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A comparison of fractional-sized to full-sized cellos

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Fractional-sized cellos (3/4, 1/2, etc.) are designed for the same musical playing range as a full-sized (4/4) cello but with scaled proportions for players for whom a full-sized cello is too large. The strings are adjusted in order to compensate for the shorter string length of the smaller instruments and obtain the correct tuning. The cello body vibration, which is strongly coupled to the internal air cavity, would not be expected to scale in the same manner as the strings. This causes the bridge impedance seen by the strings on the fractional-sized cellos to differ from the bridge impedance seen by the strings on a full-sized cello. In this paper, the physical dimensions of a 1/2 and 3/4 cello are compared with a full cello. Drive point measurements are also compared to illustrate how the strings couple differently with the body of each size cello. The fractional-sized cellos are found to exhibit a slightly different sound due to the bridge impedance mismatch.

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1. INTRODUCTION

Fractional sized cellos (3/4, 1/2, etc.) are designed for the same musical playing range as full (4/4) size cellos but have scaled proportions. These instruments are intended to accomodate physically smaller players such as young children. The cello is comparatively less well studied than other string instruments such as the violin and the guitar.[1] The body of literature on the acoustics of the cello is primarily focused on full sized instruments. The scaling of acoustic properties of stringed instruments has also been studied within the violin family. Hutchins has analyzed how the acoustical properties within the musical playing range of an instrument scale with instrument size for the violin, viola, cello, and bass.[2] As a result of that work, Hutchins designed a new family of intstruments, the violin octet, that covers the same musical range as the traditional violin family. The playing range and size of the instruments is scaled such that each member of the octet has acoustical properties similar to a violin. The baritone member of the octet is larger than a full size cello but shares the same musical playing range.

The widespread adoption of the traditional violin family of instruments has established the cello as an integral part of string ensembles and a common instrument for students to learn to play. Understanding how the acoustics of the cello scale for fractional sized instruments can inform the design of entry-level instruments and explain the range of musical sounds that a student can produce on a smaller instrument. This preliminary experimental investigation analyzes the acoustical properties over the fixed playing range of the cello as the instrument is reduced in size. It compares the scaling of fractional sized cello body vibration and air cavity resonance to that of a full sized instrument in order to understand the effects within the musical playing range of the instrument. The study examines a set of measurements conducted on a 4/4, a 3/4, and a 1/2 size cello. The physical dimensions of each instrument are compared to observe the ratios at which each scales. The measured dimensions are then used to estimate the Helmholtz resonance of the air cavity and determine how that resonance frequency scales with the instrument size. Next, the structural vibrations of the cellos are analyzed using drive-point mobility measured at a point on the top plate. A set of resonance and anti-resonance frequencies are found from the measured mobility and compared to the fundamental frequencies of the open strings for each cello. The open strings are found to couple to different resonant and anti-resonant behavior of the body as the cellos scale in size. Finally, the set of resonance frequencies are compared to a simplified model of a cello using plate theory. The resonance frequencies of the fractional sized cellos are found to scale with ratios consistent with plate theory.

2. DIMENSIONAL MEASUREMENTS

A set of five dimensions were measured on each cello in order to compare the physical size of the instruments. The first measurement, the scale length, was measured along the strings from the nut to the bridge. The scale length determines the musical playing range of the string that is tuned to a given tension. The remaining four measurements capture the physical dimensions of the cello's body. The width of the body was measured at both the upper and lower bouts. The length of the body was measured along the top of the cello. Finally, the depth of the body was measured at its greatest point. The acoustical and structural properties of the cello body determine the radiated sound of the cello over the musical playing range of the instrument.

The ratio of each of these dimensions relative to those on the full size cello are shown in Fig. 1. These results show that the dimensions on the face of the cello (the body length and width) scale with similar ratios to the strings but the body depth scales at a smaller ratio.

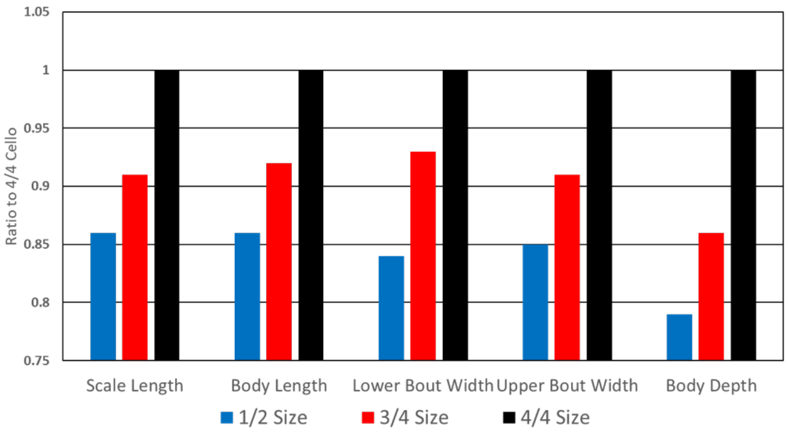

Figure 1: The ratio of physical dimensions of fractional sized cellos to a full size cello. The length and width of the cello body were found to scale at ratios similar to the scale length. The body depth scales with a smaller ratio.

3. AIR CAVITY HELMHOLTZ RESONANCE

Estimates of the cavity volume and the sound port area are required to estimate the Helmholtz resonance frequency of the body. The Helmholtz resonance frequency is a theoretical quantity in a cello and cannot be measured unless the body is mechanically blocked. The volume of the cello cavity was estimated as the surface area of the top plate, including the area of the f-holes, multiplied by the body depth at its greatest point. The surface area of the top plate was estimated by fitting two rectangles to the outer dimensions of the cello face and finding the proportion of each rectangle that contains the cello. A simple image processing algorithm that counts pixels was used to find the proportion of cello to non-cello area inside each rectangle. One rectangle was bound by the width of the cello at the upper bout and the length from the top of the cello body to the bridge. The second rectangle was bound by the width of the cello at the lower bout and the length from the bridge to the bottom of the cello body.

The surface area of the sound ports on each cello was estimated using graph paper and tracing. The outline of the sound port was traced onto graph paper with a pencil and the area was estimated by counting the number of whole and fractional squares circumscribed by the traced perimeter.

The ratios of the estimated cavity volume and the estimated sound port area for each cello relative to that of the full size cello is shown in Fig. 2. It is observed that the sound port area, S, and cavity volume, V, scale at similar ratios. The Helmholtz resonance can be found from

$$f = \frac{c}{2\pi}\sqrt{\frac{S}{V(L + end\ corr.)}} \tag{1}$$

where c is the speed of sound, and L is the thickness of the cello top plate.[3] As the cavity volume and sound port area scale at similar ratios they will together have little effect on the scaling of the Helmholtz resonance. The dominant factors in the scaling of the air cavity resonance will be the thickness of the cello top plate and the radiation impedance of the sound port, described in Eq. 1 as an end correction. The top plate thickness of each cello was similar (about 4.5 mm) but more investigation is required to understand how the reactive component of the radiation impedance for the f-hole geometry scales with cello size. The geometry of the f-hole complicates the accurate modeling of the radiation impedance.[4]

Estimates for the Helmholtz resonance of each cello, and the ratios of those frequencies to that of the full size cello, are shown in Fig. 3 where the radiation impedance of the f-holes is modeled as a baffled

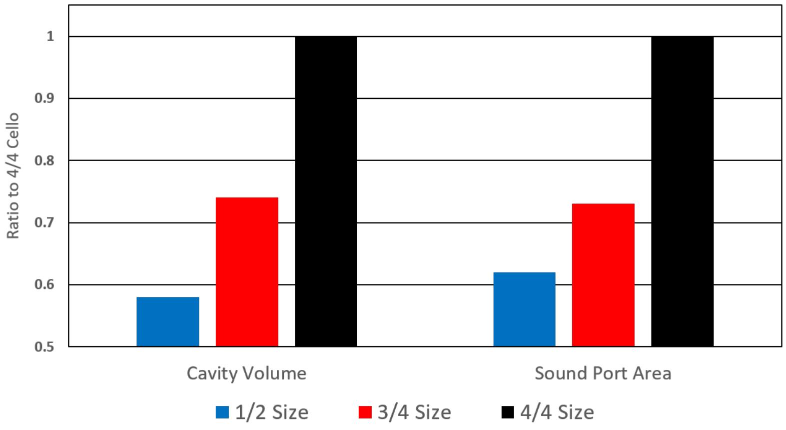

Figure 2: ***The ratio of estimated cavity volume and sound port area of fractional sized cellos to a full size cello. The estimated cavity volume and sound port area scale at similar ratios.***

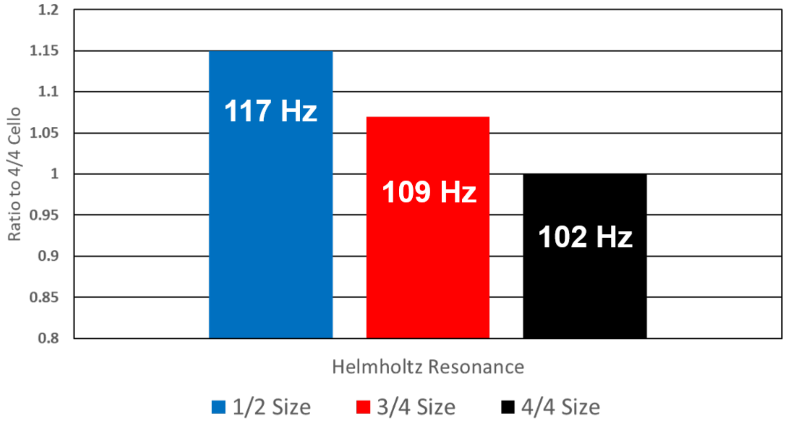

Figure 3: ***The estimated ratios of the Helmholtz resonance frequency for the cavity of three cello sizes. The estimated frequency in Hz for each size cello is shown inside the bars.***

open end of a pipe with the same radiating area. The estimates of the Helmholtz resonance frequencies are consistent with previous measurements[2] and follow the expected trend that the resonance frequency increases for decreasing cello size.

4. DRIVE POINT MOBILITY

The drive point mobility of each cello was measured to analyze the modal behavior of the body. After damping the strings with foam, the cello was tapped with an impact hammer and the response measured with an accelerometer at a point on the cello top plate between the lower bout and the tailpiece. The mechanical mobility vs. frequency for the full size cello is shown in Fig. 4 and the relevant modes to this study are highlighted. The modes in a cello are sometimes mislabeled because the relative positions in frequency differ from other instruments in the violin family.[2] The modes in Fig. 4 are identified by their nominal behavior in order to avoid confusion due to nomenclature.

Figure 5 compares the drive point mobility for the three cello sizes. The mobility of each cello is plotted in decibels relative to an arbitrary value that allows the plots to be offset vertically for visual clarity. The

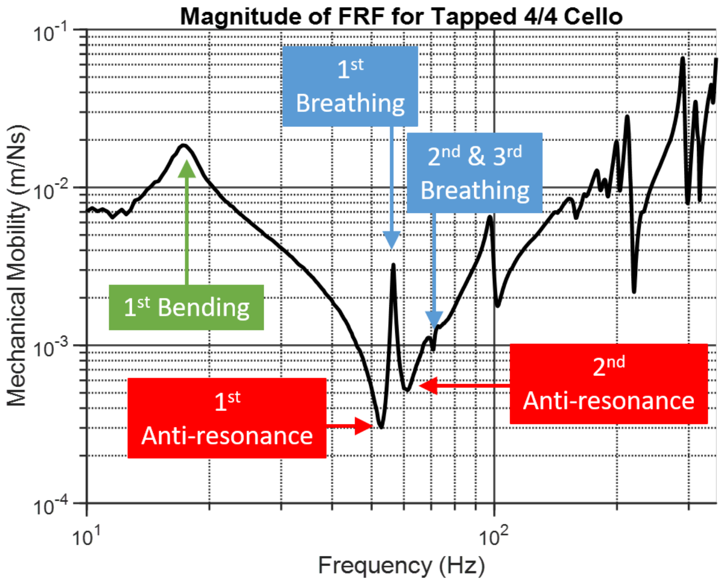

Figure 4: The magnitude of the drive point mobility vs. frequency for a full size cello measured at a point in line with the lower bound and the f-hole on the bass bar side of the instrument.

green lines in the plots indicate the fundamental frequency of the lowest two open strings on the cello. In these measurements the structural modes of the body and the resonant behavior of the air cavity are coupled so it is not possible to identify the Helmholtz resonance as a unique mode. The frequency of the estimated Helmholtz resonance for each cello is indicated by the dots on each plot. Several authors have measured the modal frequencies of the full size cello and have reported a range of values that varies with each individual instrument.[1] The measurements in this study are consistent with the range of values and the general trends in those results.

The measurements in Fig. 5 show that the shape of the drive point mobility for each cello is similar but scaled in frequency according to the cello size.

5. COMPARISON OF MODAL BEHAVIOR

The plot in Fig. 6 compares the frequency of each resonance and antiresonance of the cello body, found in Fig. 5, for the different cello sizes. The dashed green lines again indicate the fundamental frequency of the lowest two open strings. From this plot it is observed that the open strings couple to different resonant or antiresonant behavior of the cello body depending on the cello size. For example, the open C string best couples to the first antiresonance on a 1/2 size cello, the second antiresonance on a 3/4 size cello, and the second breathing mode on a full size cello. These disparities in how the strings couple to the body will create differences between the cello sizes in the radiated sound over the musical playing range. This coupling behavior of fundamental tones to body modes will also influence the production of wolf tones[5] Further analysis on the production of wolf tones in fractional sized cellos is required due to the many factors that contribute to that behavior.[6]

The ratios of the frequencies for each resonance and antiresonance compared to those for a full size cello are shown in Fig. 7 to show how the frequencies of each mode scale with cello size. It is observed that the resonant and antiresonant frequencies of the 1/2 size cello are 25% to 45% greater than the full size cello and the same frequencies on a 3/4 size cello are 15% to 25% greater than the full size cello.

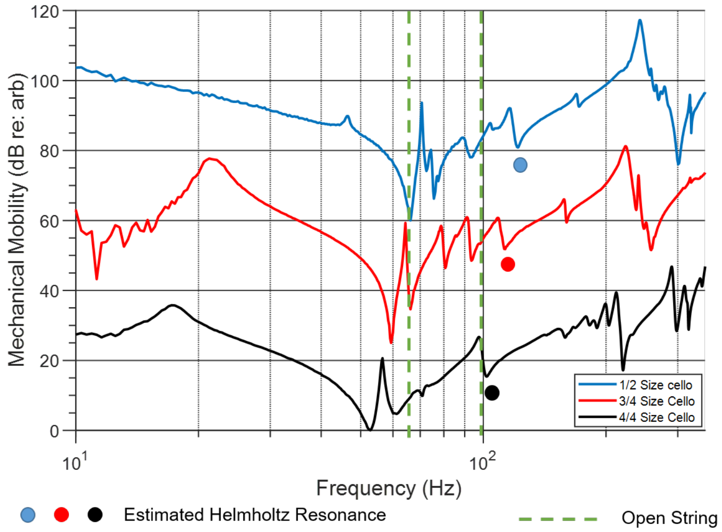

Figure 5: A comparison of magnitude of drive point mobility vs. frequency for three cello sizes. The plots are shown in decibels relative to an arbitrary value that allows each curve to be offset vertically for visual clarity. The dashed green lines represent the fundamental frequency of the lowest two open strings. The dots indicate the frequency of the estimated Helmholtz resonance frequency for each size cello. The Helmholtz resonance is estimated for rigid walls and does not appear as a distinct feature in the mobility curves because the structural and acoustical modes are coupled.

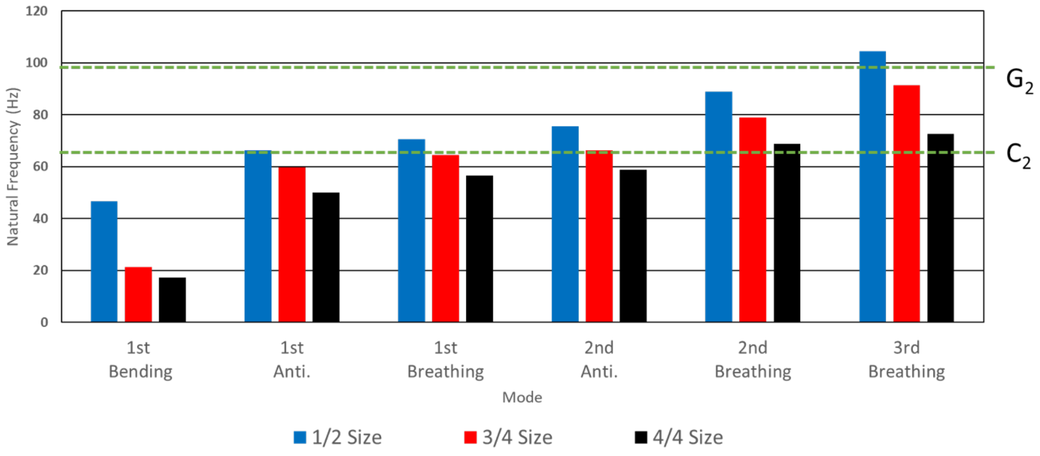

Figure 6: A comparison of the measured resonance and antiresonance frequencies for three cello sizes to the fundamental frequencies of the lowest two open strings.

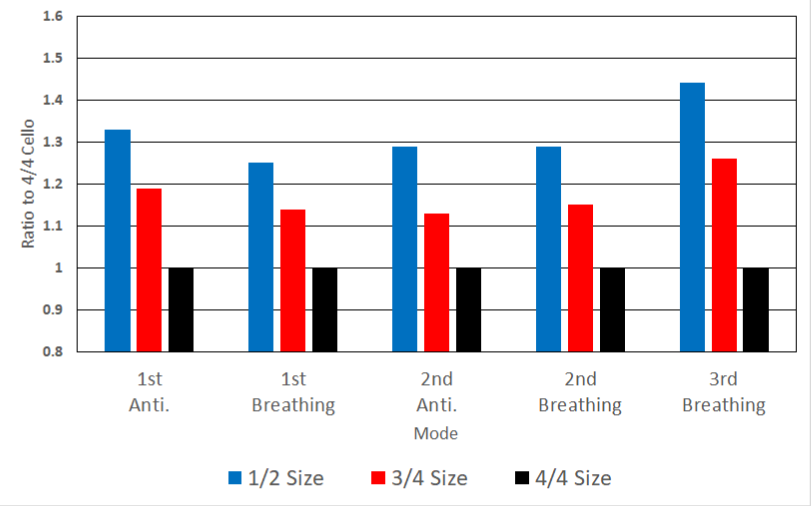

Figure 7: The ratio of the measured resonance and antiresonance frequencies for fractional size cellos to a full size cello.

6. UNKNOWN WOOD PROPERTIES

One of the significant unknown variables in this study are the properties of the wood. The cellos in this study are from different makers and unknown years. The important acoustical properties of the wood, the elasticity and the density, have been shown to vary naturally within a species as well as with moisture content.[7] This variability due to the wood is unaccounted for in this study and would impact the measurements of drive point mobility.

A simple model was constructed to see if the scaling of the modal behavior measured in Sec. 4 fits the trends expected by the scaled geometry of each cello. In this model each cello was approximated as a simply supported rectangular plate fit to the size of the cello body. The ratio of the frequencies of each mode, relative to those of a full size cello, are given by

$$\frac{f_{frac}}{f_{44}} = \frac{(\frac{m}{a_{frac}})^2 + (\frac{n}{b_{frac}})^2}{(\frac{m}{a_{44}})^2 + (\frac{n}{b_{44}})^2} \tag{2}$$

The ratio of the frequencies depend only on the particular mode, (m, n) and the dimensions of each rectangular plate a, and b. Using the dimensions measured in Sec. 2 the modeled frequency ratios for each mode, relative to a full size cello are shown in Fig. 8. The modal frequencies of a plate model of a 1/2 size cello are 35% to 40% greater than those for the model of a full size cello. The modal frequencies of a plate model of a 3/4 size cello are about 15% greater than those for the model of a full size cello. The frequency ratios in Fig. 7 are consistent with those modeled by the simply supported rectangular plates.

7. CONCLUSIONS AND FUTURE WORK

The results of this preliminary experimental study show that fractional size cellos do not scale physically with their nominal ratios, nor do they scale equally in all dimensions. The dimensions of the face of the cello scale similarly to the length of the strings. The depth of the body, however, scales at a different ratio the volume of the cavity scales with a ratio near to that for the area of the sound ports. This results in the scaling of the Helmholtz resonance of the cavity being largely controlled by the radiation impedance of the f-holes. The open strings are observed to couple to different resonant or antiresonant behavior on fractional sized

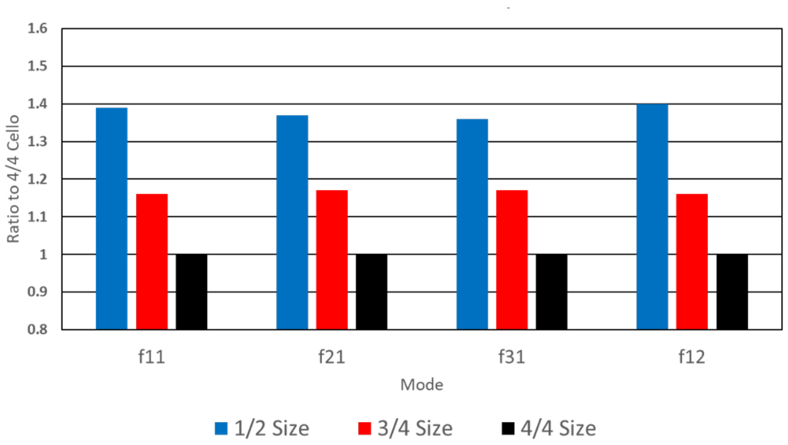

Figure 8: The ratio of the resonance frequencies for three simply supported rectangular plates sized to encompass full and fractional sized cellos. The modeled resonance frequencies of the plates are observed to scale with ratios similar to those measured on the cellos

cellos compared with their full size counterparts. Finally, the modes of fractional size cellos are shown to scale with ratios consistent with plate theory.

Further study of the scaling of fractional sized cellos should include additional fractional size cellos in the data set (1/4 size, 1/8 size). An analysis of the scaling of the sound post and of the string gauge will also give a more complete understanding of the effects on the musical playing range of the instruments. The directivity of the cello's sound radiation is also expected to change as the cello is reduced in size. Fractional-sized cellos have smaller proportions to the acoustic wavelengths over the musical playing range than full size cellos. As a result they would likely exhibit more omnidirectional radiation patterns than those of full size cellos. Experimental measurements of the changes in radiation patterns with cello size would also further develop the theoretical understanding of fraction-sized cellos.

REFERENCES

[1] T. D. Rossing, Ed., *The Science of String Instruments*. Springer, 2010.

[2] C. M. Hutchins, "A 30 year experiment in the acoustical and musical development of violin family instruments," *The Journal of the Acoustical Society of America*, vol. 92, no. 2, pp. 639–650, 1992.

[3] L. E. Kinsler, A. R. Frey, A. B. Coppens, and J. V. Sanders, *Fundamentals of Acoustics*, 4th ed. John Wiley and Sons, 2000.

[4] H. T. Nia, A. D. Jain, Y. Liu, M.-R. Alam, R. Barnas, and N. C. Makris, "The evolution of air resonance power efficiency in the violin and its ancestors," *Proc. R. Soc. A*, 2015.

[5] A. Chaigne and J. Kergomard, *Acoustics of Musical Instruments*. Springer-Verlag, 2016.

[6] J. C. Schelleng, "The violin as a circuit," *J. Acoust. Soc. Am.*, 1963.

[7] *The elastic properties of wood: The moduli of rigidity of Sitka Spruce and their relations to moisture content.* Forest Products Laboratory No. 1528-B, U.S. Department of Agriculture and University of Wisconsin, 1962.