Using Time Reversal Acoustics to Remotely Deliver Energy for Active Noise Control

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ABSTRACT

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Time reversal (TR) acoustics is capable of remotely focusing sound energy to a point in space. This thesis explores the remote delivery of a noise-canceling signal to a desired location (e.g. a patient's ears) using TR. A parameterization study testing frequency dependence, and signal length is conducted in a reverberation chamber to determine the effectiveness of using TR with active noise control (ANC). The reduction of Magnetic Resonance Imaging (MRI) noise using ANC delivered by TR (ANC+TR) is demonstrated using recordings of MRI noise. For both the parameterization study and the MRI noise experiments, the simulated noise and ANC+TR signals are broadcast from two separate sources, recorded by a microphone, and their responses are linearly superposed in post-processing to determine the noise attenuation. The parameterization study results show that TR is better at reducing noise at frequencies below 1 kHz and for narrowband signals with reductions as great as 20 dB. MRI noise is reduced by up to 18 dB in overall sound pressure level. Both the parameterization study and the MRI noise reduction study utilized a single control loudspeaker; further reductions should be possible with the use of more control sources.

Keywords: time reversal, time reversal acoustics, active noise control, magnetic resonance imaging

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I. Introduction

Noise from magnetic resonance imaging (MRI) equipment put patients at risk for hearing damage.[1] Lee *et al.* measured sound pressure levels up to 130 dB from a 3 Tesla MRI. Active noise control (ANC) systems have been implemented to reduce the noise patients perceive by using non-ferromagnetic equipment next to the patient's ears, inside the bore.[1–3] These studies cancelled sound at the patient's ears without requiring the patient to wear headphones. MRI compliant noise cancelling headphones have also been developed and produced commercially.[4] Not only do the magnetic fields from dynamic loudspeakers pose a problem, but stray eddy currents generated in conductors as a result of Lenz's law also can corrupt the imaging.[5,6] The small bore of the MRI machine also limits the size of ANC systems, further restricting usable equipment. Other issues for ANC systems applied to MRI noise include, but are not limited to, the fundamental frequency produced by a scanning sequence, the speed of a scan, and noise conduction through the body.[3,7]

Active noise control systems reduce noise by broadcasting an opposite-phase sound that cancels the noise at a desired location for the control. A typical ANC system is comprised of a reference microphone, a loudspeaker to broadcast a noise-canceling (or "control") signal, and an error microphone. The reference microphone records noise to be controlled and passes it through a filter that creates the control signal. The control signal is then broadcast through the loudspeaker and the two sounds cancel each other out by linear superposition at the error microphone. The error microphone records the superposed result, and sends a feedback signal through the original filter in order to train the system to get further noise reduction.

Time reversal is a signal processing technique capable of focusing sound energy to an arbitrary position in space from a remote location.[8–11] In a room, TR can focus sound energy by first broadcasting a signal, for the purpose of this explanation, an impulse signal, from a loudspeaker (see Fig. 1(a)). This impulsive sound follows various paths in the room, with the direct sound path arriving at the microphone first and the reflected paths arriving at later times, for a response like the one found in Fig. 1(b). This impulse response can then be flipped in time (time-reversed), and then broadcast from the original loudspeaker as seen in Fig. 1(c). The last arrivals in the impulse response are played first and the direct path arrival is played last. These emissions follow their original traversed paths, and the timing is such that they simultaneously converge at the microphone, the focus point. The resulting response is a matched signal to the original impulse signal[8,11] (compare Figs. 1(a) and 1(d)). The process just outlined is called reciprocal time reversal.[10]

TR has been applied in rooms, though not extensively. Interestingly, the first demonstration of TR was done with sound in a room.[11] Candy *et al*. used TR to deliver a focused communication signal in a highly reverberant room and tested how to improve the communication quality.[12,13] Yon *et al.* determined that TR provides better spatial focusing of sound in a room than traditional beamforming.[14] Ribay *et al.* used a numerical simulation to relate reverberation time, the number of sound sources, and the bandwidth of the source signal used to the focal amplitude of a time-reversed signal.[15] Anderson *et al.* determined that the direction of the sound source relative to the focus location and a room's reverberation time affects the temporal focus quality and spatial focus clarity.[16] Willardson *et al.* compared different signal processing techniques applied to the time-reversed impulse response from the sound sources to the focal position to determine which technique would produce the highest focal amplitude in a reverberation chamber.[17] This research builds upon knowledge gained from past work for a different application of TR of audible sound in rooms.

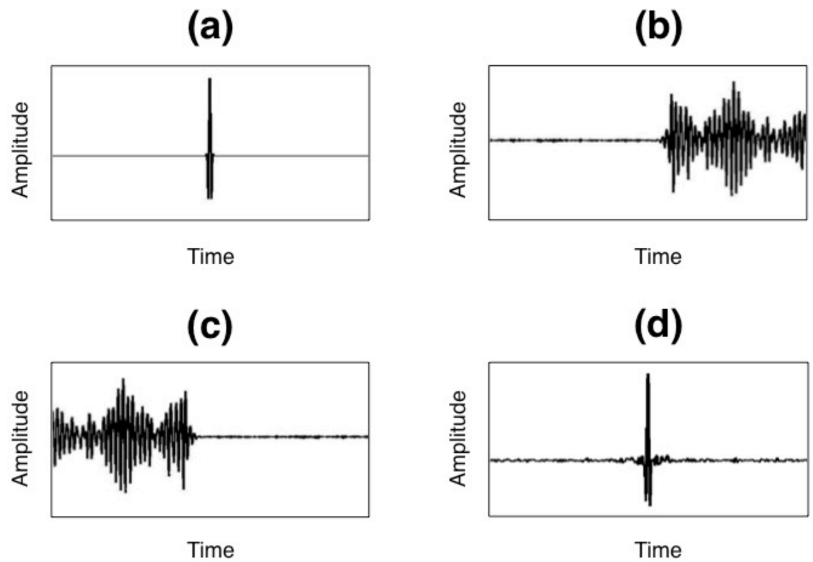

FIG. 1. Illustration of the reciprocal time reversal process: (a) A noise signal is broadcast from a loudspeaker in a room. (b) Its response is recorded at a microphone. (c) The noise response is time-reversed, then broadcast through the same loudspeaker. (d) The matched signal is recorded at the microphone.

The purpose of this thesis is to report some preliminary experiments that explore the effectiveness of using TR for the delivery of ANC (ANC+TR). To the author's knowledge, TR has never been used in conjunction with ANC. There are various parameters that need to be tested to determine how effective it may be. Some of these parameters include the frequency content, the bandwidth of the signal (whether it is broadband or narrowband in nature), and the acoustics of the room (i.e. reverberation time). Each of these parameters will be addressed in this thesis. The general process used here for proof of concept experiments involves using a microphone to record the sound emitted from one loudspeaker, the “noise” loudspeaker, and then inverting the phase of the response to allow for the creation of a control signal. The control signal comprises a convolution of the inverted noise recording and the reversed impulse response between another loudspeaker, the “control” loudspeaker, and the microphone. Using TR causes the control response to arrive later in time than desirable, compared to traditional applications of ANC, as it takes time for the time-reversed sound to traverse the room before focusing, thus the noise signal likely needs to be known *a priori*. The recorded responses are manually manipulated in post-processing to determine the optimal conditions for effective ANC+TR. The purpose of this study is to demonstrate that ANC+TR can be done, not to develop the necessary control algorithms and hardware needed for an actual implementation.

A parameterization study is conducted with ANC+TR to control the noise produced by sinusoidal pulses played in a reverberation chamber with different center frequencies and pulse lengths. Additional results using ANC+TR for the reduction of MRI noise recorded in an MRI facility and played back in a reverberation chamber and in a standard laboratory room will also be reported. Data was collected and analyzed in the Brigham Young University MRI facility and played back in other room environments for convenience. Reductions in the overall sound pressure level of MRI noise of up to 18 dB show the effectiveness of ANC+TR in handling complex sound signals.

II. Parameterization Study

In order to determine the limitations in using TR to deliver ANC, a parameterization study is conducted with pulse signals of different center frequencies and pulse lengths. These experiments are conducted in Brigham Young University's reverberation chamber with dimensions 4.96 x 5.89 x 6.98 m, as TR is expected to have a stronger amplitude focus in a reverberant environment.[15] The reverberation chamber has a reverberation time equal to 6.89 s, and utilizes hanging scattering panels to diffuse sounds throughout the room. A similar experiment utilizing recordings of MRI noise is conducted in the reverberation chamber and in a standard laboratory room to determine if ANC+TR is more effective in rooms with different reverberation times and dimensions. The laboratory room measures 7.66 x 6.44 x 3.67 m and has a reverberation time of 0.68 s. The setup for these experiments consists of two Mackie (Woodinville, WA) HR824 MK2 loudspeakers, one as the noise source and the other as the control source, and a 1.27 cm (1/2 in.) 46AQ GRAS (Holte, Denmark) random incidence microphone with a sensitivity of 53.03 mV/Pa. These are all connected to a computer-based data acquisition system for analysis. Figure 2 shows a photograph of this setup in the reverberation chamber. Note that the loudspeakers are pointed away from the microphone as suggested by Anderson *et al.* for maximum focal amplitude.[16]

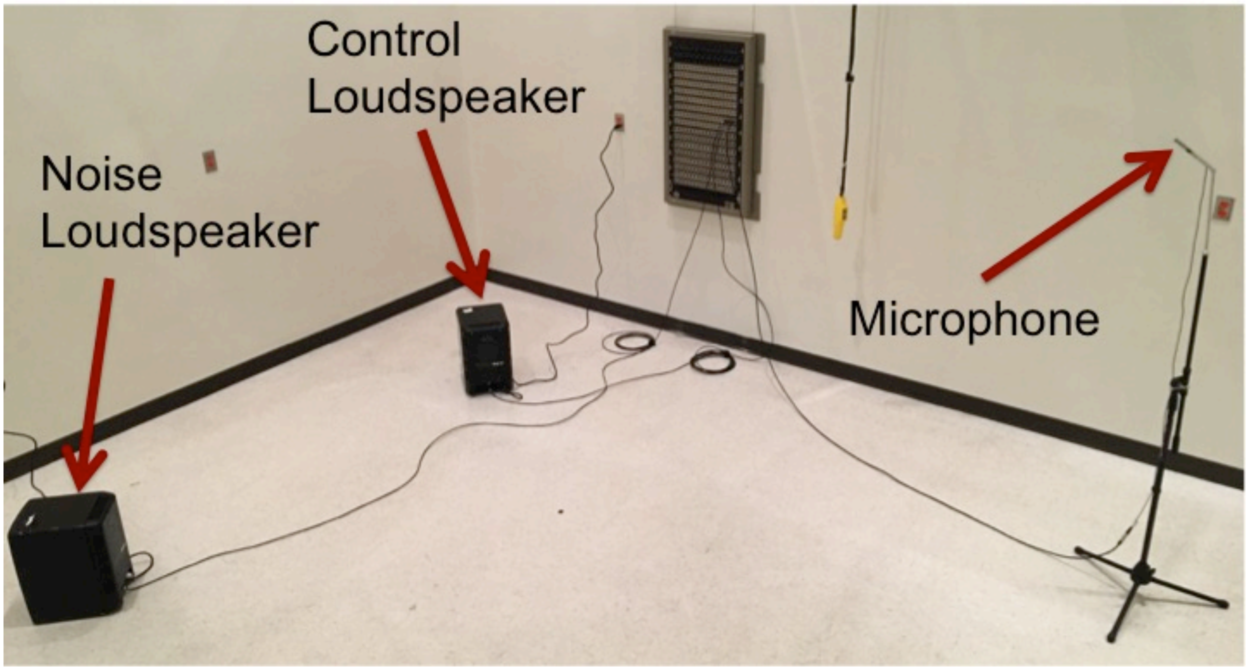

FIG. 2. Photograph of the experimental setup in a reverberation chamber with a noise loudspeaker, control loudspeaker, and microphone.

The loudspeakers receive their respective input signals from Spectrum (Grosshansdorf, Germany) M2i.6022-exp signal generation cards that have 14 bit resolution. The microphone is powered by a GRAS 12AX 4-channel CCP power supply and is then connected to a Spectrum M2i.4931-exp acquisition card with 16 bit resolution and a sampling frequency of 100 kHz. The generation and acquisition are synchronized through custom developed LabVIEW software that was designed for TR experiments and can simultaneously broadcast multiple output signals and record from multiple input signals.

For this proof of concept study, a real-time ANC system is not used, but is simulated through post-processing. This is done by broadcasting a noise signal and a control signal individually from their corresponding loudspeakers and using one microphone to record each response. The control signal consists of a time-reversed impulse response, between the control loudspeaker and the microphone, convolved with the phase inverted noise response. An appropriate time delay is then calculated between the noise response and the control response, and the amplitude of the control response is optimized to control the noise response through post-processing. This process simulates the simultaneous arrival of amplitude-matched noise and control signals at the microphone, assuming linear superposition.

It should be noted that the necessary time delay is several seconds, which depends on the length of the reversed impulse response used and Willardson *et al.* found that 2 s is sufficiently long for a reverberation chamber, thus shorter times could be used in rooms with lower reverberation times.[17] This delay is too long for real time ANC controllers to have any ability to control unknown noise signals. Fortunately, the MRI sequence is often known *a priori* and thus the noise can be anticipated by assuming a linear, time-invariant system. Thus the time delay may not present as severe of a problem for this application as it would for an ANC system attempting to control unknown noise signals. The controller should still have the ability to make slightly delayed adjustments for small system changes to update the reversed impulse response used for control signal delivery, or new measurements of the impulse response may be needed, for example with a new patient.

A. Time-Reversed Impulse Response

The TR process consists of two steps, the forward and backward steps. The purpose of the forward step is to obtain the impulse response(s) needed to focus sound. First, a chirp signal, $c_s[n]$, a linear frequency sweep over a finite period of time, is broadcast from the control loudspeaker and its response, $c_r[n]$, is recorded at the microphone. In order to avoid the problems of dividing by zero using a deconvolution operation with frequency domain spectra, we take the cross-correlation, $\star$, of $c_s[n]$ with $c_r[n]$ to obtain a scaled version of the band-limited impulse response, $h_{c,m}[n]$, of the room between the control loudspeaker and the microphone[16] (see Fig. 3),

$$h_{c,m}[n] = c_s[n] \star c_r[n]. \tag{1}$$

The impulse response is then time-reversed to obtain a time-reversed impulse response (TRIR) from the control loudspeaker, $h_{c,m}[-n]$. Deconvolution, or inverse filtering is then applied to $h_{c,m}[-n]$ to reduce the amplitude of the side lobes during TR focusing, thus making the focal signal (without the convolution of the control signal) appear more like a delta function, improving its spatio-temporal focusing capabilities.[18,19]

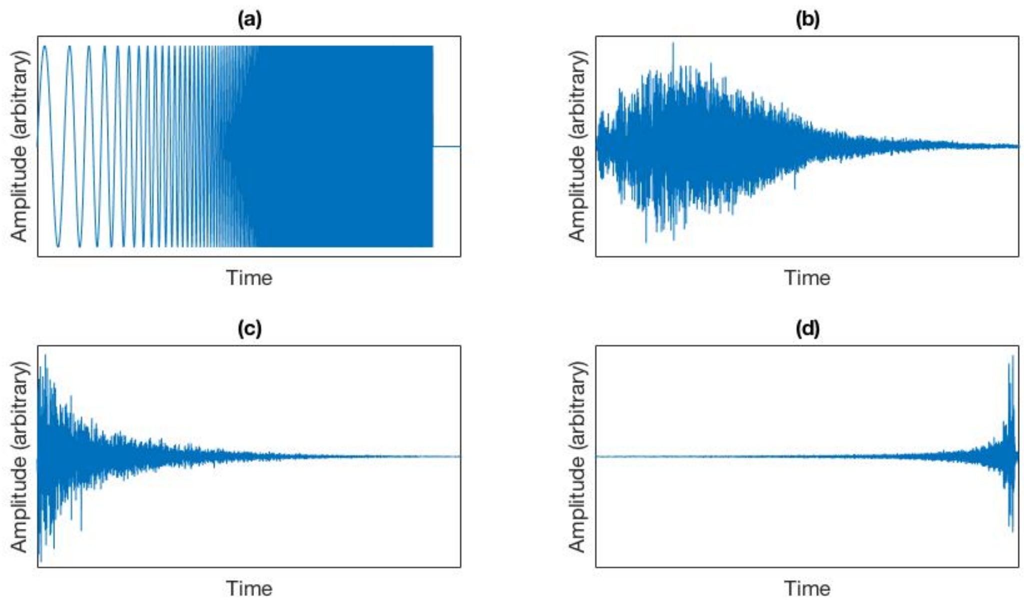

FIG. 3. Illustration of the cross-correlation of the chirp signal (a) and the chirp response (b) to obtain an impulse response (c). The impulse response is time-reversed and using the deconvolution method, a time-reversed impulse response is obtained (d). Signals have been normalized for illustrative purposes.

B. Generate Control Signal

A noise signal $x_s[n]$ is broadcast from the noise loudspeaker and its response, $x_r[n]$, is recorded at the microphone. The response is multiplied by negative one to invert the phase of the response and is then convolved with $h_{c,m}[-n]$ to create the control signal, $a_s[n]$, (see Fig. 4),

$$a_s[n] = -x_r[n] * h_{c,m}[-n], \tag{2}$$

where the * symbol denotes a convolution. $h_{c,m}[-n]$ provides the information to focus the inverted noise signal to the microphone using TR.

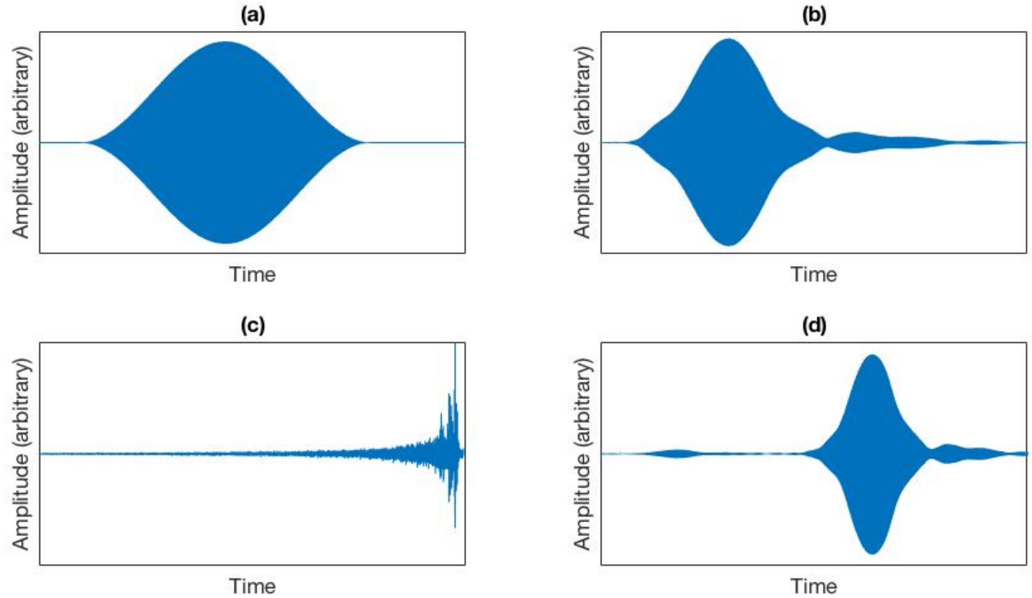

FIG. 4. A noise signal (a) is broadcast from the noise loudspeaker and its response (b) is recorded at the microphone. The control signal that will focus sound to cancel the noise at the microphones location is found by convolving the inverted noise response (b) with the TRIR from the control loudspeaker to the microphone (c) to obtain the control signal (d). Signals have been normalized for illustrative purposes.

Then, $a_s[n]$ is broadcast from the control loudspeaker and recorded at the microphone to obtain its response, $a_r[n]$. Figures 5(a) and 5(b) display $x_r[n]$ and $a_r[n]$ respectively, and while the two signals appear to be visually similar in shape, they reach the microphone at different times and with different amplitudes. This is because the TR process requires some finite amount of time for the reversed emissions to propagate along their original paths before they constructively interfere upon arrival at the microphone.

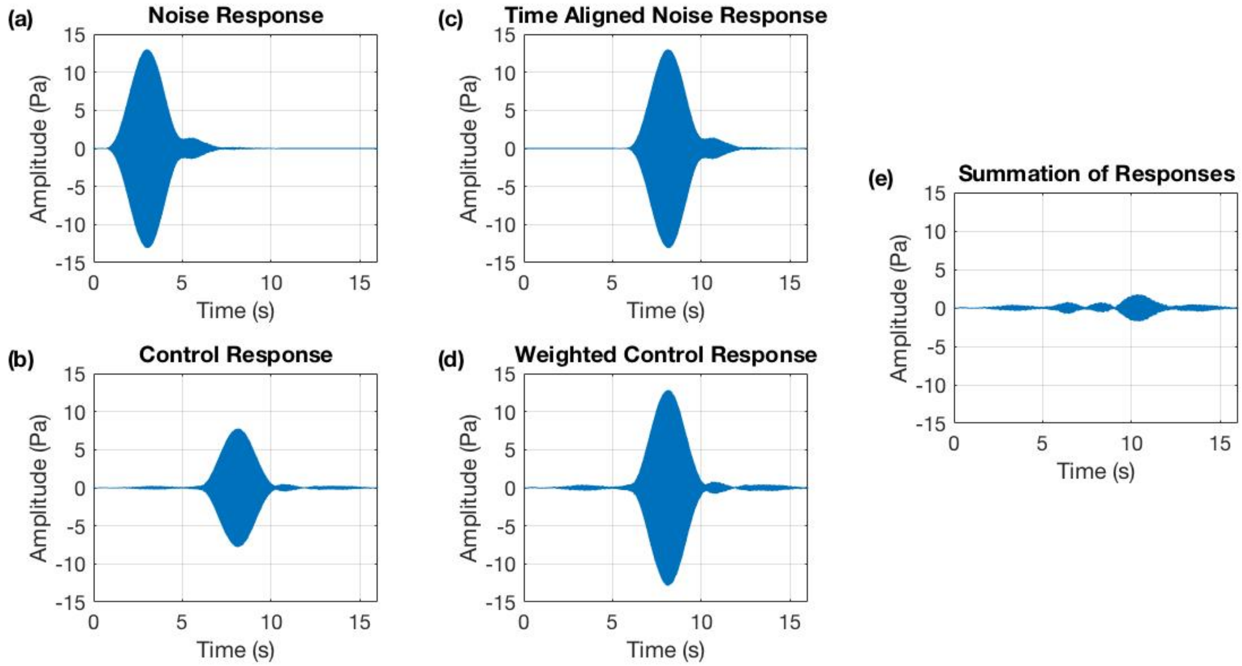

FIG. 5. Overview of the simulation of active noise control delivered by time reversal. (a) Noise response $x_r[n]$ from the noise loudspeaker. (b) Control response $a_r[n]$ from the control loudspeaker. (c) Time aligned noise response $x_{r,p}[n-k]$. (d) Amplitude weighted control response $\mu a_{r,p}[n]$. (e) The superposition of $x_{r,p}[n-k]$ and $\mu a_{r,p}[n]$ to achieve noise cancelation.

C. Analysis of Noise and Control Responses: Simulated ANC

The signals $x_r[n]$ and $a_r[n]$ need to be time aligned to achieve cancelation. One of the issues is that these are discrete time signals and it is probable that the most accurate time delay is not an integer number of time samples. To account for this, an interpolation function is applied to these signals to increase the number of samples for each signal, creating $x_{r,p}[n]$ and $a_{r,p}[n]$. A discrete time cross-correlation between $x_{r,p}[n]$ and $a_{r,p}[n]$, allows the integer number of samples between them, k, to be determined for optimal alignment, $x_{r,p}[n] \rightarrow x_{r,p}[n-k]$, as if the two signals arrived at the microphone at the same time.

With the determined k-value, $x_{r,p}[n-k]$ becomes the time delayed noise response. This is done by adding k-number of zeros in front of $x_{r,p}[n]$ (see Fig. 5(c)). Though $x_{r,p}[n-k]$ and $a_{r,p}[n]$ are time aligned, $a_{r,p}[n]$ needs to be amplitude matched to $x_{r,p}[n-k]$ in order to achieve the greatest noise reduction. An optimal weight factor, μ, is determined that, when multiplied by $a_{r,p}[n]$, will provide the greatest reduction in overall sound pressure level (OASPL). Figure 5(d) displays the weighted control response, $\mu a_{r,p}[n]$, and Fig. 5(e) displays the summation signal, $s[n] = x_{r,p}[n-k] + \mu a_{r,p}[n]$, simulating the result of the ANC+TR process. The OASPL of $s[n]$ (control on) is calculated and compared to the OASPL of $x_{r,p}[n-k]$ (control off) to determine the overall noise reduction. The OASPL for both situations is

$$L_{\mathrm{OASPL}} = 10\log_{10}\left(\frac{\sum p[n]^2}{{p_{ref}}^2}\right) \tag{3}$$

where $p[n]$ is the pressure of a signal (either $s[n]$ or $x_{r,p}[n-k]$), and p_{ref} is 20 µPa.

III. Results from Parameterization Study

Following the procedures outlined in Chapter 2, the frequency and pulse length of a pulse signal, used as $x_s[n]$, are varied to quantify the capabilities of ANC+TR to reduce different types of noise. This parameterization study is performed in the reverberation chamber where TR is expected to have the greatest focal amplitude due to its reverberation time. The pulse length, which is doubled for every successive test, ranges from a pulse consisting of simply a half cycle of a sine wave to a maximum length of 2.048 s. The center frequency of the pulse ranged from 63 Hz to 16 kHz (see Fig. 6 for example signals of different pulse durations). Center frequencies were chosen to line up with the standardized octave band center frequencies.[20] These signals help determine the ability of ANC+TR to control different frequencies and whether narrowband or broadband signals are easier to control over the audible range of hearing. The results for the OASPL noise reductions are found in Fig. 7.

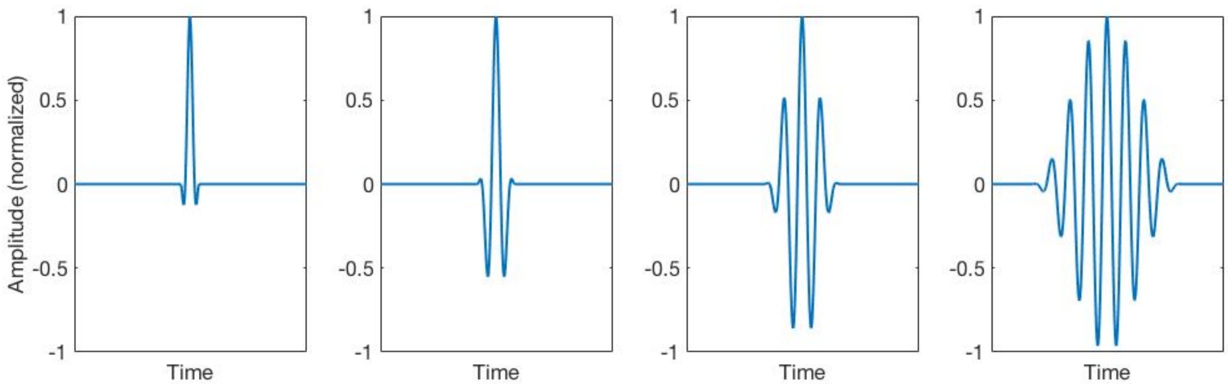

FIG. 6. An example of the pulse signals used, with a doubling of the pulse length for each signal shown.

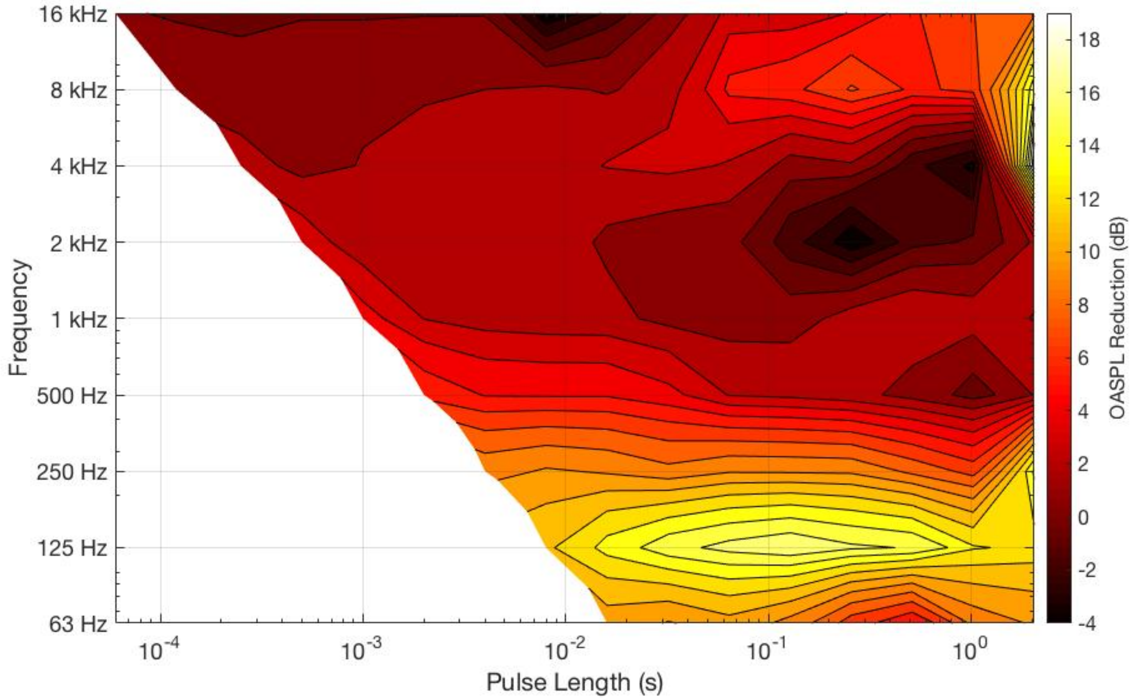

FIG. 7. The OASPL noise reduction of pulse signals played in a reverberation chamber expressed versus frequency and pulse length (s) using time reversal and active noise control. The color bar represents OASPL noise reductions in dB. The whited out area indicates no data.

It is clear from the results depicted in Fig. 7 that greater reductions are found with lower frequency signals for all pulse lengths. This is a common finding, that ANC is better able to attenuate low frequencies, because the phase mismatch errors are typically smaller. An interesting trend is observed at almost all frequencies when the pulse length exceeds 1 s, as reductions apparently increase as the pulse length increases, even at the highest frequencies. This implies that narrowband signals are easier to control, which is also commonly seen for ANC implementations.

IV. Results from Simulated Active Control of MRI Noise

The noise reducing capability of ANC+TR for MRI noise will now be explored. The noise from four MRI scanning sequences were recorded in the MRI facility on the campus of Brigham Young University. The MRI is a Siemens (Munich, Germany) TIM-Trio scanner with a magnetic field strength of 3 T. The scanning sequences used for this study are the Constructive Interference Steady State (CISS), Echo Planar Imaging (EPI) Diffusion, Magnetization-Prepared Rapid Gradient-Echo (MP-RAGE), and True Fast Imaging with Steady Precision (TrueFISP or TRUFI) sequences.

The MRI noise signals were obtained by placing the microphone outside of the MRI's active, magnetic-field cancellation zone with an approximate distance of 4.57 m (15 ft.) between the microphone and the MRI scanner. Though the measured sound level at the microphone is less than the level inside the bore, where a patient's head would be during a scanning sequence, the purpose of the current study is to quantify the performance of ANC+TR for actual MRI noise signals, irrespective of how loud they are or if it is done in the optimal location for the reductions. If reductions are achieved in one part of the room, then similar reductions should be possible to obtain in other room locations as well (though the bore presents a more confined spatial constraint). Note that these proof of concept experiments were not conducted *in situ.* It was easier to conduct these tests in facilities without equipment constraints and to avoid interruption of actual scans for medical purposes. Additionally, these experiments were conducted in two rooms with very different reverberation times.

Given that a scan sequence can run for long periods of time producing periodic noise, the noise signals used in the experiments were shortened to 6–8 s depending on the signal to contain the representative periodicity of each signal. It is expected (due to this periodicity) that any noise reductions within these time intervals should translate over into reductions over the entire duration of however long the MRI machine is actually scanning. Though the noise signals were shortened to differing lengths of time, the noise response of each signal was recorded for the same duration of time. The same holds true for the control signals' responses, and when the OASPL reductions were calculated for each MRI scan, all signals were of equal length; thus the OASPL calculation is over the same length in time for each scan. Figure 8 displays the recorded noise waveforms for each of the 4 scan sequences.

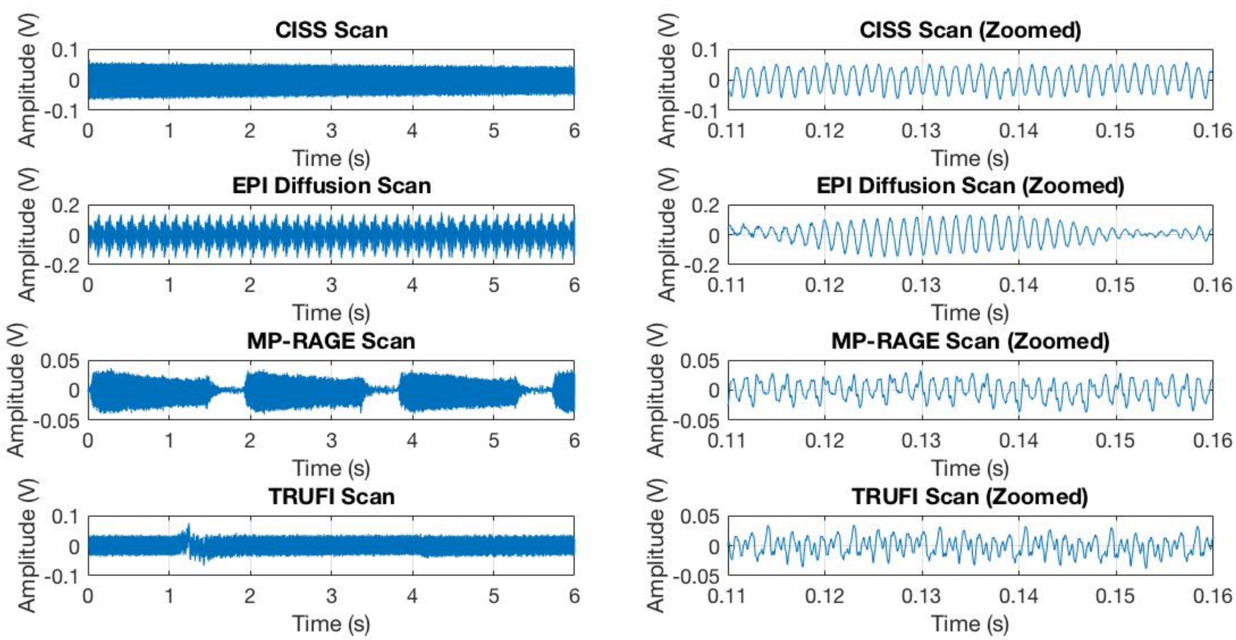

FIG. 8. Recorded MRI noises from the BYU MRI facility. The left column shows the full length of each noise signal. The right column shows zoomed in portions of the same signals to show respective signal behavior and periodicity.

Both the CISS and TRUFI scans have low frequency tones that remain throughout the duration of the scan. One of the main differences between the two is that the TRUFI scan has an audible, short-duration, buzzing sound that occurs every few seconds. The MP-RAGE scan also has the characteristic of a prominent lower frequency tone, but only for just over a second at a time, followed by brief silence. This is an important signal to analyze as well because of those silent pauses, since added noise, when there should be silence, is undesirable. The EPI Diffusion scan has very quick bursts of sound that repeat in a small amount of time. Though these are not all the signals that an MRI machine is able to produce, this set contains a variety of behaviors that are representative of typical MRI scans.

The recordings of the aforementioned MRI noise were broadcast through the noise loudspeaker in the reverberation chamber and in a standard laboratory room having a similar reverberation time as the room the MRI machine is housed in to analyze the optimal noise reduction using the procedure in Chapter 2. As before the control loudspeaker attempts to reduce the noise recorded at the microphone. The OASPL results of these experiments are shown in Fig. 9.

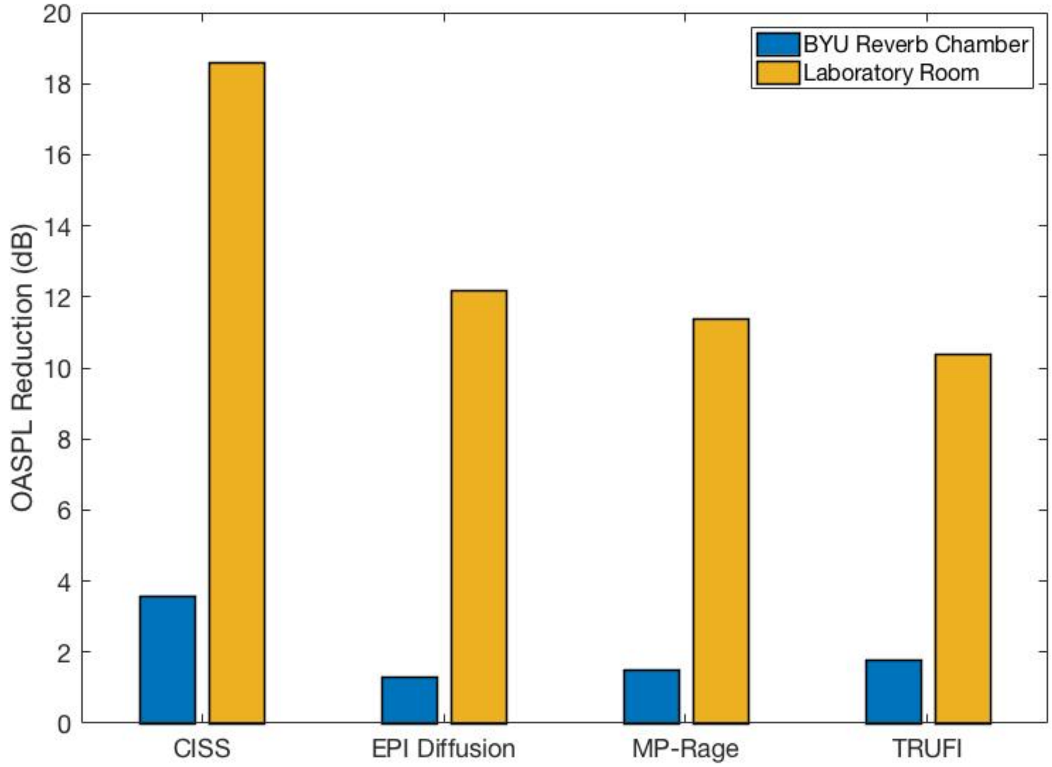

FIG. 9. Comparison of different MRI scans and their corresponding OASPL noise reductions in rooms with different reverberation times.

It is apparent that a room's reverberation time greatly impacts the effectiveness of ANC+TR in reducing MRI noise. Even though TR is expected to provide a higher amplitude focus in a more reverberant environment, it would seem that a room with a lower reverberation time provides the ability to achieve greater noise reductions with ANC+TR. These results are promising in that TR is capable of reducing complex noise signals with various behaviors. It should be noted that these results are obtained with a single control loudspeaker, but since the control loudspeakers may be placed at locations away from the MRI scanner, more loudspeakers should be able to be used to achieve greater reductions.

V. Conclusion

One of the primary motivations behind this study is to determine the feasibility of using time reversal (TR) for active noise control (ANC) purposes. A parameterization study explored the frequency dependence and signal duration (i.e. narrowband or broadband) for pulse signals in a reverberation chamber. The results show that low frequency signals allow for more consistent reductions, and longer duration signals (narrowband) also achieve greater noise reductions.

MRI noise from four different scan sequences were recorded and TR delivery of ANC (ANC+TR) signals was demonstrated in a reverberation chamber and a standard laboratory room, each with very different reverberation times. These noises are representative of various MRI scan sequences and illustrate that ANC+TR can reduce complex signals. Results from the parameterization study and the MRI noise control study show that ANC+TR is capable of reducing both simple and complex signals up to 20 dB. In the case of MRI noise, results show that a room with a smaller reverberation time yield greater noise reductions than results in a more reverberant room. This is a promising result because MRI facilities are not typically highly reverberant environments.

These results provide evidence that TR has the capability of being used for ANC purposes. Real time controllers would need to be implemented and incorporation of the controller with scan sequence information would need to be developed. The controller would automatically provide the time delay and amplitude adjustment necessary to optimize the noise control. Additionally, the required amount of reverberation to include in the TR process would need to be optimized for brevity while still maintaining adequate control at the desired location.

The control signal must be broadcast before the noise has been generated, limiting the range of applications.

References

[1] N. Lee, Y. Park, and G. W. Lee, “Frequency-domain active noise control for magnetic resonance imaging acoustic noise,” Appl. Acoust. **118**, 30–38 (2017).

[2] F. G. Pla, S. D. Sommerfeldt and R. A. Hedeen, “Active control of noise in magnetic resonance imaging,” Proceeding of Active 95, Newport Beach, CA, USA, 573-582 (1995).

[3] M. E. Ravicz, J. R. Melcher and N. Y. Kiang, “Acoustic noise during functional magnetic resonance imaging,” J. Acoust. Soc. Am. **108**(4), 1683–1696 (2000).

[4] Optoacoustics, LTD., “OptoActive II ANC Headphones” http://www.optoacoustics.com/medical/optoactive-ii (Last viewed June 22, 2018).

[5] C. B. Ahn and Z. H. Cho, “Analysis of the eddy-current induced artifacts and the temporal compensation in nuclear magnetic resonance imaging,” IEEE Trans. Med. Imaging **10**(1), 47–52 (1991).

[6] W. M. Spees, N. Buhl, P. Sun, J. J. H. Ackerman, J. J. Neil, and J. R. Garbow, “Quantification and compensation of eddy-current-induced magnetic-field gradients,” J. Magn. Reson. **212**(1), 116–123 (2011).

[7] M. E. Ravicz and J. R. Melcher, “Isolating the auditory system from acoustic noise during functional magnetic resonance imaging: Examination of noise conduction through the ear canal, head, and body,” J. Acoust. Soc. Am. **109**, 216–231 (2001).

[8] A. Parvulescu and C. S. Clay, “Reproducibility of signal transmission in the ocean,” Radio Elec. Eng. **29**, 223-228 (1965).

[9] M. Fink, “Time reversed acoustics,” Phys. Today **50**, 34-40 (1997).

[10] B. E. Anderson, M. Griffa, C. Larmat, T. J. Ulrich, and P. A. Johnson, "Time Reversal," Acoust. Today **4**(1), 5-16 (28).

[11] C. S. Clay and B. E. Anderson, “Matched signals: The beginnings of time reversal,” Proc. Meet. Acoust. **12**, 055001 (2011).

[12] J. V. Candy, A. W. Meyer, A. J. Poggio, and B. L. Guidry, “Time-reversal processing for an acoustic communications experiment in a highly reverberant environment,” J. Acoust. Soc. Am. **115**(4), 1621–1631 (2004).

[13] A. W. Meyer, J. V. Candy, and A. J. Poggio, “Time reversal signal processing in communication – A feasibility study,” Lawrence Livermore Nat. Lab. Lib. 246245 (2002).

[14] S. Yon, M. Tanter, and M. Fink, “Sound focusing in rooms: The time-reversal approach,” J. Acoust. Soc. Am. **113**(3), 1533–1543 (2003).

[15] G. Ribay, J. Rosny, and M. Fink, “Time reversal of noise sources in a reverberation room,” J. Acoust. Soc. Am. **117**(5), 2866–2872 (2005).

[16] B. E. Anderson, M. Clemens and M. L. Willardson, "The effect of transducer directivity on time reversal focusing," J. Acoust. Soc. Am. **142**(1), EL95–EL101 (2017).

[17] M. L. Willardson, B. E. Anderson, S. M. Young, M. H. Denison, and B. D. Patchett, "Time reversal focusing of high amplitude sound in a reverberation chamber," J. Acoust. Soc. Am. **143**(2), 696–705 (2018).

[18] M. Tanter, J.-L. Thomas, and M. Fink, "Time reversal and the inverse filter," J. Acoust. Soc. Am. **108**(1), 223–234 (2000).

[19] B. E. Anderson, J. Douma, T. J. Ulrich, and R. Snieder, "Improving spatio-temporal focusing and source reconstruction through deconvolution," Wave Motion **52**(9), 151–159 (2015).

[20] A. D. Pierce, *Acoustics: An introduction to its physical principles and applications*, (Acoustical Society of America, Woodbury, New York), p. 58 (1989).

Appendix A. Matlab Code

This Matlab script calculates the time delay between two signals by increasing the number of samples between data points of two input signals and taking the cross-correlation between these interpolated signals.

interpSigs.m

```
function [sq1_ts,sq2,lagDiff] = interpSigs(s1,s2,r,fs)
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Created by Trent Furlong; May 23, 2018
%%% DESCRIPTION %%%
% Interpolates a noise signal ('s1') and control signal ('s2') that were
% recorded at a sampling frequency equal to 'fs' and increases the number
% of samples between each point by 'r'. Outputs two time aligned,
% interpolated signals.
%
%%% APPLICATION %%%
% To be used for time reversal research applied to active noise control, in
% order to determine the optimal time delay between a noise response and a
% control response using time reversal.
%
%%% INPUT PARAMETERS %%%
% s1 = Input signal #1 e.g. the noise response. (double - column vector)
% s2 = Input signal #2 e.g. the control response using time reversal. (double
- column vector)
% r  = Defines an increased sampling rate; MUST BE AN INTEGER VALUE!
(integer)
% fs = Original sampling frequency of 's1' and 's2'. (double)
%
%%% OUTPUT PARAMETERS %%%
% sq1_ts  = The interpolated, time aligned noise response with 'sq2'. (double
- column vector)
% sq2     = The interpolated control response. (double - column vector)
% lagDiff = The calculated time delay (in samples) between 's1' and 's2'.
(double)
%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
dt = 1/fs;
t1 = 0:dt:(length(s1)-1)/fs; t1=t1'; % time vector correlating w/ 's1'
t2 = 0:dt:(length(s2)-1)/fs; t2=t2'; % time vector correlating w/ 's2'
```

```
tq1 = 0:dt/r:(length(t1)-1)/fs; % interpolated time vector for 's1'
tq2 = 0:dt/r:(length(t2)-1)/fs; % interpolated time vecotr for 's2'

sq1 = interp1(t1,s1,tq1,'spline'); sq1 = sq1'; % Interpolated 's1'
sq2 = interp1(t2,s2,tq2,'spline'); sq2 = sq2'; % Interpolated 's2'

[xc,lag] = xcorr(sq2,sq1); % Cross-Correlation of 's1' and 's2'

[~,I] = max(abs(xc)); % Returns the index value I of the max(abs(xc)).
lagDiff = lag(I); % The index value I of 'lag' is the difference in samples
between 's1' and 's2'.

y = -lagDiff:(length(sq2)-1)-lagDiff; % New time vector for interpolated
noise signal to map to.

sq1_ts = interp1(sq1,y); sq1_ts = sq1_ts'; % Maps 'sq1' to 'y'.

[row,~] = find(isnan(sq1_ts)); % Find NaN in sq1_ts
sq1_ts(row)=0;
```

This Matlab script calculates the weight factor needed to scale the control response such that when the noise and control responses are added together, the greatest noise reduction is achieved. It is also capable verifying the time delay value determined from the cross-correlation between the noise and control responses by moving the noise response sample by sample until a maximum reduction is calculated.

sigOpt.m

```
function x=sigOpt(signal,antisignal,ToA,j0,jf,step)


if nargin == 6
    dj=step;
elseif nargin < 6
    dj=1;
end

Lmin=500;         % High enough dB level to guarantee first iteration will be
lowest sound level
an=antisignal;    % Anti-noise signal
n=signal;         % Noise signal
count=1;          % Number of time positions with same sound level
ratio=length(antisignal)/length(signal);

tf=strcmp(ToA,'Time');
```

```
if tf==0        % ToA='Amp'
    % Amplitude optimization 'for' loop
    for mu=j0:dj:jf
        N=n;        % Noise signal
        AN=mu*an;    % New anti-noise signal
        L=calcOASPL(N+AN); % Calculate the OASPL of the two summed signals

        % Keeps track of all potential positions with same OASPL
        if L==Lmin
            x(count)=mu;
            count=count+1;
        end
        % Compare the OASPL of each iteration and identify optimal 'mu'
        if L<Lmin
            Lmin=L;
            x=mu;
        end
    end

elseif tf==1    % ToA='Time'
    % Time optimization 'for' loop
    for tau=j0:dj:jf

        z=length(an)-length(n)-tau; % Sets length(N)=length(an)

        if length(z)<0
            disp('Cannot move signal any farther!')
            x=x;
            break
        end

        N=zeropad(n,tau,z);
        AN=zeropad(an,0,length(N)-length(an));

        L=calcOASPL(N+AN);

        % Keeps track of all potential positions with same OASPL
        if L==Lmin
            x(count)=tau;
            count=count+1;
        end
        % Compare the OASPL of each iteration and identify optimal 'mu'
        if L<Lmin
            Lmin=L;
            x=tau;
        end
    end

end

% Because this can take so long, sounds a blip to let you know its done.
fs=50000; f=1000; w=2*pi*f;
t=0:1/fs:.125;
sound(sin(w*t),fs)
% disp('DONE!')
```

This Matlab script calculates the OASPL of an input signal.

calcOASPL.m

```
function L=calcOASPL(signal)

pref=20e-6;
prms=rms(signal);

L=20*log10(prms/pref);
```

This Matlab script will add zeros before or after an input signal. Used primarily to time align noise and control signals.

zeropad.m

```
function A=zeropad(signal,ndiff,mdiff)

z1(1:ndiff)=0; z1=z1';
z2(1:mdiff)=0; z2=z2';

N=cat(1,z1,signal);
A=cat(1,N,z2);
```