Constructing a Helical Resonator for a Dual-Species Ion and Atom Trap

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ABSTRACT

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Dual-species ultra-cold plasmas in our laboratory expand too quickly for measurements at times longer than 30 μs. We plan to solve this problem by trapping the plasma using a Paul trap. The trap will be loaded by photo-ionizing neutral atoms in a co-located magneto-optical trap. This will enable plasma studies over longer time periods, making it possible to measure the internal interactions within the plasma. In order to use a Paul trap, we need to deliver higher voltages to the trap at radio frequencies. We report the successful construction of a harmonic resonator with a resonant frequency of $\omega = 2\pi \times 4.0$ MHz and a Q-factor of 151. When imperfect impedance matching is included in our measurements, we find that our helical resonator amplifies our radio frequency generator's output voltage by a factor of 120 at the trap.

Keywords: Q-factor, helical resonator, Paul trap

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Contents

List of Figures

Chapter 1

Introduction

1.1 Dual-species plasmas and why we care

Transport properties of multi-species plasmas are interesting to high density plasma physics, particularly in fusion plasmas. These fusion reactions often occur at high temperatures and densities. This environment is very difficult to study, but a lot can be learned from observing plasmas at lower temperatures. At these lower temperatures, the ions in the plasma move slowly and their motion is easier to measure and characterize.

1.2 Previous work in our laboratory

We have a long history of studying ion-ion interactions in our lab. Previously, we focused mainly on studying the expansion of the ions in a dual-species plasma. We trapped neutral atoms of calcium and ytterbium using a magneto-optical trap (MOT), then ionized them using nanosecond-duration pulsed lasers. The resulting overlaying plasmas were left to expand, and we measured this expansion for both calcium and ytterbium, which lasted approximately 30 μs. These MOTs allow us to study the Ca and Yb atoms in a controlled and stable environment.

There are two main components that make up the MOT: a three-dimensional optical molasses and a magnetic quadrupole field. An optical molasses works by using counter-propagating laser beams that are red-detuned from resonance relative to the atoms. Atoms moving towards the laser will absorb more photons due to the Doppler effect. This causes the atoms, which are now in an excited state, to lose momentum in that direction. After a short time the atom will drop back to the ground state and re-emit a photon in a random direction. After many rounds of absorption and emission, the atoms moving towards the laser beam will lose momentum on-average and slow down. By using three optical molasses, we create a three-dimensional slower for our atoms.

Once the atoms are slowed down, the magnetic quadrupole field does the rest of the trapping. This field is created using anti-Helmholtz coils. The magnitude of the magnetic field increases linearly with distance from the center of the trap. This field Zeeman-shifts the energy levels, splitting the excited state into a triplet of levels. When the atom drifts far enough from the center of the trap, the energy lines split far enough so that the down-shifted level is in resonance with the MOT laser beam. Through absorption of photons, the atom is pushed back towards the center of the trap.

1.3 Motivation for creating a hybrid trap

Using our MOTs, we are only able to trap neutral atoms, not ionized plasmas. When measuring properties of the dual-species plasmas, our time scale is limited by the freely expanding ions. This motivated us to superimpose a new type of trap over our MOT, creating a hybrid trap. Our second trap would be capable of trapping our atoms after they have been ionized, allowing us to contain the plasmas and prevent them from expanding and escaping. This will allow us to extend our time scales and enable new geometries and other collisional transport effects within the dual-species plasma.

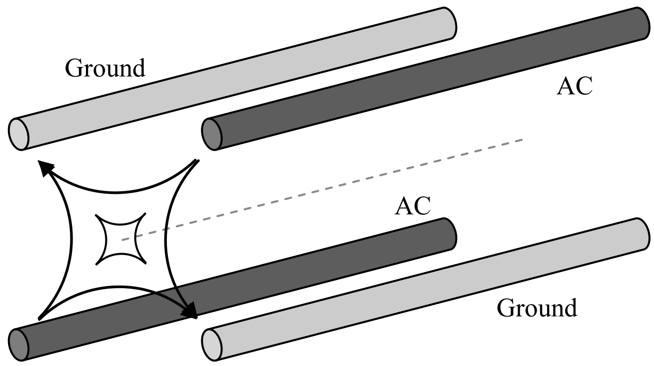

Figure 1.1 Set up for a Paul trap. This design consists of four electrodes. Two of them are held at ground, and the voltage on the other two oscillate at a predetermined frequency. Atoms are trapped in the center, along the dotted line. The atoms are pushed away from the grounded rods while being pulled towards the rods with oscillating voltage. When the field flips with the voltage, the atoms are pulled towards the grounded rods and pushed away from the other rods.

1.4 How a Paul trap works

Although easily done in one or two dimensions, it is impossible to create a three-dimensional confining force using static electric fields. This effect is known as Earnshaw's theorem. However, dynamic electric fields can be used to trap particles. One such technique for trapping charged particles, formally called the quadrupole ion trap, was developed by Wolfgang Paul and was soon nicknamed the Paul trap in his honor.

There are a few different Paul trap configurations. One such configuration, referred to as a linear quadrupole trap, has four cylindrical electrodes spaced evenly apart from each other (see Fig.1.1). Two of the electrodes are held at ground potential, while the other two are connected to an alternating voltage. This creates a saddle-shaped potential along the center of the trap, which flips orientation at the frequency of the voltage source.

When a charged particle is placed within the alternating field, it will try to "fall out" of the saddle. If the fields were stationary, it would do so. However, as the particle moves away from the trap center, the potentials flip orientation, and now the field pushes the particle back towards the center of the trap. As long as the frequency is set correctly, the particle reaches the center just as the current switches direction again, and it begins to fall back out of the trap. This back-and-forth motion is typically referred to as the particle's micromotion.

Using the Paul trap allows us to use oscillating fields to trap ions. In conjunction with our MOT, we can build a hybrid trap that works on both the neutral and charged atoms. The MOT and ion trap combination (MOTion trap) will allow us to trap our dual-species plasma for extended time periods.

1.5 The need for a helical resonator

The benefits of using a Paul trap are somewhat offset by the added complication of finding a suitable generator. Paul traps work in the radio frequency (RF) range. Finding the right generator capable of delivering large voltages at high frequencies can be both challenging and expensive. However, a helical resonator is a cost-effective way to achieve the frequency and voltage requirements that a Paul trap needs.

Just like how specific frequencies resonate by creating standing waves within a pipe or on a string, helical resonators are designed to create a standing wave with voltages. These resonators are designed to have a high ratio of output to input voltages when operated at its resonance frequency. This ratio is called the Q-factor. Instead of finding a generator capable of producing high voltages at radio frequencies, we can build a helical resonator with a large Q-factor to amplify the output of a generator that produces lower voltages at radio frequencies.

1.6 Overview

The overarching goal for my project was to design and build a helical resonator with a resonant frequency in the 1-5 MHz range and a Q-factor of a few hundred. This helical resonator can be used in conjunction with our MOTion trap to further our lab's research goals. We present designs for our helical resonator and numerical results on the Q-factor of our resonator. We also discuss building and using an antenna coil to fix impedance matching issues.

The resonator we built has a resonant frequency of $\omega = 2\pi \times 4.0$ MHz and a Q-factor of 151. Due to imperfect impedance matching, only 75% of our RF signal is injected into our coil. In total, the helical resonator amplifies the RF signal by 120 at the trap.

Chapter 2

Methods

2.1 Voltage and frequency calculations

Paul traps used to trap ion particles typically run anywhere from 100 kHz to 50 MHz [1–3]. We know that for a charge in an oscillating electric field, $E = E_0 \cos(\omega t)$, the Coulomb force acting on that charge can be expressed as $F = eE = eE_0 \cos(\omega t)$, with e being the absolute value of the charge of the particle we are trying to trap. Making use of Newton's Second Law, we find the acceleration on the charge to be

$$a(t) = \frac{eE_0}{m} \cos(\omega t),$$

where m is the mass of our particle. Integrating twice, we find the oscillation amplitude to be

$$x(t) = \frac{eE_0}{m\omega^2} \cos(\omega t). \tag{2.1}$$

When building a Paul trap, it is important to construct it in a way such that the trapped particles are stable. The stability of the particle trajectories in the trap can be determined by the Mathieu q-parameter, q_M. We can find q_M by dividing the maximum amplitude by the characteristic size of the trap. In our case, we have

$$q_M = \frac{eE_0}{m\omega^2 d},$$

which we can approximate as

$$q_M = \frac{\beta e V_0}{m\omega^2 d^2}, \tag{2.2}$$

with β being a dimensionless parameter to account for trap geometry. This derivation is intended to illustrate how the voltage, trap size, and RF frequency combine to trap the ions. A more exact derivation would use the full quadrupole field and solve the Mathieu equation. An example of this can be found in Ref. [4]. Our derivation also misses the ion motion at the so-called secular frequency. However, it illustrates the physical meaning of q_M as the ratio of the oscillation amplitude to the trap size.

For $q_M \leq 1$, Equation 2.2 gives us the range of voltages and frequencies that will allow for stable trapping of our ions given our Paul trap's configuration. In our case, we aimed to optimize our helical resonator for a resonant frequency of 5 MHz, which we will use to trap calcium ions. Letting $q_M = 0.1$, $\beta = 1$, $e = 1.6\times10^{-19}$ C, $m = 40\cdot1.67\times10^{-27}$ kg, $\omega_0 = 2\pi\cdot5\times10^6$ Hz, and $d = 0.01$ m, we find the needed voltage to be $V_0 \approx 4000$ V. When considering the full derivation of q_M, the voltage is reduced by an additional factor of 4.

2.2 Deciding parameters and approximations

Helical resonators need carefully selected dimensions in order to produce the expected results. The various dimensions, including the heights and diameters of the helical coil and outer shield, are all related to and affect each other. The basic layout of these various dimensions can be found in Figure 2.1.

Before making any official calculations, we needed to determine three parameters: resonator material, wire thickness, and coil winding pitch. These parameters were selected based on availability and cost. A publication by Siverns et al. [5] presents a detailed discussion of helical resonator design. For hand-wound coils, Siverns suggested a wire thickness of $d_0 = 5$ mm and a coil winding pitch of

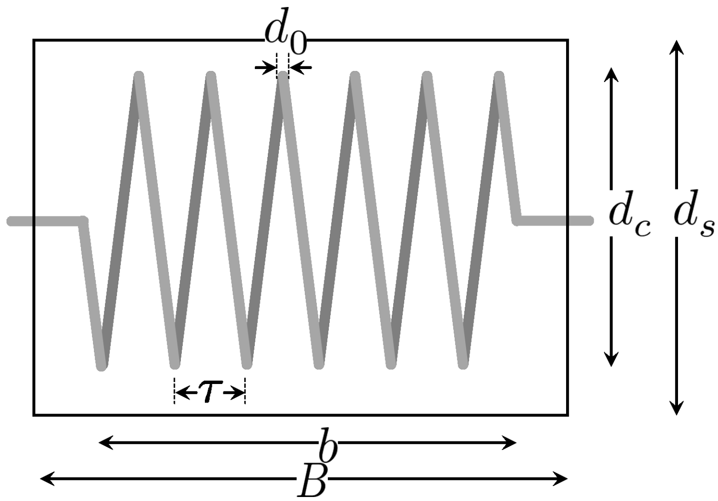

Figure 2.1 Here we see the various dimensions that are relevant to our helical resonator design. B and b are the heights of the shield and coil, respectively. Similarly, d_s and d_c are the diameters of the shield and coil. Also shown are d_0, the wire thickness, and τ, the winding pitch of the coil.

$\tau = 2d_0$ [5]. We also wanted to use a relatively inexpensive material with low resistivity. Consulting a table of resistivity values [6], we decided to make the coil out of copper ($\rho_{cu} = 1.72 \cdot 10^{-8}$ Ω·m) and the shield out of aluminum ($\rho_{al} = 2.82 \cdot 10^{-8}$ Ω·m).

We also had to approximate the resistances and capacitances of our MOTion trap and its connectors before we were able to correctly calculate the size and shape of our helical resonator. We measured the physical capacitances of each electrode for the Paul trap and found that each electrode had a capacitance of about 25 pF. Accounting for the configuration of our electrodes and the typical capacitance of a BNC connector, we estimated our MOTion trap and connecting wires would have a total capacitance of around 55 pF. We also made an order-of-magnitude estimate of 1 Ω for our trap resistance.

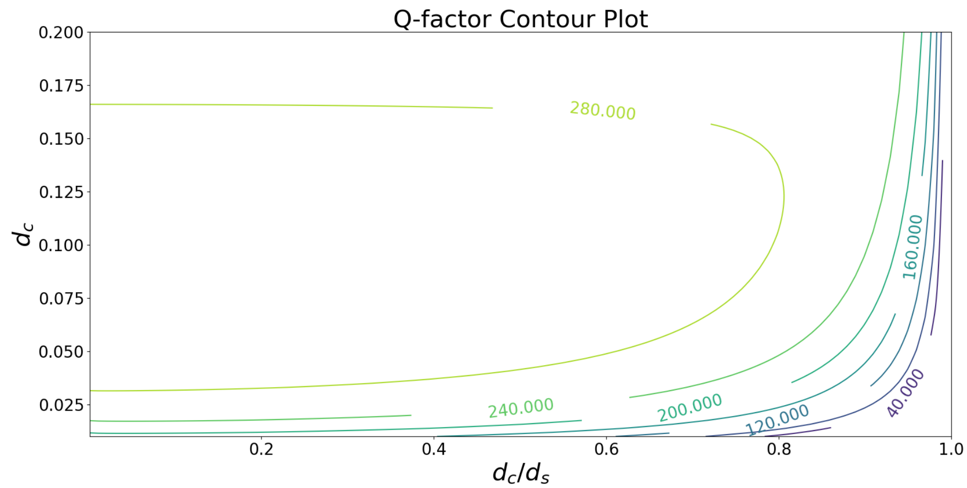

Figure 2.2 After measuring or deciding the parameters for resonant frequency, winding pitch, and the resistance and inductance of the ion trap, we can plot the theoretical Q-factor of our resonator as a function of two variables: the diameter of the coil (d_c) and the ratio of the coil and shield diameters (d_c/d_s).

2.3 Helical resonator calculations and approach

Following the derivation provided by Siverns et al. [5], we created a plot to show the optimization of the Q-factor given our above-mentioned parameters. Treating both the coil diameter (d_c) and the shield diameter (d_s) as variables, we made contour plots as a function of d_c and the ratio d_c/d_s (see Fig. 2.2). We aimed to maximize the Q-factor while minimizing the influence of errors in building the resonator. We knew that having a smaller d_c/d_s ratio would make the trap easier to build, since it would allow us to stabilize the coil in the center more easily.

We decided to build our coil out of 10-gauge copper wire with a coil diameter of about 9 cm, a winding pitch of 5.2 mm, and a little over 22 turns. The coil was formed around a plastic base to provide stability. We ordered a 12-cm-diameter, 30-cm-long aluminum shield to fit over our coil. Following the calculations provided by Siverns et al. [5], we predicted a resonant frequency of 5 MHz and a Q-factor of 250.

Chapter 3

Results

3.1 Construction and changes

We designed our helical resonator to operate at a resonant frequency of 5 MHz with a predicted Q-factor of around 250. After we finished building our resonator, we measured the Q-factor and found no resonance frequencies for our coil. Upon looking into the issue, we found that the majority of our signal was being reflected, not transmitted. In other words, the impedance between the frequency generator and our helical resonator was mismatched. Siverns et al. discussed impedance matching in their paper [5], and we decided to follow their advice and build an antenna coil to fix our issue.

We built a smaller coil using insulated 10-gauge wire with a coil diameter of about 5 cm. We modified our previous design, moving one of the main coil's connectors from the end cap to the side, and placing the antenna coil's connectors on the now-unused end cap (see Fig 3.1). We designed our antenna coil to be adjustable; the insulated coil could be stretched or compressed to alter the winding pitch and change it's impedance. By using a oscilloscope, we were able to adjust our antenna coil to minimize the percentage of the signal that is reflected back towards the source;

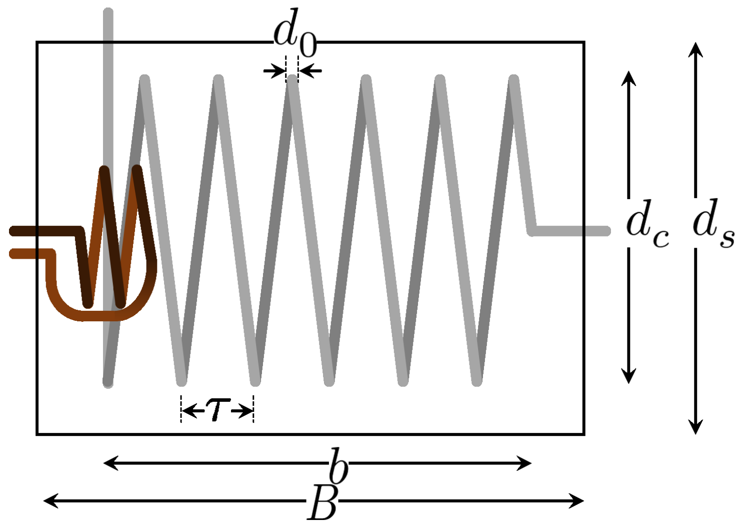

Figure 3.1 A diagram of the helical resonator after the antenna coil was inserted. Here, the left connector of our coil was moved to the side of the shield to leave room for the antenna coil.

however, we were unable to completely eliminate the reflected signal.

3.2 Resonance and Q-factor Measurement

To make our Q-factor measurements, we scanned over a range of 400 kHz in 10-20 kHz intervals and used an oscilloscope and a bi-directional coupler to measure the peak-to-peak voltage of what is reflected back from our helical resonator. On resonance, this signal is minimized. After taking our measurements, we fit our data to a Lorentzian curve (see Fig. 3.3). Based on our fit, we found that our helical resonator is optimized at 4 MHz with a full-width-half-maximum of 26 kHz. Our Q-factor is then given to be

$$Q = \frac{\omega_0}{\Delta\omega} = \frac{4 \times 10^6 \text{ Hz}}{26 \times 10^3 \text{ Hz}} = 151.$$

We also measured what percentage of our input voltage is still reflected from our trap on resonance, and found that about 75% of the input signal is being reflected, giving us $\eta = 0.75$. Overall, we find that $V_{out} = 120 \times V_{in}$ (see Appendix A).

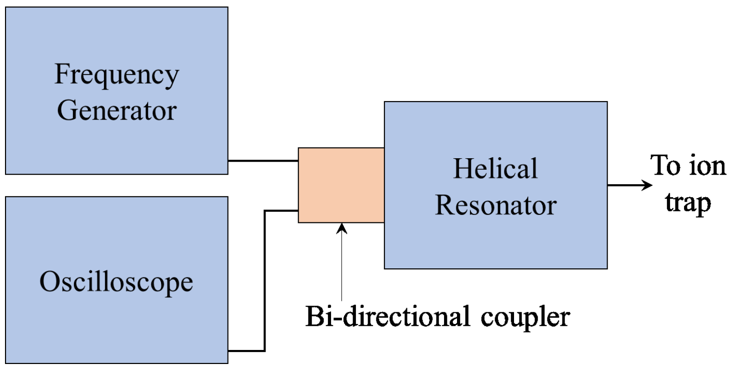

Figure 3.2 A plot of the peak-to-peak reflected voltage vs. frequency.

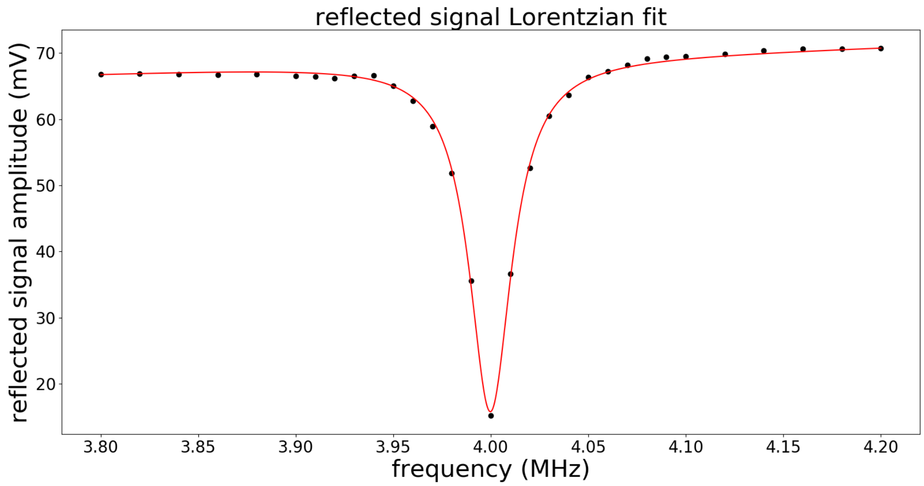

Figure 3.3 A plot of the peak-to-peak reflected voltage vs. frequency, fitted to a Lorentz curve.

3.3 Future work

Our helical resonator will be used in the future to help trap ions in our newest dual-species MOTion trap in our research lab. We hope to be able to better characterize the properties of our plasmas and develop methods of predicting their behavior in various situations. The cost- and energy-efficient helical resonator that we built will allow us to explore new configurations that were previously inaccessible to us.

In the future, we hope to build a MOTion trap that traps the plasma's electrons directly, instead of trapping the ions. This trap will be physically smaller than our other traps, and will need to run at 1 GHz. We hope to build another helical resonator in the future that is optimized for our needed frequency of 1 GHz.

Appendix A

Q-factor derivation

The helical coil and Paul trap system can be thought of as a circuit with a resistor (R), an inductor (L), and a capacitor (C), commonly known as an RLC circuit. When driven by an sinusoidal voltage at frequency ω, we can express the reactance of the inductor as

$$X_L = i\omega L, \tag{A.1}$$

where L is the inductance. Similarly we can express the reactance of the capacitor to be

$$X_C = \frac{1}{i\omega C}, \tag{A.2}$$

with C being the capacitance.

Using Ohm's law, we can calculate the current of our RLC circuit using equations A.1, A.2, and the resistance, R, to be

$$I = \frac{V}{X} = \frac{V}{R + i\omega L + \frac{1}{i\omega C}} \tag{A.3}$$

We can solve A.3 for $\frac{I}{V}$ and separate the real and imaginary parts as follows:

$$\begin{aligned}
\frac{I}{V} &= \frac{1}{R+i(\omega L - \frac{1}{\omega C})} \\
&= \frac{1}{R+i(\frac{\omega^2 LC-1}{\omega C})} \\
&= \frac{\omega C}{\omega RC + i(\omega^2 LC - 1)} \\
&= \frac{\omega^2 RC^2}{\omega^2 R^2 C^2 + (\omega^2 LC-1)^2} - i\frac{\omega C(\omega^2 LC-1)}{\omega^2 R^2 C^2 + (\omega^2 LC-1)^2}
\end{aligned} \tag{A.4}$$

While on or near resonance, ideally the system is purely resistive and has no imaginary component. Therefore, we can say that

$$\frac{I}{V} = \frac{1}{R} \approx \frac{\omega^2 RC^2}{\omega^2 R^2 C^2 + (\omega^2 LC-1)^2}. \tag{A.5}$$

From here we can calculate the half-width at half-maximum (HWHM). At the HWHM, we know that

$$\frac{1}{2R} = \frac{\omega^2 RC^2}{\omega^2 R^2 C^2 + (\omega^2 LC-1)^2}.$$

Multiplying both sides by R and then simplifying, we get

$$\frac{1}{2} = \frac{1}{1+\frac{(\omega^2 LC-1)^2}{\omega^2 R^2 C^2}}. \tag{A.6}$$

This implies that the denominator of the ride side of Equation A.6 must equal 2, or in other words,

$$1 = \frac{(\omega^2 LC-1)^2}{\omega^2 R^2 C^2}.$$

This equation simplifies to become

$$\omega^2 LC - 1 = \omega RC. \tag{A.7}$$

From here we note that our resonance frequency is defined as $\omega_0 = \frac{1}{\sqrt{LC}}$ and we assume our HWHM frequency is given by $\omega = \omega_0 + \Delta\omega$. Making these substitutions into Equation A.7, we have

$$\left(\frac{\omega_0 + \Delta\omega}{\omega_0}\right)^2 - 1 = (\omega_0 + \Delta\omega)RC. \tag{A.8}$$

Expanding the squared term using a first-order Taylor series expansion, we find that Equation A.8 becomes

$$\left(1 + \frac{2\Delta\omega}{\omega_0}\right) - 1 = \frac{2\Delta\omega}{\omega_0} = \omega_0 RC + \Delta\omega RC.$$

Solving for $\Delta\omega$, we find that

$$\Delta\omega = \frac{\omega_0^2 RC}{2 + \omega_0 RC}. \tag{A.9}$$

If we make the assumption that $R \ll \frac{1}{\omega_0 C}$, we find that

$$\Delta\omega = \frac{\omega_0^2}{2} = \frac{R}{2L}.$$

The Q-factor is given by the resonance frequency, ω_0, divided by the full-width half-maximum, which is two times the HWHM, or $2\Delta\omega$, where $\Delta\omega$ is given by Equation A.9. Therefore, we have that the Q-factor is given by

$$Q = \frac{\omega_0}{2\Delta\omega} = \frac{1}{R}\sqrt{\frac{L}{C}}.$$

At any given time, the voltage across our helical resonator can be expressed using Ohm's law. The voltage of our helical resonator, which is an inductor, is given by

$$V_L = X_L I = i\omega L\left(\frac{V_{source}}{R}\right) = i\frac{1}{R}\sqrt{\frac{L}{C}}V_{source} = iQV_{source}.$$

Hence, we find that the Q-factor amplifies the source voltage linearly. The i tells us that our resonant signal is completely out of phase with our input signal. Imperfect impedance matching reduces the RF signal injected into the coil. If we parameterize RF indection efficiency as

$$\eta = \frac{1 - V_{\text{ref}}(\omega_0)}{V_{\text{source}}},$$

we find that

$$V_L = \eta Q V_{source}.$$

Bibliography

[1] K. Chen, S. J. Schowalter, S. Kotochigova, A. Petrov, W. G. Rellergert, S. T. Sullivan, and E. R. Hudson, "Molecular-ion trap-depletion spectroscopy of $BaCl^+$," Phys. Rev. A **83,** 030501 (2011).

[2] W. G. Rellergert, S. T. Sullivan, S. Kotochigova, A. Petrov, K. Chen, S. J. Schowalter, and E. R. Hudson, "Measurement of a Large Chemical Reaction Rate between Ultracold Closed-Shell ^{40}Ca Atoms and Open-Shell $^{174}Yb^+$ Ions Held in a Hybrid Atom-Ion Trap," Phys. Rev. Lett. **107,** 243201 (2011).

[3] E. Peik, J. Abel, T. Becker, J. von Zanthier, and H. Walther, "Sideband cooling of ions in radio-frequency traps," Phys. Rev. A **60,** 439–449 (1999).

[4] R. E. March, "An Introduction to Quadrupole Ion Trap Mass Spectrometry," Journal of Mass Spectrometry **32,** 351–369 (1997).

[5] J. D. Siverns, L. R. Simkins, S. Weidt, and W. K. Hensinger, "On the application of radio frequency voltages to ion traps via helical resonators," Applied Physics B-Lasers and Optics **107,** 921–934 (2012), pT: J; NR: 21; TC: 22; J9: APPL PHYS B-LASERS O; PG: 14; GA: 964XO; UT: WOS:000305730200006.

[6] "Resistivities for Common Metals,".

Index