Photoemission of a Single-Electron Wave Packet in a Strong Laser Field

Justin Peatross,[1,2,*] Carsten Müller,[1] Karen Z. Hatsagortsyan,[1] and Christoph H. Keitel[1]

[1]*Max-Planck-Institut für Kernphysik, Saupfercheckweg 1, D-69117 Heidelberg, Germany*

[2]*Dept. of Physics and Astronomy, Brigham Young University, Provo, Utah 84602, USA*

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The radiation emitted by a single-electron wave packet in an intense laser field is considered. A relation between the exact quantum formulation and its classical counterpart is established via the electron's Wigner function. In particular, we show that the wave packet, even when it spreads to the scale of the wavelength of the driving laser field, cannot be treated as an extended classical charge distribution, but rather behaves as a pointlike emitter carrying information on its initial quantum state. We outline an experimental setup dedicated to put this conclusion to the test.

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The availability of super-intense lasers has stimulated interest in relativistic electron dynamics in strong driving fields [1]. Experimenters have observed the effects of ponderomotive acceleration, the Lorentz drift, and plasma wake fields through direct detection of electrons ejected from an intense laser focus. Photoemission from relativistically driven plasmas, in particular, nonlinear Thomson scattering [2], has also been studied.

Much theory and computational effort has been devoted to the dynamics of free-electron wave packets driven by intense fields [3–5] and the associated scattered radiation [4,6,7]. Coherent emission from many electrons can be viewed in the forward direction with the emerging laser beam. Here, we consider incoherent photoemission by free electrons out the side of a focused laser, as a means of studying electron dynamics.

A free-electron wave packet with an initial spatial size on the scale of an atom undergoes natural quantum spreading, which eventually reaches the scale of an optical wavelength, as illustrated in Fig. 1 [5]. Moreover, an electron wave packet born through field ionization is pulled from its parent atom at a finite rate, typically emerging over multiple laser cycles. This, combined with the Lorentz drift and sharp ponderomotive gradients found in a tight relativistic laser focus, can cause a single-electron wave packet to be strewn throughout a volume several laser wavelengths across [7].

The question naturally arises as to how a single-electron wave packet radiates, especially when it undergoes such highly nondipole dynamics, where different parts of the electron wave packet experience entirely different phases of a stimulating laser field. Against the background of the numerous successes of semiclassical strong-field physics, the problem has been treated so far within an intuitively appealing model [6,7] where the quantum probability current is multiplied by the electron charge to produce an extended current distribution used as a source in Maxwell's equations [8]. The intensity computed classically from the extended current distribution is then associated with the probability of measuring a photon. Because of interference, this approach can lead to dramatic suppression of radiation for many directions and to a substantial loss in the overall scattered-light energy (see Fig. 2) [9]. It is interesting to note that A. H. Compton initially proposed a "large electron" model to explain the decrease in cross section with angle for Compton scattering of harder x rays, which he later abandoned when the effect of momentum recoil was understood [10].

In this Letter, we provide a fully quantum-mechanical treatment of photoemission by a single-electron wave packet in a laser field and relate it to a classical description via the electron's Wigner function. We show that no interference occurs between emission from different parts of an initially Gaussian wave packet, even if spatially large. In a plane-wave driving field, this result holds for wave packets of any shape and size. The radiative response can be mimicked by the incoherent emission of a classical ensemble of point charges. We outline an experimental arrangement able to probe the single-electron emission behavior by combining methods from strong-field physics and quantum optics.

Quantum electrodynamics provides the general framework to calculate the radiation from a single-electron wave packet. The radiated intensity is proportional to the expec-

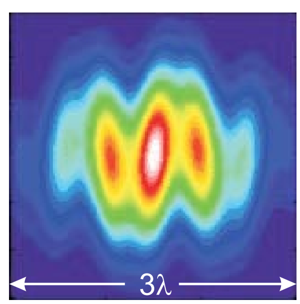

FIG. 1 (color online). An electron wave packet via Klein-Gordon equation after natural spreading from an initially Gaussian-shaped size of 1 Å. The spreading takes places during 190 cycles in a plane wave with intensity 2×10^{18} W/cm^2 and wavelength $\lambda = 800$ nm.

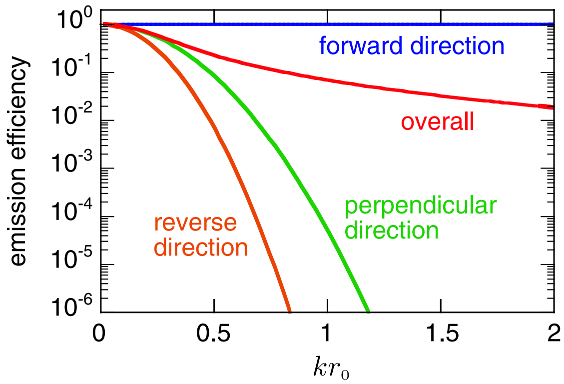

FIG. 2 (color online). Efficiency of light scattering into different directions as a function of size r_0 of a laser-driven classical Gaussian charge distribution associated with an electron wave packet. k is the laser wave vector. The forward emission does not vary from that of a single point oscillator with equal net charge. In the perpendicular direction, the intensity drops by orders of magnitude as the wave function grows to the scale of the wavelength or bigger.

tation value of the correlation function of the current density, rather than the expectation value of the current density of the wave packet [11]. From the definition of the spectral intensity of the emitted radiation $d\varepsilon_{\mathbf{k}'} = \frac{c}{4\pi^2}\cdot\langle\hat{\mathbf{H}}^{(-)}_{\omega'}\cdot\hat{\mathbf{H}}^{(+)}_{\omega'}\rangle R_0^2 d\Omega' d\omega'$, with radiation magnetic field $\hat{\mathbf{H}}^{(\pm)}_{\omega'} = i\mathbf{k}'\times\hat{\mathbf{A}}^{(\pm)}_{\omega'}$ and vector-potential $\hat{\mathbf{A}}^{(\pm)}_{\omega'} = \frac{ee^{ik'R_0}}{cR_0}\cdot\int d^4x\hat{\mathbf{J}}^{(\pm)}(x)e^{ik'x}$, together with the current-density operators $\hat{\mathbf{J}}^{(\pm)}$ in the Heisenberg representation, one obtains

$$\frac{d\varepsilon_{\mathbf{k}'}}{d\Omega' d\omega'} = \frac{e^2}{4\pi^2 c}\int d^4x\int d^4x' e^{ik'(x-x')} \times\langle[\mathbf{k}'\times\hat{\mathbf{J}}^{(-)}(x)]\cdot[\mathbf{k}'\times\hat{\mathbf{J}}^{(+)}(x')]\rangle. \quad (1)$$

Here, a superscript $(\pm)$ indicates the positive (negative) frequency parts of operators, R_0 the distance to the observation point from the coordinate center, $\mathbf{k}'$ and ω' the radiation wave vector and frequency, respectively, $d\Omega'$ the emission solid angle, e the electron charge, and c the speed of light. Equation (1) can be represented in a more familiar form via the transition current density $\mathbf{J}_{\mathbf{p}'i}(x)$ in the Schrödinger picture:

$$\frac{d\varepsilon_{\mathbf{k}'}}{d\Omega' d\omega'} = \frac{e^2}{4\pi^2 c}\int\frac{d^3p'}{(2\pi\hbar)^3}\left|\int d^4x[\mathbf{k}'\times\mathbf{J}_{\mathbf{p}'i}(x)]e^{ik'x}\right|^2. \quad (2)$$

In deriving Eq. (2), the relation between the Heisenberg $\hat{J}^{(\pm)}_\alpha(t)$ and Schrödinger $\hat{J}^S_\alpha$ representation via the time-evolution operator $U(t)$ is used: $\langle\hat{J}^{(-)}_\alpha(t)\hat{J}^{(+)}_\beta(t')\rangle = \langle\psi_i(t)|\hat{J}^S_\alpha U(t)U^{-1}(t')\hat{J}^S_\beta|\psi_i(t')\rangle = \int\frac{d^3p'}{(2\pi\hbar)^3}\langle\psi_i(t)|\hat{J}^S_\alpha|\psi_{\mathbf{p}'}(t)\rangle\langle\psi_{\mathbf{p}'}(t')|\hat{J}^S_\beta|\psi_i(t')\rangle$, and the notation $\langle\psi_{\mathbf{p}'}(t)|\hat{\mathbf{J}}^S|\psi_i(t)\rangle = \mathbf{J}_{\mathbf{p}'i}(x) = (\bar{\psi}_{\mathbf{p}'}(x)\boldsymbol{\gamma}\psi_i(x))$, where $\alpha,\beta\in\{1,2,3\}$. $\psi_i(x)$ is the initial electron wave packet in the driving field, $\psi_{\mathbf{p}'}(x)$ is a complete set of electron states in the driving field with an asymptotic momentum $\mathbf{p}'$, and $\boldsymbol{\gamma}$ are the Dirac matrices. Equation (2) indicates that the total probability of photon emission should be calculated as an incoherent sum over the final momentum states of the electron, even though in the experiment, the final electron momentum could be undetected.

First, we examine the contributions to the emitted radiation from the different momentum states of the initial electron wave packet $\psi_i(x) = \int d^3p\alpha(\mathbf{p})\psi_{\mathbf{p}}(x)$. Then, we consider contributions from different phase-space parts of the electron wave packet to the radiation.

In the case of a plane-wave driving laser field, Eq. (2) implies that there is no interference between the different momentum states of the electron wave packet. Here, $\psi_{\mathbf{p}}$ are the well-known Volkov states. Consider for simplicity the low-intensity case (i.e., one-photon Thomson scattering), where the emitted intensity adopts the structure $d\varepsilon_{\mathbf{k}'}\propto|\int d^3p\alpha(\mathbf{p})f(\mathbf{p})\delta(\varepsilon_{p'}+\hbar\omega'-\varepsilon_p-\hbar\omega)\delta(\mathbf{p}'+\hbar\mathbf{k}'-\mathbf{p}-\hbar\mathbf{k})|^2$, with $\varepsilon_p = c\sqrt{\mathbf{p}^2+m^2c^2}$ and a function $f(\mathbf{p})$ accounting for scalar products of Dirac spinors and matrices. The energy-momentum-conserving δ functions stemming from the space-time integral in Eq. (2), require a particular initial electron momentum $\mathbf{p}$ for given final electron and photon momenta $\mathbf{p}'$ and $\mathbf{k}'$ (i.e., there is a unique quantum path). The emitted intensity thus becomes an incoherent sum

$$\frac{d\varepsilon_{\mathbf{k}'}}{d\Omega' d\omega'} = \int d^3p|\alpha(\mathbf{p})|^2\frac{d\varepsilon_{\mathbf{k}'}(\mathbf{p})}{d\Omega' d\omega'} \quad (3)$$

over the contributions $d\varepsilon_{\mathbf{k}'}(\mathbf{p})$ emitted from a specific initial $\mathbf{p}$ state [12]. One can show that this argument also holds in the case of multiphoton Thomson scattering in a plane-wave field.

The situation is more complicated in a focused laser beam when the driving field contains a distribution of initial photon momenta $\mathbf{k}$. In this case, interference in the emitted radiation in principle is possible. If, for example, the initial electron is in a superposition of two momentum states $|\mathbf{p}_1\rangle$ and $|\mathbf{p}_2\rangle$, then the final electron state $|\mathbf{p}'\rangle$ with emission of a photon of certain momentum $\mathbf{k}'$ can be reached by two indistinguishable paths: either from the state $|\mathbf{p}_1\rangle$ or from $|\mathbf{p}_2\rangle$ by absorption of different photons $\mathbf{k}_{1,2} = \mathbf{k}' + (\mathbf{p}'-\mathbf{p}_{1,2})/\hbar$ from the external field, giving rise to interference. However, in the case of a Gaussian wave packet, interferences of different momentum states are suppressed by the continuous spectrum of the initial electron momenta, which allow for many paths whose interfering contributions largely cancel out.

Although Eq. (2) shows the general way to calculate the emission intensity, it is difficult to apply in a real experimental situation, as the quantum eigenstates of the electron in a focused laser beam are usually unknown. It is therefore desirable to mimic the quantum electrodynamical result of Eq. (2) by means of classical electrodynamical calculations (i.e., in the quasiclassical limit, when the photon energy is much smaller than the electron rest energy and recoil effects are negligible). We illustrate how to do this by way of the example of one-photon Thomson scattering in a focused laser beam. For the initial wave packet defined by the momentum distribution coefficients $\alpha(\mathbf{p})$, one can express the quantum-mechanical formula for spectral intensity of Eq. (2) via the Wigner function $\rho_W(\mathbf{r},\mathbf{p}) = \int d^3q\alpha(\mathbf{p}+\mathbf{q}/2)\alpha^*(\mathbf{p}-\mathbf{q}/2)e^{(i/\hbar)\mathbf{q}\cdot\mathbf{r}}$ of the initial electron wave packet as follows:

$$\frac{d\varepsilon_{\mathbf{k}'\lambda'}}{d\Omega' d\omega'} = \frac{e^2\omega^2}{4\pi^2c^3}\int d^3r\int d^3p\rho_W(\mathbf{r},\mathbf{p})M_{\mathbf{k}'\lambda'}(\mathbf{r},\mathbf{p}), \quad (4)$$

where $M_{\mathbf{k}'\lambda'}(\mathbf{r},\mathbf{p})$ gives the contribution in radiation by an electron of momentum $\mathbf{p}$, with $M_{\mathbf{k}'\lambda'}(\mathbf{r},\mathbf{p}) = (2\pi\hbar)^2\int d^3k\int d^3\tilde{k}e^{i(\mathbf{k}-\tilde{\mathbf{k}})\cdot\mathbf{r}}A_0(\mathbf{k})A_0^*(\tilde{\mathbf{k}})l_{\lambda'}(\mathbf{p}_+,\mathbf{p}',\mathbf{k},\mathbf{k}')\times l_{\lambda'}^*(\mathbf{p}_-,\mathbf{p}',\tilde{\mathbf{k}},\mathbf{k}')\delta(\varepsilon_{p_+}+\hbar\omega-\varepsilon_{p'}-\hbar\omega')\delta(\varepsilon_{p_-}+\hbar\tilde{\omega}-\varepsilon_{p'}-\hbar\omega')$; $l_{\lambda'}(\mathbf{p},\mathbf{p}',\mathbf{k},\mathbf{k}')$ is defined via $\int d^4xe^{ik'x}(\mathbf{e}_{\lambda'}^*\mathbf{J}_{p'p}) = (2\pi\hbar)^4\int d^3kA_0(\mathbf{k})l_{\lambda'}(\mathbf{p},\mathbf{p}',\mathbf{k},\mathbf{k}')\times\delta^{(4)}(p+\hbar k-p'-\hbar k')$, $\mathbf{p}_\pm = \mathbf{p}\pm\hbar(\tilde{\mathbf{k}}-\mathbf{k})/2$, $\mathbf{p}' = \mathbf{p}+\hbar(\tilde{\mathbf{k}}+\mathbf{k})/2-\mathbf{k}'$, $\mathbf{e}_{\lambda'}$ is the polarization of the emitted photon, and $A_0(\mathbf{k})$ the Fourier component of the focused laser beam. In the case of a plane-wave driving field, Eq. (4) summed over the polarizations $\mathbf{e}_{\lambda'}$ reduces to Eq. (3) since $M_{\mathbf{k}'\lambda'}$ is $\mathbf{r}$-independent then, and $\int d^3r\rho_W(\mathbf{r},\mathbf{p})\propto|\alpha(\mathbf{p})|^2$.

When the Wigner function is non-negative (e.g., for a Gaussian wave packet), it may be interpreted as the initial electron distribution in phase space. The message of the structure of Eq. (4) is that the total photoemission probability is an *incoherent* sum over the contributions of each local phase-space element of the electron distribution. If, for example, the phase-space distribution consists of two separate parts: $\rho_W = \rho_W^{(1)} + \rho_W^{(2)}$, then the intensities emitted from each part incoherently add up to yield the total radiation intensity. In the quasiclassical limit, the radiation intensity $M_{\mathbf{k}'\lambda'}$ of an electron of momentum $\mathbf{p}$ can be directly related to the known classical electrodynamical result: $M_{\mathbf{k}'\lambda'}\approx|\mathcal{M}_{\mathbf{k}'\lambda'}(\mathbf{r},\mathbf{p})|^2$, with $\mathcal{M}_{\mathbf{k}'\lambda'}(\mathbf{r},\mathbf{p}) = 2\pi\hbar\int d^3ke^{i\mathbf{k}\cdot\mathbf{r}}A_0(\mathbf{k})l_{\lambda'}(\mathbf{p},\mathbf{p}',\mathbf{k},\mathbf{k}')\delta(\varepsilon_p+\hbar\omega-\varepsilon_{p'}-\hbar\omega')$; in particular, in the nonrelativistic limit, $l_{\lambda'}(\mathbf{p},\mathbf{p}',\mathbf{k},\mathbf{k}') = (\mathbf{e}_{\lambda'}^*\mathbf{e}_\lambda)$ with the polarization of the driving field $\mathbf{e}_\lambda$. Thus, Eq. (4) describes the incoherent *average* of the radiation intensity over the initial electron probability distribution in phase space. The radiation can thus be modeled by a classical ensemble of point emitters, taken individually.

A different situation arises when the Wigner function is negative in some phase-space region, indicating intrinsic quantum behavior. For instance, this happens when the initial electron is in a superposition of two momentum states. As shown above, this leads to interference in the emitted radiation. This effect is included in Eq. (4), but cannot be modeled by classical means here.

The quantum interference effects above have to be clearly distinguished, though, from the classical interference effects arising in the *coherent* part of the radiation mentioned in the introduction. The latter is calculated employing an ensemble average of the current operator $\langle\mathbf{J}(x)\rangle$ as a source for the expectation value of the radiated field $\langle\mathbf{E}\rangle$ (i.e., the scattering amplitude is first averaged over the initial electron distribution, and afterwards squared). This way, the incoherent field with a random phase is excluded [13]. Under such a scenario, different spatial parts of an electron wave packet give strongly interfering contributions to the coherent radiation (see Fig. 2). Regarding its interference structure in momentum space, we examine the coherent part of one-photon Thomson radiation from an electron wave packet in a plane laser wave, which reads $\frac{d\varepsilon_{\mathbf{k}'}^{(\mathrm{coh})}}{d\Omega' d\omega'} = \frac{e^2A_0^2\hbar^2}{4m^4c^3}|\int d^3p\alpha^*(\tilde{\mathbf{p}})\alpha(\mathbf{p})\delta(\varepsilon_{\tilde{p}}-\varepsilon_p+\hbar\omega'-\hbar\omega)\times[\mathbf{k}'\times(\frac{(\mathbf{p}+\hbar\mathbf{k})(\mathbf{e}\cdot\mathbf{p})}{\varepsilon_{p+\hbar k}-\varepsilon_p-\hbar\omega}+\frac{\mathbf{p}[\mathbf{e}\cdot(\mathbf{p}-\hbar\mathbf{k}')]}{\varepsilon_{p-\hbar k'}-\varepsilon_p+\hbar\omega'})]|^2$. It allows all transitions $\mathbf{p}\to\tilde{\mathbf{p}}\equiv\mathbf{p}+\hbar(\mathbf{k}-\mathbf{k}')$, which fulfill the energy-momentum conservation with the initial and final momentum states available in the initial wave packet, to interfere in the coherent radiation. This is in contrast to the total emitted radiation in Eq. (3), which is free from interference in a plane-wave driving field. The coherent radiation thus qualitatively differs from the total radiation. In the case of a single electron that we are interested in, the coherent radiation represents only a small part of the total radiation, but it becomes dominant in the case of $N\gg1$ radiating electrons that simultaneously interact with an applied field. As in the case of radiation from an N-atom ensemble [11], the intensity of the phase-matched coherent radiation is multiplied by a factor of $N(N-1)$, while the incoherent radiation by N, which becomes negligible. Therefore, the main contribution in nonlinear Thomson scattering in plasma [2] comes from the coherent radiation. The same holds for high-harmonic generation from many atoms (see, e.g., [14]). Note that, in fact, the "single-atom response" which is usually calculated using the expectation value of the electron acceleration, accounts for the coherent part of the high-harmonic radiation only [11,15].

Finally, we suggest that the radiation scattered from a single free electron in a laser is detectable by modern experimental techniques. The feasibility of seeing scattered light depends crucially on whether the electron wave packet radiates with the strength of a classical point-like electron, as argued above. For linear Thomson scattering, the rate of emitted photons is proportional to the stimulating intensity. A longer pulse duration, therefore, can compensate for lower intensity. Nevertheless, relativistic intensities can aid in spectral-discrimination: The Lorentz drift pushes the electron in the forward direction of the laser, causing the scattered fundamental light to be redshifted when viewed from the side.

We envision an electron ionized from an atom during the driving pulse or prepared by a suitable prepulse (e.g., the

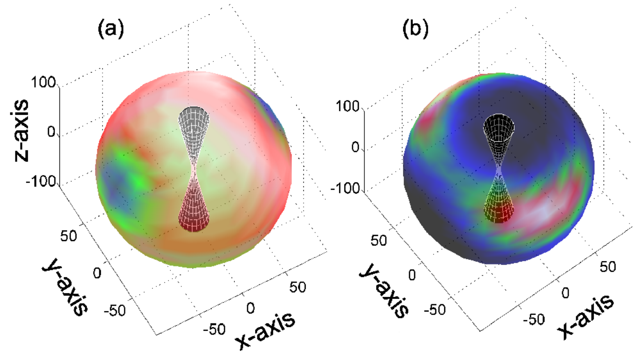

FIG. 3 (color online). (a) Far-field intensity (at 100 μm distance) of light scattered from a single electron trajectory born on axis during the early rising edge of an intense (10^{19} W/cm^2) laser pulse. Light (dark) [red (blue), respectively] scale indicates regions of high (low) intensity. The laser beam (mesh) has waist $w_0 = 3\lambda$. The total scattered energy is 0.24 eV. (b) Far-field intensity between 850 and 950 nm with total scattered energy 0.05 eV.

second electron stripped from He at $\sim 2 \times 10^{16}$ W/cm^2). Electrons given up by atoms during the leading edge of the pulse tend to be pushed out of the focus by the ponderomotive gradient. The density of donor atoms can be chosen such that on average one electron experiences the highest intensity with the accompanying forward drift and red shift. Keying in on this spectral signature may be critical for differentiating authentic scattered photon events from other noise sources. Fast timing in the photon-detection electronics can also suppress false signals, for example, delayed light scattered from the walls of the experimental chamber.

We computed a representative single classical electron trajectory in a tightly focused vector laser field with a peak intensity of 10^{19} W/cm^2, duration 35 fs, and wavelength 800 nm. Figure 3(a) shows the total radiated energy in the far field emitted from the electron as it is released on the rising edge of the pulse. The electron trajectory eventually exits the side of the focus due to ponderomotive gradients. Much of the scattered radiation emerges out the side of the laser focus. The total radiated energy (all angles and frequencies) from the single-electron trajectory is only $\sim$0.24 eV, indicating less than one photon per laser shot.

Figure 3(b) shows the spatial distribution of light with wavelengths falling between 850 nm and 950 nm. This accounts for approximately 20% of the total emitted energy. Assuming 10% collection efficiency, this would amount to an average 0.005 eV per shot, or one photon per 300 shots. This can of course be increased if more electrons are used, where the light radiated out the side of the focus adds incoherently for a random distribution. Intensities above 10^{19} W/cm^2 are not ideal for this experiment; a strong Lorentz drift redirects the photoemission into the far forward direction.

In conclusion, we have studied the amount of light that an electron scatters out the side of a laser focus. We have shown that individual electrons radiate with the strength of point emitters. The electron's initial quantum state is imprinted on the radiation spectrum via its Wigner function. In special cases, interference of different electron momentum components can arise, but it is qualitatively distinct from the classical interference in the coherent radiation of an extended charge distribution. Our results can be tested in an experiment that combines for the first time the sensitive techniques of quantum optics (e.g., single-photon detectors) with the traditionally incompatible discipline of high-intensity laser physics.

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*peat@byu.edu

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