DESIGN OF A FREQUENCY-DOUBLED 423 nm LASER FOR USE IN A CALCIUM INTERFEROMETER

by

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ABSTRACT

DESIGN OF A FREQUENCY-DOUBLED 423 nm LASER

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In this thesis I determine the optimal parameters for maximizing second harmonic generation of 423 nm light for use in an atom interferometer. The analysis is done using the nonlinear crystal Beta Barium Borate. Both critical and noncritical phase matching methods are considered. The optimal beam size for a Gaussian beam profile is calculated. Build-up cavity design is analyzed in order to maximize the intensity in the crystal. An optimal setup, one that maximizes the output power of the 423 nm beam, is proposed.

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1. Introduction

1.1. Interferometry

In this thesis I analyze the problem of optimizing a frequency doubled 423nm laser for use in an atom interferometer. The introduction contains background information on interferometry and nonlinear optical processes. In the following sections, the optimal values for the free parameters are calculated. These include phase matching, beam size, and build-up cavity design. The recommended configuration for use in the interferometer is summarized in the final section.

Over the last century, interferometers have developed as a primary tool for making precise measurements in science and industry. Interferometers are devices that split and recombine a wave so that it travels to a detector through at least two different paths or "arms." The different arms can accrue a phase shift relative to one another, due to a difference in path length, electromagnetic fields, or other differences between the paths. This results in an interference pattern at the detector. This pattern can be analyzed to determine differences between the arms of the interferometer [1].

Interferometers were initially developed using light waves, but they can be made using any substance that exhibits wave properties. More recently, particles and even entire atoms or molecules have been used [1]. Because the deBroglie wavelengths of atoms at thermal velocities are much smaller than the wavelengths of available light sources, atom interferometers can have a much higher accuracy than optical interferometers in many applications. Additionally, the rest mass and internal structure of atoms allows atom interferometers to be used in applications where optical interferometers cannot, such as measuring acceleration, gravity, or electric and magnetic fields.

1.2. Lasers and Atom Interferometry

A necessary part of an atom interferometer is a probe that can measure the number of atoms in a particular state after the two arms have interfered. Additionally, the atom beam in an atom interferometer needs to be collimated. Lasers can be utilized for both of these purposes.

A laser that is resonant with one of the atoms' transitions out of the ground state can be directed at the beam of atoms exiting the interferometer. The ground state atoms in the beam will scatter the light. The number of ground state atoms in the beam can be determined by measuring the amount of light scattered by the atom beam.

Lasers can also be used to cool, and thereby collimate, the beam. Laser cooling involves placing the atoms in the path of several counter-propagating laser beams. The frequencies of these beams are tuned slightly below resonance for an atomic transition. When the atoms are traveling against a beam, the laser's frequency is Doppler shifted up, and the transition will be in resonance. The resulting change in momentum from absorbing photons will slow the atoms down. When the atoms are traveling with the beam, the laser's frequency is Doppler shifted down and no absorption takes place [2]. Several counter-propagating laser beams aligned perpendicular to an atom beam can be used to damp the atoms' transverse motion, thereby collimating it.

1.3. Calcium Interferometry and Frequency Doubling

In this thesis, a cooling and probe laser for use in a Calcium-I interferometer will be considered. The transition that will be used to cool and probe the atoms is the $3p^6 4s^2\ {}^1S_0$ - $3p^6 4s4p\ P_1^o$ transition at 423 nm [3]. A minimum of approximately 12 mW of 423 nm light is needed for the interferometer probe, but more power will improve collimation and the probe signal.

Unfortunately, 423 nm diode lasers are not commercially available. However, diode lasers at 846 nm are widely available. A process called frequency doubling can be used to transform the 846 nm light to 423 nm light. In this thesis, several frequency doubling schemes will be analyzed to determine which will allow for the creation of a 423 nm laser beam with the required output power.

1.4. Nonlinear Processes and Frequency Doubling

Frequency doubling is a nonlinear optical process. When matter is exposed to an electromagnetic field oscillating at a frequency ω, the material becomes polarized. The polarization can be written as a power series expansion in terms of the applied field [4]: $\boldsymbol{P} = \varepsilon_0(\boldsymbol{\chi_1} + \boldsymbol{\chi_2}E + \boldsymbol{\chi_3}E^2 + \cdots)\boldsymbol{E}$. This expansion has a linear term, proportional to E, that oscillates at ω, but it will also have nonlinear terms, proportional to E^2, E^3, etc, that oscillate at 2ω, 3ω, etc. For most familiar materials and under normal circumstances, the response is predominantly linear; the higher order dielectric response tensors $(\boldsymbol{\chi})$ are vanishingly small. However, for certain materials and high field intensities, nonlinear effects can become significant. If the second dielectric response tensor is large enough, the polarization will have a significant component oscillating at twice the frequency of the applied field. This will result in the generation an electromagnetic wave at twice the applied frequency, a phenomenon known as second harmonic generation (SHG) or frequency doubling [4].

Second harmonic generation with lasers is most commonly done by focusing the laser into a crystal with significant nonlinear properties. The response of the crystal to the applied fields is determined by the second dielectric response tensor. However, for a given crystal type, input beam polarization, and angle, the effects can be summarized by a single number, called the effective nonlinear coefficient [4]. The higher the coefficient for a given crystal and direction, the higher the conversion efficiency; more of the energy in the incident beam will be transferred to the second harmonic beam. There are many different crystals available, but the one we will be investigating in this thesis is Beta Barium Borate (BBO). BBO has a high nonlinear coefficient and is a commonly used crystal type for SHG.

1.5. Parameters Affecting Frequency Doubling Efficiency

The efficiency of second harmonic generation in nonlinear crystals depends primarily on four factors: phase matching, Poynting vector walk-off, input beam size, and input beam power. Of these, phase matching, beam size, and beam power are optimizable parameters, while Poynting vector walk-off is not [5]. The following sections will determine the effects and appropriate values for these parameters in order to maximize efficiency.

2. Optimization of Second Harmonic Generation Parameters

2.1. Phase Matching

There are several problems that need to be overcome when using nonlinear crystals for SHG. One of these is the dispersive nature of nonlinear crystals; the index of refraction is frequency dependant. Due to this frequency dependence, the first and second harmonic beams will travel at different speeds. This causes the second harmonic light generated at different points in the crystal to be out of phase. The resulting destructive interference greatly reduces the conversion efficiency. This problem can be overcome by phase matching.

2.1.1. Index of Refraction

There are two primary methods of phase matching, critical and noncritical, both of which take advantage of the index of refraction's dependence on polarization direction (anisotropy). Anisotropic materials are classified by three principle indices of refraction (n_x, n_y, n_z), which define an orthogonal coordinate system. The analysis is much simpler in uniaxial crystals, such as BBO, where two of the three indices are equal. These are denoted by $n_x = n_y \equiv n_0$ and $n_z \equiv n_e$, where n_o and n_e stand for the ordinary and extraordinary index, respectively. Waves traveling in a uniaxial crystal are called ordinary if they are polarized in the x-y plane, and extraordinary if they have a component of polarization along the z-axis (optical axis). Ordinary waves experience an index of refraction n_o, while extraordinary waves experience an index of refraction $n_e(\theta)$ which depends on the angle between the wave vector and the z-axis [4].

2.1.2. Noncritical Phase Matching

In noncritical phase matching, the first harmonic travels perpendicular to the z-axis and is polarized in the x-y plane. The second harmonic, whose polarization is always generated orthogonal to the first harmonic, is polarized in the z direction. This gives the first harmonic an index of refraction n_o^{ω} and the second harmonic an index $n_e^{2\omega}$.[1]

Noncritical phase matching takes advantage of the temperature dependence of the indices of refraction. If a temperature can be found at which the ordinary index for the first harmonic equals the extraordinary index for the second harmonic, then noncritical phase matching can occur. If the crystal is heated to this temperature, both waves will travel at the same speed and will remain in phase. When the temperatures involved are reasonable, noncritical phase matching is typically preferable over critical phase matching because it avoids one of the main detriments to SHG efficiency, walk-off (see sections 2.2 and 2.3). Because of this, the feasibility of noncritical phase matching will be explored first.

The index of refraction of a material at different wavelengths (λ) can be calculated from the Sellmeier equation:

$$n^2 = A + \frac{B}{\lambda^2 + C} + D\lambda^2 \tag{2.1}$$

The Sellmeier coefficients A, B, C, and D depend on the material and temperature. The value of the coefficients as a function of temperature is typically not known so their value will be extrapolated from the room temperature value. The change in index of refraction with temperature for BBO is well approximated by a linear function [6]. Therefore, $n(T)$ can be approximated by $n(T_0) + \frac{dn}{dT}\Delta T$ where T_0 is room temperature and $\frac{dn}{dT}$ is the thermo-optic coefficient. Phase matching occurs when the ordinary index for the first harmonic equals the extraordinary index for the second harmonic, $n_o^{\omega}(T) = n_e^{2\omega}(T)$. Solving for ΔT gives:

$$\Delta T = \frac{n_e^{2\omega}(T_0) - n_o^{\omega}(T_0)}{\frac{dn_o}{dT} - \frac{dn_e}{dT}} \tag{2.2}$$

The values of the Sellmeier and thermo-optic coefficients for BBO at room temperature are given in appendix A. Using these, the phase matching temperature can be found for a first harmonic beam at 846 nm. Unfortunately, the required temperature of over $10000\ ^{o}C$ is not reasonable. Noncritical phase matching of BBO at 846 nm is not an option, so the feasibility of critical phase matching will now be explored.

2.1.3. Critical Phase Matching Angle

In critical phase matching, the first harmonic beam is sent into the crystal polarized as an ordinary ray such that it makes an angle θ with respect to the z axis in the y-z plane $(\phi = 0)$.[2] This gives the second harmonic, whose polarization is orthogonal to the first, a component of polarization along the z-axis. As a result, the second harmonic has an angular dependant index of refraction, $n_e^{2\omega}(\theta)$. Critical phase matching involves choosing θ such that the ordinary index of refraction for the first harmonic (n_o^{ω}) equals the extraordinary index of refraction for the second harmonic ($n_e^{2\omega}(\theta)$).

The Sellmeier equation (2.1) can be used to calculate $n_o^{\omega}$and $n_e^{2\omega}$. From these, $n_e^{2\omega}(\theta)$ can be calculated by[3]:

$$n_e^{2\omega}(\theta) = \left(\frac{\cos(\theta)^2}{(n_o^{2\omega})^2} + \frac{\sin(\theta)^2}{(n_e^{2\omega})^2}\right)^{-\frac{1}{2}} \tag{2.3}$$

Figure 2.1 is a polar plot of $n_o^{\omega}(\theta)$ and $n_e^{2\omega}(\theta)$. The ordinary index of refraction is independent of angle, so $n_o^{\omega}(\theta)$ is a circle. On the other hand, $n_e^{2\omega}(\theta)$, is an ellipse (2.3). For certain angles, θ_{pm}, from the z axis, the circle and ellipse intersect. At these angles $n_o^{\omega} = n_e^{2\omega}(\theta)$; critical phase matching is achieved. Using the Sellmeier parameters in appendix A along with equations 2.1 and 2.3, the phase matching angle for SHG in BBO using 846 nm light is found to be 27.6^{o}.

In this section the phase matching angle was determined under the assumption that perfect phase matching yields optimal efficiency. However, Boyd and Kleinman [5] show that for a finite crystal length, a small amount of phase mismatch is desirable. In practice, this adjustment is very small and has a negligible effect on both the indices of refraction and the walk-off angle (section 2.3). Therefore we assume perfect phase matching in calculating these quantities. The optimal phase mismatch will be analyzed quantitatively in section 2.3.

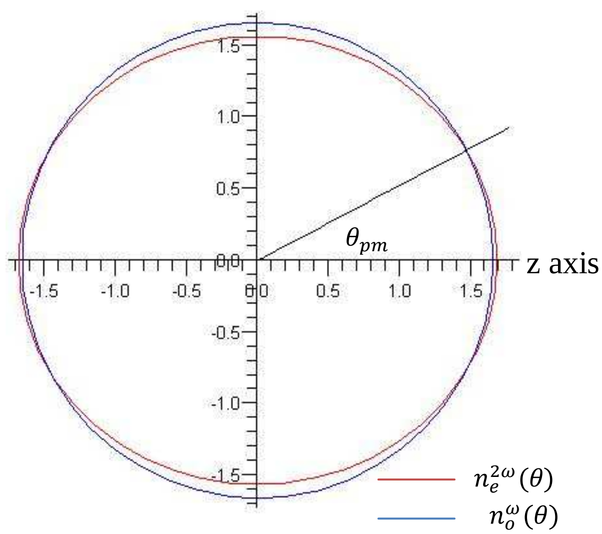

Figure 2.1: Critical Phase Matching Angle

2.2. Poynting Vector Walk-off

In a birefringent material, the wave vector and Poynting vector of an extraordinary wave are not parallel. Therefore, for critically phase matched SHG, the second harmonic's Poynting vector will make some angle ρ with the first harmonic's Poynting vector. Because the direction of energy propagation is not the same, the second harmonic beam will "walk off" the axis of wave propagation. When the two beams no longer overlap, the SHG efficiency is greatly reduced [4].

The walk-off angle ρ between the first and second harmonic is given by [4]:

$$\tan(\rho) = \frac{(n_e^{2\omega}(\theta))^2}{2}\left[\frac{1}{(n_e^{2\omega})^2} - \frac{1}{(n_o^{2\omega})^2}\right]\sin(2\theta) \qquad (2.4)$$

Using equations 2.1, 2.3, and 2.4, ρ for BBO critically phased matched at $\theta = 27.6^{\circ}$ is calculated to be 3.73°. The effect of this walk-off angle on efficiency will be determined in the following section.

2.3. Beam Waist and Phase Mismatch for a Gaussian Laser Beam

With the phase matching and walk-off angles determined, the relationship between output power and input power can now be calculated. Boyd and Kleinman [5] have developed a theory of second harmonic generation in the case of a Gaussian input beam. A Gaussian beam is an electromagnetic wave whose intensity in any plane perpendicular to the direction of propagation is a Gaussian distribution.

In order to improve efficiency, the doubling will take place in a build-up cavity. The laser will be coupled to the TEM_{00} mode of the cavity, which has a Gaussian intensity profile. This makes a Gaussian beam the preferred model for the laser. A Gaussian beam is completely characterized by the location and radius of its beam waist, the point where the beam's "radius" is a minimum. The value of the beam radius is the lateral distance from the z-axis at which the intensity drops by a factor of $1/e^2$ [7].

Boyd and Kleinman's theory provides a formula for the output power of a frequency-doubled Gaussian laser beam, $P_{2\omega}$, as a function of the input first harmonic power, P_ω, taking into account phase mismatch, beam waist, and walk-off. The following equation assumes the beam waist is in the center of the crystal and absorption by the crystal is negligible:

$$P_{2\omega} = E_{nl} P_\omega^2 \tag{2.5}$$

From this equation, one can see that the output power depends on the input power and the effective nonlinearity, E_{nl}. Because the effects are multiplicative, they can be optimized separately. A build-up cavity will be used to increase the input power into the crystal. This is described in section 2.4. This section will be concerned with maximizing the effective nonlinearity.

The effective nonlinearity, which determines the efficiency of SHG, is given by the following.

$$E_{nl} = K l k_1 h(\sigma, \mathrm{B}, \xi) \tag{2.6}$$

Where:

$$K = \frac{2\omega^2 d_{eff}^2}{\varepsilon_0 n_\omega^2 n_{2\omega} c^3 \pi} \tag{2.7}$$

Here, l is the crystal length, k_1 is the first harmonic wave number, n_ω and $n_{2\omega}$ the first and second harmonic indices of refraction, and d_{eff} the effective nonlinear coefficient. These parameters are fixed for a given phase matching angle and are not considered optimizable.

$h(\sigma, \mathrm{B}, \xi)$ contains the dependence of E_{nl} on the optimizable parameters, σ, B, and ξ. These variables represent the phase mismatch, walk-off angle, and beam waist respectively (see Glossary for exact functional form). The assumption that the crystal is cut for complete phase matching fixes the value of the indices of refraction, along with the walk-off angle (and therefore B). This reduces the problem of maximizing E_{nl} to one of maximizing $h(\sigma, \mathrm{B}, \xi)$ for a given value of B where $h(\sigma, \mathrm{B}, \xi)$ is the following integral [5]:

$$h(\sigma, \mathrm{B}, \xi) = \frac{1}{4\xi} \int_{-\xi}^{\xi} \int_{-\xi}^{\xi} \frac{e^{i\sigma(\tau-\tau')} e^{-B^2(\tau-\tau')^2/\xi}}{(1+i\tau)(1-i\tau')} d\tau d\tau' \qquad (2.8)$$

A Matlab program that will maximize $h(\sigma, \mathrm{B}, \xi)$ for a given value of B and output the optimal beam waist is given in appendix B. Using this program it was determined that, for a 10 mm long BBO crystal cut at 27.6^o ($B = 11.4$), the maximum of $h(\sigma, \mathrm{B}, \xi)$ is:

$$h_{max} = 0.0619 \qquad (2.9)$$

This corresponds to ξ=1.43 and $\sigma = 0.75$ (Figure 2.2). The optimal beam waist parameter value, $\xi = 1.43$, corresponds to a beam waist $w_0 = 23.8\ \mu m$. To maximize

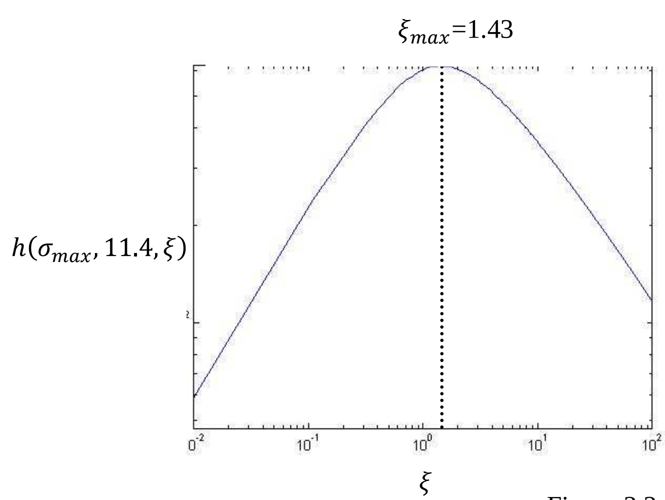

Figure 2.2: Optimal Beam Waist

SHG, the input laser beam should be focused into the crystal such that the beam waist is $23.8\ \mu m$.

The phase mismatch parameter value, $\sigma = 0.75$, indicates that incomplete phase matching is optimal. Although we assumed complete phase matching when determining the crystal cut, indices of refraction, and B, the difference between the optimal and complete phase matching angles is so small that in practice, the crystal can be cut for perfect phase matching [5]. The incomplete phase matching condition can be met experimentally by slight adjustments of the crystal alignment. As mentioned in 2.1.3, the changes in angle necessary are too small to significantly affect B or $n_{2\omega}$.

2.4. Cavity Design

The heavy dependence of frequency doubling efficiency on input power means that a build-up cavity is needed to achieve the necessary output power. A build-up cavity is an arrangement of mirrors that allows the electromagnetic power inside to build up to a higher value than the input power. This higher power will yield a better SHG efficiency.

The two most common cavity configurations are linear and bowtie. A linear cavity is composed of two mirrors, with the nonlinear crystal between them (Figure 2.3). The beam travels both directions as it reflects off of the mirrors. A bow tie cavity is composed of two curved mirrors and two flat mirrors (Figure 2.4) with the nonlinear

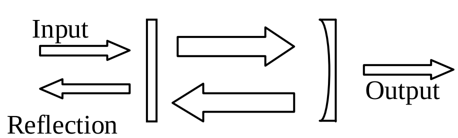

Figure 2.3: Linear Cavity

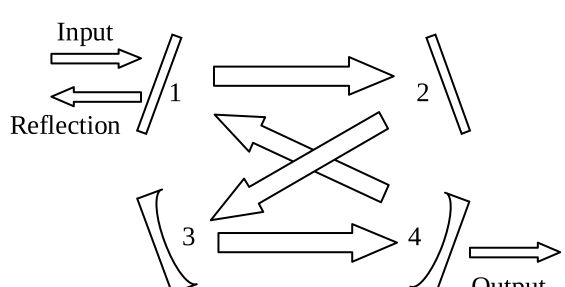

Figure 2.4: Bow Tie Cavity

crystal between mirrors 3 and 4. In a bow tie cavity the beam travels in one direction only. In both configurations the output mirror is designed to be reflective to the first harmonic but transparent to the second harmonic. Additionally, the curvature of the mirrors is chosen to provide the appropriate beam waist and the reflectivity of the mirrors is chosen to maximize the intensity in the cavity.

The bow tie cavity requires twice as many mirrors as the linear cavity, making it more expensive and more difficult to align. In addition, the laser hits the mirrors at an angle, which can cause astigmatism. However, the linear cavity has more significant drawbacks. First, the counter propagating first harmonic beams create a standing wave pattern of high and low intensity spots. This degrades beam quality. Secondly, the second harmonic is generated in both directions, leading to power loss out of the input mirror. Third, reflections off of the input mirror are coupled back into the laser, which can lead to instability [8]. These significant drawbacks associated with linear cavities suggest that a bow-tie cavity would be preferable.

2.4.1. Calculation of Optimal Reflectivity

Designing a bow tie cavity requires a calculation of the optimal reflectivity of all four mirrors, the optimal focal length of the two curved mirrors, as well as a stability assessment. We will assume the mirrors have no scattering or other losses and ignore the absorption in and reflection off the crystal, as these are typically small compared to mirror transmission and SHG losses. Let t_n and r_n denote the field transmission and reflection coefficients of the n^{th} mirror, respectively (Figure 2.4) and ℓ_c the field losses in the crystal due to the transfer of power to the second harmonic. A laser beam with electric field amplitude E_0 is incident on the first mirror. After the first mirror the amplitude (ignoring phase) is:

$$E = t_1 E_0 \tag{2.10}$$

The beam then propagates to the second mirror and reflects again. After this second reflection, the amplitude is:

$$E = r_2 t_1 E_0 \tag{2.11}$$

Similarly, after reflecting off the third and fourth mirrors and traveling through the crystal, the amplitude is:

$$E = r_2 r_3 r_4 \ell_c t_1 E_0 \tag{2.12}$$

The beam then reflects off the first mirror and combines it's amplitude with the transmitted incident beam (assuming the beams are in phase):

$$E = r_1 r_2 r_3 r_4 \ell_c t_1 E_0 + t_1 E_0 \tag{2.13}$$

So after n round trips, the field amplitude just after the input mirror is:

$$E = \sum_{i=0}^{n} (r_1 r_2 r_3 r_4 \ell_c)^i t_1 E_0 \tag{2.14}$$

In the limit as $n \rightarrow \infty$, this geometric series converges to:

$$E = \frac{t_1 E_0}{1 - r_1 r_2 r_3 r_4 \ell_c} \tag{2.15}$$

The amplitude at the crystal's position, between mirrors 3 and 4, is:

$$E_\omega = \frac{r_2 r_3 t_1 E_0}{1 - r_1 r_2 r_3 r_4 \ell_c} \tag{2.16}$$

Mirrors are more commonly characterized by their power reflectance and transmittance, R and T respectively. These are related to the field reflectance and transmittance, r and t, by $R = r^2$ and $T = t^2$. Assuming the mirrors are lossless, $R_n + T_n = 1$ giving

$t_n = \sqrt{1 - R_n}$. In terms of R:

$$E_\omega = \frac{\sqrt{R_2 R_3 (1 - R_1)} E_0}{1 - \sqrt{R_1 R_2 R_3 R_4} \ell_c} \tag{2.17}$$

The beam's power is proportional to the square of the field amplitude. To maximize the power, the square of the amplitude must be maximized.

$$E_\omega{}^2 \propto P_\omega = \frac{R_2 R_3 (1 - R_1) P_0}{(1 - \sqrt{R_1 R_2 R_3 R_4} \ell_c)^2} \tag{2.18}$$

The field loss due to SHG in the crystal is $\ell_c = \sqrt{1 - E_{nl} P_\omega}$ where E_{nl} is the effective nonlinearity, as defined in section 2.3. Substituding this result into equation 2.18 gives a implicit equation for P_ω:

$$P_\omega = \frac{R_2 R_3 (1 - R_1) P_0}{(1 - \sqrt{R_1 R_2 R_3 R_4 (1 - E_{nl} P_\omega)})^2} \tag{2.19}$$

From equation 2.5 it is evident that SHG efficiency is dependent on the input power into the crystal. Therefore, the reflectivities that maximize P_ω for a given input power, P_0, should be found. The closer R_2, R_3, and R_4 are to 1, the less power will be lost out those mirrors as the beam circulates, so those mirror reflectivities should be as high as possible. The input mirror is different, however. A higher value for R_1 reduces the losses in the circulating beam, but limits the light coming into the cavity. The value of R_1that maximizes the power (denoted R_{1max}) can be calculated by solving:

$$\left. \frac{dP_\omega}{dR_1} \right|_{R_{1max}} = 0 \tag{2.20}$$

Which gives:

$$R_{1max} = (1 - E_{nl} P_\omega) R_2 R_3 R_4 \tag{2.21}$$

Typical values for highly reflective mirrors are $R \approx 0.99$. Letting $R_2 = R_3 = R_4 = 0.99$ turns equations 2.19 and 2.21 into two implicit functions, for P_ω and R_{1max} respectively, in terms of P_0. For SHG in BBO, configured as specified in the previous sections, $E_{nl} = 1.1 \times 10^{-4}$. This value of E_{nl} indicates that the single pass conversion efficiency is low enough to neglect crystal losses when designing the cavity. We therefore use the approximation $1 - E_{nl}P_\omega \approx 1$. Making the approximation $1 - E_{nl}P_\omega \approx 1$ is equivalent to selecting $R_1 = R_{1max}(P_0 = 0)$ where $R_{1max}(P_0)$ is the optimal value of R_1 as a function of the input power. Figure 2.5 shows that $R_{1max}(P_0)$ is approximately constant over the range of powers we are interested in; therefore approximation doesn't significantly affect the chosen value of R_1. Making the approximation $1 - E_{nl}P_\omega \approx 1$ gives $R_{1max} \approx R_2R_3R_4 = 0.97$.

The relationship between P_ω and P_0 is approximately linear (Figure 2.6), and will be approximated by:

$$P_\omega = 30P_0 \tag{2.22}$$

This is a conservative estimate over the range $P_0 \epsilon [0,1]\ W$, an appropriate range for typical diode laser and amplifier output powers. A more accurate value of P_ω for a

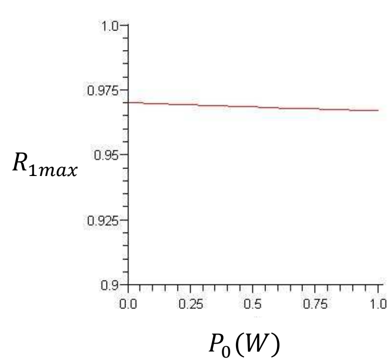

Figure 2.5: Optimum Input Reflectivity versus Input Power

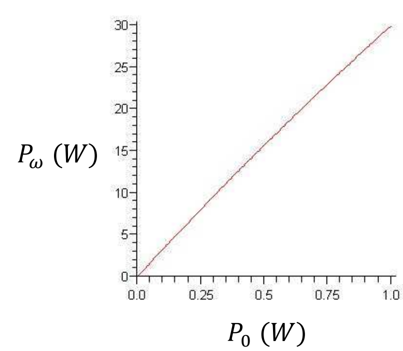

Figure 2.6: Power in Crystal versus Input Power

particular value of P_0 can be calculated from equation 2.19. From equation 2.22 it is evident that the power circulating in the cavity is significantly greater than the input power. This greatly increases the SHG efficiency.

2.4.2. Calculation of Mirror Focal Length

In section 2.5 it was shown that the optimal beam waist for doubling 846 nm light in a 10 mm BBO crystal is $23.8\ \mu m$. The radii of curvature of mirrors 3 and 4 can be chosen to provide this beam waist in the center of the crystal. SNLO [9] is a computer program designed to assist in cavity design. By inputting the wavelength, the crystal length and index of refraction, the cavity mirror curvatures, and the distances between mirrors, the program will calculate the beam waist in the crystal and assess the cavity's stability. By varying the distances between mirrors and their curvatures, a combination can be found that provides the desired beam waist[3] in the crystal. Using SNLO, it was determined that a $1/e^2$ beam waist of $23.8\ \mu m$ can be achieved using two mirrors with radii of curvature of 75 mm, with 103 mm of separation between mirrors 3 and 4, and a total cavity path length of 405 mm.

2.5. Second Harmonic Power

All the necessary parameters have been calculated to determine the second harmonic output power for the proposed frequency doubling scheme. Substituting the results from equations 2.9 and 2.22, along with the appropriate constants, into the equation for the second harmonic power (2.5) gives:

$$P_{2\omega} = 0.095 P_0^2\ W^{-1} \tag{2.23}$$

Where P_0 is the input laser power into the cavity.

3. Conclusion

423 nm laser light is an integral part of building a Calcium interferometer. In this thesis the optimum parameter values were calculated and used to determine the output power at 423 nm of a frequency-doubled 846 nm Gaussian beam. Phase matching, Poynting vector walk-off, and build-up cavity design were taken into account in the analysis. The values of the optimized parameters for a 10 mm long Beta Barium Borate crystal in a bow tie build up cavity were calculated to be the following: critical Phase matching angles $\theta = 27.6^{\circ}$ and $\phi = 0$, beam waist $w_0 = 23.8\ \mu m$, mirror radii of curvature of 75 mm, input mirror reflectivity of $R = 0.97$ (assuming R =0 .99 for the other three mirrors), curved mirror separation of 103 mm, and total cavity path length of 405 mm.

The second harmonic output power of this configuration was calculated to be $0.095P_0^2\ W$, where P_0 is the output power of the 846 nm laser in watts. To achieve the necessary 12 mW for use in the interferometer, an input laser power of 356 mW is necessary. This is well within the range of available 846 nm diode laser and amplifier setups, making a frequency-doubled 846 nm laser a feasible source of 423 nm light for a Calcium interferometer.

Notes

1. Superscripts on indices of refraction that contain ω indicate what frequency the index corresponds to, the fundamental or the first harmonic,.
2. This describes Type I (ooe) critical phase matching. Type I indicates that a first harmonic beam with a single polarization direction is used. This is opposed to Type II, in which the first harmonic is composed of two orthogonally polarized beams. ooe indicates the first harmonic is an ordinary ray, while the second harmonic is an extraordinary ray.
3. See *Physics of Light and Optics* [10] pp. 95-99 for a derivation.
4. SNLO uses a FWHM definition of beam waist, whereas in this thesis the $1/e^2$ definition is used. To translate FWHM beam waist to the $1/e^2$ beam waist, multiply the former by $1/\sqrt{2\ln 2}$.

Appendix

Appendix A: Beta Barium Borate Crystal Properties

1. Sellmeier parameters (room temperature):

	A	B (μm^2)	C (μm^{-2})	D (μm^{-2})
n_e	2.3730	0.0128	-0.0156	-0.0044
n_0	2.7405	0.0184	-0.0179	-0.0155

(Davis et al., 1987) [6]

2. Nonlinear coefficients:

Type I Critical Phase Matching (ooe): $d_{eff} = d_{15}\sin(\theta) - d_{22}\cos(\theta)\cos(3\phi)$

Type II Critical Phase Matching (eoe, oee): $d_{eff} = d_{22} cos\,(\theta)^2 cos(3\phi)$

$d_{15} = 0.08$ pm/V $d_{22} = 2.3$ (pm/V)

(Sutherland, 2003, p. 72, 296) [4], (Smith, 2007) [9]

3. Thermo-optic coefficients:

$$\frac{dn_o}{dT} = -16.6 \times 10^{-6}/\,^oC \qquad \frac{dn_e}{dT} = -9.3 \times 10^{-6}/\,^oC$$

(Davis et al., 1987) [6]

Appendix B: Matlab Program for Calculation of Optimal Beam Waist

```
clear; close; clc;
N=200; m=0; n=0;
rho=.0651;   %walk-off angle
no=1.659;    %ordinary index of refraction
lambda=846e-9;  %first harmonic wavelength
l=.01;   %crystal length
B=rho*sqrt(pi*no*l/2/lambda);      %value for B based on walk-off

zetamin=1; zetamax=4; zetastep=10^-2;   %range for zeta
j=ceil((zetamax-zetamin)/zetastep+1);

sigmamin=-.1; sigmamax=1; sigmastep=10^-2;   %range for sigma
k=ceil((sigmamax-sigmamin)/sigmastep+1);

Sigma=zeros(1,k); z=zeros(1,j); H=zeros(j,k); f=zeros(N,N);

for sigma=sigmamin:sigmastep:sigmamax     %step through sigma
   n=n+1;
   for zeta=zetamin:zetastep:zetamax      %step through zeta
      m=m+1;
      h=2*zeta/(N-1);
      t=-zeta:h:zeta;
      [T,T1]=meshgrid(t);
      f=exp(i*sigma*(T-T1)).*exp(-B^2*(T-T1).^2/zeta)./(1+i*T)./(1-i*T1);
                %integrand of h(sigma,B,zeta)
      H(m,n)=1/4/zeta*real(sum(sum(f))*h^2);   %numerically integrate to get
                                                %h(sigma,B,zeta)
      z(m)=zeta;
   end
   m=0;
   Sigma(n)=sigma;
end

[X,Y]=meshgrid(z,Sigma);

surf(X,Y,H.')
xlabel('\zeta')
ylabel('\sigma')
zlabel('h(\sigma,B,\zeta)')

[hm,m_max]=max(H);
[hmm,n_max]=max(max(H));

fprintf('max h')
```

```
max(max(H))  %maximum value of h(sigma, B, zeta)
fprintf('optimal zeta ')
z(m_max(n_max))
fprintf('optimal sigma')
Sigma(n_max)
fprintf('optimal w0')
sqrt(lambda*l/2/pi/no/z(m_max(n_max))) %value of beam waist corresponding to
optimal zeta

figure

loglog(z,max(H.')) %plot of h(sigma_max,B,zeta)
axis([zetamin zetamax min(min(H)) max(max(H))])
xlabel('\zeta')
ylabel('h')
title('h(\sigma, B, \zeta) optimized in \sigma')
```

Appendix C: Glossary

Anisotropy: Directional dependence

B: A parameter in the optimization of SHG that represents the walk-off angle. $\mathrm{B} = \rho\sqrt{\pi n_{\omega} l/2\lambda}$

Beam waist: The point in a Gaussian beam where the $1/e^2$ beam diameter is a minimum.

Build up cavity: An arrangement of mirrors that allows for a circulating beam power greater than the input beam power.

Critical Phase Matching: A process where the angle between the crystal's optical axis and the first harmonic wave is adjusted to achieve phase matching.

Effective nonlinear coefficient: A constant describing the strength of a given nonlinear process.

Effective nonlinearity (E_{nl}): The coefficient relating the output beam power to the square of the input beam power.

First harmonic beam: The input beam into the nonlinear crystal.

Frequency doubling: See second harmonic generation

Gaussian Beam: An electromagnetic wave who intensity profile in any plane perpendicular to the wave vector is a Gaussian distribution.

Index of refraction: The ratio of speed of light to wave speed of an electromagnetic wave in a material.

Interferometer: A device that splits and recombines a wave so that it travels to a detector through at least two different paths or "arms".

Noncritical Phase Matching: A process where the nonlinear crystal's temperature is adjusted to achieve phase matching.

Optical Axis: The direction in a uniaxial crystal for which all polarizations have the same index of refraction, n_o. Typically denoted the z-axis.

$P_{2\omega}$: The power in the second harmonic beam exiting the nonlinear crystal.

P_{ω}: The power in the first harmonic beam entering the nonlinear crystal.

Phase matching: Any process that allows the first and second harmonic beams to remain in phase.

Poynting vector: A vector indicating the direction of energy propagation of an electromagnetic wave.

Poynting vector walk-off: A phenomenon in birefringent materials where the Poynting vector of the first harmonic is not parallel to the Poynting vector of the second harmonic.

Second harmonic: The output beam from the nonlinear crystal with twice the input frequency.

Second harmonic generation (SHG): A process whereby the power in an input electromagnetic wave is transferred to another electromagnetic wave at twice the input frequency. Also known as frequency doubling.

σ: A parameter in the optimization of SHG that represents the phase matching of the beams. $\sigma = n_\omega \pi w_0^2 \Delta k/\lambda$ where Δk is the phase mismatch, given by $\Delta k = |2k_1 - k_2|$.

Thermo-optic coefficient: The first derivative of the index of refraction with respect to temperature. Usually given at room temperature.

Uniaxial (birefringent): A material in which two of the three principle indices of refraction are equal.

Wave vector: A vector indicating the direction of wave propagation with magnitude equal to the wave number.

ξ: A parameter in the optimization of SHG that represents the beam waist of the first harmonic. $\xi = \lambda l/(2\pi n_\omega w_0^2)$

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