Comparing holography and beamforming inverse methods 
applied to jet noise radiation near a high-performance military aircraft

Jacob A. Ward, Kent L. Gee and Tyce Olaveson
Department of Physics and Astronomy, Brigham Young University, Provo, UT 84602, USA; jacob.ward@live.com; kentgee@byu.edu; tyceolaveson@gmail.com

Alan T. Wall
Air Force Research Laboratory, Wright-Patterson Air Force Base, OH, USA; alan.wall.4@us.af.mil

Multisource statistically-optimized near-field acoustical holography and hybrid beamforming are two inverse techniques that have been successfully used to reproduce sound fields from limited measurements of military aircraft. These methods solve the inverse problem through different means but arrive at the same conclusion. In this paper, the performance of each method is compared to the same baseline measurement. It is found that while both perform well at mid-range frequencies, holography excels at lower frequencies and beamforming at higher frequencies. The spatial Nyquist frequency imposes a soft limit on the accuracy of field reconstructions and limits the usable frequency range.

Distribution A: Approved for public release; distribution unlimited. Cleared 07/12/2022, JSF22-0057.
1. INTRODUCTION

Due to the potential impacts on military personnel, residential communities, and the environment, the jet noise produced by high-performance military aircraft has been the subject of study for many years.

Jet noise sound field data are useful in adding to the understanding of the sound radiated by heated, supersonic jets, particularly since direct flow measurements that permit extraction of acoustic sources are not currently feasible, especially at afterburner. Obtaining sound field data can be challenging, however, as measuring the sound pressure levels at every point in the radiated sound field is prohibitively expensive due to the time required, the setup complexity, and number of microphones needed. Inverse array methods can be helpful in overcoming these limitations.

Two well-known inverse array methods are near-field acoustical holography (NAH) and beamforming. A specific variant of NAH, namely multisource statistically-optimized near-field acoustical holography (M-SONAH) has been used previously to analyze the sound field of a static F-35 aircraft using complex pressure data obtained from a 71 microphone input array. Likewise, a variant of beamforming called hybrid beamforming has been used to analyze the sound field of the same aircraft. These inverse array methods are detailed in the following section.

This paper offers a direct comparison of the two methods by using the same microphone data as input. In doing so, the relative strengths and weaknesses of both methods can be easily seen. This more complete understanding of the capabilities of each method will help to guide future work. Knowing which method gives the most accurate results for a specified measurement geometry or frequency regime can improve the quality of jet noise analyses and also guide improvements to both methods.

2. INVERSE ARRAY METHODS

A. MULTISOURCE STATISTICALLY-OPTIMIZED NEAR-FIELD ACOUSTICAL HOLOGRAPHY

Near-field acoustical holography (NAH) has long been used for three-dimensional imaging of noise fields, however one limitation of NAH is that large truncation errors result when the measurement aperture is not significantly larger than the source. Statistically-optimized near-field acoustical holography (SONAH) reduces the required spatial aperture by avoiding the use of spatial Fourier transforms. Because the noise from tethered military aircraft seen at any point in space consists of noise from the direct path as well as noise from the ground reflection, SONAH had to be altered in order to handle multiple sources. Consequently, multisource statistically-optimized near-field acoustical holography (M-SONAH), developed by Wall et al., uses a two-source model consisting of one source above the ground along the jet centerline and another source along the reflected image of the jet centerline below the ground to account for ground reflections. Originally, M-SONAH utilized a two-dimensional measurement array but was then adapted to use the input from a one-dimensional microphone array. This method has been used to reconstruct the sound field of an F-35 aircraft, as well as an F-22 and the T-7A. A detailed treatment of the M-SONAH algorithm can be found in ref. The M-SONAH output is an equivalent wave model (EWM) which is used to obtain pressures for field predictions.

B. HYBRID BEAMFORMING

Beamforming methods use data from microphone arrays in the noise field to obtain the source distributions responsible for the noise. While traditional beamforming assumes sources can be represented as incoherent monopoles, hybrid beamforming (HBF) is an advanced beamforming algorithm that is able to image both coherent and incoherent sources. This is important because the dominant jet noise sources are partially coherent.
This method has been applied to high-performance military aircraft\textsuperscript{12,13} and shown to be accurate for frequencies within the array design frequency. This method has been used to study the noise fields surrounding the F-35\textsuperscript{14} and the T-7A trainer aircraft.\textsuperscript{15} A detailed treatment of the hybrid beamforming algorithm can be found in ref. 12. Unlike the EWM produced by holography, beamforming produces an equivalent source model (ESM) of monopoles whose volume-velocities (i.e., source strengths) are then repropagated to predict sound fields.

3. MEASUREMENT AND METHODS

A. MEASUREMENT

In order to test the effectiveness of both array processing methods, a 71-microphone array was used to measure complex pressure data of the noise from an F-35B aircraft operating at military power, that is, 100\% engine thrust request (ETR). These microphones were evenly spaced 18 inches apart and were placed approximately 8 to 10 meters from and parallel to the shear layer. Acoustic pressure waveform data were acquired simultaneously at a sampling rate of 204.8 kHz for 30 seconds. The data recorded by these microphones were used as the input for both the M-SONAH and HBF methods. These data have been analyzed for other purposes in Refs 16-18.

Several arcs of microphones at radii of 19 m, 29 m, 38 m and beyond as well as a near-field line array were used for model validation. The positions of these microphone arrays in relation to the F-35 aircraft are shown in Fig. 1 with angle measured relative to the engine inlet and the measurement array reference point (MARP) which was 7.5 m downstream from the jet nozzle. The jet nozzle and arc arrays were elevated while the input array was positioned on the ground; this distinction becomes important when discussing ground reflection nulls seen in the reconstructed sound fields in Section 4.

![Microphone locations](image.png)

Figure 1: Microphone locations (marked in blue) in relation to the F-35 aircraft. Angles are measured from the MARP 7.5 m downstream from the jet nozzle (marked by the red x).

B. COMPARISON METHODS

After obtaining the EWM from the M-SONAH method and the ESM from the HBF method, both models are used to predict the sound field. The EWM and ESM are repropagated to each of the points along a fine
two-dimensional polar grid surrounding the aircraft to obtain a reconstructed sound field map, like the one shown in Fig. 2, for each frequency. This two-dimensional polar grid is in the plane parallel to the ground that contains the jet centerline, which in this case is at a height of 2 m off the ground. In order to perform source comparisons, the EWM and ESM are also used to reconstruct sound pressure levels along the jet lipline.

To validate the model’s accuracy, the sound pressure levels measured at each microphone location are compared to the sound pressure levels produced by the models. The decibel difference between the measured and predicted levels is calculated for each microphone position. These differences are indicated on the total sound field reconstruction maps using green circles, seen in Fig. 2. The size of the circle indicates the magnitude of the error in decibels. By looking at the size of the green circles at different locations, the accuracy of the method in predicting the sound levels can be gauged.
4. RESULTS/DISCUSSION

A. LOW FREQUENCY RESULTS (50 HZ)

Figure 2 shows the reconstructed sound pressure level map for the areas surrounding the jet at a frequency of 50 Hz. The upper plot shows the results for the M-SONAH method and the lower plot shows the results for the HBF method. When comparing the low frequency results from both methods, the M-SONAH method generally is more accurate in predicting the radiated sound field levels, especially in the peak radiation direction. In this figure, larger errors are seen in the HBF reconstruction map in the maximum radiation region where the majority of the sound energy is concentrated. This is true along all of the microphone arcs as well as along the near-field array. Slightly larger errors exist in the M-SONAH reconstruction map outside of the angular aperture of the array; however, in the peak radiation direction, errors are extremely low.

While the error circles only show the absolute value of the errors, the sign of the errors can be seen by plotting the differences in level across each microphone array individually. Specifically, Fig. 3 shows the levels predicted by both methods compared to the actual recorded levels for the microphones along the 19m arc. While both methods perform similarly at predicting the levels at lower angles, the HBF method fails to capture the energy at the farther aft angles resulting in an underprediction of about 4 dB. This omission is problematic as the sound levels in the maximum radiation region (located at approximately 150° for this frequency) are often what is most important to jet noise researchers. The likely reason for this error is because as frequency decreases, the peak radiation lobe shifts in the aft direction until the input array no longer captures the entire lobe. That missing energy from the parts of the lobe that were not captured in the input array measurements results in a failure to reproduce the correct sound levels in the peak radiation direction. The M-SONAH method eventually has these issues as well, but they occur at a lower frequency.
Figure 4: Total sound field reconstructions for M-SONAH (top) and HBF (bottom) at 150 Hz.
Figure 5: Total sound field reconstructions for M-SONAH (top) and HBF (bottom) at 300 Hz.

Figure 6: Sound pressure levels recorded along the 19m arc at 150 Hz along with predictions from M-SONAH and HBF.
Figure 7: Sound pressure levels recorded along the 19m arc at 300 Hz along with predictions from M-SONAH and HBF.

B. MID-FREQUENCY RESULTS (150 AND 300 HZ)

For mid-frequencies such as 150 and 300 Hz, both methods are able to accurately predict the radiated sound field levels within 1 dB in most cases. The reconstructed sound pressure level maps for 150 and 300 Hz are shown in Figures 4 and 5, respectively. Even though these methods are very different mathematically, they both produce results that are consistent with the measured levels for these mid-frequencies. This agreement is seen along the 19m arc as well, as shown in Figures 6 and 7. The levels predicted by both methods are almost identical to the actual levels, except for an overprediction at 140° for the HBF method at 150 Hz and an underprediction of levels at 40° for both methods at 300 Hz.

The one major difference between the two methods at mid-frequencies is the behavior at the farthest aft microphone locations. Looking at Figures 4 and 5, the M-SONAH method tends to have much higher errors at the most aft microphone locations along the 38m arc while the HBF method is able to capture much of that energy. Errors outside the angular aperture of the input array have been seen in other numerical validations of M-SONAH as well and are "due to [M-SONAH’s] lack of an effective aperture extension protocol." For mid frequencies, both methods successfully recover the sound pressure levels in the peak-radiation region, but HBF more accurately recovers aft locations outside the angular aperture of the array.

C. HIGH-FREQUENCY RESULTS (450 HZ)

The Nyquist frequency for this input array (determined by when the microphone spacing is equal to half the wavelength) is 375 Hz. At frequencies above this Nyquist frequency limit, grating lobes begin to interfere with the sound field reconstructions. Of the two, M-SONAH is the most impacted resulting in many large errors, while HBF is affected less. This behavior is observed in Fig. 8 where M-SONAH has larger errors along the input array than HBF. Significant errors in the peak radiation region are seen for the M-SONAH method, and especially high errors are seen along the 19m arc. Some error is seen for the HBF case in the maximum radiation region, but these are several decibels less than the corresponding errors seen in the case of M-SONAH. At high enough frequencies, aliasing eventually causes both of these methods to break down, but this breakdown occurs at a much lower frequency for the M-SONAH method.

Another factor also influences the larger error seen along the 19m arc for the M-SONAH method. Figure 9 shows the predicted and actual levels along the 19m arc, and the M-SONAH method gives very large errors...
Figure 8: Total sound field reconstructions for M-SONAH (top) and HBF (bottom) at 450 Hz.

Figure 9: Sound pressure levels recorded along the 19m arc at 450 Hz along with predictions from M-SONAH and HBF. Note the general underprediction of both methods.
Figure 10: Spectra for the microphones along the 19m arc. Note the dip in sound pressure level at approximately 500 Hz due to the ground reflection null.

at almost all angles. At this frequency (450 Hz), a ground reflection null occurs at approximately 19m, as seen in the measured spectra in Fig. 10. The M-SONAH method tends to overestimate how deep ground reflection nulls should be, resulting in an underprediction of sound pressure levels along the 19m arc, hence the larger errors. A closer prediction of this ground reflection null is provided by HBF. The reason for this better performance is unclear. It would be expected that the cylindrical wavefunctions and special care for an image source present in M-SONAH would account for an interference null better than a series of monopoles. Perhaps more partial fields (see ref. 7) are required to match these high frequencies. Further investigation is needed.

D. SOURCE RECONSTRUCTIONS

Because HBF gives source volume velocity and M-SONAH gives near-field pressure, it is difficult to directly reconcile their behavior at the source; but, with the accuracy of the field reconstructions as a benchmark, it can still be instructive to look at the source reconstructions given by both methods. The ESM obtained from the HBF centerline monopoles can also be propagated to the lipline of the jet so that the same physical quantity (near-field pressure) can be compared with the results from M-SONAH. Figure 11 shows the near-field pressures at the lipline versus frequency given by HBF and M-SONAH.

Mirroring the field reconstructions, both methods give very similar lipline pressures for mid-frequencies and show a maximum region that moves upstream as frequency increases. Major differences are seen at high frequencies as the 6 dB-down region extends about 100 Hz higher for the HBF results. This underprediction in high-frequency energy by M-SONAH is consistent with the underprediction in the sound field reconstruction levels seen at higher frequencies. On the other hand, at the lowest frequencies M-SONAH predicts more energy than HBF which again parallels HBF’s underprediction in the sound field reconstruction levels.
seen at lower frequencies.

Lastly, we see that HBF does not predict energy upstream of the jet nozzle. This also matches the sound field reconstruction maps where HBF predicts very low levels upstream of the jet while M-SONAH predicts slightly increased levels. It is not immediately apparent that this difference in the prediction of upstream source sound levels leads to better accuracy in predicting upstream field levels for either method. Neither method excels at predicting upstream levels according to the field reconstruction results, although upstream microphone locations to compare against are limited.

Because the trends in source reconstructions match those seen in the field reconstructions, it is reasonable that the source reconstructions share the same errors that are produced by each method in the field reconstructions. Thus, the source reconstruction produced by M-SONAH will be more accurate at low frequencies while that of HBF will be more accurate at higher frequencies. Still, the fact that each method produces similar source reconstructions at mid-frequencies instills confidence in the general accuracy of both of these methods in properly capturing the propagation of jet noise.

5. CONCLUSION

Two inverse array methods, multisource statistically-optimized near-field acoustical holography and hybrid beamforming, have been compared in their ability to recreate the radiated sound field of a military aircraft using measurements taken at an input array in the near-field of the jet. The M-SONAH method was shown to be more accurate at low frequencies where the main radiation lobe is not entirely captured by the input array; however, at frequencies beyond the spatial Nyquist, M-SONAH breaks down. On the other hand, HBF gave more accurate results at high frequencies above the Nyquist and was also shown to better predict levels within a ground reflection null. At mid frequencies, HBF was also better able to recreate the sound field outside of the angular aperture of the array, whereas future work is needed to give the M-SONAH method better aperture extension capabilities. Source reconstruction trends obtained from both of these methods mimic the characteristics of their sound field reconstructions, and therefore likely mimic their errors as well. These strengths and weaknesses should be taken into account when deciding which method to use for sound field or source reconstruction.
ACKNOWLEDGEMENTS

This paper was revised and submitted under Office of Naval Research Grant N00014-21-1-2069

REFERENCES


