturbation theory in the Furry picture. The full amplitude

$$\langle \mathbf{p}';\mathbf{k}'|S|\Psi_{\text{in}}\rangle \tag{41}$$

can be computed from the Furry-Feynman diagrammatic expansion shown in Fig. 4. We note that the higher-order terms of (41) introduce only internal particle lines, as the bra and ket have only 0 and 1 for occupation numbers. This is a beneficial consequence of treating the incident field nonperturbatively.

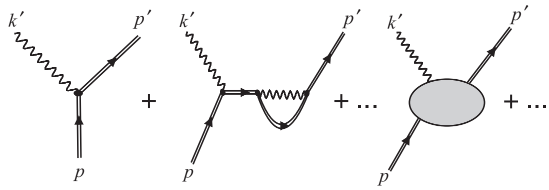

FIG. 4. Furry-Feynman diagrammatic expansion for the photoemission amplitude.

The presence of dressed field operators in the interaction Hamiltonian density (A4) changes the explicit calculation of internal fermion lines, but not the general structure thereof [34]. The dressed fermion propagator, a 4×4 matrix, is still computed as the time-ordered product of field operators

$$S_L(x,x') = \langle 0|T\psi_L(x')\bar{\psi}_L(x)|0\rangle, \tag{42}$$

where T is the time-ordering operator and $\psi_L(x)$ is defined by (A8). Inserting the expression for $\psi_L(x)$ yields

$$\begin{aligned} S_{L\alpha\beta}(x,x') &= \theta(t-t')\langle 0|\psi_{L\alpha}(x)\bar{\psi}_{L\beta}(x')|0\rangle \\ &\quad -\theta(t'-t)\langle 0|\bar{\psi}_{L\beta}(x')\psi_{L\alpha}(x)|0\rangle \\ &= \theta(t-t')\sum_{\mathbf{p}r}\psi^{v+}_{\mathbf{p}r\alpha}(x)\bar{\psi}^{v+}_{\mathbf{p}r\beta}(x') \\ &\quad -\theta(t'-t)\sum_{\mathbf{p}r}\psi^{v-}_{\mathbf{p}r\alpha}(x)\bar{\psi}^{v-}_{\mathbf{p}r\beta}(x'), \end{aligned} \tag{43}$$

where we have included spinor indices α and β. The laser-dressed electron propagator has received considerable attention in the literature [29,35,36].

The spacetime dependence of (43) is thus equal to a sum of products of two Volkov functions of identical parameters $\mathbf{p}$ and r, but different argument x. We showed in Sec. IV that products of Volkov functions can be expanded as sums of complex exponentials. In this case, the generic exponential term has the form

$$e^{\pm iq\cdot(x-x')}e^{i\Delta k_{z1}\eta}e^{i\Delta k_{z2}\eta'} \tag{44}$$

for some suitably chosen Δk_{z1} and Δk_{z2}. When these exponentials are integrated over d^4x and d^4x' in Eq. (10), kinematic delta functions appear. Hence, energy-momentum is still conserved at each vertex (where the dressed momentum q^ν represents the electron), the Δk_{zi} specifying a net exchange of laser photons between vertices. The overall energy-momentum conservation for the entire amplitude must take account of these local net exchanges with a global net exchange of laser photons. In the end, one may still define a global Δk_z that may be substituted away as done in connection with (38) and (39).

The conclusion is that the sum over $\mathbf{p}$ in higher-order amplitudes will always collapse to the same value $\bar{\mathbf{p}}$ (for a given bra), dictated by the delta functions (34) and (40). These same arguments also apply to amplitudes that reflect multiphoton emission since the external lines from scattered photons enter the kinematic constraints in the usual way, as shown in Eq. (15). Reference [37] computes the amplitude corresponding to Fig. 5, in which two photons are emitted by the (monochromatic) laser-dressed electron. In agreement

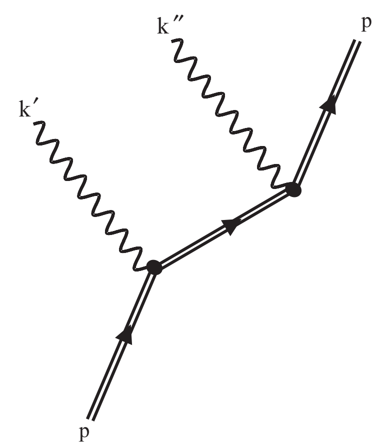

FIG. 5. Furry-Feynman diagram for the emission of two photons.

with our discussion, they find that the kinematic constraints predictably include the dressed momenta, emitted photons, and a global net exchange of laser photons.

We therefore conclude that, to all orders in a high-intensity picture, the detection of scattered photons does not depend on the phases of $\beta_{\mathbf{p}}$. This result holds for the emission of arbitrary numbers of radiative photons.

VI. SUMMARY

In summary, we have investigated the radiation scattering from a single-electron wave packet that is stimulated by a unidirectional pulse. In contrast to our previous work [3], the present analysis considers the possibility of interference at *all* orders of perturbation theory and for *all* numbers of emitted photons. We moreover treat intense-field stimulation with multimode pulses instead of single-mode plane waves, because this makes the electron wave packet's "size" less ambiguous during the interaction. This makes the present analysis more relevant to a high-intensity laser experiment [11].

When describing the incident light pulse as a coherent state, we showed that the relative phases of momenta that compose the initial electron wave packet have no influence on the scattered radiation, owing to energy-momentum conservation and Born's probability interpretation of quantum mechanics. This implies that wave-packet size cannot influence light scattering. We have shown this to be true even for high-intensity pulses of *arbitrary* spectral content, taking account of nonperturbative effects by working in the Furry picture of QED. On the other hand, classical electrodynamics dictates that the emissions from different regions of a charge current add coherently. The intuition gleaned from such classical scenarios clearly does not carry over to single-particle, quantum probability currents.

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APPENDIX: FURRY PICTURE OF QED

In the external-field approximation, we separate the interaction Lagrangian density as follows:

$$\mathcal{L}_{\rm int}(x) = -e\bar{\psi}(x)\gamma_\mu\psi(x)\big[A^\mu(x) + A^\mu_{\rm ext}(x)\big], \tag{A1}$$

where $A^\mu_{\rm ext}(x)$ represents the classical external potential (a c-number function) and $A^\mu(x)$ becomes the free photon field operator upon second quantization [34]. In the Furry picture [25], we absorb the interaction with the external field into the free electronic Lagrangian density:

$$\mathcal{L}_{\rm Dirac} = \bar{\psi}(i\gamma\cdot\partial - m)\psi \to \bar{\psi}_L(i\gamma\cdot\partial - e\gamma\cdot A_{\rm ext} - m)\psi_L. \tag{A2}$$

The quantized matter field operator must therefore satisfy

$$(i\gamma\cdot\partial - e\gamma\cdot A_{\rm ext} - m)\psi_L = 0. \tag{A3}$$

The interaction Hamiltonian density in this laser-dressed picture is

$$\mathcal{H}_{\rm int}(x) = e : \bar{\psi}_L(x)\gamma_\mu\psi_L(x)A^\mu(x) : . \tag{A4}$$

If $A^\mu_{\rm ext}(x)$ is a function of only $\eta = n\cdot x = x^0 - \hat{\mathbf{n}}\cdot\mathbf{x}$, then the Volkov functions $\psi^{v\pm}_{\mathbf{p}r}$ [38] are an orthonormal solution basis for (21) [36,39]. Explicitly, these c-number solutions are

$$\begin{aligned}\psi^{v+}_{\mathbf{p}r}(x) &= \sqrt{\frac{m}{VE_p}}\left[1 + \frac{e}{2p\cdot n}\gamma\cdot n\gamma\cdot A_{\rm ext}(\eta)\right]u_{\mathbf{p}r}e^{-ip\cdot x - i\int_{-\infty}^{\eta}S_p(\eta')d\eta'},\\ \psi^{v-}_{\mathbf{p}r}(x) &= \sqrt{\frac{m}{VE_p}}\left[1 - \frac{e}{2p\cdot n}\gamma\cdot n\gamma\cdot A_{\rm ext}(\eta)\right]v_{\mathbf{p}r}e^{ip\cdot x - i\int_{-\infty}^{\eta}S_{-p}(\eta')d\eta'},\end{aligned} \tag{A5}$$

where

$$S_p(\eta') = \frac{ep\cdot A_{\rm ext}(\eta')}{p\cdot n} - \frac{e^2A_{\rm ext}(\eta')\cdot A_{\rm ext}(\eta')}{2p\cdot n}, \tag{A6}$$

and $u_{\mathbf{p}r}$ and $v_{\mathbf{p}r}$ are Dirac spinors satisfying

$$(\gamma\cdot p - m)u_{\mathbf{p}r} = 0, \quad (\gamma\cdot p + m)v_{\mathbf{p}r} = 0. \tag{A7}$$

for $p^0 > 0$. We expand the laser-dressed matter field operator $\psi_L(x)$ in creation and annihilation operators of Volkov functions, rather than free-particle plane waves:

$$\psi_L(x) = \sum_{\mathbf{p}r}\big[b_{\mathbf{p}r}\psi^{v+}_{\mathbf{p}r}(x) + d^\dagger_{\mathbf{p}r}\psi^{v-}_{\mathbf{p}r}(x)\big]. \tag{A8}$$

We then impose the standard fermionic anticommutation relation, $\{\psi_{L\alpha}(\mathbf{x},t),\pi_{L\beta}(\mathbf{x}',t)\} = i\delta_{\alpha\beta}\delta^3(\mathbf{x}-\mathbf{x}')$, between the new field operator and its conjugate momentum π_L. The creation (annihilation) operators in Eq. (A8) create (annihilate) particles in Volkov states. These operators also satisfy the usual anticommutation relations pertinent to fermions.

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