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High amplitude time reversal focusing of sound and vibration

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Time reversal (TR) is a signal processing technique that can be used to focus high amplitude sound or vibration at a desired location. TR focusing can be done with sources placed far from the desired focal location and the technique excels in complex environments. The impulse response between each source and the desired focal location must be obtained prior to the focusing and the environment must remain relatively unchanged for successful focusing. Multiple scattering or reverberation of waves off of many reflecting surfaces in the environment can actually be used advantageously by exploiting those reflections as additional image sources. This talk will provide an introduction to TR and then focus on the use of TR to provide high amplitude focusing of sound and vibration. Applications of high amplitude TR include lithotripsy of kidney stones, histotripsy of lesions, and locating cracks and defects in structures such as in human teeth, spent nuclear fuel storage casks, airplane wings, and automotive bearing caps. Recently, high amplitude TR of airborne sound has been studied in a reverberation chamber to generate a focused difference frequency and to study the nonlinear acoustics of the peak sound levels of 200 dB that have been attained.

1. INTRODUCTION

Time Reversal (TR) may be used to focus sound or vibration waves for three main purposes: high amplitude focusing, source localization/characterization, and secure communications.^{1,2,3} Initially TR was demonstrated for underwater SONAR applications and it was found that the technique, originally called matched signal processing, provided a means of reproducible transmission of signals in the varying ocean environment.^{4,5} Extensive additional development of underwater TR communications was subsequently done.^{6,7,8,9} TR has been explored for air-borne communications in rooms as well.^{10,11,12,13,14} It has also been used to for vibrational communications in metal pipes in three orthogonal directions.¹⁵ TR has also been used in various applications for source localization and characterization, including touchpads,^{16,17} earthquakes,^{18,19,20,21} and gun shots.^{22,23} Often the backward step of the TR process is done in a numerical model for these source localization applications. When the waves interfere with each other at the original source location, information may be available from the timing and phasing of how the source evolved over time (with reversed timing). Finally, TR has been used to generate high amplitude focusing of sound and vibration for various applications. Early applications of high amplitude TR focusing include using it for lithotripsy of kidney stones²⁴ and histotripsy of brain tumors.^{25,26} Montaldo *et al.*²⁷ demonstrated that shock waves could be generated with TR focusing. Localized focusing of high amplitude waves with TR has also enabled its use for nondestructive evaluation of cracks and defects.^{3,28,29,30,31} Additional high amplitude TR studies will be reviewed in more detail later on in this paper.

Figure 1 depicts a cartoon schematic image of how the TR process may be done (see also reference 32 for an animated version of this figure). Initially an impulse is emitted from point A into a room or environment. This wave energy spreads out spherically in general, but here we simply trace a few of the paths traversed by the wave as it travels from point A to point B. A receiver placed at point B thus records the impulse response of the room, containing the timing information of the direct sound arrival and the reflected paths. The impulse response may be reversed in time and broadcast from the original source location, termed reciprocal TR.² This backward step of the TR process emits energy corresponding to the later reflected arrivals first, followed by energy corresponding to the earlier reflected arrivals, and finally the energy corresponding to the direct sound is emitted. Assuming linear time invariant conditions, a portion of each of these energy emissions will arrive at point B simultaneously, providing constructive interference (or coherent wave superposition). A recording made at point B will include an impulsive focal event that approximates the initial impulse that was emitted, though reversed in time. In fact this focal signal recording has been shown to be equivalent to an autocorrelation.³³

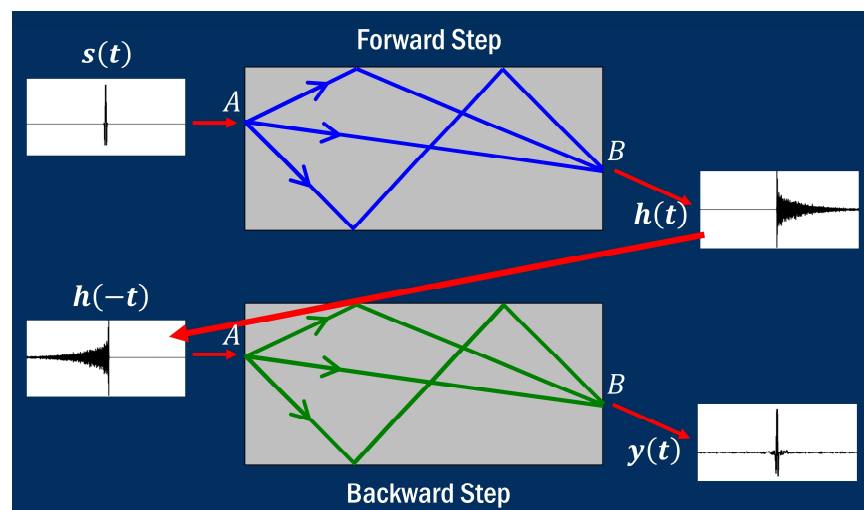


Figure 1. Cartoon illustration of the time reversal process. During the forward step, an impulse $s(t)$ is emitted from point A and the impulse response, $h(t)$, is recorded at point B. The reversed impulse response, $h(-t)$, is then emitted from point A during the backward step and focusing occurs at point B, producing a focal signal $y(t)$.

Reference 32 also shows an animated video depicting the TR process when multiple sources are used with circular wave front emissions representing the emitted energy during the forward and backward steps. Snapshots of that animation are included in Fig. 2. Note that the bold-line circles depict the emissions that arrive simultaneously but that there are many other circular wave fronts at other locations in the room, illustrating that the time reversal process is not perfect (wave energy arrives before the intended time of focusing [termed side lobes] and exists elsewhere in the room during the time of focusing).

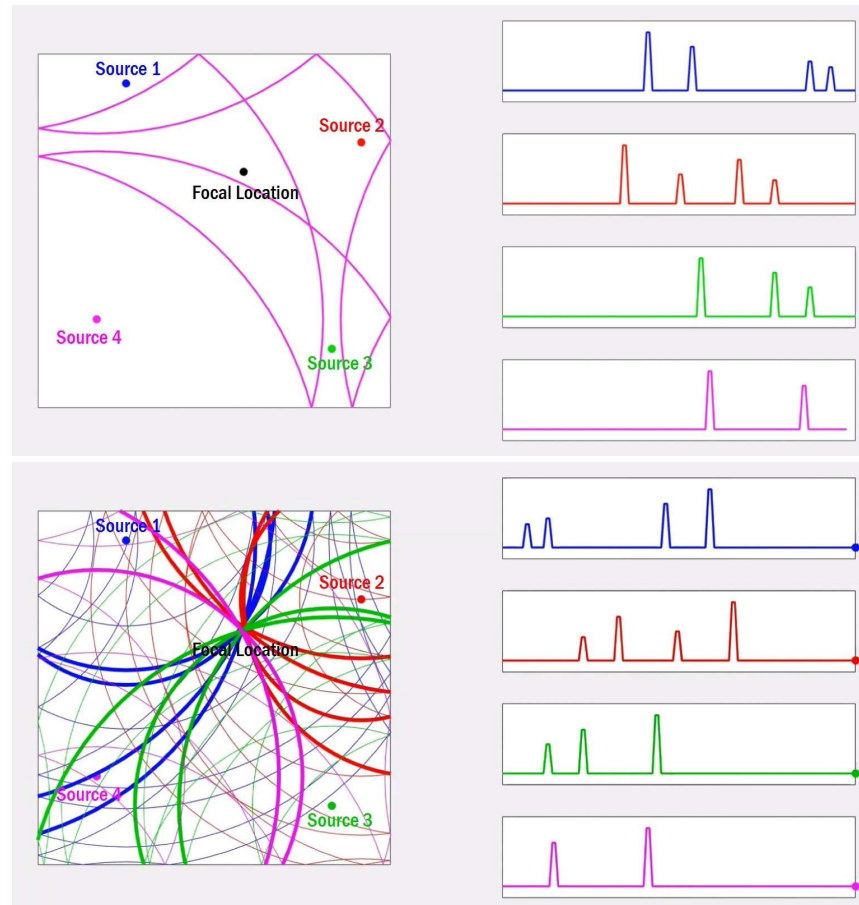


Figure 2. (Upper image) Illustration of the forward step of time reversal. Each of four sources emits a pulse sequentially and the impulse response from each is recorded at the “focal location”. (Lower image) Illustration of the backward step of time reversal. The four reversed impulse responses are broadcast simultaneously producing constructive interference at the focal location, shown by the bolded circular wave fronts.

2. HIGH AMPLITUDE FOCUSING OF VIBRATION

To visually illustrate the high amplitude focusing of vibration, a LEGO minifigure demonstration was invented by the author. Research was done exploring this demonstration and additional similar demonstrations by Heaton *et al.*³⁴ Later, Barnes *et al.*³⁵ further perfected the demonstration in order to develop it for a Wave Propagation Museum exhibit hosted by ETH University in Zurich, Switzerland. Once impulse responses are obtained from sources to a target location, a LEGO figure may be placed at that target location and during the backward step the constructive interference launches the LEGO into the air and it falls over when hitting the ground. Again, reference 32 provides some video content showing the LEGO demonstration.

The Time Reversed Elastic Nonlinearity Diagnostic (TREND) was developed to enable nondestructive evaluation of cracks and defects in structures.³⁶ Because TR provides a focusing of high amplitude

vibrations, the local nonlinear properties at the focal location may be explored, assuming that the vibrations located elsewhere vibrate linearly (with lower relative amplitude). TREND is often done with a Scanning Laser Doppler Vibrometer (SLDV) as the receiver. The laser is shined at a location of interest and the entire TR process is carried out. The focal signal may be generated using different levels of linear amplification from the sources such that a linear scaling and subtraction of the focal signals^{37,38} (or their spectra^{39,40}) can yield some indication of the relative increase in nonlinear content generated in the low and high amplitude focal signals (the Scaling Subtraction Method). Alternatively, the focal signal may be generated with normal reversed impulse responses and then also with phase inverted reversed impulse responses (multiplying the reversed impulse responses by -1 prior to their broadcast). The original focal signal and its phase inverted version, if done at the same amplitude, should sum to zero if everything was linear, but to the degree that the subtraction results in a finite signal, this can be used as an indicator of nonlinearity (termed the Phase Inversion or Pulse Inversion Method).^{41,42,43} Finally, a method was developed in which the reversed impulse responses are broadcast with relative phases of 0° , 120° , and 240° and the focal signals can be compared to quantify the degree of nonlinearity present (called Third Order Phase Symmetry Analysis).⁴⁴

Two metal disks were diffusion bonded together and Ulrich *et al.*⁴³ used standard, linear, ultrasonic C-scan imaging in addition to nonlinear ultrasonic imaging using TREND and Phase Inversion. They showed that the linear C-scan imaging could detect voids, or regions that were significantly disbonded. In the TREND image they were able to identify locations of small cracks in the bond that were not visible in the linear image. TREND was unable to detect the larger voids that the linear C-scan imaging was easily able to detect. This showed the advantage of using linear C-scan and nonlinear TREND techniques in tandem to locate different types of damage, though linear imaging with TREND is possible as well.

Anderson *et al.*⁴⁰ used TREND, along with a frequency domain version of the Scaling Subtraction Method, to image stress corrosion cracking in a steel sample that had been exposed to corrosion and as a result had developed cracking in the heat affected zone near a weld in the sample. When they used a higher excitation frequency and observed second harmonic generation, they were able to detect the surface expression of a prominent crack. When a lower excitation frequency was used, the second harmonic generation was maximum at that surface expression of the crack, but it gradually decreased in amplitude as the focusing was moved towards the weld region. It was found that in fact the crack initially ran perpendicular to the surface, but then it curved under the surface to be nearly parallel to the surface. The excitation using different frequencies was thus able to provide some imaging of this crack.

Ulrich *et al.*⁴⁵ and Van Den Abeele *et al.*⁴⁶ showed that if a sensor is used that detects a particular Cartesian component of the velocity response at a target location, then the TR process may be used to focus energy principally in that direction and not in the other two Cartesian directions. The focusing direction is determined by which velocity component is used to measure the impulse response. This technique was used by Anderson *et al.*¹⁵ for the three dimensional TR communications among pipes mentioned in the introduction. Anderson *et al.*⁴⁷ and Remillieux *et al.*⁴⁸ used this three component TR focusing along with TREND to show that the orientation of a crack or defect could be determined if the crack ran perpendicular to the orientation of the TR focusing.

When TREND techniques are done, typically the transducers used are contact based ones, such as piezoelectric transducers bonded onto the structure or sample. Researchers at Los Alamos National Laboratory developed a noncontact acoustic source in the 20-100 kHz range using TR principles.^{49,50,51,52,53} TR was used to focus sound waves at the point of contact with a structure to enable efficient coupling of the sound waves into vibrations of the structure. This concept was called the Time Reversal Acoustic NonContact Excitation (TRANCE). The device that was built relied on the efficient coupling of piezoelectric transducers to sound in air, which was accomplished through the use of wedges of power law profiles (sometimes called acoustic black holes).^{54,55,56}

Le Bas *et al.*³⁹ combined the use of TREND, the frequency domain version of the Scaling Subtraction Method, the three component TR focusing, and the TRANCE device to image a carbon fiber plate. This plate had a delamination below the plate surface that ran parallel to the surface along with a crack that ran perpendicular to the surface. These two defects were imaged successfully by Le Bas *et al.* using out of plane

excitation and in plane excitation, respectively. These results were verified with other imaging techniques as well.

3. HIGH AMPLITUDE FOCUSING OF SOUND

As mentioned in the introduction, Montaldo *et al.*²⁷ demonstrated that shock waves could be generated with TR focusing with underwater sound waves. Their results showed that, relative to linear scaling of the focused waveform at a lower amplification level, the high amplitude generation focused waveform exhibited waveform steepening and a lowering of peak amplitudes.

Willardson *et al.*⁵⁷ generated high amplitude focusing of airborne, audible-frequency sound waves in a reverberation chamber and observed some curious effects. As the amplified broadcast levels of the reversed impulse responses were increased linearly in a TR experiment, the focused waves had compression peaks that were slightly nonlinearly amplified relative to linear scaling and the rarefaction troughs were slightly nonlinearly suppressed. They achieved peak sound pressure levels of 173.1 dB. They also observed a nonlinear increase in higher frequency content.

Patchett and Anderson⁵⁸ expanded upon the work of Willardson *et al.* by using various techniques to increase the amplitude of the focusing of airborne, audible sound further. These techniques included pointing the loudspeaker sources away from the intended focal location,⁵⁹ using the clipping TR processing technique,^{34,57} focusing in the corner of the room,⁶⁰ focusing in a smaller room,⁶¹ and placing the sources and microphone in the same plane.⁶² Patchett and Anderson were able to achieve focal amplitudes of up to 200.6 dB peak sound pressure level. These results were extensively verified with multiple microphone types. The focal signals in this work also showed evidence of waveform steepening as Montaldo *et al.* had seen but the compression peak amplitudes were dramatically nonlinearly amplified at these higher focal levels and the rarefaction peaks were nonlinearly suppressed. In a subsequent numerical modeling study, Patchett *et al.*⁶³ showed that free-space Mach stems are created when high amplitude sound waves are focused and that they cause a nonlinear amplification of compression peaks and waveform steepening was also observed. It is suspected that the nonlinear suppression of compression peaks observed by Montaldo *et al.* was due to the fact that their transducers were arranged in an array and that the reversed impulse responses principally utilized the direct sound emissions only and thus the focused sound principally came from very similar directions, which the numerical work of Patchett *et al.* showed should not exhibit significant Mach stem formation.

Finally, Wallace and Anderson⁶⁴ explored what happens when high amplitude airborne ultrasound is focused. They also utilized beam blockers, which are known to decrease the directionality of highly directional sources, in order to scatter sound in more directions.⁶⁵ The beam blockers and the use of many of techniques mentioned in the previous paragraph to increase the amplitude of the TR focusing, allowed Wallace and Anderson to focus up to 134 dB peak amplitude sound pressure level of ultrasound in the 35-40 kHz range. Interestingly, when Wallace and Anderson focused two different primary frequencies, they were able to observe a difference frequency generated at the common focal location. This difference frequency increased in amplitude quadratically as the amplitude of the primary frequencies were linearly increased, which was expected for difference frequency generation often seen in parametric arrays.

4. CONCLUSION

This paper has reviewed several Time Reversal (TR) studies where TR was used to generate high amplitude focusing of vibration and sound, using audible and ultrasonic frequencies in both cases. High amplitude TR has shown promise in biomedical applications for lithotripsy and histotripsy, for nondestructive evaluation of cracks and defects in structures, and exhibits some interesting physical phenomena when sound in air is focused. These developments have greatly benefitted from the work done by various research groups previously.

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