Universal Quantum Circuitry: Deutsch Gate Construction

Using GaAs/InAs Quantum Dots

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ABSTRACT

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I propose to incorporate two GaAs/InAs quantum dots in a larger circuit comprised of linear optical elements to create a spin-spin-photon polarization three-qubit Deutsch gate. Since the Deutsch gate is a universal quantum logic gate, any quantum computing task can be completed using a combination of Deutsch gates. I argue the significance of the spin-spin-photon Deutsch gate protocol, given that no Deutsch gate has been experimentally realized and the only other published Deutsch gate proposal does not use a flying qubit. The use of a flying photonic qubit facilitates quantum communication applications. In addition, I show that the versatility of my protocol can lead to two different constructions of the Toffoli gate either by fixing a parameter of the Deutsch gate or by taking a sub-circuit of the Deutsch gate. Within my Deutsch gate circuit is a smaller Toffoli gate circuit. I calculate the fidelity of the Toffoli gate circuit for different material parameters of the GaAs/InAs quantum dots. I display a schematic of the Deutsch gate circuit with removable mirrors that allow the circuit to switch into a Toffoli gate. Finally, I discuss how appropriate Toffoli gates can be adapted into Deutsch gates using a sub-circuit of the original Deutsch gate circuit.

Keywords: Deutsch gate, Toffoli Gate, GaAs/InAs quantum dot, universal logic gate, flying photonic qubit

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Chapter 1

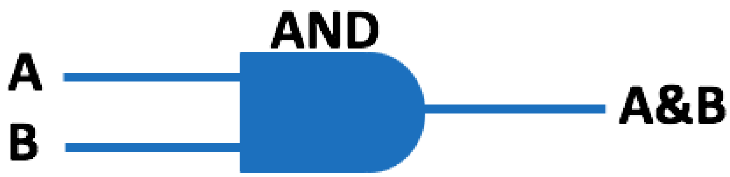
Introduction

1.1 Universal Logic Gates

Quantum computers are made up of quantum logic gates. Building a fully functional quantum computer requires a universal set of quantum logic gates. This thesis focuses on a proposal to build the Deutsch gate [1], which is a universal logic gate. Since the Deutsch gate is a universal quantum logic gate, one could build a quantum computer just out of Deutsch gates rather than using many different gates. Aside from universality, the Deutsch gate has other advantages. As I will outline in this thesis, the Deutsch gate proves more useful for particular quantum tasks than the universal sets typically used in quantum computing today.

But first, what is a logic gate? A logic gate is a physical realization of a logical operator. In classical computing logic gates act on bits, which can hold the value of either 0 or 1. A common classical logic gate is the AND gate represented in Fig. 1. The AND gate takes in two bits and returns a single bit in the state 1 if both input bits are in a state 1 and a 0 otherwise.

For quantum logic gates, information is stored in qubits rather than bits. Qubits are in a superposition of the basis states $|0\rangle$ and $|1\rangle$. Additionally, in quantum logic gates, no information

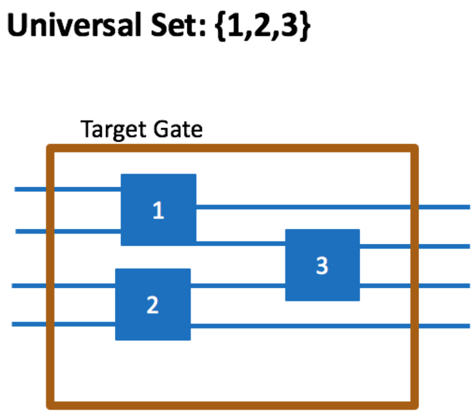


A	B	A&B
1	1	1
1	0	0
0	1	0
0	0	0

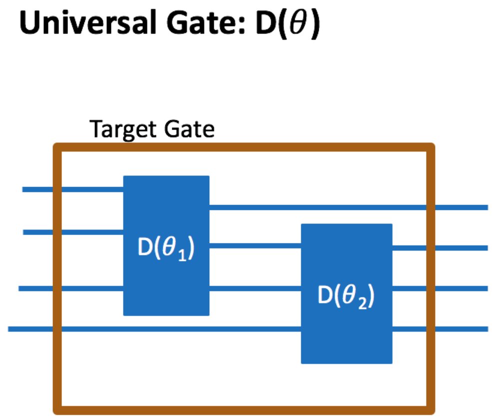
Table 1.1 AND gate schematic and truth table. The AND gate returns a 1 if both inputs are 1 and a 0 otherwise.

can be destroyed: every operation is reversible. In the classical AND gate, given the output 0, the inputs could have been 0 and 1, 1 and 0, or 0 and 0. The input state of the AND gate gets erased following the operation. In contrast, quantum logic gates retain information. Given an output state of an operation, the input state can not only be determined but recovered with an inverse of that operation. What this thesis truly concerns itself with, however, is a universal quantum logic gate.

A universal set of gates is a set of gates that can create any other gate through some combination of the set. As an example, Fig. 1.1a shows a universal set and how it might be used to create some desired target gate. Since any gate can be created with a universal set, any quantum computing task can be completed with a universal set. This thesis examines a universal three-qubit gate called the Deutsch gate [1], which has an angular dependence and is notated $D(\theta)$. A universal gate is simply a universal set of size one. So, a combination of Deutsch gates $(D(\theta_1), D(\theta_2), D(\theta_3), ...)$ can create any quantum logic gate. It then follows that a combination of Deutsch gates can complete



(a) An example of some target gate being create with a universal set of gates. A universal set with gates 1, 2, and 3 can create any gate if combined correctly.



(b) An example of the same target gate being created with a universal Deutsch gate. Deutsch gates combined together can yield any gate.

Figure 1.1 An example of how some target gate could be created with (a) a universal set of gates and (b) the universal Deutsch gate.

any quantum computational task. How the Deutsch gate might be used to create some desired target gate is shown in Fig. 1.1b.

Typically, quantum computing is done with a universal set of logic gates. Despite the broad use of universal sets, the use of a single universal gate rather than a set has several advantages. Using a single universal gate bypasses the challenge of achieving a high fidelity on each gate in a set. The fidelity of a gate is a measure of how accurate the gate is. Specifically, the fidelity of a gate is how often, on average, the output of the gate is the desired output. Uncontrollable noise like decoherence is inherent to all systems and limits how high a fidelity gates can attain. So, using the Deutsch gate would only require making sure that the Deutsch gate had a high fidelity. On the other hand, a universal set requires you to assess how high a fidelity several different gates have. Additionally, known universal gates—like the Deutsch gate examined in this thesis—have an angular dependence (described in more detail in Section 1.2) that allows for simple changes in the operation completed

Deutsch Gate Protocol	Target Qubit	Control Qubit 1	Control Qubit 2
Shi [5]	Rydberg	Rydberg	Rydberg
Bailey	Flying Photon	Quantum Dot	Quantum Dot

Toffoli Gate Protocol	Target Qubit	Control Qubit 1	Control Qubit 2
Shi [5]	Rydberg	Rydberg	Rydberg
Wei *et al.* [6]	Quantum Dot	Quantum Dot	Flying Photon
Kim *et al.* [7]	Quantum Dot	Quantum Dot	Quantum Dot
Bailey	Flying Photon	Quantum Dot	Quantum Dot

Table 1.2 Deutsch gate and Toffoli gate protocols comparison by qubits used.

by the gate [1]. This could help in designing quantum computers. One last advantage is that most universal sets used today are comprised of at most two-qubit gates [2]. A two-qubit gate is a gate that has two input and two output qubits. It turns out that two-qubit gates sometimes run into challenges when used to create higher-qubit gates. As an example, Duan *et al.* [3] found that a minimum of five two-qubit gates are required to implement the Toffoli gate, a useful three-qubit gate for quantum error correction [4]. The Deutsch gate, meanwhile, can implement the Toffoli gate if the right angle is chosen as described in Section 1.3. The Deutsch gate may not always be more efficient than other universal sets for a particular quantum computational task. However, the Deutsch gate can complete some quantum computational tasks more efficiently than universal sets.

In the remaining sections of this chapter, I will describe the operations of the Deutsch gate and Toffoli gate then outline protocols for each gate that are contained in this thesis and in the literature. Table 1.2 compares Deutsch gate and Toffoli gate protocols by the type of qubits used.

1.2 Deutsch Gate

The Deutsch gate is a three-qubit universal quantum logic gate first proposed by David Deutsch in 1989 [1]. The Deutsch gate performs the following operation on an input state $|\Psi_{in}\rangle = |a\rangle|b\rangle|c\rangle$:

$$|\Psi_{in}\rangle \mapsto \begin{cases} i\cos(\theta)|a\rangle|b\rangle|c\rangle + \sin(\theta)|a\rangle|b\rangle|1-c\rangle & a=b=1 \\ |a\rangle|b\rangle|c\rangle & otherwise, \end{cases} \tag{1.1}$$

where $|a\rangle$, $|b\rangle$, and $|c\rangle$ each are in a superposition of basis states $|0\rangle$ and $|1\rangle$. The ket $|1-c\rangle$ represents the state following a NOT operation on $|c\rangle$, which switches $|0\rangle$ to $|1\rangle$ and $|1\rangle$ to $|0\rangle$. So, if $|c\rangle = \alpha|0\rangle + \beta|1\rangle$ then $|1-c\rangle = \alpha|1\rangle + \beta|0\rangle$. The kets $|a\rangle$ and $|b\rangle$ are control qubits meaning that the gate's operation depends on their states. The ket $|c\rangle$ functions as the target qubit that undergoes a transition if the control qubits are in the right state. Here, the target qubit $|c\rangle$ undergoes a transition only if $|a\rangle = |b\rangle = |1\rangle$. The angle θ allows the Deutsch gate to complete many different operations. So, in a sense, the Deutsch gate is an uncountable set of gates—a different gate at each angle. In Section 1.3, I discuss how choosing $\theta = \pi/2$ results in the Toffoli gate. The angular dependence of the Deutsch gate creates difficulty for coming up with Deutsch gate protocols. Possibly because of this, a physical realization of the Deutsch gate has yet to be achieved and there exists only one other protocol for the Deutsch gate, proposed by Shi [5], which uses Rydberg blockade of neutral atoms. Shi's protocol tunes θ to any value between 0 and π by adjusting the strengths of external control fields. In my proposal for the Deutsch gate described in Section 2.3, I use two GaAs/InAs quantum dots alongside linear optical elements to build a circuit. I then send a flying photonic qubit through the circuit to realize the Deutsch gate. A flying qubit is a qubit that connects different gates. The spins of each electron in the quantum dots and the polarization of the photon function as the physical degrees of freedom to encode the qubits. My proposal sets the angle θ using half-wave plates resulting in fast and simple changes to the Deutsch gate's operation.

1.3 Toffoli Gate

The Toffoli gate is a universal gate for classical reversible computing [8], a subset of classical computing. Thus, any reversible classical logic gate can be created with a combination of Toffoli gates. Some classical computing operations like the AND operation are not reversible; they destroy information. A reversible operator has an inverse operation associated with it. So, the inputs can be used to find the outputs and vice versa. Any reversible operation can be enacted by a quantum logic gate. So, the Toffoli gate can be used in quantum computing. It has the following operation with input state $\Psi_{in} = |a\rangle\,|b\rangle\,|c\rangle$:

$$|\Psi_{in}\rangle \mapsto \begin{cases} |a\rangle\,|b\rangle\,|1-c\rangle & a=b=1 \\ |a\rangle\,|b\rangle\,|c\rangle & otherwise. \end{cases} \tag{1.2}$$

Aside from classical computing, the Toffoli gate is used for quantum error correction [4]. Quantum error correction aids in initializing, manipulating, and measuring quantum states—three important parts of quantum information.

For a universal set containing only two qubit gates, a minimum of five two-qubit gates are necessary for implementing the Toffoli gate [3]. As mentioned, setting $\theta = \pi/2$ for the Deutsch gate results in the Toffoli gate. Thus, using the three-qubit Deutsch gate rather than a universal set of two-qubit gates results in a simpler implementation of the Toffoli gate. Additionally, our proposed circuit contains the Toffoli gate as described in Section 2.4. By taking out many of the optical elements in the Deutsch gate protocol, a simple protocol for the Toffoli gate remains. Since, including more optical elements increases decoherence, this would result in a higher fidelity for the Toffoli gate than using the Deutsch gate with $\theta = \pi/2$.

In Section 2.4, for this Toffoli gate found within the Deutsch gate, I calculate the fidelity based on relevant material parameters of the quantum dots. I find that a fidelity of 0.952 can be reached using values for the material parameters that have been achieved in the lab. Kim *et al.* [7] proposed to create a Toffoli gate with the same GaAs/InAs quantum dots presented here and reach a higher fidelity for the same values of material parameters. However, they do not show that their protocol can transition into a Deutsch gate protocol as the Toffoli gate I present can.

1.4 Prior Work

The Deutsch gate has never been experimentally realized. However, Shi [5] has proposed creating a Deutsch gate using Rydberg Blockade of Neutral Atoms. While there is not much published work on creating a Deutsch gate, the GaAs/InAs quantum dots used in my protocol have been used for many quantum information applications following the work of Hu *et al.* [9, 10], who showed that GaAs/InAs quantum dots could be used as entanglement beam splitters. Bonato *et al.* [11] proposed using these quantum dots to create both a CNOT gate and a Bell state analyzer. Their proposal for a CNOT gate uses a flying photonic qubit that interacts with one quantum dot. Whereas quantum dots are stationary in the circuit, the photonic qubit travels through the circuit and can go on to connect with other gates, hence the term flying. Wei *et al.* [6] created protocols for the CNOT, Toffoli, and Fredkin gates. Their protocols use a flying photonic qubit as a control qubit rather than as a target qubit like the protocols in this thesis and in [11]. Kim *et al.* [7] also proposed creating a CNOT gate, Toffoli gate, and a Fredkin gate. Their CNOT gate differs from the CNOT gate proposed by Bonato *et al.* in that the qubits used are only the spins of the electrons in two separate quantum dots. Likewise, Kim *et al.* [7] Toffoli gate differs from my own in the same manner; only the spins of electrons inside three separate quantum dots are used. Thus, the gates proposed by Kim *et al.* are more geared toward storage. Whereas the gates proposed by Bonato *et al.* [11], by Wei *et al.* [6], and by myself are more useful for communication networks since we include a flying photonic qubit that can connect different gates together. Table 1.2 shows how the different Deutsch gate and

Toffoli gate protocols referenced here compare based on the qubits used.

Additionally, using these GaAs/InAs quantum dots, Wang *et al.* [12] created a protocol for a quantum repeater and Wang *et al.* [13] created protocols for a two-qubit phase gate and teleportation of a CNOT gate. These schemes and others similar to them can be found in a review by Reiserer *et al.* [14]. The majority of systems considered in [14] involve systems other than these GaAs/InAs quantum dots. The review covers cavity-based quantum networks that involve flying photonic qubits.

1.5 Proposal

I propose creating a Deutsch gate by combining two singly charged GaAs/InAs quantum dots with linear optical elements as seen in Fig. 2.2 and described in Section 2.3. I expand upon schemes by Bonato et al. [11] who proposed using GaAs/InAs quantum dots to create a CNOT gate. One qubit will be an incoming photon's polarization state while the other two qubits will be the state of the electrons' spins inside the quantum dots. In my Deutsch gate protocol, a variable angle θ can be controlled by changing the angle of half-wave plates. This allows the Deutsch gate to quickly and easily change its angle and perform different operations. The three qubits used in my Deutsch gate protocol are the spins of the electrons confined to the quantum dots and the photon that interacts with the quantum dots and passes through the optical elements. In my scheme, the two electron spins take on the role of control qubits while the photonic qubit functions as the target qubit. Only when both electrons are spin-up does the photonic qubit undergo a transition. The flying photonic qubit also allows for quantum communication since it can go on to interact with other gates.

1.6 Overview

Before the Deutsch gate circuit can be understood, the individual components must be explained. First, in Section 2.1, I describe the interaction between the quantum dots and an incoming photon. In Section 2.2, I describe the operation of different optical elements employed in my Deutsch gate protocol. With the description of the quantum dot transitions and optical elements, the Deutsch gate circuit can be understood. In Section 2.3, I walk through the Deutsch gate construction step by step. I discuss how the input states transition into intermediate states and then to a final output state. Following the description of the Deutsch gate, in Section 2.4, I show how a Toffoli gate can be created by removing optical elements from our protocol. I calculate the fidelity of this Toffoli gate in terms of operating parameters of the quantum dots. In Section 2.5, I show how appropriate Toffoli gates can be turned into Deutsch gates using a sub-circuit of my Deutsch gate design. Finally, in Chapter 3, I discuss my results, future work, and compare my Deutsch gate proposal with Shi's [5] proposal, the only other existing Deutsch gate proposal.

Chapter 2

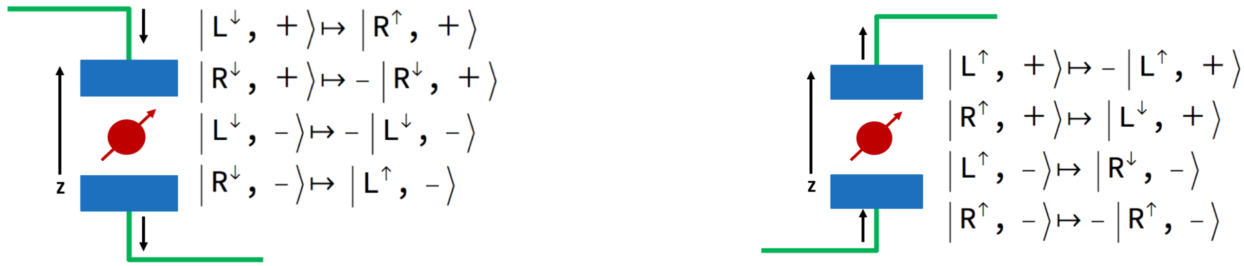
Deutsch Gate Proposal

2.1 Quantum Dot Transitions

My Deutsch gate protocol makes use of GaAs/InAs quantum dots and various linear optical elements. Before outlining the protocol, I will describe the effect of each component separately—starting with the quantum dots.

Circularly polarized photons undergo transitions after interacting with the electron in a singly charged GaAs/InAs quantum dot as described by Bonato *et al.* [11]. I will only present these transitions in the amount of detail needed for my purpose, which is to describe the state of a photon before and after interacting with a GaAs/InAs quantum dot. A more detailed description of these transitions can be found in [11].

I assume an arbitrary state for the electron in the GaAs/InAs quantum dot, $|e\rangle = k_1 |+\rangle + k_2 |-\rangle$, where $|+\rangle$ represents the electron spin-up state with respect to the axis of the incoming photon, $|-\rangle$ represents the spin-down state with respect to that same axis as seen in Fig. 2.1, and k_1 and k_2 are normalized coefficients. The incoming photon is in some superposition of right circularly polarized light $|R\rangle$ and left circularly polarized light $|L\rangle$. The incoming photon will either reflect



(a) Photon traveling in $-\hat{z}$ direction.

(b) Photon traveling in $+\hat{z}$ direction.

Figure 2.1 Photon transition rules after interacting with the GaAs/InAs quantum dot. The arrow superscripts indicate the direction of travel for the photon.

off the quantum dot and invert its polarization, go from $|R\rangle$ to $|L\rangle$ or vice versa, or will transmit through the quantum dot picking up a phase shift of π while keeping its polarization state. In either case, the state of the electron remains unchanged. The transformation of the photon's polarization state differs depending on whether the photon is incident from above (traveling in the $-\hat{z}$ direction) or below (traveling in the $+\hat{z}$ direction) as can be seen in Fig. 2.1a and b.

2.2 Optical Elements

Alongside GaAs/InAs quantum dots, my Deutsch gate protocol uses half-wave plates , quarter-wave plates , mirrors, π phase shiftters, linear polarizing beam splitters, and circularly polarizing beam splitters. Jones matrices describe the effect of each element—aside from the beam splitters—on the polarization of light. The Jones matrices of each element are listed in Table 2.1. These matrices act on two-dimensional Jones vectors representing the polarization of light. For example, right and left circularly polarized light are represented by the following Jones vectors:

$$|R\rangle = \begin{bmatrix} 1/\sqrt{2} \\ i/\sqrt{2} \end{bmatrix} \tag{2.1}$$

and

$$|L\rangle = \begin{bmatrix} 1/\sqrt{2} \\ -i/\sqrt{2} \end{bmatrix}. \tag{2.2}$$

The top entry in the Jones vector represents the horizontal component of the polarization while the bottom entry represents the vertical component. For a more complete treatment of Jones calculus, see Peatross and Ware [15].

Element	Jones Matrix
Half-Wave Plate	$\begin{bmatrix} \cos 2\phi & \sin 2\phi \\ \sin 2\phi & -\cos 2\phi \end{bmatrix}$
Quarter-Wave Plate	$\begin{bmatrix} \cos\phi^2 + i\sin\phi^2 & \sin\phi\cos\phi - i\sin\phi\cos\phi \\ \sin\phi\cos\phi - i\sin\phi\cos\phi & \sin\phi^2 + i\cos\phi^2 \end{bmatrix}$
Mirror	$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$
π Phase Shiftter	$\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$

Table 2.1 Jones matrices for optical elements.

The linear polarizing beam splitters send the horizontal component of light in one direction and vertical light in the other. Similarly, circular polarizing beam splitters send right circularly polarized light one way and left circularly polarized light another. In this work and the diagram of the Deutsch gate seen in Fig. 2.2, horizontally polarized light passes through linear polarizing beam splitters, while vertically polarized light is reflected. For circularly polarized beam splitters, the right circularly polarized light transmits, while the left circularly polarized light reflects.

2.3 Deutsch Gate Circuit

The Deutsch gate protocol is displayed in Fig. 2.2. The two GaAs/InAs quantum dots are labeled QD1 and QD2. The green lines represent the paths the incident photon may take. Mirrors are represented by diagonal black lines. The red squares indicate paths phase shifted by π. Red lines represent half-wave plates, while blue lines represent quarter-wave plates. The orange beam splitters are circularly polarizing, while the white beam splitters are linearly polarizing. Notice that each quantum dot has a different reference axis indicated by an arrow labeled z, which affects the transitions that happen between it and an incoming photon. The black dashed lines are removable mirrors that divert beams represented by green dashed lines to implement the Toffoli gate as described in Section 2.4.

The protocol starts with an incident photon in the polarization state $|\gamma_{in}\rangle = \alpha|R\rangle + \beta|L\rangle$, where $|R\rangle$ represents a right circularly polarized state, $|L\rangle$ represents a left circularly polarized state, and α and β are normalized constants. The electrons in the quantum dots start out in the spinor states $|e_1\rangle = c_1|+\rangle + c_2|-\rangle$ and $|e_2\rangle = d_1|+\rangle + d_2|-\rangle$, where $|+\rangle$ and $|-\rangle$ represent the electron in each quantum dot being in the spin-up or spin-down state respectively with respect to the z axes labeled in Fig. 2.2 and c_1, c_2, d_1, and d_2 are all normalized constants. Note again that each electron state is defined according to different axes as indicated in Fig. 2.2. To achieve the Deutsch gate we want to transform our three-qubit input state $|\Psi_{in}\rangle = |\gamma_{in}\rangle|e_1\rangle|e_2\rangle$, according to Eq. (1.1) with $|a\rangle = |e_1\rangle, |b\rangle = |e_2\rangle$, and $|c\rangle = |\gamma_{in}\rangle$, into a new three-qubit state

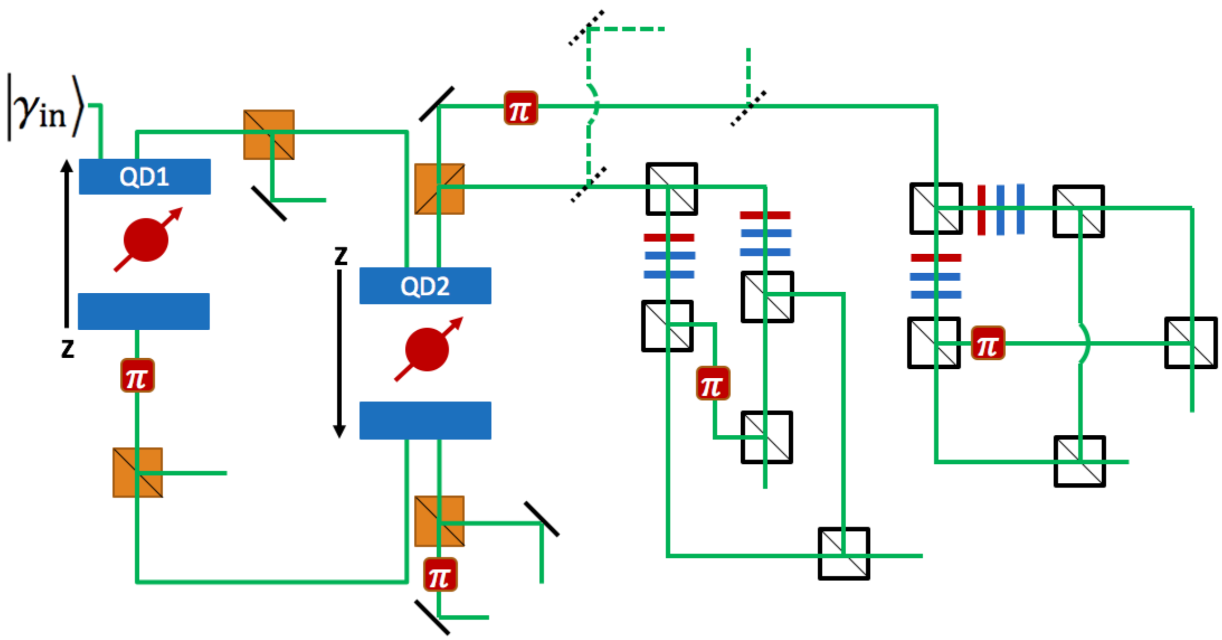


Figure 2.2 Schematic of the proposed circuitry for a universal Deutsch gate. The meaning of symbols and icons is detailed in the text. Dashed lines correspond to diverting beams to create a Toffoli gate as detailed in Section 2.4.

$$\begin{aligned}|\Psi_{out}\rangle = c_1|+\rangle d_1|+\rangle\big[i\cos\theta\big(\alpha|R\rangle+\beta|L\rangle\big)\\ +\sin\theta\big(\alpha|L\rangle+\beta|R\rangle\big)\big]\\ +c_1|+\rangle d_2|-\rangle\big(\alpha|R\rangle+\beta|L\rangle\big)\\ +c_2|-\rangle\big(d_1|+\rangle+d_2|-\rangle\big)\big(\alpha|R\rangle+\beta|L\rangle\big),\end{aligned} \tag{2.3}$$

or equivalently

$$\begin{aligned}|\Psi_{out}\rangle = c_1|+\rangle d_1|+\rangle\big(i\cos\theta\,|\gamma_{in}\rangle+\sin\theta\,|\gamma_{in}-1\rangle\big)\\ +c_1|+\rangle d_2|-\rangle|\gamma_{in}\rangle+c_2|-\rangle|e_2\rangle|\gamma_{in}\rangle.\end{aligned} \tag{2.4}$$

We see that $|\gamma_{in}\rangle$ is unchanged except for when each quantum dot's electron is in the $|+\rangle$ state, in which case it undergoes a transition described by Eq. (1.1). Here, $|\gamma_{in}-1\rangle$ represents the ket of $|\gamma_{in}\rangle$

having undergone a NOT operation where the polarization is inverted. Thus, $|\gamma_{in}-1\rangle = \alpha|L\rangle + \beta|R\rangle$. Note, also, that each electron stays in its initial spinor state as can be seen from the GaAs/InAs quantum dot transition rules in Fig. 2.1.

Now I will describe the transitions which occur as the photon goes through the circuit. I will follow how the input state $|\Psi_{in}\rangle$ transitions with each step until it becomes $|\Psi_{out}\rangle$. First, $|\Psi_{in}\rangle$ will interact with QD1 transforming into $|\Psi_{QD1}\rangle$. Then $|\Psi_{QD1}\rangle$ will interact with some optical elements transforming into $|\Psi_2\rangle$. The photon then interacts with QD2 resulting in $|\Psi_{QD2}\rangle$. Then the photon will interact with the optical elements prior to the wave plates resulting in $|\Psi_4\rangle$. Finally, $|\Psi_4\rangle$ will traverse the series of wave plates and the rest of the circuit to transition into the output state $|\Psi_{out}\rangle$.

The incident photon first interacts with the first quantum dot labeled QD1, which transforms the input state $|\Psi_{in}\rangle = |\gamma_{in}\rangle|e_1\rangle|e_2\rangle$ into

$$|\Psi_{QD1}\rangle = \left[c_1|+\rangle\left(-\alpha|R\rangle+\beta|R\rangle\right)+c_2|-\rangle\left(\alpha|L\rangle-\beta|L\rangle\right)\right]\otimes|e_2\rangle. \tag{2.5}$$

Following the schematic in Fig. 2, the bottom output beams from QD1 goes through a phase shift of π and is split by a circularly polarized beam splitter while the top output beam is split by another circularly polarized beam splitter and the left circularly polarized half is flipped to right circularly polarized light by a mirror. This transforms $|\Psi_{QD1}\rangle$ into

$$|\Psi_2\rangle = \left[c_1|+\rangle\left(\alpha|R\rangle+\beta|R\rangle\right)+c_2|-\rangle\left(\alpha|R\rangle+\beta|L\rangle\right)\right]\otimes\left|e_2^-\right\rangle. \tag{2.6}$$

Next, the right circularly polarized terms associated with the first quantum dot being spin up interact with QD2 while the spin-down terms are left untouched yielding

$$\begin{aligned} |\Psi_{QD2}\rangle = c_1 |+\rangle [d_1 |+\rangle (-\alpha |R\rangle + \beta |L\rangle) \\ +d_2 |-\rangle (\alpha |L\rangle - \beta |R\rangle)] \\ +c_2 |-\rangle (d_1 |+\rangle + d_2 |-\rangle)(\alpha |R\rangle + \beta |L\rangle). \end{aligned} \tag{2.7}$$

Both output beams following QD2 are then split according to their circular components and each right circularly polarized component is phase shifted by π. Then all beams other than the left circularly polarized component associated with QD1 and QD2 both being spin up have their polarization flipped by mirrors resulting in

$$\begin{aligned} |\Psi_4\rangle = c_1 |+\rangle [d_1 |+\rangle (\alpha |L\rangle + \beta |L\rangle) \\ +d_2 |-\rangle (\alpha |R\rangle + \beta |L\rangle)] \\ +c_2 |-\rangle (d_1 |+\rangle + d_2 |-\rangle)(\alpha |R\rangle + \beta |L\rangle). \end{aligned} \tag{2.8}$$

Half-wave plates introduce the angular dependence of the Deutsch gate—represented by red lines in Fig. 2.2, each at an angle $\phi = \theta/2$. Quarter-wave plates are represented by blue lines and always occur in pairs. In each series of wave plates that the beams pass through in Fig. 2.2, the first quarter-wave plate is set at an angle $\theta_1 = \pi$ and the second quarter-wave plate is at an angle $\theta_2 = \pi/2$. We will be taking the left circularly polarized components of the $c_1 |+\rangle d_1 |+\rangle$ term, $\alpha |L\rangle$ and $\beta |L\rangle$, and transform them to take on the form of $|\Psi_{out}\rangle$. We do this by splitting each beam into its linear components and pass all beams through the series of wave plates, which transforms $\alpha |L\rangle$ according to

$$\alpha |L\rangle \mapsto \alpha(\sin\theta |H\rangle + i^2 \cos\theta |V\rangle) + \alpha(i\cos\theta |H\rangle + i\sin\theta |V\rangle), \tag{2.9}$$

where $|H\rangle$ and $|V\rangle$ represent the horizontal and vertical components respectively. Similarly, $\beta |L\rangle$ is transformed according to

$$\beta\left|L\right\rangle \mapsto \beta(\sin\theta\left|H\right\rangle + i^2\cos\theta\left|V\right\rangle) + \beta(i\cos\theta\left|H\right\rangle + i\sin\theta\left|V\right\rangle). \tag{2.10}$$

Each grouping of terms corresponds to a different beam path, which are each split again into their linear polarization components. The $\alpha i\sin(\theta)\left|V\right\rangle$ term in Eq. (2.9) and the $\beta i^2\cos(\theta)\left|V\right\rangle$ term in Eq. (2.10) are each then phase shifted by π. Beams with identical α/β and cosine/sine terms are then recombined using linear polarizing beam splitters to again produce circularly polarized light. This all results in the original $\alpha\left|L\right\rangle$ and $\beta\left|L\right\rangle$ terms transforming into

$$\begin{aligned} \alpha\left|L\right\rangle &\mapsto \alpha(i\cos\theta\left|R\right\rangle + \sin\theta\left|L\right\rangle) \\ \beta\left|L\right\rangle &\mapsto \beta(i\cos\theta\left|L\right\rangle + \sin\theta\left|R\right\rangle). \end{aligned} \tag{2.11}$$

This results in the desired final output state of

$$\begin{aligned} \left|\Psi_{out}\right\rangle = c_1\left|+\right\rangle d_1\left|+\right\rangle (i\cos\theta[\alpha\left|R\right\rangle + \beta\left|L\right\rangle] \\ + \sin\theta[\alpha\left|L\right\rangle + \beta\left|R\right\rangle]) \\ + c_1\left|+\right\rangle d_2\left|-\right\rangle (\alpha\left|R\right\rangle + \beta\left|L\right\rangle) \\ + c_2\left|-\right\rangle (d_1\left|+\right\rangle + d_2\left|-\right\rangle)(\alpha\left|R\right\rangle + \beta\left|L\right\rangle), \end{aligned} \tag{2.12}$$

where each of the eight circularly polarized terms correspond to a separate output beam in Fig.(2.2).

2.4 Toffoli Gate Circuit

The Toffoli gate is a three-qubit gate useful for quantum error correction [4]. As previously mentioned, taking $\theta = \pi/2$ in the Deutsch gate yields the Toffoli gate. However, my protocol can achieve the Toffoli gate by other means. My Deutsch gate protocol can implement the Toffoli gate by diverting the beams before they reach the wave plates and linear beam splitters as shown in Fig. 2.2.

Each additional element added to a protocol introduces decoherence, which lowers the fidelity of a gate. Since this Toffoli gate protocol has fewer elements than using the Deutsch gate to create a Toffoli gate, it should have a higher fidelity. The fidelity of a quantum logic gate refers to how close, on average, the output state matches the intended output state. For example, if a gate has a fidelity of 0.95 then it has a 95% chance of achieving the desired output state. The Toffoli gate's fidelity for different operating parameters of the quantum dots is found in Fig. 2.3. This plots the fidelity in terms of the normalized coupling strength g/κ and normalized side leakage κ_s/κ, which I calculated following the methods of Hu *et al.* [10]. This involves taking into account that the transition rules described in Section 2.1 are not entirely accurate in a real-world setting. There is a small chance of getting the opposite and unwanted transition. All elements in the circuit aside from the quantum dots are assumed to be ideal in my calculations, however. From Fig. 2.3, we see that we can reach a fidelity of 0.928 or higher when operating the GaAs/InAs quantum dots with a high coupling strength and low side leakage. Experimentally, researchers have been able to operate these quantum dots with a normalized coupling strength of $g/\kappa = 2.4$ and a normalized side leakage of $\kappa_s/\kappa = 0.01$ as quoted in [7] . With these value the Toffoli gate would reach a fidelity of 0.952 based on my calculations.

Another Toffoli gate proposal by Kim *et al.* [7] also uses GaAs/InAs quantum dots. However, in that proposal, three quantum dots are used as the qubits and no flying qubit is used. The Toffoli gate proposed by Kim *et al.* can reach a fidelity of 0.966 with $g/\kappa = 2.4$ and $\kappa_s/\kappa = 0.01$, a value 0.014 higher than the Toffoli gate presented here at those operating parameters. My proposal has additional uses, however. The protocol by Kim *et al.* does not use a photonic qubit and there is no indication that they can implement the Deutsch gate using their Toffoli gate. Their proposal is better geared toward storage, whereas the photonic qubit in my proposal allows for easier quantum

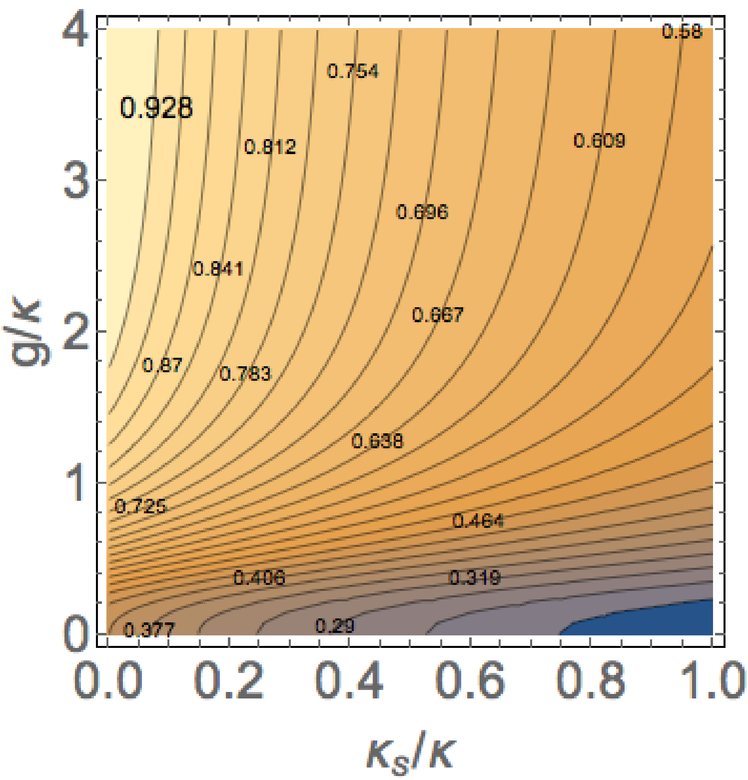


Figure 2.3 The calculated fidelity of the Toffoli gate in terms of the normalized coupling strength g/κ and normalized side leakage κ_s/κ.

communication. Other Toffoli gate protocols and their fidelities can be found for comparison in [7]. The considerably more involved fidelity calculattion for the Deutsch gate is not included in this work.

2.5 Deutsch Gate Design Generalization

I introduced the Toffoli gate circuit in Section 2.4 as the Deutsch gate circuit with some elements taken away as shown in Fig. 2.2. However, there are many ways to construct a Toffoli gate. In this section, I suggest how appropriate Toffoli gates can have the latter half of my Deutsch gate protocol added to yield a Deutsch gate. Any Toffoli gate protocol with a flying photonic target qubit whose different $|R\rangle$ and $|L\rangle$ terms can be separated could have the proper outputs run through the latter

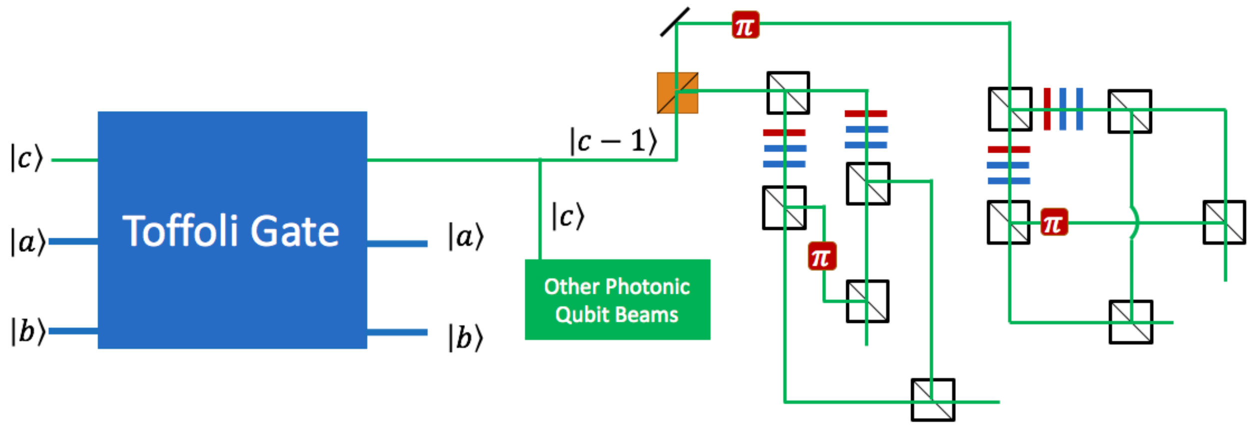


Figure 2.4 Schematic of the Deutsch gate implemented by adding optical elements to an appropriate Toffoli gate. By taking the proper beams from a Toffoli gate with a flying photonic target qubit—namely those asociated with $a = b = 1$—and running them through the latter half of my Deutsch gate protocol, the Deutsch gate can be implemented.

part of the Deutsch gate circuit and thus create a Deutsch gate. I show in Fig. 2.4 how to implement a Deutsch gate using an appropriate Toffoli gate.

More precisely, assume that the input of a Toffoli gate is $|\Psi_{in}\rangle = |a\rangle\,|b\rangle\,|c\rangle$ where $|a\rangle = k_1\,|0\rangle + k_2\,|1\rangle$, $|b\rangle = d_1\,|0\rangle + d_2\,|1\rangle$, and $|c\rangle$ is a photonic target qubit with $|c\rangle = \alpha\,|R\rangle + \beta\,|L\rangle$. Then the output state of the Toffoli gate is

$$\begin{aligned} |\Psi_{out}\rangle = {} & k_1\,|1\rangle\,d_1\,|1\rangle\,(\alpha\,|L\rangle + \beta\,|R\rangle) + k_1\,|1\rangle\,d_2\,|0\rangle\,(\alpha\,|R\rangle + \beta\,|L\rangle) \\ & + k_2\,|0\rangle\,d_1\,|1\rangle\,(\alpha\,|R\rangle + \beta\,|L\rangle) + k_2\,|0\rangle\,d_2\,|0\rangle\,(\alpha\,|R\rangle + \beta\,|L\rangle). \end{aligned} \tag{2.13}$$

To create a Deutsch gate from this Toffoli gate the $|L\rangle$ and $|R\rangle$ beams of the photon associated with the control qubits both in state $|1\rangle$ must be separated from the rest of the beams. If this is possible, these beams can pass through the optical elements in Fig. 2.4 to yield the Deutsch gate. So, although there are not many Deutsch gate protocols, there are many Toffoli gate protocols and some could easily change into a Deutsch gate protocol.

Chapter 3

Discussion

3.1 Results

I showed in Chapter 2 that a Deutsch gate could be created with GaAs/InAs quantum dots and linear optical elements. The Deutsch gate is a universal quantum logic gate and a series of Deutsch gates can achieve any quantum computational task. This means a fully functional quantum computer could be created with Deutsch gates. The calculated fidelity of the Toffoli gate in Section 2.4 also indicates that the Deutsch gate would likely have a high fidelity. Additionally, as shown in Section 2.5, the latter part of my Deutsch gate circuit could integrate other Toffoli gate circuits into a Deutsch gate circuit.

The Deutsch gate has several advantages over universal sets currently used in quantum computing. First, the universal sets typically used consists of at most two-qubit gates. This creates difficulty when implementing operations that use more than two qubits. Duan *et al.* [3] showed that a minimum of five two-qubit gates is required to implement the Toffoli gate. On the other hand, a single Deutsch gate with $\theta = \pi/2$ results in the Toffoli gate. So, the Deutsch gate is better suited for some purposes than current universal sets. Second, the Deutsch gate also simplifies quantum computing since, in contrast with a universal set, it requires only achieving a high fidelity for one type of gate rather than for several.

A proposal by Shi [5] to create a Deutsch gate using Rydberg blockade of neutral atoms is the only other Deutsch gate proposal currently available. Shi's protocol tunes the angular dependence θ of the Deutsch gate by adjusting the laser field strengths. In contrast, my protocol just requires changing the angles of two half-wave plates to affect θ. Shi's protocol does not use flying qubits like mine does. This means that connecting gates may be more difficult using Shi's protocol. On the other hand, Shi's protocol may be better suited to storage than my own.

Shi [5] also made protocols for the Toffoli gate and CNOT gate in addition to the Deutsch gate protocol. So, both Shi's system and my own (namely, the GaAs/InAs quantum dots alongside optical elements) can produce all three gates. So, the matter of being able to produce these gates is not a determining factor in which system to use. Instead, the intended use of the gate may determine which system to select.

3.2 Future Developments

Before the Deutsch gate can be effectively implemented, more work needs to be done. Aside from my proposal and Shi's [5] proposal, there are no other proposals for creating the Deutsch gate. There are many other quantum dots and systems that have similar transitions when interacting with circularly polarized light [14]. Many of these could potentially lead to a new Deutsch gate protocol. As described in Section 2.5, if a Toffoli gate can be made from one of these systems, then it can likely be converted into a Deutsch gate in the same way the Toffoli gate circuit can that I displayed in Section 2.4. This could be realized by sending a few particular outputs through the same system of half-wave plates, quarter-wave plates, linear polarized beam splitters, and phase shiftters as shown in Fig. 2.4. The GaAs/InAs quantum dots examined in this thesis are obviously useful for quantum information, but there are many other systems that could lead to Deutsch gate protocols. Many may be able to use the latter half of my Deutsch gate circuit to achieve this.

Possibly due to the fact that the Deutsch gate has not been feasible in the past, there is a lack of algorithms that utilize the Deutsch gate. Deutsch gates can complete any quantum computational task, but how? The Deutsch gate is a solution waiting for a problem. Some computations are likely simpler to implement using the Deutsch gate but few have been identified. Significant progress could result from additional algorithms that utilize the Deutsch gate. Finding smaller quantum computational tasks fitting for the Deutsch gate would take advantage of its universality. This could take the form of finding new gates that the Deutsch gate implements quickly aside from the Toffoli gate.

3.3 Conclusion

A universal Deutsch gate can be created with quantum dots and optical elements. This gate could lead to more efficient and more accessible quantum computing. The Deutsch gate could at least prove more useful than current universal sets for certain quantum computational tasks; implementing the Toffoli gate serves as an example of this. In addition, the lack of multiple gates simplifies the need for each type of gate to reach a high fidelity.

The Deutsch gate described in this thesis can also switch back and forth between a simple Toffoli gate and a Deutsch gate with just the use of removable mirrors. The Toffoli gate we described in Section 2.4 can reach high fidelities under realistic conditions. This indicates that the Deutsch gate has the potential to reach high fidelities as well.

This thesis also further verifies the usefulness of GaAs/InAs quantum dots for quantum computing. Bonato *et al.* [11] and Kim *et al.* [7] found efficient applications in quantum computing with these GaAs/InAs quantum dots. However, the fact that a Deutsch gate can be created with these quantum dots means that any quantum computational task can be completed with them. Since the Deutsch gate is universal, the elements that make up a Deutsch gate can be considered—in some sense—a "universal set of elements."

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