Connecting Physics and Finance

Nick Steil

Physics 492R Capstone Project Report

Brigham Young University

December 23, 2019

Advisor: Dr. David Allred, Dr. Tim Leishman

Abstract

This paper delves into the connections between physics and finance, at the surface two seemingly unrelated fields. Central to the nature of the scientific method as an integral part of the physics education, I will connect elements of the method to the field of finance. The skills of building models to mathematically describe a law of physics based on certain assumptions is a skill that translates to build financial models to price assets. Both fields rely on the need for data to work in harmony with derived models to better understand the subject.

Introduction

Often times as I would tell individuals that I am studying both physics and economics for my college education, many were surprised to hear of that combination that I choose. A common response I would receive was, “How are those two subjects related?”. At the beginning after my choice to pursue these two subjects, I was unsure how to best respond since I had a difficult time to see how the two fields were connected. Physics, a hard science that studies the immutable laws that govern the physical universe, seems to be in stark contrast to economics, a social science that studies both individual and market behavior given scarcity of capital and goods. However, as I have gone through the process of completing both majors, I have come to realize that there is a core process that runs through the heart of these two disciplines: develop a mathematical model to describe a situation and then find data to verify the model. In essence, the scientific method, which has been engrained into me based on my education in physics, applies just as much in the field of economics and finance, one of its subfields.

This paper will show examples of how I have been able to apply the principles and skills of my physics education into the field of finance, a field I am deeply passionate about and in which I am building my career. The processes of making a mathematical model based on assumptions, collecting data, and using complex mathematics to describe certain phenomena will be shown. Elaborating on how the skillset of a physics education can be applied into the field of finance could help current or prospective students of physics see one of many possible routes to take with their education. The knowledge of problem-solving, trial and error, critical thinking in the realm of studying the laws of the universe are applicable in many fields and can be used for a successful career. My goal is to show how combining a skillset provided by a physics education with a passion, in my case finance, creates a more enriching experience in the pursuit of truth.

Making a Model

The happenings of the physical world can be difficult to describe, even more so when one tries to do it mathematically. A mathematical model of a physical phenomenon, derived using intuition of everyday experience and mathematics, is how one conveys mathematically what happens physically. One of the first models that is introduced to a first-year physics student is projectile motion. Given the initial velocity of a particle and an angle indicating the direction of the initial velocity, the model of projectile motion can provide useful information about what will happen in this system. One can know how far out the particle will land from the initial launch point, how high up the particle will travel relative to the ground, how long will the particle be in the air and so forth. A curious student could then devise their own experiment to test the validity of the model of projectile motion. Knowing what the model predicts will happen given certain inputs, the student can do an experiment where they, for example, build a contraption to launch a ball at a given angle, measure how far the ball lands, and compare what is experimentally observed to the predicted landing distance. If there is a discrepancy between what is observed and what the model predicts, something must be wrong. One can troubleshoot the experimental design to make sure the inputs are correct; if that is the case, then something must be wrong with the model.

One of the most prevalent flaws used in models to describe the laws of physics are the assumptions being made in the formulation of said model. The simplest models used in physics rely on key assumptions about a given system without losing a general core idea of what will happen. In the case of introductory projectile motion, the key assumptions being made are that the particle does not have a volume and the particle is traveling in a vacuum. It does not take long to realize that those assumptions are not accurate, explaining the difference in results; however, those assumptions still provide a good picture of the laws of physics at work. In order to create a better model, the assumptions need to be altered and other facets need to be taken into account. If one takes into account the drag force on the projectile as it moves through the air, the density of the air, cross-sectional area of the object, and drag coefficient need to be known. In certain cases, the Coriolis force might be appropriate to be accounted for. The result is a more complicated model with sophisticated mathematics in order to describe the physics of projectile motion. So there exists this trade-off in how fine does one want the model to describe whatever it is they are trying to describe: the model can be simple to provide strong intuition without losing generality, or more complex requiring greater knowledge to understand what is going on and stronger mathematical rigor to have more predicting power.

The power of making assumptions to produce models that provide a lot of knowledge and insight is found in finance. As one can imagine, trying to mathematically describe the workings of the financial market with all of its economic agents involved can seem like an impossible task. One of the first models used in finance is the Capital Asset Pricing Model, or CAPM. Developed in the 1960s, the model is used to determine what the expected rate of return of an asset is. The expected excess return of an asset *i* is

$$E[r_i] - r_f = \beta_i \left(E[r_M] - r_f \right)$$

where r_f is the risk-free rate, r_M is the return of the market portfolio, and β_i is defined as $\frac{Cov(r_i,r_M)}{\sigma_M^2}$. Some of the main assumptions used to derive this model to find the expected return of an asset are all investors have an identical holding period, all investors are risk-averse and are mean-variance optimizers, and there are no transaction costs or taxes. Not surprisingly, these are very strong assumptions that are not really reflective of what actually exists in financial markets creating a model that does not provide strong predictive capabilities for the return of an asset. Yet, the CAPM does provide a valuable piece of insight in that it helps understand the market risk of a given asset and how exposed it is to fluctuations in the stock market. The assumptions used in the derivation allow for a clear and concise mathematical expression where one can gain astute intuition in financial economics. Further research has been done to build upon the ideas presented in the CAPM and more advanced models with relaxed assumptions such as the Fama-French Three Factor Model have been shown to have more predictive power when it comes to asset pricing.

The CAPM is one of many models that is used in the field of finance. The assumptions used in its derivation can add fantastic economic understanding while at the same time provide short comings in its actual representation of financial markets and asset pricing. This is just like many of the models that a physics student is exposed to during their course of studies that balance these kinds of trade-offs. Both disciplines rely on the power and skill to make good assumptions, whatever the circumstance calls for, in order to produce a model for the system, environment, physical process, or economic behavior an individual is looking to understand.

Gathering Data

As previously mentioned, a model is not a good picture of reality if there is no evidence to support it. In physics, scientific experiments are conducted in order to test the validity of a given model and its underlying assumptions. In a sense, a model is an extension of a hypothesis of what one thinks will happen, and the data collected can either support or refute that hypothesis. This same process is used in finance. Instead of running experiments, however, a common tool used in finance is using regression analysis with historical data. In the case of testing the CAPM and its ability to price a certain asset, an individual can go collect, as an example, historical time-series monthly return data of a given asset *i*, the monthly return of thirty day U.S. Treasury Bill as a proxy for the risk-free rate of return, and the monthly return of the S&P 500 as a proxy for the market portfolio. The regression for the CAPM would look like

$$r_{it} - r_{ft} = \alpha_i + \beta_i(r_{Mt} - r_{ft}) + e_{it}$$

where r_{it} is the return of asset *i* at time *t*, r_{ft} is the risk-free rate at time *t*, r_{Mt} is return at time *t* of the market, and e_{it} is a residual term for asset *i* at time *t*. α_i and β_i are the coefficients of interest in the test of the CAPM. The hypothesis of the CAPM is that α_i is equal to zero and β_i is non-zero. Using the ordinary least squares method for a regression to find α_i and β_i and a t-test used in statistics to see if the coefficients are equal to zero, statistical evidence can be produced to either validate or refute the hypothesis. Just like in physics, models used in finance are scrutinized and put through the test of experimentation.

I would like to walk through an example of how I used data to answer a question I was interested in. A couple of years ago during the massive rally in the bitcoin market, I was interested in seeing if the price of gold had any effect on the price of bitcoin. Using econometrics, or the study of quantifying casual relationships, I looked to develop a model to try to explain if the price of gold had any explanatory power in the price of bitcoin. Looking through the research to see what variables to control for so as to isolate the price effect of gold, my model looked like $\Delta LNXBTUSD_t = \beta_0 + \beta_1 LNXAUUSD_t + \beta_2 LNINDU_t + \beta_3 LNSPX_t + \beta_4 LNUSDEUR_t + \beta_5 LNXAGUSD_t + \beta_6 LNGC1_t + \beta_7 TIME_t + u_t$

where $\Delta LNXAUUSD_t$ is the change in log-price of gold, $LNINDU_t$ is the log-price of the Dow Jones Industrial Average, $LNSPX_t$ is the log-price of the S&P 500, $LNUSDEUR_t$ is the log-price US Dollar-Euro exchange rate, $LNXAGUSD_t$ is the log-price of silver, $LNGC1_t$ is the log-price of gold futures, $TIME_t$ is dummy variable for time to account for seasonality, and u_t is a general error term. These control variables are strongly correlated with the gold market and accounting for them and finding data on them acted to better isolate the casual effect between the price of gold and the price of bitcoin. My null hypothesis was that β_1 was equal to zero and my alternative hypothesis was that β_1 was not equal to zero. After gathering historical data on all the variables in my model and running an ordinary-least squares regression on my dataset, I found that there was not enough statistical evidence to say that β_1 was not equal to zero. Based on my findings, I was able to know that my model did not correctly capture the effects of the market for gold on the market for bitcoin.

Experimentation and testing hypothesis are at the center of a physics education. Finance teaches a student to do the exact same thing. Both disciplines teach about how to test a model and what data one would need to gather in order to prove/disprove a theory. Having been exposed to that key philosophy of the critical nature of having data in both the physical and social sciences has been a tremendous benefit for me.

Complex Mathematics

This past summer, I had the chance to intern at PIMCO, the premier fixed-income asset management firm working on their pension solutions team. The team focuses on developing investment strategies for clients who oversee massive pension plans. The core strategy that the team focuses on is Liability-Driven Investing, or LDI. At its core, Liability-Driven Investing is factoring into the investment calculus both the timetable and the magnitude of future expected benefit obligations that a plan sponsor will need to make. LDI is using assets that a sponsor has at its disposal today and investing them in such away so that one can consistently pay for their expected liabilities that extend to fifty, sixty, or seventy plus years into the future. That can be a very imposing task and the analysis of it is very technical in nature and highly quantitative. With my mathematical background that has been provided by my physics education, I was able to quickly pick-up and understand some of the basics of LDI.

A central tenet of LDI is both duration and convexity matching of a plan sponsors assets and liabilities. The schedule of expected benefit obligations for a plan sponsor in the future are discounted to order to find the present value P of the liabilities for a given plan. The present value of liabilities is

$$P = \sum_{t\,=\,1}^{T} \frac{CF_t}{(1+y)^t}$$

where CF_t is the cash-flow representing the obligation the pension plan sponsor is expected to pay in year *t.* Taking the derivative of *P* with respect to *(1+y),*

$$\frac{\partial P}{\partial(1+y)} = \sum_{t=1}^{T} \frac{-tCF_t}{(1+y)^{t+1}}$$

$$= -\frac{1}{1+y} \sum_{t=1}^{T} \frac{tCF_t}{(1+y)^t}.$$

Multiplying both sides by $\left(\frac{1+y}{P}\right)$,

$$\frac{\partial P}{\partial(1+y)} \left(\frac{1+y}{P}\right) = -\frac{1}{1+y} \sum_{t=1}^{T} \frac{tCF_t}{(1+y)^t} \left(\frac{1+y}{P}\right)$$

$$\frac{\partial P}{\partial(1+y)} \left(\frac{1+y}{P}\right) = -\frac{\sum_{t=1}^{T} \frac{tCF_t}{(1+y)^t}}{P}.$$

The term on the left, is defined as an elasticity in economics. In general, an elasticity can capture how one variable affects the other. Mathematically, this can be shown as

$$\frac{\%\Delta P}{\%\Delta(1+y)} = \frac{\frac{\Delta P}{P}}{\frac{\Delta(1+y)}{(1+y)}} = \frac{\Delta P}{\Delta(1+y)} * \frac{(1+y)}{P} = \frac{\partial P}{\partial(1+y)} \frac{(1+y)}{P}$$

where %Δ is denoting a percentage change. In fixed-income mathematics, this elasticity is called duration. In the case of a bond, the duration of a bond is a metric capturing how the price of a bond will change of interest rates [1].

$$Duration = -\frac{\sum_{t=1}^{T} \frac{tCF_t}{(1+y)^t}}{P}$$

In the case of liabilities for a plan sponsor, the duration of liabilities show by how much the present value of liabilities will change as interest rates change, a very useful thing to know depending on the overall direction of interest rates in the economy and the amount of assets the plan sponsor has in order to better fund those changes in liabilities. The negative sign is strictly to capture the inverse relationship between present value of liabilities and interest rates due to discount future value of expected benefit obligations. Normally the absolute value of duration is quoted.

Just like with a Taylor Series Approximation, the second derivate with respect to *(1+y)* is useful to know to better approximate for larger changes in interest rates.

$$\frac{\partial^2 P}{\partial(1+y)^2} = \sum_{t=1}^{T} \frac{t(t+1)CF_t}{(1+y)^{t+2}}$$

$$\frac{\partial^2 P}{\partial(1+y)^2} = \frac{1}{(1+y)^2} \sum_{t=1}^{T} \frac{t(t+1)CF_t}{(1+y)^{t}}$$

$$\frac{\partial^2 P}{\partial(1+y)^2} = \frac{1}{(1+y)^2} \sum_{t=1}^{T} \frac{t(t+1)CF_t}{(1+y)^{t}} \left(\frac{1}{P}\right) P$$

In fixed-income mathematics, convexity is defined as [2]:

$$Convexity = \frac{\sum_{t=1}^{T} \frac{t(t+1)CF_t}{(1+y)^{t}}}{P(1+y)^2}$$

$$\frac{\partial^2 P}{\partial(1+y)^2} = (Convexity)P$$

$$Convexity = \frac{\partial^2 P}{\partial(1+y)^2} \frac{1}{P}.$$

The goal of duration and convexity matching in the case of LDI is to calculate these metrics for one's liabilities and create a benchmark portfolio whose duration and convexity as similar, if not exactly the same as the liabilities. The idea is that if interest rates change by a given amount, the plan's assets are invested in such a way that the value of the assets would change by the same amount as the liabilities. We were trying to hedge the plan sponsor from interest rate risk so that the invested assets would act as an immunizer to this risk.

Throughout the course of the summer, I got the chance to perform a lot of these types of analyses. Given an expected benefit schedule of a plan sponsor, I would calculate the present value of the liabilities, calculate the duration and convexity of the liabilities, and create a benchmark portfolio in such a way that it has the same duration and convexity as the liabilities. The benchmark would often consist of a combination of different fixed income indices with their respective duration and convexity comprised of bonds with different ranges in maturities. Since the derivative operator is linear, the duration of a portfolio of different indices is a linear combination of each item's duration in the portfolio, whose weights represent the fraction of the portfolio that is invested in each item. The same is true for convexity.

The analyses that I conducted for the myriad of clients that the team had was actually used in presentations and phone meetings that senior leaders of the team had with clients. These analyses were very helpful to clients as they tried to navigate the difficult challenges they are facing trying to figure out the best way to invest their assets to fund their pension plan in the future. I know that my math skills allowed me to quickly pick-up, understand, and provide strong analyses in LDI that led to success on my team throughout the summer.

Conclusion

While two seemingly unrelated subjects and disciplines, physics and finance/economics share more similarities than one may think. The skillset the one develops studying physics is completely relatable to the soft sciences of finance/economics. Principles that are central tenets to the training in a physics education such as building models based on assumptions, the necessity of experimentation and gathering data to prove/disprove a hypothesis, and using complex mathematics are pertinent to strong empirical research skills in finance. The combinations of these two disciplines has been extremely instrumental in my understanding on how to pursue truth. As I graduate from BYU and move-on to start my career at PIMCO, I know that I am in a much better place because of my decision to have studied both physics and economics.

References

[1] Britten-Jones, M. Chapter 2 – Imperfect Replication: Immunization and Duration. (1998). In *Fixed Income and Interest Rate Derivative Analysis* (pp. 20-37). Butterworth-Heinemann. https://doi.org/10.1016/B978-075064012-1.50016-9

[2] Zipf, Robert. Chapter 17 – The Mathematical Development of Duration, Convexity, and the Equation to Predict New Bond Prices Given Yield Changes. (2003). *Fixed Income Mathematics* (pp. 211-218). Academic Press. http://www.sciencedirect.com/science/article/pii/B9780127817217500171

Acknowledgments

I would like to thank my parents for encouraging me to follow my path and pursue my passions. I would like to thank Kate Tan, Kulin Chheda, and other members of the pension solutions team at PIMCO for their willingness to help me out and teach me about LDI and how to be successful in the asset management industry. Thank you to those involved at PIMCO for giving me a chance to intern with the company for the summer and giving me an offer to return full-time after graduation. I want to thank all my physics and economics professors at BYU for teaching me about these two incredible subjects and keeping me fascinated in both throughout my time at BYU.