Teaching the Practical Principles of Fourier Optics

in an Undergraduate Laboratory

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in partial fulfillment of the requirements for the degree of

Bachelor of Science

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ABSTRACT

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With an evolving world, pressured by events such as the outbreak of COVID-19, educational laboratory courses need to evolve to fit a more online-based teaching regimen. Presented here is an affordable and portable teaching laboratory setup that will provide students with hands-on experience with diffraction patterns in the far-field region of space, digital signal processing, sampling and signal-to-noise using a double pinhole diffraction apparatus. The inexpensive ($150) optical setup includes a laser, a single pinhole for spatial filtering, a double pinhole, a camera, and a pair of lenses, all attached to a 1 meter long optical rail. This simple apparatus allows for easy packaging and shipping. To capture and analyze the images, the free software ImageJ is used. Though aimed for undergraduate university laboratory classes, this laboratory can easily be adjusted to accommodate pre-secondary and secondary school classes.

Keywords: [Fourier Optics, Undergraduate, Secondary, Pre-Secondary, Laboratory Procedures, ImageJ, Diffraction Patterns]

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I would also like thank Dr. Sandberg and Nick Porter for their mentoring and help over the past year.

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Introduction

1.1 Background and Motivation

Classes have been slowly taking a more online approach for many years, but COVID-19 accelerated the transition. To meet with the ever-changing demands of education, I helped with the development of a portable and inexpensive lab that can be adjusted to meet the needs of students from middle school through college. It is a simple and portable experiment that allows students to explore the basic principles of coherent imaging and Fourier optics in a hands-on setting. Students will attempt to capture the diffraction pattern created from a double pinhole aperture and then use the diffraction pattern to determine the spacing and diameter of the aperture. The simple apparatus consists of a laser, double pinhole, pair of lenses, and a camera, all aligned on a basic optical rail as shown in Figure 1.1. The low cost and high portability of the apparatus makes it ideal for a take-home-lab, as part of a remote or virtual outreach effort, or used at schools with lower budgets. The portability of this lab was first tested during the fall semester of 2020 on the Physics 245 class when classes were completely remote after Thanksgiving break at Brigham Young University. Students were sent home with the kit and interacted with professors and teaching assistants via

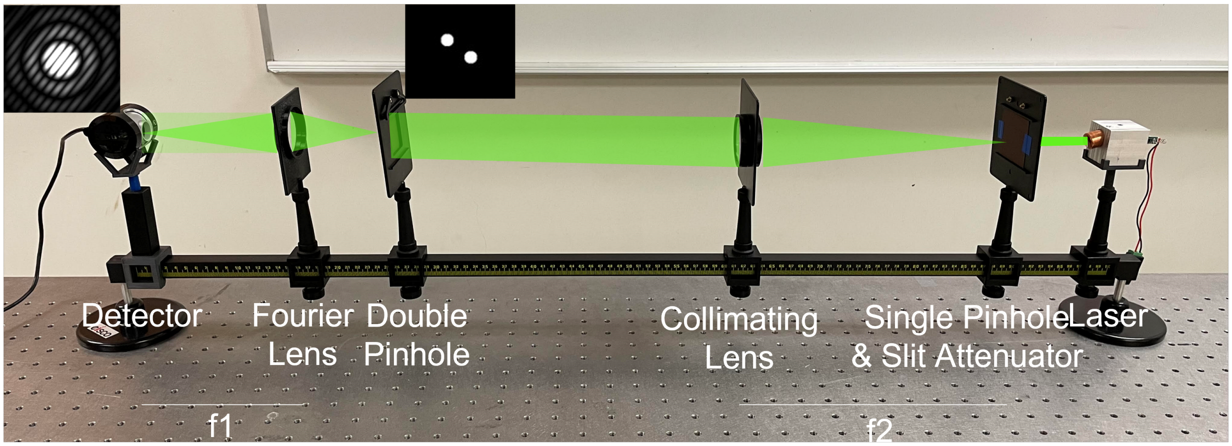


Figure 1.1 Labeled experimental setup, green color is to depict laser trajectory. Also included are additions of the focal lengths f1 and f2 and images resembling the double pinhole aperture and the desired image on the detector.

Zoom at Brigham Young University after the Thanksgiving Holiday.

Physics students should get a wide range of exposure and practice in many different fields while pursuing their education. In an undergraduate's first-year classes, they get taught the concept of diffraction patterns and have labs that provide them with the opportunity to produce these patterns using a double slit pattern and a laser. Largely these experiments do not delve into the advanced analysis of diffraction imaging and do not prompt students to research diffraction imaging further. [7]

Diffraction imaging is an important topic that spans across multiple educational fields. Biologists use macro-molecular crystallography, a diffraction-based technique similar to coherent diffraction imaging to obtain he structure of proteins and use phase retrieval programs to reconstruct the image [1]. In physics, diffraction imaging is used in x-ray crystallography [4]. Each of the aforementioned applications require varying proficiency of software and equipment, but each need a similar, solid understanding of imaging and image processing.

With this laboratory, we hope to spark the interest of students in this ever-growing field of research.

1.2 Principles

This laboratory requires an understanding of three important principles: a Fourier transform model of diffraction, dynamic range and noise reduction. As this lab is meant to reach many different levels of students, only the basics of each of these principles will be reviewed.

1.2.1 Fourier Optics

A diffraction pattern forms when coherent light passes through an aperture or as it passes around an object. When light reaches the far-field, the electric field can be approximated using the Fraunhofer equation:

$$\mathbf{E}(\mathbf{r},z) \approx -\frac{ike^{ikz}e^{i\frac{k}{2z}\mathbf{r}^2}}{2\pi z}\int \mathbf{E}(\mathbf{r}',0)e^{-i\frac{k}{z}(\mathbf{r}\cdot\mathbf{r}')}\,d\mathbf{r}', \tag{1.1}$$

where $\mathbf{r}$ and $\mathbf{r}'$ are respectively the positions in the diffraction and aperture plane, z is the propagation distance from the diffracting object, $k = \frac{2\pi}{\lambda}$ is the wavenumber, and $E(\mathbf{r}',0)$ is the electric field immediately after passing through the aperture which is at times referred to as the exit wave [5].

This approximation can further be simplified. When the light is coherent, the exit wave is simply the aperture function which will be called $\mathbf{A}(\mathbf{r}')$.

$$\mathbf{A}(\mathbf{r}') = \mathbf{E}(\mathbf{r}',0) \tag{1.2}$$

Second, imaging detectors do not measure the electric field, but instead measure the intensity as it is received by the detector. This means that ($\mathbf{E}$) can be written as the norm squared ($|\mathbf{E}^2|$) which will get rid of some of the more complicated elements of the equation:

$$|\mathbf{E}(\mathbf{r},z)|^2 \approx |\frac{k}{2\pi z}\int \mathbf{A}(\mathbf{r}')e^{-i\frac{k}{z}\mathbf{r}\cdot\mathbf{r}'}\,d\mathbf{r}'|^2. \tag{1.3}$$

The final simplification is applied by combining the Fourier transform integral

$$\mathscr{F}[f(u)]_{u\rightarrow v} = \frac{1}{\sqrt{2\pi}}\int f(u)e^{-iuv}\,du, \tag{1.4}$$

with the diffraction intensity as

$$|\mathbf{E}(\mathbf{r},z)|^2 \approx |\frac{k}{z}\mathscr{F}[\mathbf{A}(\mathbf{r}')]_{\mathbf{r}'\to\frac{k}{z}\mathbf{r}}|^2. \tag{1.5}$$

This order of equations shows that the intensity of the far-field diffraction field is equivalent to the norm square of the Fourier transform of the aperture function. Or in other words, given the amplitude (simply the square root of the intensity) and phase at each point in the diffraction pattern, one can perform an inverse Fourier transform and attempt to retrieve the aperture or object from which the light was diffracted.

When a lens is introduced after the aperture, then the diffraction pattern in the lens' back focal plane is proportional to a Fourier transform of the aperture. [2] This principle is important because for many aperture sizes and for visible light, the far field can be multiple meters away which would require a large experimental setup. This would also lead to a very large diffraction image which would be hard to fit on a common image detector such as a CCD.

From this relationship, we can derive the equations for double and single aperture diffraction:

$$2\Delta \approx \frac{f_1 * \lambda}{s}, \tag{1.6}$$

where 2Δ is the distance between two slits, s is the spacing between two adjacent fringes, λ is the wavelength of light, and f_1 is the distance from the lens to the detector, and

$$W_{\text{Airy}} \approx \frac{1.22 * f_1 * \lambda}{w}, \tag{1.7}$$

where W_{Airy} is the diameter of the first-order fringes (the inner-most ring) and w is the diameter or width of the slit (as shown in Figure 1.2.

For the full derivation and explanation of these principles refer to references: [2, 11].

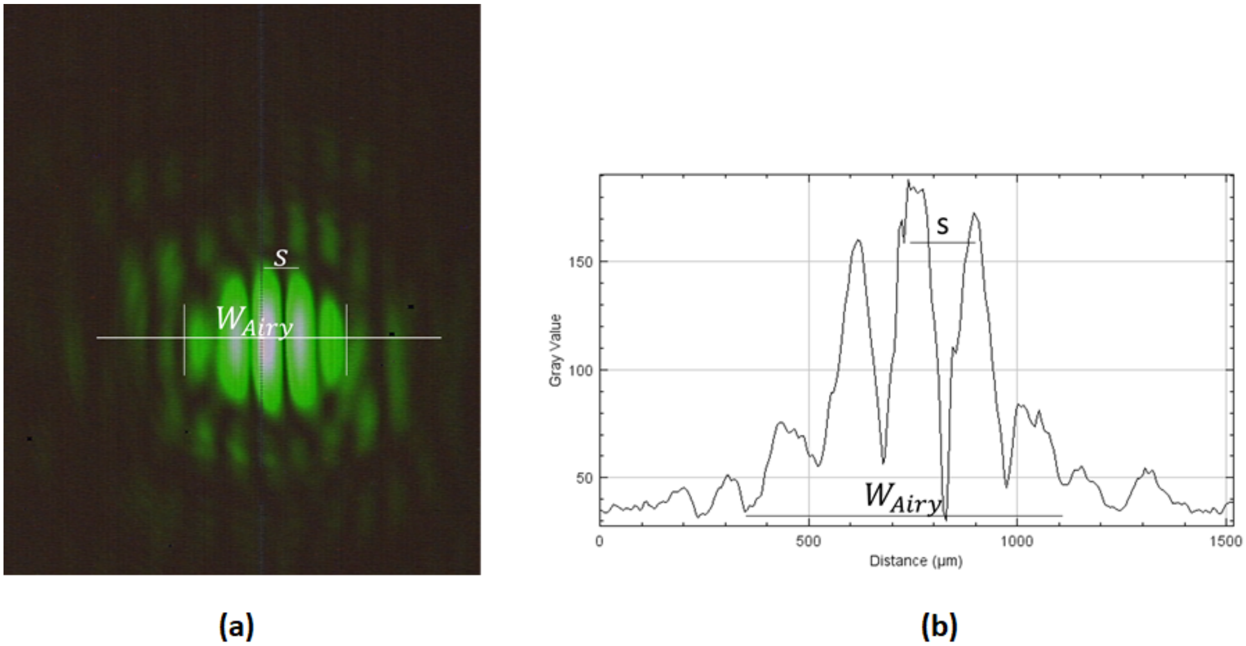


Figure 1.2 The overlay of what was measured as the Airy disc (W_{Airy}) and fringe spacing (s) as seen on my diffraction pattern (Figure 3.3a) and the respective lineout (Figure 3.3b). Exact measurements were of the distances were made using ImageJ.

1.2.2 Measuring Techniques

The above equations can only be used on a diffraction pattern if said diffraction pattern is reliable. In order to produce a reliable diffraction pattern, students need to understand the principles of dynamic range and noise reduction and how they relate to a diffraction patterns both quantitatively and qualitatively.

Dynamic Range

All image detectors have a maximum and minimum intensity that can be read, and the ratio of these two values is known as the dynamic range. A common topic when referring to dynamic range is saturation. Saturation is when the intensity of the signal is higher than what can be read by the

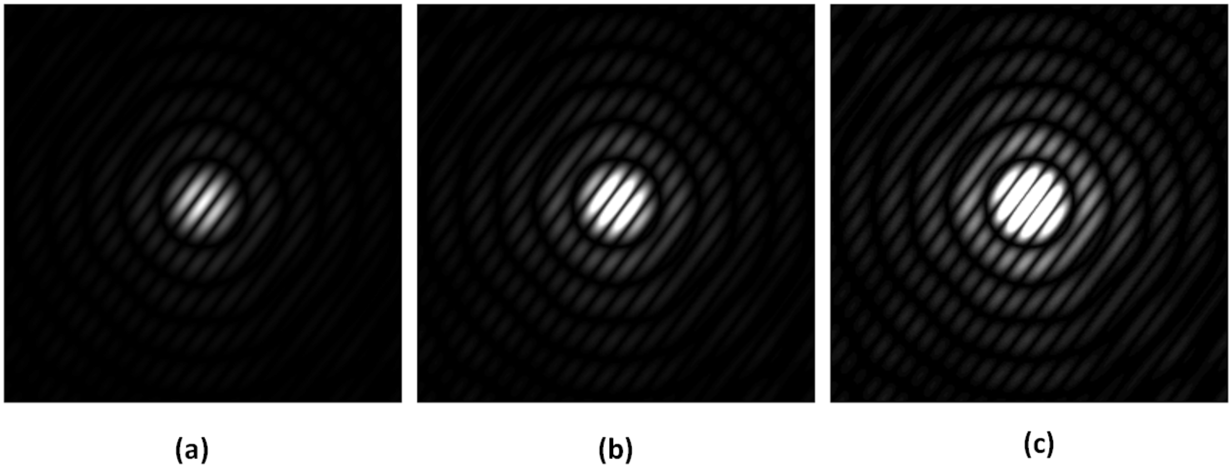


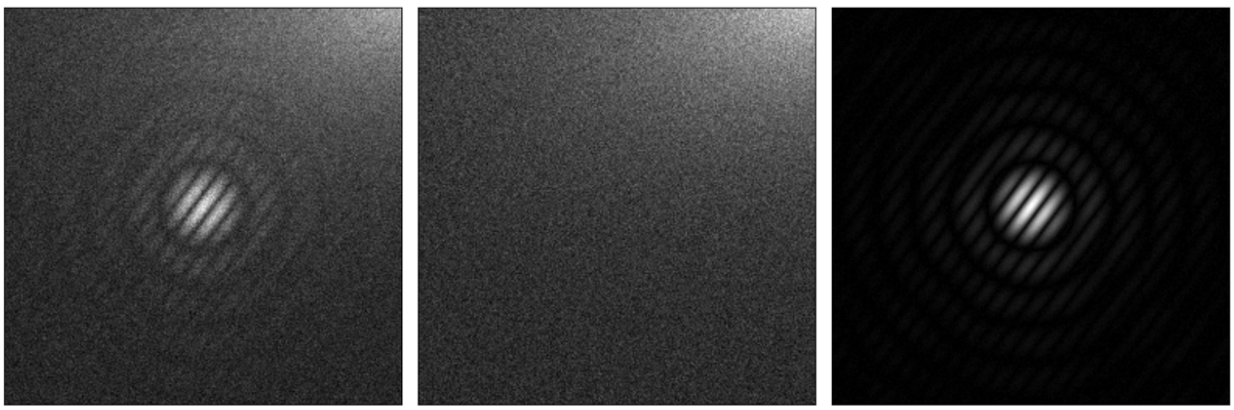
Figure 1.3 Simulation showing the effects of saturation levels on a diffraction image. The saturation increases from left to right and with it, dimmer higher-order fringes come into view. Simulations courtesy of Nick Porter.

detector. Saturation can be avoided by a number of different physical methods including the use of: LEE filters, a slit attenuator, or a potentionmeter. Another option for controlling the intensity of the signal is to decrease the exposure of the detector or to adjust the gain, though these options are generally reserved for more expensive detectors.

In general, saturation is considered bad when it comes to imaging as it decreases the reliability of the captured signal and can even damage the detector. Figure 1.3 shows the simulated benefits of increasing the overall intensity of a diffraction image despite saturating the Airy disc. The higher overall intensity brings out the dimmer features of the higher-order fringes of the diffraction pattern.

Noise Reduction

Noise reduction is critical to coherent imaging because it increases the signal-to-noise ratio and therefore increases the reliability of a captured diffraction pattern. There are two primary methods involved in noise reduction, background subtraction and image summation. Background subtraction is the process of subtracting the constant noise captured by a detector at any point in time. This is done by capturing an image where the detector is not receiving the diffraction signal and then subtracting it from the captured signal. Image summation reduces the external noise captured by a detector between signal capture. This external noise can include a fluctuation in light intensity, dust falling in front of the detector or a slight bump of the table. Figure 1.4 demonstrates the effectiveness of the two aforementioned processes.



(a) Raw Image **(b)** Background **(c)** Analyzed Image

Figure 1.4 Simulated data showing the effects of background subtraction and image summation. Image *a* depicts the raw image with noise inclusion. Image *b* is the background which includes the noise and background signal. Image *c* is the result of subtracting the average of Image *b* from Image *a* then summing those images. Simulation courtesy of Nick Porter

Chapter 2

Methods

2.1 Equipment and Software

The assignment for the student is to produce a diffraction image from a double pinhole aperture and use the image to calculate the diameter and spacing of the pinholes. This is done using the apparatus in Figure 1.1 which was adapted from reference [3]. The optical rail can be purchased from an Eisco Labs kit which also comes with the Fourier lens, collimating lens, a lens mount, and the sample holders. Mounts were 3D printed for the laser and detector along with a second lens mount. The laser was purchased online and is a 532 nm diode mounted in an aluminum cube with an external IR filter.

The experiment requires a double and a single pinhole aperture. These apertures can be made by piercing a dark, thick piece of paper with a thin needle. One constraint on the size and spacing of the pinholes is that they must be small and close enough to allow for Nyquist sampling. To follow this constraint, the fringes must be larger than two pixels wide on the detector. Note should be made that the largest constraint on the double pinhole aperture is the beam size. The collimated light should be larger than the pinhole spacing. Students would also benefit from having LEE filters or a slit attenuator, in our experiment a rudimentary slit attenuator was created, to control the intensity of the laser. Filters and attenuators should be placed before the first pinhole. In our experiment, we had a laser module with adjustable power via a screw that could be tightened or loosened.

The placement of the pinhole apertures is rather arbitrary, though it was found to produce higher quality images when the double pinhole aperture was placed closer to the Fourier lens. Students were left to determine the focal lengths of the collimating and Fourier lenses. Using the focal lengths, the collimating lens is placed one focal length away from the single pinhole aperture and the Fourier lens should be placed one focal length from the detector. At one focal length, the Fourier lens produces an image in the Fourier plane of the detector.

The image detector can be any digital camera barring that the bit depth and pixel size are known, the lens can be removed, and an uncompressed image file can be exported. Data provided for this paper was obtained using the Philips NPC900 webcam, as depicted in Figure 1.1, though success was also found in using the Logitech C500 webcam.

For the experiment we used, and recommend using, the free software ImageJ developed by National Institute of Health and Webcam Capture plugin developed by Jerome Mutterer [9, 10]. Students should download ImageJ and not Fiji as the Webcam Capture plugin does not work with Fiji. The Webcam Capture plugin allows for images to be taken with and uploaded directly into ImageJ rather than having to import the images from a secondary program. The number of images and time between images taken can be adjusted using the plugin. The downside of using this plugin is that it does not allow for changes in exposure or gain, meaning all controls on saturation must be done via physical means. ImageJ allows for the writing of macros which can be created by professors and sent out to students as a means to help with the analyzing process.

In total, the experimental setup used in capturing the data in this paper was less than \$150, refer to Table 2.1 for a full breakdown of equipment pricing. All pieces of equipment can be substituted for higher quality pieces if an institution has access to such.

Equipment Price	
Eisco Optical Rail	$60.00
Philips NPC900 Webcam	$10.00
or Logitech C500 Webcam	$10.00
Extra Fine Pin Needles	$3.50
Dark Construction Paper	$2.00
Fourier and Collimating Lenses	$40.00
532 nm Green Laser	$8.00
Aluminum Box and Infrared Filter	$10.00
3D Printed Mounts	<$1.00
Total Cost ≈ $ 144.00	

Table 2.1 The prices included in this table are all based on when the equipment was bought in the fall of 2020.

2.2 Procedure

The steps outlined in this procedure are done with the idea that this lab will be performed in-home, though they can be easily changed to fit for an in-lab environment.

Emphasis should be made on the need for safety when handling lasers and safety goggles should be provided. The lasers used in this laboratory were either laser pointers (no safety goggles needed if the power is less than 5 mW) or commercial green laser pointer modules (20 mW of power, safety goggles recommended). It is recommended that students go over the IEC Laser Classification Chart and watch any laser safety video that the instructor recommends. Students should also understand that laser power is not always labeled correctly (the inexpensive green laser used for the experiment emitted infrared light requiring an infrared filter). To protect others, before beginning the experiment all windows should be covered with blinds or drapes and make sure that a sign is placed on the outside of the door notifying of the use of a laser. The laser used in this experiment is a Class 3B laser and thus proper eye ware must be worn at all times the laser is on.

ImageJ is free and fairly easy to download, but students should make sure that it and all necessary drivers are installed before taking the experiment home. Instructors should understand how to install ImageJ and its plugins on multiple operating systems in order to trouble shoot issues that students run into. This will allow the experiment to run smoother once taken home.

Once all these elements are gathered, the optical rail then should be put together as shown in Figure 1.1. The laser will be placed at one end of the optical rail. Any LEE filters and/or slit attenuators should be placed in the same holder with the single pinhole aperture which should be placed close to the laser. LEE filters are absorbing thin film attenuators available at [8]. Following the single pinhole, the collimating lens will be placed one focal length away. Next the double pinhole aperture should be placed on the rail (students will need to experiment with the best location for it in reference to the lenses on either side). The Fourier lens should then be placed exactly one focal length away from the detector which will finally be placed at the end of the optical rail. The only distances that are essential are that the collimating lens be placed one focal length after the single pinhole and that the Fourier transforming lens be placed one focal length before the detector. Students are encouraged to experiment and play with the distances of the other pieces of the optical setup.

It is best to place any LEE filters and attenuators before turning on the laser and then slowly take them away to increase to the desired intensity to avoid damaging the detector with the direct laser light. Because the mounts are not all the same it can be difficult to align the detector and laser and students may have to get creative in order to align them. The quality of the lenses must also be taken into account as they can slightly shift the direction of the laser. We found that it was easiest to align the laser to the detector with the lenses in place and then add in the apertures afterwards. Note, the laser will not hit the center of the pinhole apertures and pinholes will need to be placed accordingly.

Once the equipment is set up, students will strive to capture the most intense diffraction image possible. The first concern with this will be saturation. LEE filters or a slit attenuator can be used to prevent saturation of the detector. As the intensity measured by the detector increases, the fringes of the Airy disc will begin to merge together, while higher-order fringes come more into view. Students will need to experiment with saturation to produce the clearest image they can.

Noise will then need to be addressed. The background of a detector is the inherent noise of an image and will be found in all images taken with said detector. To eliminate the inherent noise, we advice to take 10-100 background images and 10-100 diffraction pattern images. Then use ImageJ to subtract the average of the background from each of the individual diffraction pattern images. After getting rid of the inherent noise, sum all the diffraction images together which will get rid of any external noise. These processes help achieve a greater signal-to-noise ratio and strengthen the signal of the higher-order fringes of the diffraction pattern.

Chapter 3

Results and Discussion

3.1 Experimental Results

To test the benefits of this lab I, an undergraduate physics student with minimal coherent imaging experience, started with a double pinhole aperture of unknown spacing and diameter and a single pinhole aperture with a rudimentary slit attenuator fitted (Figure 3.1). I put together the experimental setup as explained in Section 2.2 and shown in Figure 1.1. The green light laser was assumed to output its stated wavelength of λ = 532 nm, and I measured the focal length of the Fourier lens to be $f_1 = 27.2 \pm .1$ cm.

Using the double pinhole aperture, I produced the diffraction pattern shown in Figure 3.2, which I have compared to simulated data of similar dimensions. From the diffraction pattern I measured the Airy disc to have a $W_{\text{Airy}} = 761 \pm 1$ μm and a fringe space: $s = 146 \pm 1$ μm (Figure 1.2). Plugging these values into Equation 1.7 and Equation 1.6, I was able to determine that the pinhole diameter $w = 0.47 \pm 0.1$ mm and a pinhole separation $2\Delta = 0.90 \pm 0.1$ mm. These calculations were later verified when I checked the pinhole aperture using an imaging microscope where I got that the actual pinhole diameter was 0.52 mm and the pinhole spacing was 1.02 mm. Though different, the

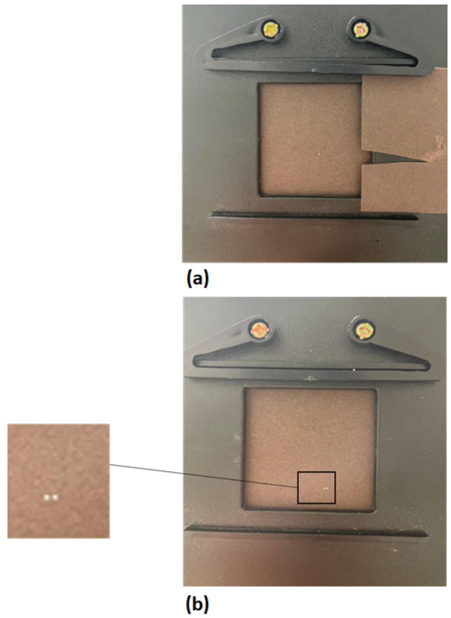
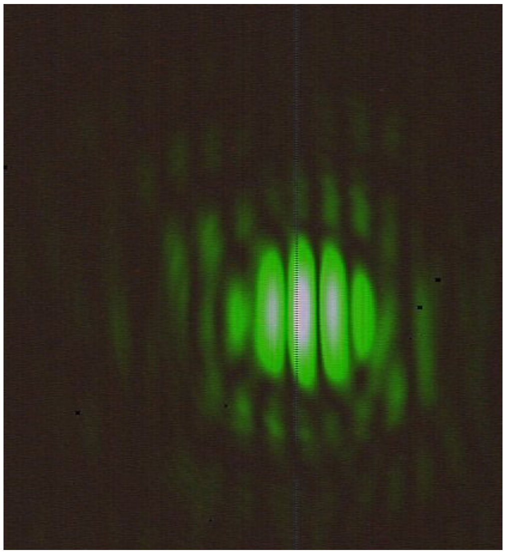


Figure 3.1 Figrue 3.1a shows the single pinhole aperture with rudimentary slit attenuator and Figure 3.1b shows the double pinhole aperture.

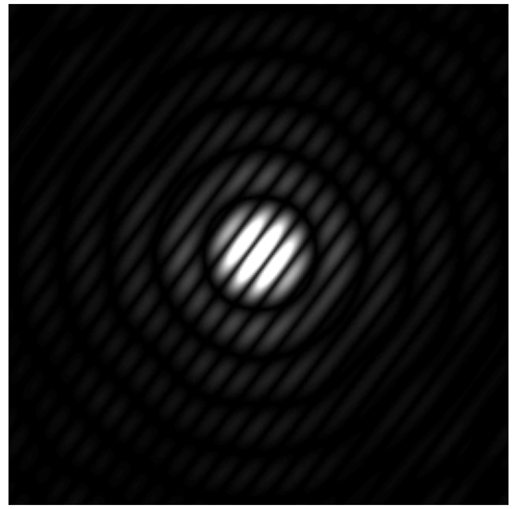
calculated values were very close to the measured values and had similar ratios.

To produce the image shown in Figure 3.2, I had to experiment with the correct plotted intensity range. To adjust the power of the laser in accordance with the saturation, I situated the slit attenuator and adjusted the power of the laser using the built-on potentiometer. Figure 3.3 shows the differences in saturation and how it affected the Airy disc and the higher-order fringes surrounding the Airy disc.

Along with correctly measuring the pinhole spacing and diameter, I was able to learn and apply noise reduction in order to isolate and strengthen my image. The lineout and images representing the application of noise reduction are shown in Figure 3.4.

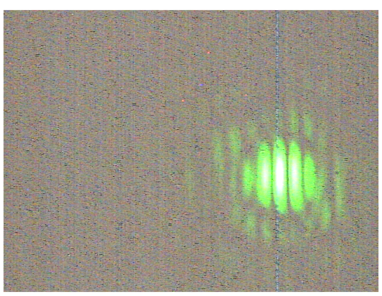


(a) Actual Diffraction Pattern

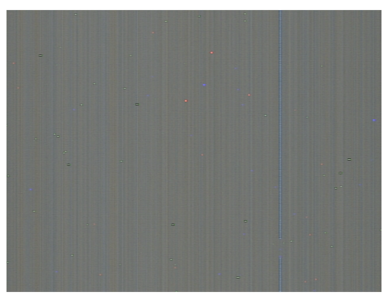


(b) Simulated Diffraction Pattern

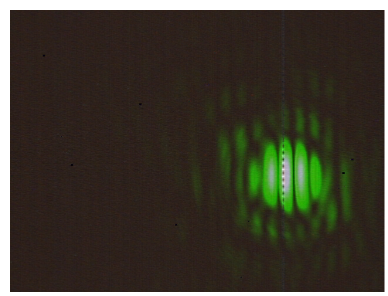
Figure 3.2 The diffraction image we were able to produce (Figure 3.2a) compared to a simulated diffraction image using similar parameters (Figure 3.2b)



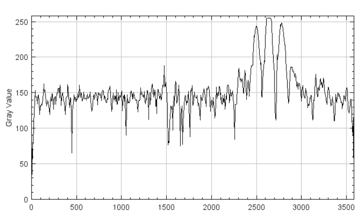
(a) Raw Image



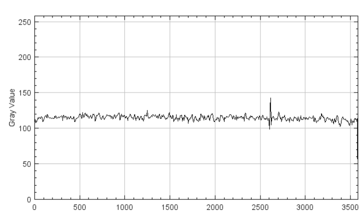
(b) Background



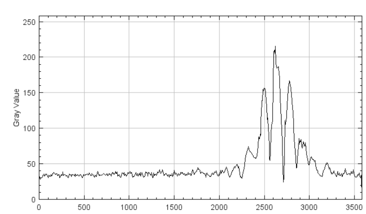
(c) Analyzed Image



(d) Raw Lineout

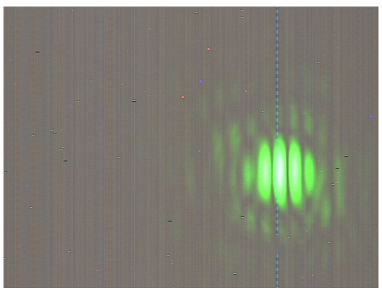


(e) Background Lineout

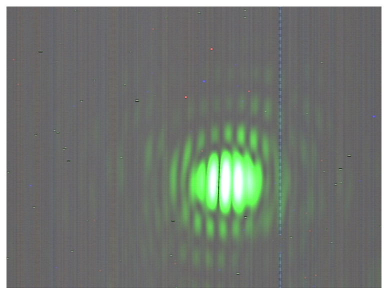


(f) Analyzed Lineout

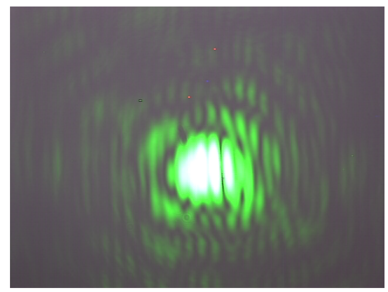
Figure 3.4 Figure 3.4c was created by subtracting Figure 3.4b from Figure 3.4a and then summing those images. This increases the signal-to-noise ratio of <1 in Figure 3.4e to a ratio of almost 5 in Figure 3.4f.



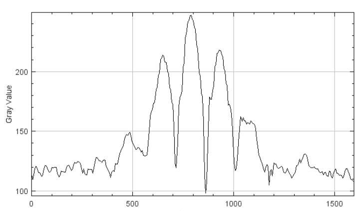
(a) Low Intensity Pattern



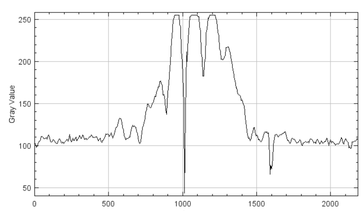
(b) Medium Intensity Pattern



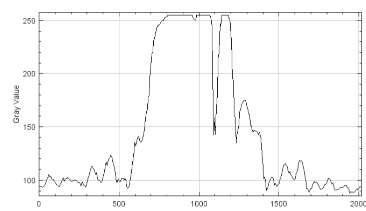
(c) High Intensity Pattern



(d) Low Intensity Lineout



(e) Medium Intensity Lineout



(f) High Intensity Lineout

Figure 3.3 Figure 3.3a depicts an unsaturated Airy disc with Figure 3.3d showing a max intensity less than 255 counts. The second order fringes are barely visible. Figure 3.3b depicts where the detector has become saturated and the Airy disc intensity has been capped at 255 counts (Figure 3.3e. Figure 3.3c shows up to fourth-order fringes and the lineout (Figure 3.3f) shows that the frignes of the Airy disc can not be distinguished.

3.2 Conclusion

The portability and low price of this lab means that a larger pool of students can have the opportunity to experiment with coherent imaging and Fourier analysis in a way that is enjoyable and applicable to their future endeavors. This laboratory can be built upon and adjusted to meet the needs of both professors and students. For example, iterative phase retrieval ([6]) can be implemented to provide students with more coherent diffraction imaging exposure. Or, if desired, image processing can be done through Python, or the creation of simulated images can be taught to help students learn programming skills.

Overall, this laboratory is an unique experience for students. It provides practical hands-on applications of coherent imaging and Fourier analysis which can prove important for future careers in research and industry.

Appendix A

Terminology

Background - the background is the noise that is always present when taking images. This noise comes from the heat of the camera, or defects in the sensor. Can be measured by capturing an image in darkness.

Bit Depth - based on the dynamic range of the camera an 8-bit camera would have a dynamic range of 256 which can be found by $2^8 = 256$.

Diffraction Pattern - interference pattern as a result of light waves spreading out after passing through a narrow opening

Dynamic Range - determined by Eq 2, it is the amount of steps available between the maximum and minimum counts allowed to each pixel.

Exposure - the amount of time in which a detector captures light

Fourier Image - representation of the object or slit in the reciprocal or frequency domain

Nyquist Sampling - sampling rate required for the image to accurate. Must have 2 pixels per fringe

Rose Criterion - Albert Rose determined that if the SNR is greater than 5 then the image will almost always be detectable.

Saturation - is the intensity, or the number of counts, of a particular image. An image is considered saturated when the intensity is higher than can be measured by the detector

Signal-to-Noise Ratio (SNR) - ratio of the signal to the background noise found in an image and can be determined using Eq 1.

Bibliography

[1] A. McPherson, "Introduction to protein crystallization", Methods **34**, Macromolecular Crystallization, 254–265 (2004).

[2] J. W. Goodman, *Introduction to Fourier Optics* (Roberts & Company Publishers, 2005).

[3] P. Thibault and I. C. Rankenburg, "Optical diffraction microscopy in a teaching laboratory", Am. J. Phys. **75**, 827–832 (2007).

[4] D. A. Keen, "Crystallography and physics", Physica Scripta **89**, 128003 (2014).

[5] M. W. Justin Peatross, *Physics of light and optics* (Justin Peatross and Michael Ware, 2015).

[6] G. Situ, J. Suo, and Q. Dai, "Generalized iterative phase retrieval algorithms and their applications", in 2015 ieee 13th international conference on industrial informatics (indin) (2015), pp. 713–720.

[7] *Phys21: Preparing Physics Students for 21st-Century Careers*, in collab. with P. Heron, L. McNeil, D. Arion, W. Buell, S. J. Gates, S. Giri, E. McCormack, H. Quinn, Q. Williams, and L. Woolf (American Physical Society, 2016).

[8] L. Filters, "Masters of light", https://www.leefilters.com/ (Accessed April 8, 2022).

[9] N. I. of Health, "Imagej", https://imagej.nih.gov/ij/download.html (Accessed April 8, 2022).

[10] J. Mutterer, "Webcam capture", https://imagej.nih.gov/ij/plugins/webcam-capture/index.html (May, 2016).

[11]W. F. Schlotter, "Lensless fourier transform holography with soft x-rays", Ph.D. dissertation (Stanford University, Stanford, California, 2007).