

H α color index observations of open-star clusters

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Abstract

Joner & Hintz (2015) presented a mono-wavelength color index centered on the $H\alpha$ spectral line designed to be a companion to Crawford's $H\beta$ color index (1960). I confirm that the standard index values obtained spectrophotometrically by Joner & Hintz can be recovered photometrically. I present $H\alpha$ color index values for 25 stars in the field of NGC 752. Additionally, I rule out two possible explanations for a second population of stars adjacent to the main sequence in the $H\alpha$ color magnitude diagrams of H & Chi Persei.

Keywords: $H\alpha$ index, Star Clusters and Associations, H & Chi Persei, ngc752

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Introduction

Stars can be modeled as blackbodies. Therefore they emit light at pretty much all wavelengths, but emit most of their light in a smaller range. It thus makes sense that astronomers study starlight using photometric filters that filter out all but a certain range of wavelengths of light. Comparing the brightness of stars measured using different filters by taking the difference in brightness is another useful practice astronomers have used for years to determine things like the color of a star (and all the properties you can figure out from a star's color). This is why this difference in brightness between two filters is known as a color index. While a color index normally uses filters of two different wavelength bandpasses, astronomers also use color indices of filters centered on the same wavelength, with one filter's bandpass wider than the others. These mono-wavelength color indices are typically centered on the wavelength of an important spectral feature. Centering these filters on a particular spectral feature allows for a measure of the equivalent width of that spectral feature. Additionally, because both atmospheric and interstellar extinction are dependent on wavelength and because both filters in these color indices are centered on the same wavelength, the effects of extinction are canceled out when we take the difference in brightness of the two filters in the color index. Using these filters thus provides both an observational advantage and additional insight of astrophysical interest.

Probably the most commonly used mono-wavelength color index is the $H\beta$ color index standardized by Crawford (1960).¹ This color index uses two filters, one with a wider bandpass than the other, centered on the second Balmer, or $H\beta$, transition. This color index has been used in conjunction with other filters to study the night sky for many years, such as some of the early studies Crawford and collaborators did on nearby open star clusters such as the Hyades (Crawford & Perry (1966)),² the Coma cluster (Crawford & Barnes (1969)),³ and the Pleiades (Crawford & Perry (1976)).⁴ With how useful the $H\beta$ color index has proven over the years, Jones & Hintz (2015) published 136 standard color index values for an $H\alpha$ color index, including standard values for stars in many common star clusters like the Hyades, the Pleiades, the Coma cluster, and NGC 752. As is typical with mono-wavelength color indices centered on a particular spectral feature, they also published relationships that users of the $H\alpha$ color index can use to find the equivalent width of the $H\alpha$

1 D. Crawford, "Early-type stars used as standards in photoelectric hbeta photometry," 1960.

2 D. Crawford *et al.*, "Four-colour and h-beta photometry of open clusters. i. the hyades," 1966.

3 D. Crawford *et al.*, "Four-color and h-beta photometry of open clusters. ii. coma and ursa major," 1969.

4 D. Crawford *et al.*, "Four-color and h-beta photometry for open clusters. xi-the pleiades," 1976.

spectral line and the effective temperature of the observed star for main sequence stars of a range of spectral types. As an $H\alpha$ equivalent and partner to the classic $H\beta$ index, this photometric system has potential to be very useful in astrophysical studies.

It is thus the aim of my research to better establish this $H\alpha$ color index and use it to observe objects of astrophysical interest. My research can be split into two parts: better establishing the $H\alpha$ color index through observations of NGC 752 and investigating the color-magnitude diagrams that use the $H\alpha$ color index through observations of H & Chi Persei (or NGC 869 and NGC 884, respectively).

The $H\alpha$ color index values published by Joneer & Hintz (2015)⁵ were obtained using spectrophotometry: the practice of obtaining brightness values in different filter bandpasses by convolving them over spectra rather than observing the object with said filters. It is important to confirm that these same standard color index values can be recovered photometrically, or in other words, when the filters are actually used to observe the night sky. In confirming this, the claim that these filters are not subject to atmospheric or interstellar extinction will also be tested photometrically. NGC 752 was one of the open star clusters that Joneer & Hintz used for standard stars, so observations of NGC 752 using the $H\alpha$ color index will be used to confirm that the system works photometrically. Because I am already using observations of NGC 752, I will also be seeking to establish more standard stars in the field.

Figure 1.1 and Figure 1.2 come from Andrew Hernandez's senior thesis at Brigham Young University (BYU). Hernandez found that when he used the $H\alpha$ color index to create a color-magnitude diagram of H & Chi Persei (NGC 869 and NGC 884, respectively), two populations of stars appeared. Normally in such young clusters as H & Chi Persei (Kharchenko et al. (2013)),⁶ we would expect to only see a single population, the main sequence. As we can see in Figures 1.1 and 1.2, however, there are two.

The other portion of my research is thus to investigate this serendipitous phenomenon. There are many possibilities as to what this second population of stars adjacent to the main sequence could be. In this thesis, I present two possibilities that I tested: background contamination from field stars and large error bars.

The following sections of this thesis will be as follows. In section 2 I will detail the observations done to take the data that I used for the research in this thesis. In section 3 I will detail the process of calibration and photometric measurements made to properly prepare the data for scientific analysis. In section 4 I discuss my findings for the first part of my research, better establishing the $H\alpha$ color index and establishing more standard stars. In section 5 I discuss my findings for the second part of my research, the tests I ran to investigate the $H\alpha$ color-magnitude diagram for H & Chi Persei. Section 6 is the conclusion of this thesis.

⁵ M. D. Joneer *et al.*, "Standard Stars and Empirical Calibrations for $H\alpha$ and $H\beta$ Photometry," 2015.

⁶ N. Kharchenko *et al.*, "Global survey of star clusters in the milky way-ii. the catalogue of basic parameters," 2013.

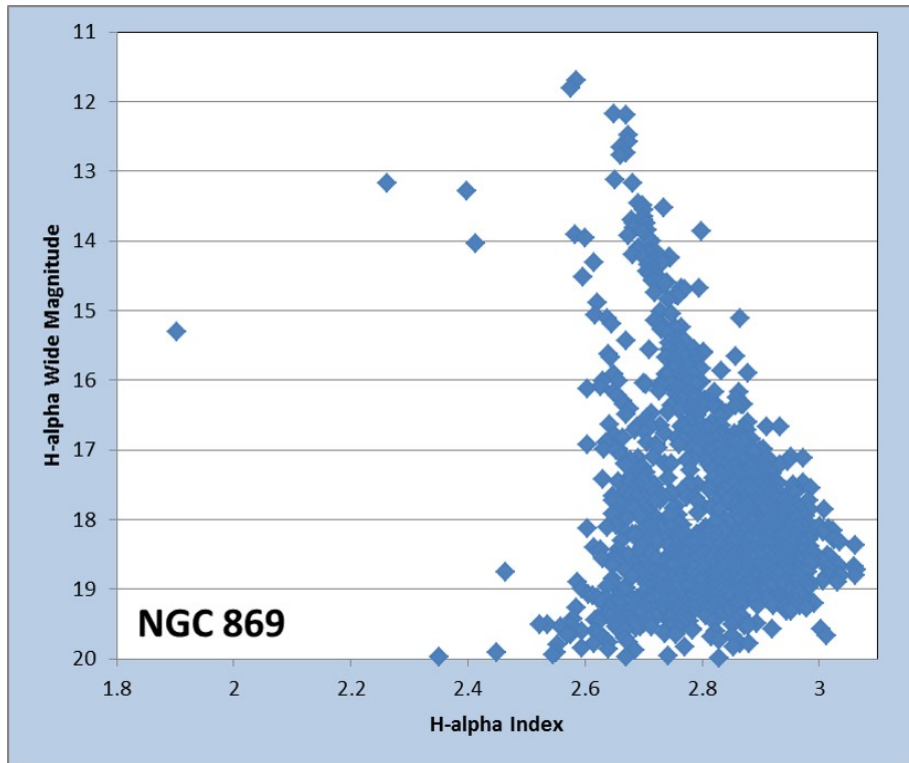


Figure 1.1: $H\alpha$ color-magnitude diagram for H-Persei from Andrew Hernandez's BYU senior thesis.

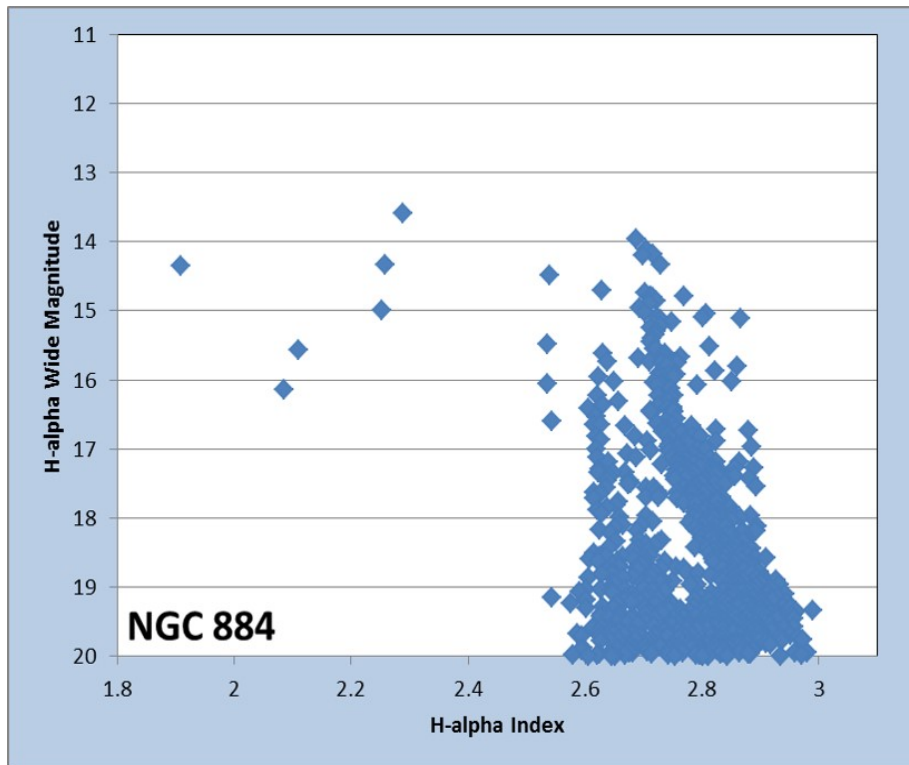


Figure 1.2: H α color-magnitude diagram for χ -Persei from Andrew Hernandez's BYU senior thesis.

Observations

To properly meet the aims of my research, the choice of observatory and telescope was important. Since open star clusters are typically crowded fields of stars when observed, the telescope used and the location of the observatory have to be able to provide sufficient resolution to be able to see all the stars in a cluster. Air pollution and light pollution are two important factors to consider for the location of an observatory. The correct combination of instruments also contributes to whether a telescope, even under ideal circumstances, can properly resolve the stars in a star cluster.

When seeking to establish standard stars for a system, high quality observations are needed. Once again, environmental factors like light pollution and air pollution can lessen the quality of observations and increase the error on brightness measurements. Luckily, since we are observing with a mono-wavelength color index, the effects of atmospheric extinction are canceled out. This does not mean that we can totally ignore observing conditions as we are making the assumption that the amount of extinction due to the atmosphere is the same when we observe with the narrow filter as when we observe with the wide filter. If these observations are done one after another and weather conditions are not entirely unideal (for example, lots of cirrus clouds, or observing at large airmass), then this is not a bad assumption to operate under. Additionally, a large sample size of observations is desirable when establishing standard stars for a system. Since the error per observation of a sample is inversely proportional to the square root of the number of observations, the larger the sample size, the smaller the standard deviation of our standard star brightness measurements will be.

When color-magnitude diagrams are our goal, being able to see as many stars that are members of our cluster as possible becomes the objective. This includes being able to see even the dim stars in a star cluster. To get observations of the dim stars in a cluster, you have to either increase your exposure time on your telescope cameras or observe on a larger telescope. Standard star measurements are also necessary to properly standardize the brightness values needed for a color-magnitude diagram, so to get proper color-magnitude diagrams for NGC 869 and NGC 884, standard stars from the list established by Joneer & Hintz (2015)⁷ need to also be observed in the same night.

The last thing to keep in mind when making decisions about how

⁷ Joneer *et al.*, "Standard Stars and Empirical Calibrations for H α and H β Photometry."

to observe my targets was that I was limited to BYU facilities. If I was observing in common photometric systems I would be able to apply for time on other telescopes, but seeing that I was using a photometric color index that was custom ordered and standardized by BYU astronomers, I had to stick to BYU facilities that had the filters that make up the H-alpha color index on hand.

With all of this in mind, I decided to use data taken at BYU's West Mountain Observatory (WMO) in 2016, 2017, and 2018. WMO is located away from the light and air pollution present in Provo, thus providing good observing conditions. This gives me 56 nights of observations for NGC 752, among which NGC 869 was also observed 22 nights and NGC 884 was observed on 21 nights. This gives me a large sample size for NGC 752 to reduce the error bars on our color and brightness measurements as well as standards to create proper color-magnitude diagrams for NGC 869 and NGC 884.

Data processing and analysis

Since all of these observations were done with charge-coupled devices (CCD), I needed to make sure that the proper calibration image frames were taken. Since our images come in the form of FITS files that count photons that hit each pixel on our chip, we need to make sure that the counts are coming from the star we're observing and not other sources of noise that are inherent to CCD's.

The first of these calibration frames is commonly known as either a zero frame or a bias frame. CCD's utilize an analog-to-digital converter which floods the chip with an offset number of counts on each pixel to avoid negative numbers. CCD's also have a certain amount of read noise that creates extra counts on the pixels of your chip. Bias frames are images taken with an exposure time of 0 seconds. These images thus record the number of counts on each pixel that are the result of both the analog-to-digital converter offset and the read noise. We can thus remove these false signals by subtracting bias frames from our images of star clusters.

The next of these calibration frames is the dark frame. Since CCD's rely on electronics to run, electrons from the electronics will be falsely counted as signal from our objects. Taking images with an equal exposure time to the exposure time of our objects, but with the shutter shut, results in an image whose signal is the result of only electronic noise. We can thus remove this false signal by subtracting dark frames from our images of star clusters.

Another technique often utilized by some observatories (including WMO) is designating a region of the chip as an overscan region. This is a region that is covered on the chip that will receive no light, thus also acting as a measurement of dark current and bias noise. The region on the image is then trimmed after the correction is applied. All of the frames taken at WMO used for this research utilized an overscan correction.

The last of the calibration frames to make note of is the flat frame. Because CCD's use an array of pixels to record signal from light sources, it is important to account for the individual bias of each pixel in how efficiently it responds to light. By applying an equal amount of light to each pixel in the chip, we can see how much of that light each pixel records. By normalizing the flat frames and dividing our images by this normalized image, we "flatten" the individual pixel response and

remove the individual pixel bias. There are many methods astronomers use to apply an equal amount of light to each pixel. The data from WMO designates our flat frames as “twilight flats” since we use the light from the Sun to apply an equal amount of light to all of the pixels, but wait until after twilight so that our chip is not saturated with signal from the Sun.

All of the images taken at WMO for this research applied bias, dark, overscan, and flat corrections for every single image frame. I used the CCDPROC package in IRAF to apply all of these corrections.

Once calibration frames were all applied, I did photometry on these images of open clusters to get brightness values for stars in both the H α narrow and H α wide filters. I did aperture photometry on the images of NGC 752 and point spread function (PSF) photometry on the images of NGC 869 and NGC 884.

Aperture photometry involves placing an annulus (or an aperture) around the target star. You integrate all of the photon counts inside of the annulus and then integrate the photon counts in the band of the annulus. Ideally, the light inside of the annulus is the sum of the flux from the star and from the sky background and the light from the band of the annulus comes from only the sky background. This allows us to get the light from the star by subtracting the light from the band of the annulus from the light inside of the annulus. Dividing the total integrated photon counts by the exposure time then gives you a measure of the flux of the star in units of photons per second. Astronomers then typically convert from photons per second to units of magnitudes by the following formula:

$$m = -2.5 * (\text{flux}) + \text{Zeropoint} \quad (3.1)$$

This measurement of brightness is known as an instrumental magnitude, and the zeropoint choice is somewhat arbitrary in our case since it’ll cancel out when we take a difference in magnitudes for our color index.

Aperture photometry is an effective method when the field of stars you’re doing photometry on is not crowded. This is the case for the stars we’re observing in NGC 752 as shown in Figure 3.1. Aperture photometry starts to become ineffective when you’re trying to do photometry in a crowded field of stars. Such is the case for NGC 869 and NGC 884 as can be seen in Figures 3.2 and 3.3. If the field of stars is so crowded that the aperture for one star crosses into another star, aperture photometry no longer works. In this case, we use PSF photometry. PSF photometry aims to construct a more detailed model of the PSF of the stars in the image and use that PSF to know how much of the light in each pixel to assign to each star when integrating counts for a brightness measurement. I used the DAOPHOT package in IRAF to do the PSF photometry for NGC 869 and NGC 884.

After obtaining instrumental magnitudes in both the H α narrow filter and the H- α wide filter, I subtracted the wide filter magnitude from the narrow filter magnitude to get our instrumental H α color index for each

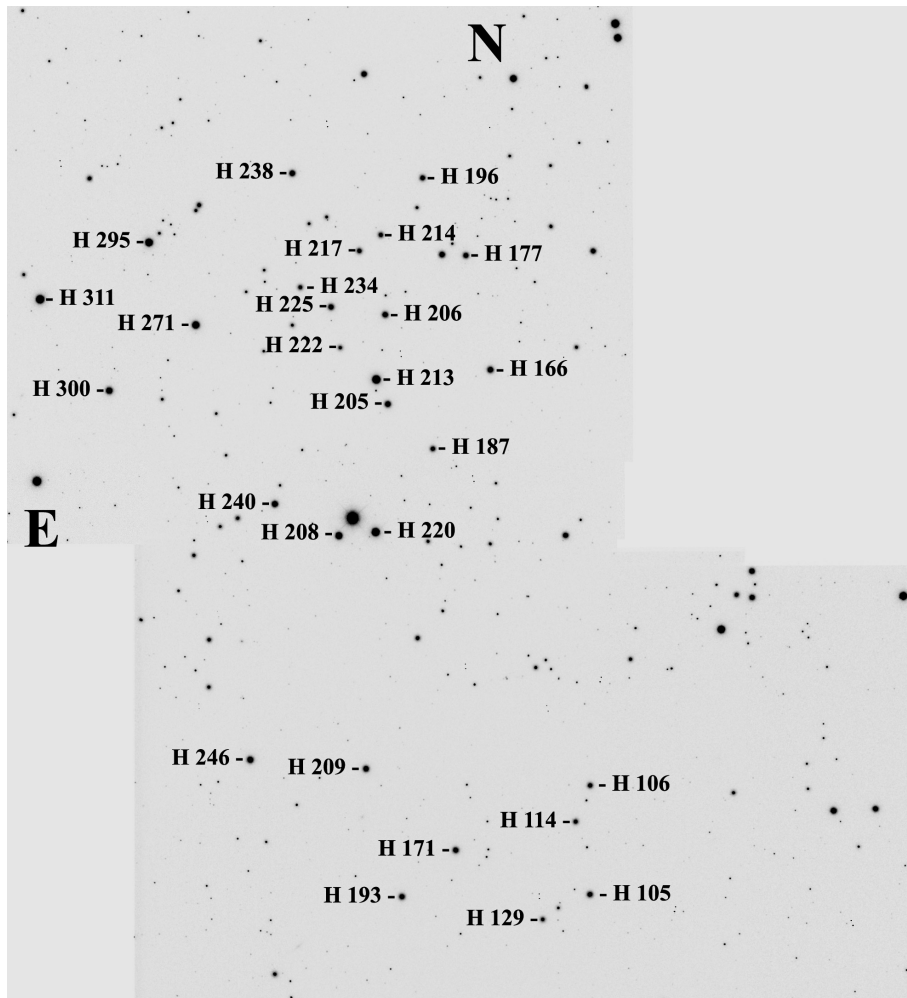


Figure 3.1: Map of NGC 752 with stars observed for this project marked by their Heinemann number.

star being observed in each cluster. We can now apply a transformation equation to convert from instrumental to apparent magnitudes, or in our case, instrumental to standardized color. Transformation equations require standard brightness or color values to be able to transform to. They normally include terms that account for atmospheric and interstellar extinction, but since we're using a mono-wavelength color index those terms get canceled out when we take a difference between the brightness of stars in the narrow and the wide filters. The only term that remains to transform from instrumental to standardized color index values is a zeropoint term. I thus used the $H\alpha$ values observed by Jøner & Hintz (2015)⁸ for 6 stars in the field of NGC 752 to determine the zeropoint term for each night that I observed each cluster. I then, for each night, added the appropriate zeropoint to the color index value for each star in the clusters observed that night to obtain standardized $H\alpha$ color index values for my targets in NGC 752, NGC 869, and NGC 884.

⁸ Jøner *et al.*, "Standard Stars and Empirical Calibrations for $H\alpha$ and $H\beta$ Photometry."



Figure 3.2: Image of the field of H-Persei being used for the color-magnitude diagrams in this project.

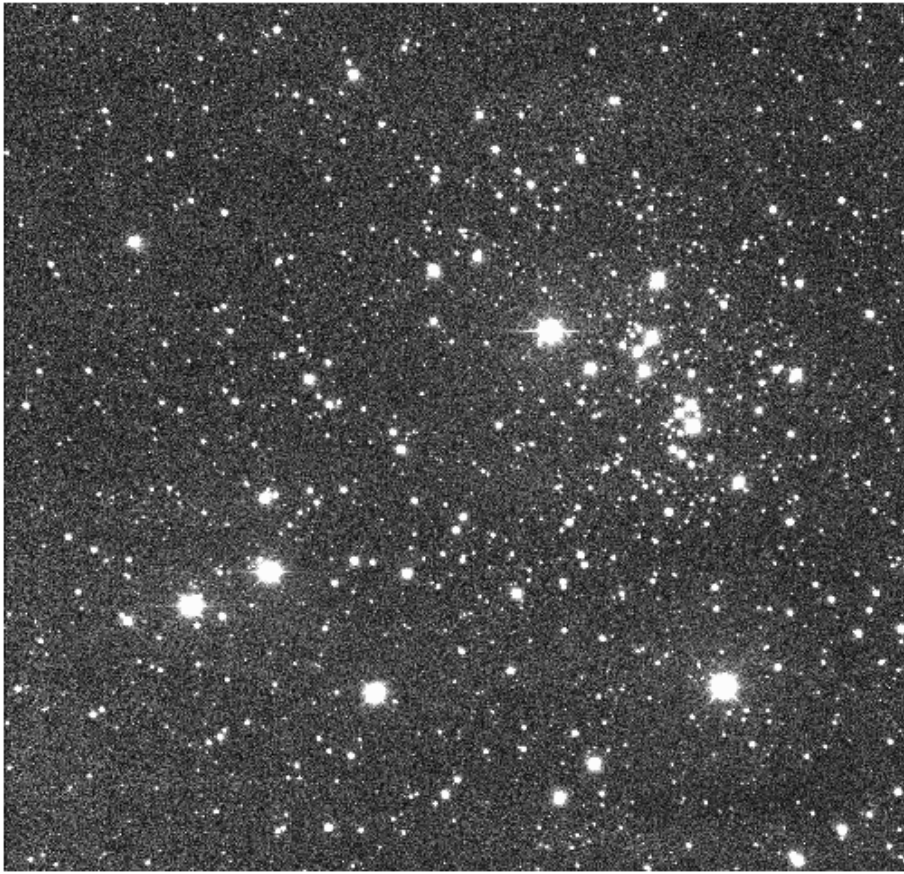


Figure 3.3: Image of the field of χ -Persei being used for the color-magnitude diagrams in this project.

4

Standard stars and photometric observations of NGC 752

Table 4.1 contains the $H\alpha$ index values for the 25 targets in the field of NGC 752 that I used to test the $H\alpha$ index photometrically and to establish as standard stars.

I was able to recover the same $H\alpha$ index on all of the standards within 1 standard deviation. Figure 4.1 further shows a $y = x$ relation between the spectrophotometric $H\alpha$ index values and the photometric $H\alpha$ index values that I measured showing that they are equal within 1 standard deviation.

This result is encouraging and shows that the $H\alpha$ color index works photometrically as it does spectrophotometrically. The error bars on these measurements, however, are unacceptable for standard stars. In future work I need to take more care in doing the aperture photometry on these stars to reduce the error on these measurements. Only then, with significantly lower standard deviations, could these stars be presented as additional standards in the field of NGC 752.

The other important piece of future work in properly testing the photometric capabilities of the $H\alpha$ index is to test the assumption that only a zeropoint is necessary in standardizing the index values and that atmospheric and interstellar extinction can, in fact, be ignored. A bayesian statistical test can be run to determine this. We start with the assumption that the transformation equation is a linear equation with a slope and y-intercept term. We then add in data of non-transformed $H\alpha$ index values together with standard $H\alpha$ index values as the likelihood function. Normal distributions with a mean of 0 and large standard deviations should work well as prior distributions for the slope and y-intercept terms of the transformation equations as we are testing to see whether the slope term is 0 or not. We can then use a Monte Carlo Markov Chain Gibbs sampler to sample the posterior distribution. If we find that the slope term's posterior distribution contains 0 within an appropriate credible interval and that the y-intercept's posterior distribution does not, then we can conclude that interstellar and atmospheric extinction can be ignored and that only a zeropoint is necessary to transform instrumental color index values to standardized color index values.

⁹ M. D. Jonev *et al.*, "Standard Stars and Empirical Calibrations for $H\alpha$ and $H\beta$ Photometry," 2015.

Table 4.1: Table showing the $H\alpha$ color index values found for 25 stars in the field of NGC 752. Standard color index value published in Jonev & Hintz (2015)⁹ is included for reference. Error per observation included to show that standardized α values measured are within one error bar of standard values published. The Heinemann number for each star is also included.

ID number	$H\alpha$ value	error	number of observations	Heinemann number	standard value
1	2.76	.087	15	196	
2	2.75	.096	15	238	
3	2.79	.053	15	271	
4	2.65	.028	7	208	
5	2.62	.055	11	220	2.63929
6	2.76	.034	15	187	
7	2.74	.050	15	240	
8	2.75	.029	15	205	2.75632
9	2.63	.082	15	213	2.63922
10	2.76	.029	15	166	2.75469
11	2.74	.072	15	177	
12	2.77	.064	15	214	
13	2.75	.077	15	217	
14	2.76	.065	15	234	
15	2.72	.052	15	225	
16	2.76	.083	15	222	
17	2.73	.039	15	206	2.73336
18	2.63	.135	14	246	
19	2.89	.112	14	209	2.89697
20	2.74	.123	14	171	
21	2.77	.135	14	193	
22	2.76	.100	12	114	
23	2.76	.084	8	106	
24	2.80	.283	7	105	
25	2.77	.117	14	129	

Color-magnitude diagrams of H & Chi Persei

I had some initial guesses to test when investigating why there is more than one population of stars present in the $H\alpha$ color-magnitude diagrams of NGC 869 and NGC 884. Hernandez when creating the color-magnitude diagrams in figure 1 plotted every detectable star in the fields of NGC 869 and NGC 884. While most of the stars in those fields are likely to be members, many could be background (or foreground) stars contaminating the sample. Additionally, I wanted to verify that this duality of populations was not due to the error of the $H\alpha$ values of the stars in these clusters.

Ruling out background stars involves running star cluster membership tests, which is a bit more complicated. Open Star Clusters are defined as a cluster of stars that are gravitationally bound and that formed from the same cloud. This would mean that the stars are in close proximity to each other. This means the distance from the Earth to each star in the cluster should not be radically different. Because the stars are gravitationally bound, this also means their movement across the sky should also be similar. Thus, taking parallax and proper motion data, observational data astronomers use to measure distance and movement across the night sky, should show us which stars are members of the cluster and which are not.

The astrometric data I used I downloaded from the second data release (DR2) of the European Space Agency's (ESA) Global Astrometric Interferometer for Astrophysics (Gaia) mission. Gaia DR2 gave me one to one parallax and proper motion data (in both the right ascension and declination directions) for every star in the fields I observed for NGC 869 and NGC 884. I then implemented a 3-parameter nearest-neighbor algorithm known as DBSCAN (Ester et al. (1996)).¹⁰ This algorithm takes data for any number of parameters as well as a nearest-neighbor distance as inputs. It then looks within a radius of each data point equal to the input nearest-neighbor distance for other data points. If two data points fall within a nearest-neighbor distance within the parameter space being analyzed, those points are considered part of a "group." DBSCAN thus outputs group numbers for each data point. Since stars within a star cluster should have similar distance values and proper motion values, stars should clump together in parallax-proper motion parameter space, thus all being part of the same DBSCAN group. Thus, in order to classify stars in the fields of NGC 869 and NGC 884 as star cluster members, I ran

¹⁰ M. Ester *et al.*, "A density-based algorithm for discovering clusters in large spatial databases with noise," 1996.

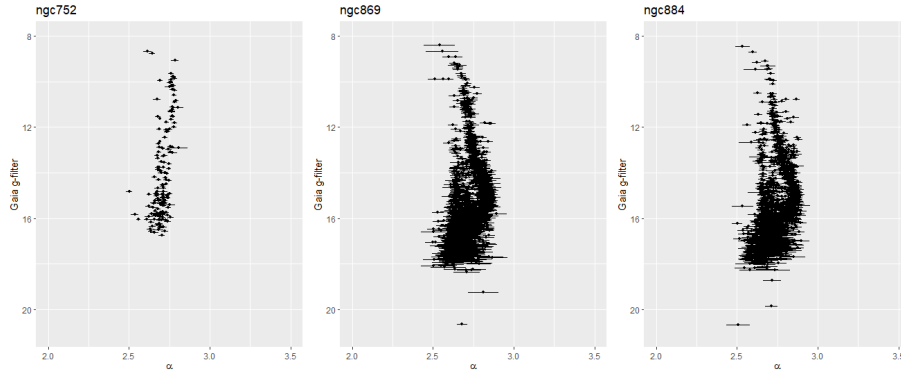


Figure 5.1: $H\alpha$ color-magnitude diagrams of NGC 752, NGC 869, and NGC 884. Gaia’s g -filter is used as the magnitude for these color-magnitude diagrams. Notice that, even after decontaminating the sample from background stars and overlaying error bars, the two populations of stars in Hernandez’s color-magnitude diagrams are still present and distinct from each other.

their parallax and proper motion data through DBSCAN and classified every data point that was part of the largest group in parameter space as a member of that respective star cluster.

Figure 5.1 shows the background star decontaminated $H\alpha$ color-magnitude diagrams for NGC 752, NGC 869, and NGC 884 with error bars overlaid for their $H\alpha$ color value and Gaia g -filter brightness values used for the y -axis of the diagram. As Joneer & Hintz (2015)¹¹ only standardized $H\alpha$ values below 2.9, I remove any stars with an H-alpha index above 2.9. I also removed stars below H-alpha values of 2.5 as those stars have H-alpha lines in emission and are not part of the two populations being analyzed. As we can see, the error bars of the two populations do not overlap, thus eliminating error per observation as a reason for two separate populations in the diagram. The diagram has also been corrected for background field stars, thus eliminating that possibility. Thus, the question of what these stars are remains unanswered and further tests are needed to determine this duality of populations in the $H\alpha$ color-magnitude diagrams of NGC 869 and NGC 884.

¹¹ Joneer *et al.*, “Standard Stars and Empirical Calibrations for $H\alpha$ and $H\beta$ Photometry.”

Conclusions

In this thesis I have reviewed my work on testing and further standardizing the $H\alpha$ color index through observations of stars in NGC 752 as well as investigating a dual-population in the $H\alpha$ color-magnitude diagrams of NGC 869 and NGC 884, or H & Chi Persei, respectively.

In the testing and standardization of the $H\alpha$ color index through the observation of stars in NGC752, it is clear that much more careful aperture photometry remains to be done to reduce the error per observation of the stars in my sample. I already have a large sample size to reduce the error per observation, and the observing conditions of the site of the observatory and the individual nights were good. It is thus clear that with more careful photometry, I'll be able to reduce the error bars to have acceptable standard deviations for standard stars. This will greatly expand the number of standard stars in the field of NGC 752 (6 to 25), thus greatly improving the practicality of using this color index. I hope to also observe and establish standard stars in the fields of star clusters observable throughout different parts of the year to allow the $H\alpha$ color index to have many usable standard stars in every part of the year.

Once I have done better photometry on the stars in my sample, I'll then be able to use a Bayesian MCMC sampler to test whether only a zeropoint is needed when standardizing $H\alpha$ color index values. A smaller error per observation to use for my prior distribution will result in a more precise posterior distribution that will allow me to make a more confident claim on whether the transformation equation for our $H\alpha$ color index only needs a zeropoint term or not.

I need to run further tests to determine the reason for a second population in the $H\alpha$ color-magnitude diagrams of NGC 869 and NGC 884. There are several more tests I can yet run to determine if there is a physical difference between these two populations. Using other color indices I could compare the color values between the two populations and use those color values to determine other astrophysical properties unique to those color indices. Since the $H\alpha$ color index value is related to the equivalent width of the $H\alpha$ spectral line, it is possible that other spectral line features are causing this difference in $H\alpha$ color value. Thus comparing the spectra of stars in each of these two populations could yield insight into the difference between these two populations.

References

- 1 D. Crawford, "Early-type stars used as standards in photoelectric hbeta photometry.," *The Astrophysical Journal*, vol. 132, p. 66, 1960.
- 2 D. Crawford and C. Perry, "Four-colour and h-beta photometry of open clusters. i. the hyades.," *The Astronomical Journal*, vol. 71, pp. 206–215, 1966.
- 3 D. Crawford and J. Barnes, "Four-color and h-beta photometry of open clusters. ii. coma and ursa major.," *The Astronomical Journal*, vol. 74, pp. 407–412, 1969.
- 4 D. Crawford and C. Perry, "Four-color and h-beta photometry for open clusters. xi-the pleiades," *The Astronomical Journal*, vol. 81, pp. 419–426, 1976.
- 5 M. D. Joner and E. G. Hintz, "Standard Stars and Empirical Calibrations for $H\alpha$ and $H\beta$ Photometry," *The Astronomical Journal*, vol. 150, no. 6, p. 204, 2015.
- 6 N. Kharchenko, A. Piskunov, E. Schilbach, S. Röser, and R.-D. Scholz, "Global survey of star clusters in the milky way-ii. the catalogue of basic parameters," *Astronomy & Astrophysics*, vol. 558, A53, 2013.
- 7 M. Ester, H.-P. Kriegel, J. Sander, X. Xu, *et al.*, "A density-based algorithm for discovering clusters in large spatial databases with noise.," *kdd*, vol. 96, 1996, pp. 226–231.