Helical Resonator for a Dual-Species Ion Trap

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ABSTRACT

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Ion collisions in a strongly coupled ultracold neutral plasma are useful for simulating collisions and properties found in high energy dense plasmas. Ultracold plasmas can be created from atoms that are cooled and trapped in a magneto-optical trap. After ionization, the resulting ions can be trapped using a linear quadrupole ion trap, making it possible to measure ion collisions for times scales greater than 10 μs. We report on designing and constructing a 5 MHz helical resonator for use in our dual species magneto optical ion trap. We obtained a quality factor of 153 with a coupling efficiency of 75%. We measured the density of trapped Ca atoms using florescence measurements.

Keywords: Harmonic Oscillator, Ion Trap, Magneto-Optical Trap

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Chapter 1

Introduction

1.1 Ultra-cold Neutral Plasmas as High Energy Density Plasma Simulators

High energy density plasmas (HEDP) occur in extreme environments such as within stars or during phases of nuclear fusion. In these extreme environments, the potential energy of the plasma can exceed the kinetic energy, creating a strongly coupled plasma. Due to the extreme environments needed to produce HEDPs,they are both hard to produce and difficult to study. Therefore, many of the properties of these plasmas are not well known.

Ultra cold neutral plasmas (UNPs) are produced by ionizing laser-cooled trapped atoms. With temperatures as low as a few microkelvin, they can reach a strongly coupled state while also maintaining low densities. The low temperatures of these plasmas makes them easy to contain and study. Ultra-cold plasmas can be using a magneto optical trap (MOT) and pulse lasers. A MOT uses lasers that are red-shifted from atomic resonance frequency to cool and trap atoms [1]. After atoms are trapped within a MOT, they can be pulse-ionized into a plasma. Because UNPs exist in a strongly coupled state, they have similar properties and are effective simulators of strongly coupled

HEDPs [2].

1.2 Ion Traps

In addition to MOTs, ion traps are also effective for studying ultracold plasmas. Ion traps use high-frequency, high-voltage oscillating electric fields to create an electric psuedo-potential capable of trapping charged ions. After neutral atoms within a MOT are ionized, the resulting ions are no longer optically trapped. An ion trap can keep ions contained and therefore increase the lifetime of ion collision experiments. The motion of ions in the trap can be described and calculated from its oscillating frequency and voltage using Matheui's equation [3].

1.3 Helical Resonators

To create a more power efficient ion trap, a helical resonator can be used to power the ion trap. Using a large inductive coil combined with the trap capacitance, a helical resonator creates an LRC circuit that amplifies the voltage of an input signal proportional to the quality factor (Q-factor) and the overall coupling efficiency of the resonator. The Q-Factor is the ratio between the resonance frequency and the full-width at half max (FWHM) of an input signal at resonance [4]. With a sufficient Q-factor and coupling efficiency, helical resonator can be used to drive ion traps that are capable of trapping a variety of ions. Some helical resonators also make use of an antenna coil in conjunction with the main resonator coil to decouple from the power source. This makes the resonator easier to impedance match to the power source, increasing the overall coupling efficiency of helical resonator [5].

1.4 Prior Work at BYU

Our group at BYU currently produces Ca and Yb UNPs by first cooling and trapping atoms within a dual-species MOT and then pulse-ionizing them into a plasma. This has allowed for ion collision measurements and measurements of the coulomb logarithm, an important value for calculating plasma collisions that is relatively unknown for strongly coupled plasmas [6]. With our current setup, after trapped atoms are pulse-ionized from our MOT, collision measurements are limited to about 6 μs before they expand out of viewing range. We are in the process of completing a second dual species MOT that has a linear quadrupole trap superimposed over the MOT to extend the lifetime of ion collision experiments.

1.5 Overview

This thesis reviews the design and construction of a helical resonator for use in our dual species MOT ion trap (MOTion trap). The design calculations and considerations are discussed that allowed us to achieve a 4 MHz resonant frequency and a Q-factor of 153. An overview is given of the physical construction process as well as an analysis of our finished trap. The resonant frequency and Q-factor was measured using signal reflection measurements.

Chapter 2

Helical Resonator Design and Construction

To extend the lifetime of ion collision experiments, a linear quadrupole ion trap will be used in conjunction with a MOT. This chapter reviews calculations and design considerations that were used to create a helical resonator capable of driving our ion trap. After a resonant frequency was chosen, the resonator dimensions were chosen and calculated based on the materials available and resonance frequency of our resonator. A small overview of the construction process is given.

2.1 Resonant Frequency and Voltage

To calculate the ideal resonant frequency needed for our ion trap we used Mathieu's equations for a quadrupole ion trap [3]. The Mathieu q parameter (q_M) describes the ratio of the oscillation amplitudes to trap size for ions within the trap. This is given by [3]:

$$q_M = \frac{\beta e V_0}{m\omega^2 d} \tag{2.1}$$

Where β is a constant based on the geometric shape of the trap. For our trap, we assume $\beta = 1$. Ion traps are stable for $q_M \leq 1$ (where oscillations are smaller than the size of the trap). A common choice for stable trapping is $q_M = 0.1$ [3].

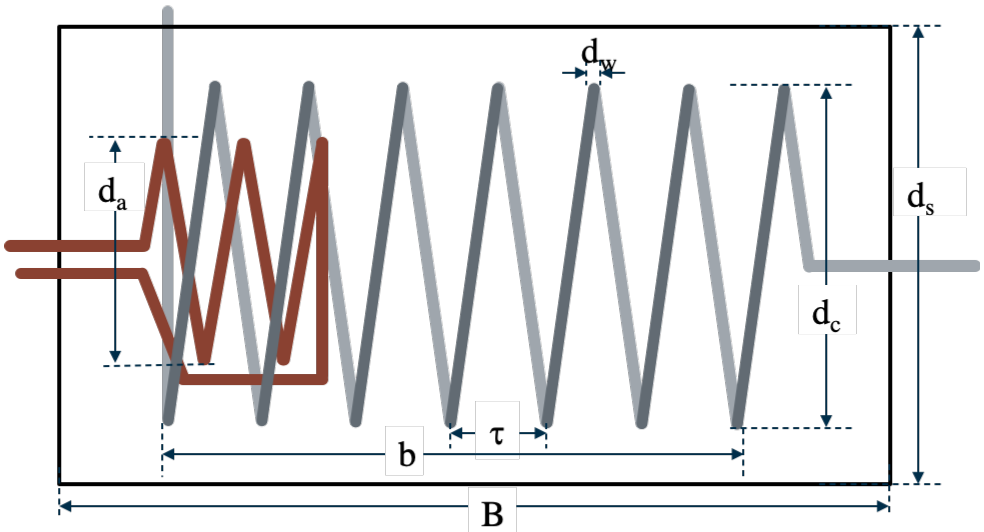

Figure 2.1 Diagram of Helical Resonator. The helical resonator is composed of a primary inductive coil (in gray) and an antenna coil (in brown) that couples the resonator to a power source. The dimensions of the resonator are marked above as: shield diameter (d_s), coil diameter (d_c), antenna coil diameter (d_a), copper wire diameter (d_w), coil spacing (τ), coil height (b), and shield height (B).

We can use Eq. 2.1 to find the frequencies that will allow for stable trapping based on the limits of our power source. A power supply with a 20 W limit in a 50 Ω system can provide about 31 V to the helical resonator [7]. Based on prior research, a target resonator Q-factor of approximately 200 was reasonable [5]. With a Q-factor of 200 and an ideal 100% coupling we would be able to supply a maximum of about 6,200 V to our ion trap [8]. With $q_M = 0.1$, the mass of of Ca being $m = 40 * 1.67 * 10^{-27}$ kg, $V_0 = 6200$ V, and a trap size of d = 1 cm, our frequency was limited to $\omega \geq 2\pi$ x 0.2 MHz. We this in mind, we chose a target resonant frequency of $\omega = 2\pi$ x 5.0 MHz.

2.2 Design Considerations

The resonant frequency of a helical resonator is determined by the inductance and capacitance of the resonator itself as well as the ion trap that it is coupled to. To calculate the resonator dimensions and parameters needed to a achieve our selected 5MHz resonant frequency with a Q-factor of around 200, we followed work by a Severns, *et al.* [5].

The primary dimensions and adjustable parameters of the resonator were the shield height, shield width, coil width, coil winding pitch, number of coil turns, wire thickness, and materials used. Due to its low cost and high conductivity, the resonator shield was made out of aluminum. All of the wiring is copper. Aluminum pipe with an outside diameter of 12 cm and copper wire with a thickness of $d_w = 2$ mm was readily available, so the rest of the resonator dimensions were chosen after fixing those parameters. Severns, et al suggested a winding pitch that was about twice the diameter of the copper wire, so the winding pitch was also fixed at $\tau = 5$ mm.

After fixing these parameters, the rest of the coil width, coil height, coil diameter, and shield height were decided by calculated the Q-factor and using work done by other groups (see Appendix A) [5]. To choose coil dimensions that would maximize our Q-factor, a contour plot of the theoretical Q-factor of our resonator is made as a function of the ratio between the shield diameter (d_s) and the coil diameter (d_c). This is seen in Fig 2.2.

We also had PVC pipes of varying diameters for wrapping our helical coil. Because we desired a Q-factor above 200, we chose a PVC pipe that gave d_c = 8.89 cm. With d_s = 12 cm, the ratio d_c/d_s = 0.74, meaning an ideal theoretical Q-factor of approximately 280.

2.3 Impedance matching the Resonator to the Trap

Rather than connecting the resonator directly to the ion trap, Siverns *et al.* [5] suggested using an antenna coil to inductively transfer an rf signal to the ion trap as (See Fig. 2.1). This decouples the resonator from the power source. Changing the dimensions of the antenna coil allows the resonator to be impedance matched to the power source without adjusting the rest of the helical resonator. This change also improves the Q-factor of the helical resonator. Our first helical resonator design

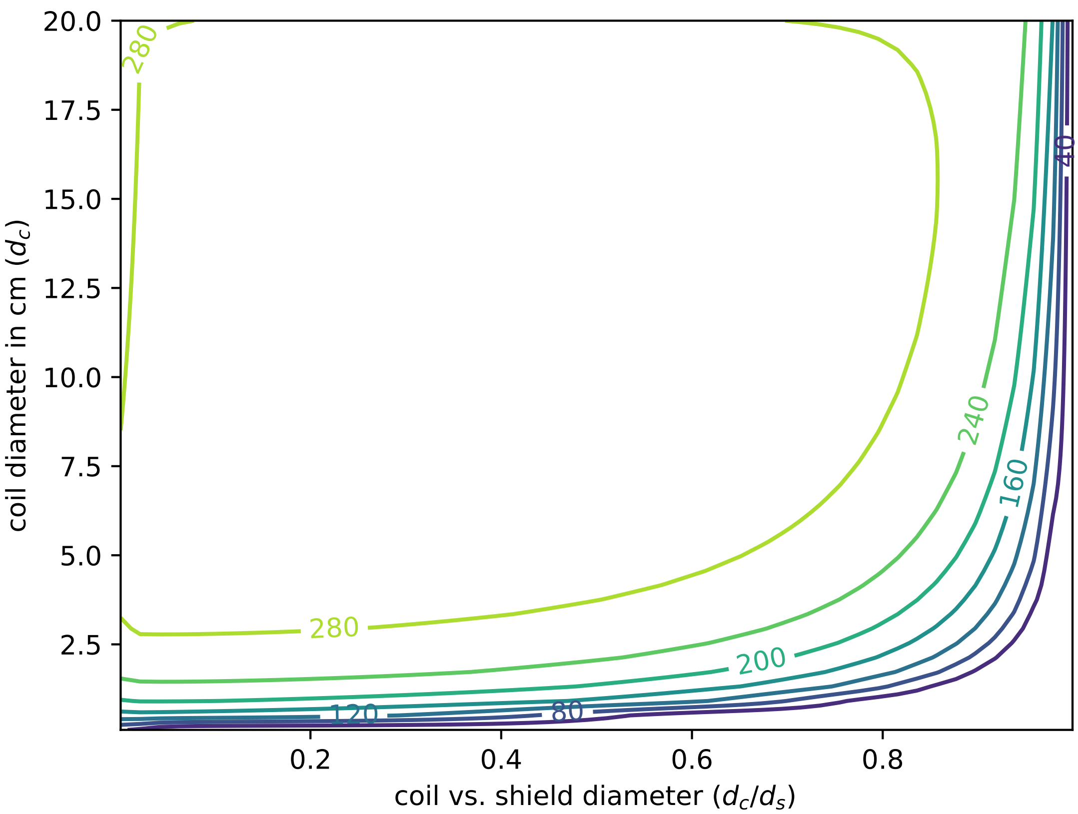

Figure 2.2 Theoretical Q-factors of a helical resonator. Given an aluminum shield diameter of d_s = 12 cm, copper wire diameter of d_w = 2 mm, wire spacing of τ = 5 mm, and a resonant frequency of $\omega = 2\pi$ x 5 MHz the Q-factor is calculated as a function of the ratio between d_c and d_s.

did not include an antenna coil. Initial results with this resonator showed that the spectral width of a reflected input signal (see Section 3.1) was very broad, or that the Q-factor was very low.

To calculate the impedance of the overall system and coupling efficiency, we chose an antenna coil diameter (d_a) just smaller than d_c and calculated theoretical impedances and coupling efficiencies between the two coils by altering the winding pitch and number of turns of the antenna coil. These calculations are given in python code of Appendix A. A winding pitch and antenna coil turn number was chosen that would provide an impedance 50 Ohms, matching the impedance of the power source, and an optimal coupling efficiency close to 1.

2.4 Construction of the Resonator

The resonator shield was ordered and cut from a section of aluminum pipe. We wound the helical coil using a PVC pipe. After winding the coil, we decided to leave it on the PVC pipe for structural support. This may effect the quality of the trap due to the dielectric properties of the PVC, but the provided stability provided a more effective resonator. Aluminum end-caps were fashioned to screw on to each end of the resonator, allowing easy removal for tuning or other adjustments. To hold the PVC pipe fixed within the resonator, we used nylon fittings on the end of the two resonator caps, which were shaped to fit inside each of the resonator coil.

The antenna coil was shaped in a similar manner as the main helical resonator coil. We used the same 2 mm wire that was stiff and could easily be adjusted to tune the impedance of the resonator for matching to our power source. Similar to the main resonator coil, we used a PVC pipe for winding and stability of the antenna coil.

Chapter 3

Results and Conclusions

After a helical resonator as described in Section 2 was built, the Q-factor and coupling efficiency was then measured. The design was then validated by trapping Ca ions.

3.1 Measuring the Quality factor

After constructing our helical resonator, we connected it to our linear quadrupole trap. To measure the coupling efficiency and Q-factor, we used a bi-directional RF coupler as diagrammed in Fig. 3.1 to measure the reflected signal. An input signal is sent through a bi-directional coupler to the helical resonator. Due to any impedance mismatch, some of the signal is reflected back to the bi-directional coupler. This reflected signal is sent to an oscilloscope for measurement.

To locate the resonant frequency, we adjusted the frequency of the input signal until we located the minimum reflected signal frequency at 4 MHz. We then took several measurements while stepping through the frequency at 20 kHz intervals near the resonant frequency.

The measured reflected signal amplitudes are shown in Fig. 3.2. The maximum reflected signal is approximately 66 mV, and the minimum is 16 mV at $\omega_0 = 2\pi$ x 4.00 MHz. A curve fit is used to fit these measurements to an inverted lorentzian. For this curve, the FWHM is 0.026 MHz. The

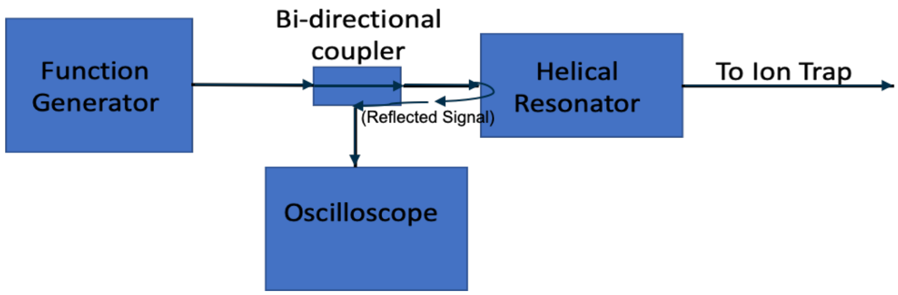

Figure 3.1 Setup for measuring the Q-factor and coupling efficiency of a helical resonator. By placing a bi-directional coupler in between a function generator and a helical resonator, the reflected signal can be measured. The reflected signal gives information about how much power is being sent to the ion trap (coupling efficiency) and the spectral width of the peak resonance frequency to calculate the Q-factor.

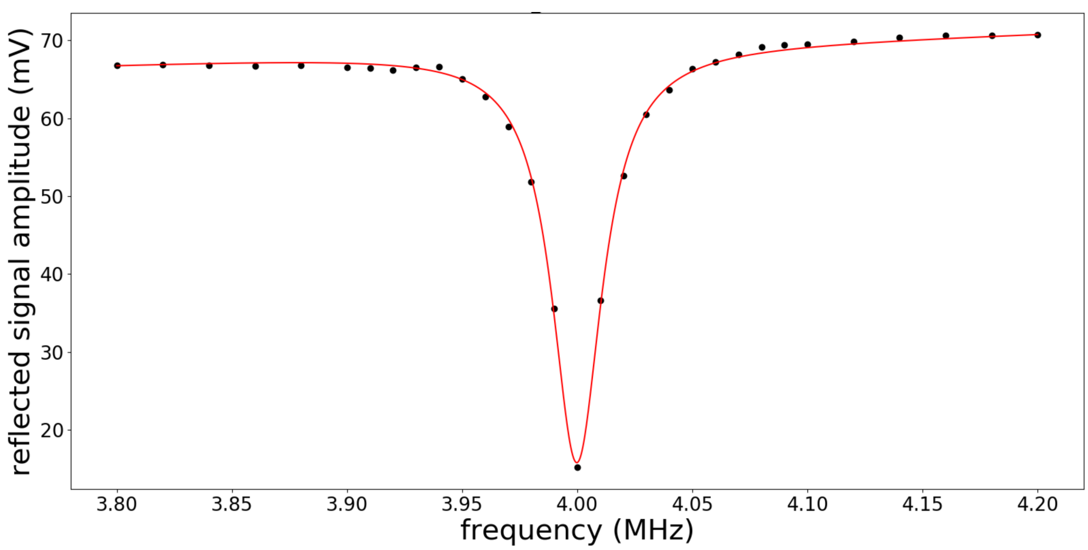

Figure 3.2 Helical resonator reflected signal measurements [8]. The blue dots represent reflected signal measurements from the helical resonator. The red curve represents a curve fit to an inverted lorentzian function. Using the curve fit, the FWHM and Q-factor can be calculated. The coupling efficiency, or percentage of the input signal sent to the ion trap can also be calculated. The reflected signal measurements of our helical resonator as shown on this graph show that the Q-factor is 153, and the coupling efficiency is approximately 76%.

Q-factor is calculated by dividing the resonant frequency by the FWHM:

$$Q = \frac{\omega_0}{\omega_{\mathrm{FWHM}}} = \frac{2\pi \times 4.00 \times 10^6\,\mathrm{Hz}}{2\pi \times 2.6 \times 10^4\,\mathrm{Hz}} = 153. \tag{3.1}$$

With the system shown in Fig. 3.1, coupling efficiency (η) was also measured. In Fig. 3.2, approximately 24% of the voltage is always reflected from the resonator. So, the resonator has a coupling efficiency of about η =76%. The voltage amplification is calculated by multiplying the Q-factor by η [8]. Thus, our resonator amplifies input voltages by $V_{\mathrm{out}} = (153 \times 0.76)\, V_{\mathrm{in}} = 116\, V_{\mathrm{in}}$.

3.2 Trapping Calcium Ions

Using our helical resonator driven ion trap, we trapped calcium ions from our magneto optical trap. We have yet to make density measurements. Early results suggest that loading our ion trap directly from the MOT filled our ion trap too much for our current setup to measure.

3.3 Conclusion

Our goal was to construct an efficient helical resonator with a resonant frequency of around 5 MHz with a Q-factor around 200. Our resonant frequency was $\omega_0 = 2\pi$ x 4.00 MHz with a Q-factor of 153 and coupling efficiency $\eta = 76\%$. The lower frequency was likely due to errors in our resonator design. The lower Q-factor most likely resulted from unaccounted for resistance in the trap and wires of our system. In addition, the inclusion of PVC pipes within the resonator may have negatively affected the Q-factor of the trap.

We were still able to ionize calcium atoms and load our ion trap directly from our magneto optical trap.

Future work may include trying different types of resonators such as quarter wavelength coaxial resonators. Such resonators have smaller trap depths are shown to achieve Q-factors above six thousand [9]. We may also attempt to build a resonator in the GHz frequency range. This would be used for trapping electrons after atoms are photo ionized from the MOT. This kind of trap may have better properties for the ion collision experiments we would like to do.

Appendix A

Code used to calculate theoretical Q-Factor

```
import numpy as np
from numpy import sqrt, pi, exp, log
import matplotlib.pyplot as plt

###(*measurement constants*)
cm = 1e-2
mm = 1e-3
pF = 1e-12
Mhz = 1e6
Kcd = lambda dc: 35*dc*1e-12 ##(*p.926*)

Kcb = 11.26*1e-12  ##(*p.926*)

mtoinch = 39.37 ##(*meters to inches converter*)
conv = .025
```

```
Rt = 1.0 ##(*resistance of the trap*)
Rho = 1.72e-8 ##(*resistivity of copper*)
Rhos = 2.82*1e-8
eps0 = 8.854187817e-12
epsr = eps0
eps = eps0*epsr
Rj = 1. ##(*helical coil to shield junction resistance, p. 925,
    fig. 6*)
mu0 = 1.25663706*1e-6
mur = 0.999991
mu = mur*mu0

### The measurements below corresponds to the capacitance
### of our ion trap
# #(*blue electrodes*)
pF = 1e-12
Ncap = 27.5*pF
Scap = 25.3*pF
Ecap = 28.2*pF
Wcap = 24.9*pF

# #(*orange electrodes*)
NEcap = 26.6*pF
NWcap = 26.3*pF
SEcap = 25.3*pF
```

```
SWcap = 23.7*pF
Ct = (Ncap + Scap) + (NEcap + SWcap) # #(*total trap capacitance
    *)

Ct = 50*pF
Cw = 5*pF
Ctot = Ct + Cw

"""#(*dc is the diameter of the coil, ds is the diameter of the
    shield, \
tau is the space between each coil winding, w0 is the \
resonant frequency in radians/s, and d0 is the thickness of the \
copper coil, all calculations are made in standard units, or
    meters. All page
and equation numbers are references to the paper by Severns*)"""

KLc= lambda dc, ds, tau: mtoinch*(conv*dc**2*(1 - (dc/ds)**2))/
    tau**2*1e-6 #(*p. 926*)

KCs = lambda dc, ds: mtoinch*0.75/log(ds/dc)*1e-12 #(*p. 926*)

Cs  = lambda dc, ds, tau, w0: b(dc, ds, tau, w0)* KCs(dc, ds) #(*
    Capacitance between coil wires and outer shield, p. 926*)
def b(dc, ds, tau, w0):
    """(*coil height, p.927*)"""
```

```
    first = (Ctot + Kcd(dc))/(KCs(dc, ds) + Kcb)
    second = sqrt((KCs(dc, ds) + Kcb)/((Ctot + Kcd(dc))**2*KLc(dc
        , ds, tau)*w0**2) + 1/4) - 1/2
    return first*second

Lc = lambda dc, ds, tau, w0: b(dc, ds, tau, w0)*KLc(dc, ds, tau)
    #(*Inductance of coil, p. 926*)

a = lambda dc, ds, tau, w0: Ct/(Cs(dc, ds, tau, w0) + Cw);
alpha = lambda dc, ds, tau, w0: a(dc, ds, tau, w0)/(a(dc, ds, tau
    , w0) + 1);#(*ratio of trap capac vs. combined capac due to
    connecting wires \& coil shield, p. 926, below (24)*)
lc = lambda dc, ds, tau, w0: sqrt((b(dc,ds,tau,w0))**2 + (pi*dc)
    **2) #(*length of helical coil*)

Ns = lambda dc, ds, tau, w0:(b(dc, ds, tau, w0)*lc(dc, ds, tau,
    w0))/(4*pi*(ds - dc)**2)
ls = lambda dc, ds, tau, w0: Ns(dc, ds, tau, w0)*sqrt((pi*ds)**2
    + (b(dc, ds, tau, w0)/Ns(dc, ds, tau, w0))**2) #(*length of
    conduction along \the shield, p. 927*)

Rs = lambda dc, ds, tau, w0:(Ns(dc, ds, tau, w0)*Rhos*ls(dc, ds,
    tau, w0))/b(dc, ds, tau, w0)#(*shield resistance, p. 927*)
delta = lambda w0: sqrt((2*Rho)/(w0*mu))*sqrt(sqrt(1 + (Rho*w0*
    eps)**2) + Rho*w0*eps) #(*skin depth of copper vs. frequency,
```

```
see wikipedia*)

Rc = lambda dc, ds, tau, w0, d0:(Rho*lc(dc, ds, tau, w0))/(d0*pi*
    delta(w0)) #(*resistance of helical coil, p. 927*)

Resr = lambda dc, ds, tau, w0, d0: Rj + Rc(d0, dc, ds, tau, w0) +
     Rs(dc, ds, tau, w0) + Rt*(alpha(dc, ds, tau, w0))**2; #()!p.
    \927*)
XLc = lambda dc, ds, tau, w0: Lc(dc, ds, tau, w0)*w0 ##(*p. 925,
    below (15)*)

Q = lambda dc, ds, tau, w0, d0: XLc(dc, ds, tau, w0)/Resr(dc, ds,
     tau, w0, d0); ##(*Q factor, bandwidth vs. frequency(higher =
    better), p. 926*)

def CoilParameters(dc, ds, tau, f0, d0):
     w0 = 2*pi*f0
     coilheight = b(dc, ds, tau, w0)
     shieldheight = coilheight + ds/2
     Qfactor = XLc(dc, ds, tau, w0)/Resr(dc, ds, tau, w0, d0)
     ratio = coilheight/dc
     validity = True if ratio > 1 else False
     SkinDepth = delta(w0)
     coils = coilheight/tau
```

```
    print(f"Valid␣Dimensions?:␣{validity};\nCoil␣Height:␣{
        coilheight}␣m;")
    print(f"Sheild␣Height:␣{shieldheight}␣m;\nQ␣value:␣{Qfactor};
        ")
    print(f"Skin␣Depth:␣{SkinDepth};\n#␣of␣coils:␣{coils}")
    return
CoilParameters(8.89*cm, 12*cm, 5.1816*mm, 5*Mhz, 2.5908*mm)

################ Q-Factor Calculations #################

def Qplot(f0, tau, d0):
    w0 = 2*pi*f0
    n_arr = np.linspace(0.001, .999)
    dc_arr = np.linspace(1*mm, 20*cm)
    N, DC = np.meshgrid(n_arr, dc_arr)

    fig, ax = plt.subplots()
    CS = ax.contour(N, DC*100, Q(DC, DC/N, tau, w0, d0))
    ax.clabel(CS, inline=1, fontsize=10,fmt='%1.f')
    ax.set_xlabel("coil␣vs.␣shield␣diameter␣($d_c/d_s$)")
    ax.set_ylabel("coil␣diameter␣in␣cm␣($d_c$)")
    ax.set_title('Q-factor')
    fig.savefig('figure_1.eps', format='eps')
    plt.show()
```

```
    # plotb =
    # RegionPlot((b(dc, dc/n, \(Tau), w0)/dc < 1), {n,
    #   0.001, .999}, {dc, 1 mm, 40 cm});
    return

Qplot(5*Mhz, 5.1816*mm, 2.5908*mm)

################### Impedance Calcualtion ###################
Xct = lambda w0: 1/(Ct*w0) #(*trap reactance*)
Xcw = lambda w0: 1/(Cw*w0) #(*trap reactance*)
Xcs = lambda dc, ds, tau, w0: 1/(Cs(dc, ds, tau, w0)*w0) #(*trap
   reactance*)
Ze = lambda dc,ds,tau,w0,d0: (1/(1j*Xct(w0) + Rt) + 1/(1j*Xcw(w0)
   ) + 1/(1j*Xcs(dc, ds, tau, w0)))**-1 #(*impedance of resonator
    + the trap*)
print(Ze(8.89*cm, 12*cm, 5.1816*mm, 5*Mhz*2*pi, 2.5908*mm))

################### More Impedance Calculations
   ###################
A = pi*(.05/2)**2; #(*cross sectional area of antenna coil*)

n = 5; #(*antenna coil # of turns*)
n2 = 21.303808534674385; #(*helical coil # of turns, (see above)
   *)
La = lambda tau:(mu0*n*A)/tau #(*inductance of antenna coil*)
```

```
l = lambda tau:(n + n2)*tau #(*height of antenna coil*)
P = lambda tau:(mu0*A)/l(tau) #(*Permeance of space that the flux
    occupies*)
Lm = lambda tau: n*n2*P(tau) #(*mutual inductance, see coupling
    constant wikipedia page*)
kl = lambda dc, ds, tau, w0: Lm(tau)/sqrt(Lc(dc, ds, tau, w0)*La(
    tau)) #(*coupling constant, see wikipedia*)
Cc = lambda dc, ds, tau, w0: Kcb*b(dc, ds, tau, w0) + Kcd(dc) #(*
    helical coil self capacitance*)
XCc = lambda dc, ds, tau, w0: 1/(Cc(dc, ds, tau, w0)*w0)#(*
    helical coil self reactance*)
Zl = lambda dc, ds, tau, w0, d0: Rc(dc, ds, tau, w0, d0) + ((Ze(
    dc, ds, tau, w0, d0) + Rs(dc, ds, tau, w0) + Rj)**-1 + (1j*XCc
    (dc, ds, tau, w0))**-1)**-1#(*impedance of the load*)

Zin = lambda dc, ds, tau, w0, d0: La(tau)*(1j*w0 + (kl(dc, ds,
    tau, w0)**2*Lc(dc, ds, tau, w0)*w0**2)/(1j*w0*Lc(dc, ds, tau,
    w0) + Zl(dc, ds, tau, w0, d0))) # (*impedance of the antenna*)

def ImpedanceCalculations(dc, ds, tau, f0, d0):
    w0 = 2*pi*f0
    coupling = kl(dc, ds, tau, w0)
    Zmatch = Zin(dc, ds, tau, w0, d0)
    print(f"Coupling Constant (k): {coupling};\nImpedance (Z_in):
        {Zmatch}")
```

```
    return
ImpedanceCalculations(8.89*cm, 12*cm, 5.1816*mm, 5*Mhz, 2.5908*mm
    )
```

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