Landau, Lifshitz, and weak Lifshitz conditions in the Landau theory of phase transitions in solids

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Landau theory provides a group-theoretical method for determining which symmetry changes can occur in second-order phase transitions in solids. The irreducible representations of the space group of the higher-symmetry phase must satisfy certain conditions. For transitions to commensurate crystalline structures, the Landau and Lifshitz conditions must be satisfied. For transitions to incommensurate structures, the Landau and "weak Lifshitz" conditions must be satisfied. The irreducible representations of each of the 230 crystallographic space groups that violate these conditions are listed.

Landau theory[1,2] uses group-theoretical methods to describe second-order phase transitions in crystalline solids. In this theory, a transition is driven by an order parameter $\boldsymbol{\eta}$. The components of $\boldsymbol{\eta}$ transform like basis functions of an irreducible representation (IR) of the space group of the higher-symmetry phase.

Near the transition, the free energy of the crystal is expanded about $\boldsymbol{\eta} = \mathbf{0}$. The space-group symmetry of the crystal requires that only invariant polynomials in this expansion can contribute. The invariant terms in the expansion determine the nature of the phase transition. If the transition is to be second order, then certain kinds of terms must *not* appear in the free-energy expansion. These terms are forced not to appear by symmetry. These considerations have resulted in two conditions which must be satisfied if the transition is to be second order: the Landau and Lifshitz conditions.

The Landau condition states that no third-degree terms of the form $\eta_i\eta_j\eta_k$ can be allowed to appear in the free-energy expansion. This condition is satisfied if the symmetrized triple Kronecker product of the IR does not contain the identity representation.

The Lifshitz condition states that no terms of the form $\eta_i(\partial\eta_j/\partial x_k)$ can be allowed to appear in the free-energy expansion. Invariant polynomials in the expansion which contain terms of this form are called "Lifshitz invariants." The Lifshitz condition is satisfied if the antisymmetrized double Kronecker product of the IR does not contain the vector representation.

If we consider second-order transitions to incommensurate phases, the Lifshitz condition need not be satisfied. Instead, the so-called "weak Lifshitz condition," introduced by Michelson[3,2] must be satisfied. This condition states that the number of Lifshitz invariants in the free-energy expansion must equal the number of degrees of freedom of the **k** vector associated with the IR. Bradley and Cracknell[4] classify **k** vectors as belonging to either points of symmetry, lines of symmetry, planes of symmetry, or general points. Points of symmetry have zero degrees of freedom; lines of symmetry have one degree of freedom; planes of symmetry have two degrees of freedom; and general points have three degrees of freedom.

TABLE I. IR's and PIR's which violate the Landau condition. The trivial case of the identity IR's (Γ_1, Γ_1^+) has been omitted from the table. All of the identity IR's violate the Landau condition.

Space groups	IR's, PIR's
Hexagonal R	
146	Γ, Σ, F
148	$\Gamma_2^+\Gamma_3^+, \Sigma, F_1^+$
155	$\Gamma_3, \Sigma_1\Sigma_1, F_1$
160,161	Γ_3, Σ, F_1
166,167	$\Gamma_3^+, \Sigma_1, F_1^+$
Hexagonal P	
143–145	$\Gamma, \Lambda, \Sigma, K, M, B$
147	$\Gamma_2^+\Gamma_3^+, \Lambda, \Sigma, K, M_1^+, B$
149,151,153,156,158	$\Gamma_3, \Lambda, \Sigma_1\Sigma_1, K, M_1, B$
150,152,154,157,159	$\Gamma_3, \Lambda_1\Lambda_1, \Sigma, K_1K_1, K_3K_3, M_1, B$
162–165	$\Gamma_3^+, \Lambda_1, \Sigma_1, K_1, K_3, M_1^+, B$
168–173	$\Gamma_3\Gamma_5, \Lambda, \Sigma, K, M_1, B$
174	$\Gamma_3\Gamma_5, \Lambda_1\Lambda_1, \Sigma_1\Sigma_1, K_1K_1, K_3K_3,$ K_5K_5, M_1, B_1B_1
175,176	$\Gamma_3^+\Gamma_5^+, \Lambda_1, \Sigma_1, K_1, K_3K_5, M_1^+, B_1$
177–186	$\Gamma_5, \Lambda_1, \Sigma_1, K_1, K_3, M_1, B$
187,188	$\Gamma_5, \Lambda_1, \Sigma_1\Sigma_1, K_1, K_3, K_5, M_1, B_1B_1$
189,190	$\Gamma_5, \Lambda_1\Lambda_1, \Sigma_1, K_1K_1, K_5K_5, M_1, B_1B_1$
191–194	$\Gamma_5^+, \Lambda_1, \Sigma_1, K_1, K_5, M_1^+, B_1$
Cubic P	
195,198	Γ, Σ, M
200	$\Gamma_2^+\Gamma_3^+, \Gamma_4^+, \Sigma, M_1^+, M_2^+, M_3^+, M_4^+$
201,205	$\Gamma_2^+\Gamma_3^+, \Gamma_4^+, \Sigma, M$
207	$\Gamma_3, \Gamma_5, \Sigma_1, M_1, M_4, M_5$
208	$\Gamma_3, \Gamma_5, \Sigma_1, M_2, M_3, M_5$
212,213	$\Gamma_3, \Gamma_5, \Sigma_1, M_1M_4, M_5$
215,218	$\Gamma_3, \Gamma_4, \Sigma, M_1, M_3, M_5$
221,223	$\Gamma_3^+, \Gamma_5^+, \Sigma_1, \Sigma_4, M_1^+, M_4^+, M_5^+$
222,224	$\Gamma_3^+, \Gamma_5^+, \Sigma_1, \Sigma_4, M_1, M_3$
Cubic F	
196	Γ, Σ, X
202	$\Gamma_2^+\Gamma_3^+, \Gamma_4^+, \Sigma, X_1^+, X_2^+, X_3^+, X_4^+$
203	$\Gamma_2^+\Gamma_3^+, \Gamma_4^+, \Sigma, X$
209	$\Gamma_3, \Gamma_5, \Sigma_1, X_1, X_4, X_5$
210	$\Gamma_3, \Gamma_5, \Sigma_1, X_2, X_3, X_5$
216,219	$\Gamma_3, \Gamma_4, \Sigma, X_1, X_3, X_5$
225,226	$\Gamma_3^+, \Gamma_5^+, \Sigma_1, \Sigma_4, X_1^+, X_4^+, X_5^+$
227,228	$\Gamma_3^+, \Gamma_5^+, \Sigma_1, \Sigma_4, X_1, X_3$
Cubic I	
197,199	Γ, Σ, N
204	$\Gamma_2^+\Gamma_3^+, \Gamma_4^+, \Sigma, N_1^+, N_2^+$
206	$\Gamma_2^+\Gamma_3^+, \Gamma_4^+, \Sigma, N$
211,214	$\Gamma_3, \Gamma_5, \Sigma_1, N_1, N_3$
217	$\Gamma_3, \Gamma_4, \Sigma, N_1, N_4$
220	$\Gamma_3, \Gamma_4, \Sigma, N$
229	$\Gamma_3^+, \Gamma_5^+, \Sigma_1, \Sigma_4, N_1^+, N_3^+$
230	$\Gamma_3^+, \Gamma_5^+, \Sigma_1, \Sigma_4, N_2$

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TABLE II. IR's and PIR's which violate the weak Lifshitz condition.

Space groups	IR's, PIR's
Monoclinic P	
4,11	C, D, E, Z, G
7,13	A, B, D, E, U, V
14	A, B, C, Z, U, V, G
Monoclinic A	
9,15	A, M, U
Orthorhombic P	
17,26	$Z, T, U, R, A, B, P, E, W$
18,55,59	$X, Y, T, U, C, D, P, E, G, H, L, N$
19	$X, Y, Z, R, A, B, C, D, G, H, L, N, W$
27,49	Z, T, U, R, A, B, P, E
28	X, S, U, R, D, P, Q, G
29	$X, Z, S, T, U, R, A, B, D, P, Q, E, G, W$
30	$Y, Z, S, T, U, R, A, B, C, P, Q, H$
31,53	$X, Z, S, T, A, B, D, P, Q, E, G, W$
32,50	$X, Y, S, T, U, R, C, D, P, E, G, H$
33	$X, Y, Z, S, U, R, A, B, C, D, P, E, G, H, W$
34,48	$X, Y, Z, S, T, U, A, B, C, D, G, H$
51	$X, S, U, R, D, P, Q, G, L$
52	$X, Y, Z, S, U, R, A, B, C, D, Q, E, G, H, N$
54	$X, Z, S, T, U, R, A, B, D, P, Q, E, G, L$
56	$X, Y, Z, T, U, A, B, C, D, P, E, G, H, L, N$
57	$Y, Z, S, T, U, R, A, B, C, P, Q, E, H, N, W$
58	$X, Y, Z, A, B, C, D, P, E, G, H, L, N$
60	$X, Y, Z, T, U, R, A, B, C, D, P, Q, E, G, H, L, W$
61	$X, Y, Z, S, T, U, A, B, C, D, P, Q, E, G, H, L, N, W$
62	$X, Y, Z, S, R, A, B, C, D, Q, G, H, L, N, W$
Orthorhombic C	
20,36,63	Z, R, T, A, B, Q
37,40,66	Z, T, A, B
39,67	R, S, D
41,68	Z, R, S, T, A, B, D
64	Z, S, T, A, B, D, Q
Orthorhombic I	
24	W
45,72	S, R, Q, D, W
46	R, Q, W
73	S, R, T, Q, P, D, W
74	T, P, W
Orthorhombic F	
43,70	Z, T, Y, A, B, H
Tetragonal P	
75	$\Gamma_3\Gamma_4, \Sigma, \Delta, M_3M_4, A_3A_4, Z_3Z_4, Y, S, T, U, B, C, F$
76,78	$\Gamma_3\Gamma_4, \Sigma, \Delta, M_3M_4, A, R, Z, Y, S, T, U, B, C, E, F$
77	$\Gamma_3\Gamma_4, \Sigma, \Delta, M_3M_4, A_1A_2, Z_1Z_2, Y, S, T, U, B, C, F$
81,83	$\Sigma, \Delta, Y, S, T, U, B, C, F$
84	$\Sigma, \Delta, A_1, Z_1, Y, S, T, U, B, C, F$
85	$\Sigma, \Delta, M_1, X, A_1, R, Y, W, S, T, U, B, C, F$
86	$\Sigma, \Delta, M_1, X, R, Z_1, Y, W, S, T, U, B, C, F$
89,93	Γ_5, M_5, A_5, Z_5
90,94	$\Gamma_5, X, R, Z_5, Y, W, T, F$
91,95	$\Gamma_5, M_5, A, R, Z, S, T, U, E$
92,96	$\Gamma_5, X, A, Z, Y, W, S, U, E, F$
100	$M_1M_3, M_2M_4, X, A_1A_3, A_2A_4, R, Y, W, T$
101	$A_1A_3, A_2A_4, R, Z_1Z_3, Z_2Z_4, T, U$
102	$M_1M_3, M_2M_4, X, R, Z_1Z_3, Z_2Z_4, Y, W, U$
103,124	A, R, Z, S, T, U
104	$M_1M_3, M_2M_4, X, A_1A_2, A_3A_4, R, Z, Y, W, S, U$
105	$A_1A_2, A_3A_4, Z_1Z_2, Z_3Z_4, S$
106	$M_1M_3, M_2M_4, X, A, R, Z_1Z_2, Z_3Z_4, Y, W, S, T$
112	A_5, Z_5, S
113,127,129	X, R, Y, W, T, F
114	$X, A_5A_5, R, Z_5, Y, W, S, T, F$
116	A_5, R, Z_5, T, U
117	M_5, X, A_5, R, Y, W, T
118	M_5, X, R, Z_5, Y, W, U
125	$M_3, M_4, X, A_3, A_4, R, Y, W, T$
126	$M_3, M_4, X, A_3, A_4, R, Z, Y, W, S, U$
128	$X, A_3A_4, Z, Y, W, S, T, U, F$
130	$X, A, R, Z, Y, W, S, T, U, F$
131	A_1, A_2, Z_1, Z_2, S
132	$A_1, A_2, R, Z_1, Z_2, T, U$
133	$M_1, M_2, X, A, R, Z_1, Z_2, Y, W, S, T$
134	$M_1, M_2, X, R, Z_1, Z_2, Y, W, U$
135	$X, A, R, Z_1, Z_2, Y, W, S, T, F$
136	$X, Z_1, Z_2, Y, W, T, U, F$
137	$X, A_3A_4, R, Z_1, Z_2, Y, W, S, T, F$
138	$X, R, Z_1, Z_2, Y, W, T, U, F$
Tetragonal I	
79	$\Gamma_3\Gamma_4, \Sigma, \Delta, M_3M_4, P_2P_2, Q, Y, A, B$
80	$\Gamma_3\Gamma_4, \Sigma, \Delta, M_1M_2, P, Q, Y, A, B$
82	$\Sigma, \Delta, Q, Y, A, B$
87	$\Sigma, \Delta, P_3P_4, Q, Y, A, B$
88	$\Sigma, \Delta, X, M_1, P, Q, W, Y, A, B$
97	Γ_5, M_5, P_3P_4
98	Γ_5, M_5, P
107	P_3P_4
108,120,140	N, P, Q
109	$X, M_1M_2, M_3M_4, P, W, Y$
110	$X, M_1M_2, M_3M_4, N, P, Q, W, Y$
121	P_5P_5
122	X, M_5, P, W, Y
139	P_5
141	X, M_1, M_2, P, W, Y
142	$X, M_1, M_2, N, P, Q, W, Y$
Hexagonal R	
146	$\Gamma_2\Gamma_3, \Sigma, T_2T_3, Y, C$
148	$\Lambda_2\Lambda_3, \Sigma, Y, C$
155	Γ_3, T_3
160	$\Lambda_3\Lambda_3$
161	$\Lambda_3\Lambda_3, L, T, Y$
166	Λ_3
167	Λ_3, L, T, Y
Hexagonal P	
143,145	$\Gamma_2\Gamma_3, \Lambda, \Sigma, A_2A_3, H, K, Q, R, U, B, C, D, E$
144	$\Gamma_2\Gamma_3, \Lambda, \Sigma, A_1A_3, H, K, Q, R, U, B, C, D, E$
147	$\Delta_2\Delta_3, \Lambda, \Sigma, H, K, P_2P_3, Q, R, U, B, C, D, E$
149,151,153	$\Gamma_3, \Lambda, A_3, H, K, Q, U, B, D, E$
150,152,154	$\Gamma_3, \Sigma, A_3, H_3H_3, K_3K_3, R, U, B, C, E$
156	$\Delta_3\Delta_3, \Sigma, R, U, B, C, E$
157	$\Delta_3\Delta_3, \Lambda, H, K, P_3P_3, Q, U, B, D, E$
158	$\Delta_3\Delta_3, \Sigma, A, H, L, Q, R, U, B, C, E$
159	$\Delta_3\Delta_3, \Lambda, A, H, K, L, P_3P_3, Q, R, U, B, D, E$
162	$\Delta_3, \Lambda, H, K, P_3, Q, U, B, D, E$
163	$\Delta_3, \Lambda, A, H, K, L, P_3, Q, R, U, B, D, E$
164	$\Delta_3, \Sigma, H_3, K_3, P_2P_3, R, U, B, C, E$
165	$\Delta_3, \Sigma, A, H, K_3, L, P_2P_3, Q, R, U, B, C, E$
168,171	$\Gamma_3\Gamma_5, \Gamma_4\Gamma_6, \Lambda, \Sigma, A_3A_5, A_4A_6, H_2H_3, K_2K_3, Q, R, C, D$
169,170,173	$\Gamma_3\Gamma_5, \Gamma_4\Gamma_6, \Lambda, \Sigma, A, H, K_2K_3, L, Q, R, C, D, E$
172	$\Gamma_3\Gamma_5, \Gamma_4\Gamma_6, \Lambda, \Sigma, A_1A_5, A_2A_6, H_1H_3, K_2K_3, Q, R, C, D$
174	$\Lambda, \Sigma, Q, R, C, D$
175	$\Lambda, \Sigma, H_3H_5, H_4H_6, K_3K_5, K_4K_6, P_2P_3, Q, R, C, D$
176	$\Lambda, \Sigma, A, H_1H_2, K_3K_5, K_4K_6, L, P_2P_3, Q, R, C, D, E$
177,180,181	$\Gamma_5, \Gamma_6, A_5, A_6, H_3, K_3$
178,179,182	$\Gamma_5, \Gamma_6, A, H, K_3, L, Q, R, E$
183	H_3, K_3, P_3P_3
184	$A, H, K_3, L, P_3P_3, Q, R$
185	$A, H, K_3, L, P_3P_3, Q, R, E$
186	$A, H_1H_2, K_3, L, P_3P_3, Q, R, E$
188	A, H, L, Q, R
189	$H_5H_5, H_6H_6, K_5K_5, K_6K_6, P_3P_3$
190	$A, H_3H_3, K_5K_5, K_6K_6, L, P_3P_3, Q, R$
191	H_5, H_6, K_5, K_6, P_3
192	$A, H, K_5, K_6, L, P_3, Q, R$
193	$A, H, K_5, K_6, L, P_3, Q, R, E$
194	$A, H_3, K_5, K_6, L, P_3, Q, R, E$
Cubic P	
195	$\Gamma_4, \Sigma, R_4, S, C$
198	$\Gamma_4, \Sigma, R, X, S, Z, C, B$
200	$\Lambda_2\Lambda_3, \Sigma, S, C$
201	$\Lambda_2\Lambda_3, \Sigma, X, M, S, Z, C$
205	$\Lambda_2\Lambda_3, \Sigma, X, M, S, T, Z, C, B$
207,208	$\Gamma_4, \Gamma_5, R_4, R_5, X_5, M_5$
212,213	$\Gamma_4, \Gamma_5, R, X, S, Z, B$
215	$\Lambda_3\Lambda_3$
218	$\Lambda_3\Lambda_3, R_4R_5, X_5, S$
221	Λ_3
222	$\Lambda_3, R_4, X, M_3, M_4, S, Z$
223	$\Lambda_3, R_4, X_1, X_2, S$
224	$\Lambda_3, X_1, X_2, M_1, M_2, Z$
Cubic F	
196	$\Gamma_4, \Sigma, L_2L_3, W, Q, C$
202	$\Lambda_2\Lambda_3, \Sigma, W, Q, C$
203	$\Lambda_2\Lambda_3, \Sigma, X, W, Q, V, C$
209	$\Gamma_4, \Gamma_5, L_3, X_5, W_3W_4$
210	$\Gamma_4, \Gamma_5, L_3, X_5, W$
216	$\Lambda_3\Lambda_3$
219	$\Lambda_3\Lambda_3, L, W, Q$
225	Λ_3, W_5
226	Λ_3, L, W, Q
227	$\Lambda_3, X_1, X_2, W, V$
228	$\Lambda_3, L, X_1, X_2, W, Q, V$

TABLE II. (*Continued*).

Space groups	IR's, PIR's	Space groups	IR's, PIR's
Cubic I			
197	$\Gamma_4, \Sigma, H_4, P_4P_4, G, C$	214	$\Gamma_4, \Gamma_5, H_4, H_5, P$
199	$\Gamma_4, \Sigma, H_4, P, G, C$	217	$\Lambda_3\Lambda_3, P_4P_4, P_5P_5$
204	$\Lambda_2\Lambda_3, \Sigma, P_4, G, C$	220	$\Lambda_3\Lambda_3, H_4H_5, N, P_3P_3, D, G$
206	$\Lambda_2\Lambda_3, \Sigma, N, P, D, G, C$	229	Λ_3, P_4, P_5
211	$\Gamma_4, \Gamma_5, H_4, H_5, P_4$	230	$\Lambda_3, H_4, N, P, D, G$

Second-order transitions to commensurate phases must be driven by an order parameter belonging to an IR associated with a **k** point of symmetry. Hence we have the Lifshitz condition as a special case of the weak Lifshitz condition: The number of Lifshitz invariants must be zero.

The Landau and weak Lifshitz conditions are entirely determined by the IR. Among the 230 crystallographic space groups, there are 10285 IR's which need to be considered. These IR's include all those associated with each **k** point of symmetry and also those associated with an arbitrary **k** vector on each inequivalent **k** line of symmetry and on each inequivalent **k** plane of symmetry, and those associated with an arbitrary general **k** vector. We list in Table I the IR's which violate the Landau condition and in Table II the IR's which violate the weak Lifshitz condition. In Table I we omit the trivial case of the identity IR's which always violate the Landau condition. Note the surprising result that none of the IR's (except the identity IR's) of the space groups in the triclinic, monoclinic, orthorhombic, or tetragonal systems violate the Landau condition. The IR's in Table II which are associated with **k** points of symmetry are also those which violate the Lifshitz condition. The data in both of these tables were generated by computer calculations using the database from Ref. 5. The mathematical equations used in these calculations are given in Ref. 6.

We use the IR labeling from Miller and Love.[7,5] (Note that Ref. 5 is an updated version of Ref. 7 and contains some additional **k** vectors. Reference 8 contains a useful table giving the corresponding labeling of IR's by Kovalev,[9] Bradley and Cracknell,[4] and Zak[10] for **k** points of symmetry. Also, a forthcoming English edition of Kovalev[11] gives the mapping of all **k** vectors from Miller and Love to Kovalev.) The IR symbols in Tables I and II are represented by a letter $(\Sigma, \Lambda, A, B, \ldots)$, which denotes the **k** vector, and a subscript number, which denotes the IR associated with that **k** vector. If the subscript is absent, then all of the IR's associated with that **k** vector are implied. For example, we see in Table I that all of the Σ IR's of space group 146 violate the Landau condition, whereas only the one IR Σ_1 of space group 166 violates the Landau condition.

In Landau theory, only physically irreducible representations (PIR's) are considered. If an IR is not real, the PIR is formed from the direct sum of the IR with its complex conjugate. This is indicated in the tables by combining two IR symbols to form a single symbol. For example, in Table I, we see that PIR $\Sigma_1\Sigma_1$ of space group 155 violates the Landau condition. This PIR is formed from the direct sum of the IR Σ_1 with its complex conjugate. In Table II, we see that PIR P_2P_3 of space group 147 violates the weak Lifshitz condition. This PIR is formed from the direct sum of P_2 with its complex conjugate. In this case, the complex conjugate of P_2 happens to be equivalent to the IR P_3. Hence we use the label P_2P_3.

Some of the **k** vectors listed in Refs. 7 and 5 are equivalent to each other. These are listed in Table III. We do not list IR's with equivalent **k** vectors in Tables I and II. For example, from Table I we see that IR Σ_1 of space group 166 violates the Landau condition. In Table III we find that **k** vectors Σ and Q are equivalent for the hexagonal R lattice. Therefore, IR Q_1 of space group 166 also violates the Landau condition, even though it is not listed in Table I.

In Ref. 8, the Landau and Lifshitz conditions have been previously determined for all IR's associated with **k** points of symmetry. In Ref. 12, the Landau condition for all real IR's can be easily obtained from the tables of symmetrized triple Kronecker products. Other papers[13–16] have listed IR's which satisfy the Landau and/or Lifshitz conditions for various space groups, but these lists contain errors.[17] The tables in Refs. 8 and 5 were typeset by computer directly from data files and therefore almost certainly do not contain any errors. Also, Tables I and II in this Brief Report were composed directly by the computer and therefore also should not contain any errors.

Using our data, we can comment on some assertions made by Michelson[3] regarding the weak Lifshitz condition. First of all, he asserts that all IR's associated with general **k** points satisfy the weak Lifshitz condition. There are no general **k** points listed in Table II, in agreement with his assertion.

Second, he asserts that IR's associated with **k** planes and **k** lines of symmetry satisfy the weak Lifshitz condition if the small representation is one dimensional. We

TABLE III. Equivalent **k** values in the Miller and Love labeling of IR's. Groups of symbols separated by semicolons are labels for equivalent **k** values.

Lattice	**k** values
Orthorhombic C	Δ, F; Σ, C; A, E; B, G; K, L; M, N
Orthorhombic I	Δ, U; Λ, G; Σ, F; A, K; B, L; C, M
Orthorhombic F	Δ, R; Λ, Q; Σ, U; A, C; B, D; H, G; E, F; J, K; M, N
Tetragonal I	Σ, F; Λ, V; Y, U; A, E; C, D
Hexagonal R	Λ, P; Σ, Q; Y, B; C, D, E
Hexagonal P	Λ, T; Q, S; C, F
Cubic P	C, J
Cubic F	Σ, S; C, J; A, B
Cubic I	Λ, F; C, J, B

have found two ways in which this rule can be violated. If the IR is not real, the PIR may not satisfy the weak Lifshitz condition. For example, the PIR A_1A_2 of space group 17 is associated with a **k** line of symmetry. The small representations A_1 and A_2 are both one dimensional. However, there are three Lifshitz invariants in the free-energy expansion for the PIR A_1A_2. Since the **k** vector has only one degree of freedom, this PIR violates the weak Lifshitz condition.

The other way in which Michelson's rule can be violated is illustrated by the following example. The IR Δ_1 of space group 75 is associated with a **k** line of symmetry (one degree of freedom). The small representation Δ_1 is one dimensional. However, there are two Lifshitz invariants in the free-energy expansion, and the weak Lifshitz condition is violated. There is a special circumstance here that allows Michelson's rule to be violated. If we consider the full symmetry of the tetragonal P lattice, the point group of every **k** vector on Δ is $mm2$. If we consider only the symmetry of the space group, then the point group of **k** vectors on Δ is 1, the same as the point group of a general **k** point. One might argue that **k** vectors on that line do not really belong to a **k** line of symmetry in space group 75, but are actually general **k** points and have three degrees of freedom. However, *by mathematical definition*, the **k** vectors that lie on Δ have only *one* degree of freedom, and therefore, if there is more than one Lifshitz invariant in the free energy, the weak Lifshitz condition is violated. The minimum of the free energy cannot fall on that line. In general, Michelson's rule may be violated in cases where **k** vectors on a **k** line or plane of symmetry do not have more symmetry than surrounding **k** vectors.

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