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A Colorfully Natural Twin Higgs Model

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ABSTRACT

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Twin Higgs models have the potential to explain phenomena outside of the Standard Model and provide an natural explanation of the Higgs mass. Here I explore a new realization of the twin Higgs concept with an $\mathrm{SU}(4)_c$ symmetry that is spontaneously broken in the Standard Model sector, but remains unbroken in the Twin sector. I detail the phenomenological results produced by this model. I show that this construction leads to a qualitatively new behavior regarding the fine-tuning of the Higgs mass due to the top-quark sector.

Keywords: Twin Higgs, Dark Matter, symmetry breaking, Higgs physics, SU(4), Lie Groups

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Chapter 1

Introduction

Particle physics seeks to describe the universe in terms of its most fundamental, indivisible elements. This process consists of two parts: theory and experiment. Theory works within the constraints of a mathematical framework to provide predictions of potential new phenomena, which may or may not be experimentally observed. Experiment measures data to identify new phenomena, which may or may not match an existing theory. When the two coincide to describe a new phenomenon with high accuracy and precision, a discovery is made.

The accepted framework for particle physics model building is Quantum Field Theory, the combination of special relativity and quantum mechanics. This framework describes every particle as a fluctuation in a field. Couplings between fields result in all particle interactions. These fields can exhibit global and/or local symmetries. Global symmetries lead to conserved charges, like baryon number, that are not associated with a force. Local or “gauge" symmetries, described by Lie groups, are related to the fundamental forces and their associated charges. An additional consideration is Lorentz invariance, the relativistic requirement that space have no preferred direction and that all inertial frames be equivalent. A model must account for each of these considerations and describe phenomena in a testable way.

Particle physics experiments are usually carried out in particle colliders such as the Large

Hadron Collider. Collisions are recorded, and the output particles from each collision are analyzed for patterns that match theoretical expectations. When a detected signal varies from expected background values by five standard deviations (5σ) or more, it is considered a discovery. Although this benchmark is somewhat arbitrary, it is well agreed-upon and encourages rigorous verification of claims.

The last century of nuclear and particle physics has provided evidence for a set of particles and forces now known as the Standard Model (SM) of particle physics. This model has been compiled over time to describe fundamental particles: quarks, leptons, the photon, the gluon, and the $W^{\pm}$, Z, and Higgs bosons. Quarks make up protons and neutrons and experience the strong nuclear force, which is communicated by gluons. Leptons include electrons and neutrinos. The photon is a massless particle which communicates electromagnetism. The $W^{\pm}$ boson can be either positively or negatively charged, and along with the neutral Z boson carries the weak force. The Higgs boson is not directly evident in atomic structures, but is essential in providing mass to other fundamental particles in a way we explore later on.

Each of these particles and the interactions between them have been described by theory and observed experimentally. At times, such as for the discovery of electrons, experiment has provided evidence which was later described by theory. More recently however, with the top quark and the Higgs boson, experiment has taken decades to confirm what was theoretically expected. The discovery of the Higgs Boson [1, 2] completed the SM as the final experimental verification of the model's predicted particles.

The SM contains several sets of particles and three fundamental forces. Each of these particles is represented by a field. Each field is represented in Lorentz space according to its spin as a scalar, vector, or tensor. Fermions, including quarks and leptons, are spin-$\frac{1}{2}$ particles. The photon and the $W^{\pm}$ and Z bosons have spin 1. The Higgs field, having zero spin, is the only Lorentz scalar. Each term of the Lagrangian must be invariant under Lorentz transformations.

Quarks and leptons are both paired into two types. Quarks are either up-type or down-type, depending on their electric charge. Leptons are either electrically charged (electron-type) or neutral (neutrino). Each of these types is further subdivided into three generations. Two particles of the same type but different generations have the same charge properties, but different masses. The three generations of up-type quarks are, in order of increasing mass: up, charm, and top. The three down-type quarks are, similarly: down, strange, and bottom. The three generations of charged leptons are: electron, muon, and tau. The three neutral lepton generations are: electron neutrinos, muon neutrinos, and tau neutrinos. When differentiating between these particles, we often refer to the quarks and leptons as each having six flavors.

The “gauge bosons" are those which carry fundamental forces. The photon and gluon are massless spin-1 bosons which carry the electromagnetic force and strong force, respectively. The $W^{\pm}$ and Z bosons are massive spin-1 bosons which carry the weak force.

The Higgs boson is unique as the only spin-0 boson in the SM. This has an effect related to the requirement of Lorentz invariance given by Quantum Field Theory. Lorentz invariance requires space-time to have no preferred direction or velocity; all inertial frames of reference are equivalent. Thus, any field with a non-zero spin must have an average value of zero throughout space. The Higgs field, with zero spin, can have a non-zero average value without disrupting Lorentz invariance. This average value is referred to as the vacuum expectation value, or VEV.

Each field exhibits gauge symmetries, invariances under Lie group transformations, which are related to the observed fundamental forces of nature. There are three gauge groups that act on SM fields: $SU(3)_c$ for fields with color charge, $SU(2)_L$ for fields that experience the weak nuclear force, and $U(1)_Y$ for fields with hypercharge. A field will interact with each of these symmetries depending on the charges it has under each. For more detail on Lie groups, see Appendix A.

The primary equation describing the SM is its Lagrangian:

$$\begin{aligned}\mathscr{L}_{SM} = &-\frac{1}{4}G^a_{\mu\nu}G^{a\mu\nu} - \frac{1}{4}W^a_{\mu\nu}W^{a\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} \\ &+ i\overline{L}\not{D}L + i\overline{E}\not{D}E + i\overline{Q}\not{D}Q + i\overline{D}\not{D}D + i\overline{U}\not{D}U \\ &+ (\lambda_l)^i_j\overline{L}_iHE^j + (\lambda_d)^i_j\overline{Q}_iHD^j + (\lambda_u)^i_j\overline{Q}_i\tilde{H}U^j + \text{H.c.} \\ &+ (D_\mu H)^\dagger D_\mu H + M_H^2|H|^2 - \lambda_H|H|^4 \ .\end{aligned} \tag{1.1}$$

The first line of Eq. (1.1) contains terms unique to each of the three gauge groups SU(3)$_c$, SU(2)$_L$, and U(1)$_Y$. The terms $G^a_{\mu\nu}$, $W^a_{\mu\nu}$, and $B_{\mu\nu}$ represent the field strength tensors of each gauge group, respectively. A field strength tensor $F^a_{\mu\nu}$ corresponding to a Lie group SU(N) is defined as

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - gf^{abc}A^b_\mu A^c_\nu \ , \tag{1.2}$$

where A^a_μ is a gauge field associated to the a^{th} generator of SU(N), g is the gauge coupling that determines the strength of its interactions, and f^{abc} gives the structure constants. The group SU(3)$_c$ has eight generators, the Gell-Mann matrices T^a, which correspond to the eight varieties of gluon fields. The group SU(2)$_L$ has three generators, proportional to the three Pauli matrices. The group U(1)$_Y$ has only one generator, B_μ. For more detail on Lie groups and their generators, see Appendix A.

The terms in Eq. (1.1) that contain D_μ and $\not{D}$ are the kinetic terms, which describe the kinetic energy of each field individually. $\not{D}$ or D_μ represents the covariant derivative. This is a derivative which has added terms to make it transform in the same way as the field under gauge transformations. For a field ψ that acts under all three gauge groups, the complete gauge covariant derivative is given by

$$D_\mu\psi = \left(\partial_\mu - ig'Y_\psi B_\mu - i\frac{g}{2}\sigma^a A^a_\mu - i\frac{g_s}{2}T^a C^a_\mu\right)\psi \ , \tag{1.3}$$

and $\not{D} = D_\mu\gamma^\mu$ to account for the spinor nature of fermion fields. The constants g', g, and g_s are the gauge couplings corresponding to the gauge symmetries U(1)$_Y$, SU(2)$_L$, and SU(3)$_c$ respectively. These factors characterize the strength of the associated force.

The field E represents right-handed charged leptons from each of the three generations via a suppressed flavor index. Expanding it in flavor space would yield:

$$E = \begin{pmatrix} e_R \\ \mu_R \\ \tau_R \end{pmatrix} . \tag{1.4}$$

The field L represents left-handed lepton doublets of $\mathrm{SU}(2)_L$. These fields, not E, experience $\mathrm{SU}(2)_L$ interactions. Each doublet contains a charged lepton and a neutrino from the same generation. For example, the first generation of L is

$$L = \begin{pmatrix} \nu_e \\ e^- \end{pmatrix} . \tag{1.5}$$

The fields D and U represent right-handed up-type and down-type quarks respectively, of all three generations. Just like E, these two are singlets under $\mathrm{SU}(2)_L$ and do not experience weak force interactions. Q represents left-handed pairs of up and down quarks which transform under $\mathrm{SU}(2)_L$. We represent these in an $\mathrm{SU}(2)_L$ doublet structure, similar to that of L:

$$Q = \begin{pmatrix} U_L \\ D_L \end{pmatrix} . \tag{1.6}$$

The field H represents the Higgs field, which is unique as the only scalar field in the model. The terms in the third line of Eq. (1.1) describe the interactions of H with the fermions. The coupling of a scalar to a fermion bilinear is a Yukawa interaction, and λ is the Yukawa coupling. The $|H|^2$ and $|H|^4$ terms are the Higgs potential, which describe the mass and self-interactions of the Higgs field. H.c. stands for Hermitian conjugate, indicating that the conjugate of the preceding terms in the line are also included.

1.1 Color Physics

We use color physics as a brief introduction to gauge symmetries in the Standard Model. SU(3)$_c$ is the symmetry which governs colored particles, i.e. quarks and gluons. The strong force is the fundamental force which acts on color charges. The strong force, much like electromagnetism, can bind together complementary charges or separate like charges. In contrast to electric charge, however, there are three variations of color charge, each with its own anti-charge. Because these three charges combine in ways reminiscent of color, they are assigned the values red, blue, and green. Similar to electric charge, a stable, neutral configuration of color charges can consist of a charge and its opposite, e.g. red and anti-red. However, color charge can also be neutral when all three colors are present. This happens when a red, a blue and a green quark bind to form a proton or neutron. Just as red, green and blue light mix to show white, the three color charges together are neutral.

These combination rules are the result of the SU(3)$_c$ symmetry. SU(3) is the Lie group composed of special unitary 3×3 matrices. A special matrix has a determinant of 1. A unitary matrix times its Hermitian conjugate is the identity. Thus, any matrix U in SU(3) has the properties

$$\mathrm{Det}[U] = 1\ , \quad U^\dagger U = \mathbb{I}\ . \tag{1.7}$$

The group elements are generated by the exponentiation of a related set of matrices, $\mathfrak{su}(3)$, which are a basis of the associated Lie algebra. When the Lie group is special and unitary, the elements of the Lie algebra are traceless and Hermitian. Thus, any matrix T in $\mathfrak{su}(3)$ has the properties

$$\mathrm{Tr}[T] = 0\ , \quad T^\dagger = T\ . \tag{1.8}$$

In this discussion we prefer to work directly with the algebra. The set of $3{\times}3$ matrices with these constraints can be specified by eight real parameters. These become the eight gluon fields, which we can label as G^a, with a running from 1 to 8. Rather than writing each of the matrices separately, we can display all eight simultaneously with the use of superscripts and a summation over a as

$$G^aT_a = \begin{pmatrix} G^3+\frac{1}{\sqrt{3}}G^8 & G^1-iG^2 & G^4-iG^5 \\ G^1+iG^2 & -G^3+\frac{1}{\sqrt{3}}G^8 & G^6-iG^7 \\ G^4+iG^5 & G^6+iG^7 & -\frac{2}{\sqrt{3}}G^8 \end{pmatrix} . \tag{1.9}$$

Although quarks and gluons both interact with the Lie group $\mathrm{SU}(3)_c$, they do so in different ways. A quark is in the fundamental representation of the group $\mathrm{SU}(3)_c$, the **3** representation, and as such can be written as a triplet in color space as

$$q = \begin{pmatrix} q_{red} \\ q_{green} \\ q_{blue} \end{pmatrix} . \tag{1.10}$$

Because this triplet is in the fundamental representation, action by any element U^a of the group $\mathrm{SU}(3)_c$ transforms the triplet as

$$q^a \to U^a_b q^b . \tag{1.11}$$

A quark-antiquark pair being acted on by the group would transform as

$$\overline{q}^a q_a \to \overline{q}^a (U^\dagger)^c_a U^b_c q_b \to \overline{q}^a \mathbb{I}^b_a q_b \to \overline{q}^a q_a , \tag{1.12}$$

and is thus invariant under action by the color group. This result is equivalent to the quark-antiquark pair being neutral under color charge, as described earlier. Describing this multiplication in representation notation shows the same neutral result

$$\overline{\mathbf{3}} \otimes \mathbf{3} = \mathbf{8} \oplus \mathbf{1} . \tag{1.13}$$

This multiplication is expanded using representation theory, as described in Appendix A. Because the result contains the singlet representation **1**, it has a charge neutral part.

A gluon, on the other hand, is in the eight-dimensional adjoint representation, **8**, of the color group. This means that instead of a vector-like triplet with 3 degrees of freedom, we represent it as a matrix with 8 degrees of freedom, in the same form as the generators of the group elements. This correctly implies that there are 8 unique varieties of charge that a gluon can take on. A gluon has one color and one anti-color of charge, with $\bar{r}r+\bar{g}g+\bar{b}b=0$. Thus, we can map out the gluon types in the following matrix:

$$G^a_b = \begin{pmatrix} g_{r\bar{r}} & g_{g\bar{r}} & g_{b\bar{r}} \\ g_{r\bar{g}} & g_{g\bar{g}} & g_{b\bar{g}} \\ g_{r\bar{b}} & g_{g\bar{b}} & -g_{r\bar{r}}-g_{g\bar{g}} \end{pmatrix} . \tag{1.14}$$

An object G^a_b in the adjoint representation of a group such as SU(3) transforms as

$$G^a_b \to U^a_c G^c_d (U^\dagger)^d_b . \tag{1.15}$$

This leads to different requirements for gauge invariance and charge neutrality than for the quark color triplets. For example, a gluon interacting with a quark-anti-quark pair is color invariant, as:

$$\bar{q}^a G^b_a q_b \to \bar{q}^b (U^\dagger)^a_c U^b_d G^d_e (U^\dagger)^e_a U^f_b q_f \to \bar{q}^a G^b_a q_b , \tag{1.16}$$

or, using representation theory:

$$\mathbf{\bar{3}} \otimes \mathbf{8} \otimes \mathbf{3} = \mathbf{27} \oplus \mathbf{10} \oplus \mathbf{\overline{10}} \oplus \mathbf{8} \oplus \mathbf{8} \oplus \mathbf{8} \oplus \mathbf{1} . \tag{1.17}$$

Because the result contains a singlet under $SU(3)_c$, it is invariant under color charge transformations. It transforms only by the identity element, i.e. not at all, when acted on by the group.

These principles of group interactions also hold for $SU(2)_L$ and $U(1)_Y$, though in simpler ways. The generator for $U(1)_Y$ is a number, a rank one matrix. States that are charged under $U(1)_Y$ transform as $e^{i\theta}$. The fundamental states of SU(2) are doublets, **2**, rather than triplets, and its generator matrices can be written as $\frac{1}{2}\sigma^a$, where σ^a are the three Pauli matrices

$$\sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} , \quad \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} , \quad \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} . \tag{1.18}$$

1.2 Spontaneous Symmetry Breaking

An aspect of the SM which is important throughout this discussion is the spontaneous symmetry breaking caused by the Higgs. Because the Higgs field is a Lorentz scalar, it is allowed to have a non-zero VEV without breaking Lorentz invariance. The VEV interacts with gauge transformations in such a way that some of the symmetry is lost. The Higgs, being charged under $\mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y$, breaks these two symmetries down to a new, smaller symmetry, $\mathrm{U}(1)_{\mathrm{E\&M}}$. We now examine the mathematical method by which these symmetries break. Because the Higgs is not charged under color, the color symmetry does not appear in this calculation. We express the Higgs field as a doublet under $\mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y$, which is

$$H = \begin{pmatrix} H^+ \\ H_0 \end{pmatrix} . \tag{1.19}$$

The terms in the SM Lagrangian defining the Higgs field are

$$\mathscr{L} = (D_\mu H)^\dagger D_\mu H - V_H , \tag{1.20}$$

where the Higgs potential, V_H, is

$$V_H = -\mu^2 |H|^2 + \lambda |H|^4 . \tag{1.21}$$

In order to see how the Higgs field's VEV disrupts symmetries, we first find its value. The VEV is the value it takes when energy is at a minimum, so we minimize the Higgs potential and find

$$\langle H \rangle_{VEV} = \frac{\mu}{\sqrt{2\lambda}} . \tag{1.22}$$

Since H transforms under $\mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y$ as a two component object, we can choose a convenient gauge transformation to put the entire magnitude of the VEV in the lower component. This becomes

$$\langle H \rangle_{VEV} = \begin{pmatrix} 0 \\ \frac{\mu}{\sqrt{2\lambda}} \end{pmatrix} \text{, or using conventional notation, } \langle H \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix} , \tag{1.23}$$

where $v = \frac{\mu}{\sqrt{\lambda}}$.

We now can use this form of H and the covariant derivative in Eq. (1.3) to evaluate the kinetic term containing $D_\mu H$. We can also assign to Y_ψ the Higgs' hypercharge value in the SM, $\frac{1}{2}$.

$$\begin{aligned}
\frac{1}{\sqrt{2}} D_\mu \begin{pmatrix} 0 \\ v \end{pmatrix} &= \frac{1}{\sqrt{2}} (\partial_\mu + i\frac{g}{2}\sigma^a A^a_\mu + i\frac{g'}{2}\mathbb{I} B_\mu) \begin{pmatrix} 0 \\ v \end{pmatrix} \\
&= \frac{1}{\sqrt{2}} \begin{pmatrix} \partial_\mu + \frac{i}{2}(g' B_\mu + g A^3_\mu) & \frac{ig}{2}(A^1_\mu - iA^2_\mu) \\ \frac{ig}{2}(A^1_\mu + iA^2_\mu) & \partial_\mu + \frac{i}{2}(g' B_\mu - g A^3_\mu) \end{pmatrix} \begin{pmatrix} 0 \\ v \end{pmatrix} \\
&= \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{ig}{2}(A^1_\mu - iA^2_\mu) \\ \partial_\mu + \frac{i}{2}(g' B_\mu - g A^3_\mu) \end{pmatrix} v \,. \qquad (1.24)
\end{aligned}$$

From this result, we expand the full kinetic term, noting that $\partial_\mu v = 0$:

$$\left[\frac{1}{\sqrt{2}} D_\mu \begin{pmatrix} 0 \\ v \end{pmatrix}\right]^\dagger \frac{1}{\sqrt{2}} D_\mu \begin{pmatrix} 0 \\ v \end{pmatrix} = \frac{v^2 g^2}{8}(A^1_\mu + iA^2_\mu)(A^{1\mu} - iA^{2\mu}) + \frac{v^2}{8}(g' B_\mu - g A^3_\mu)^2 \,. \qquad (1.25)$$

When the symmetry $SU(2)_L \otimes U(1)_Y$ breaks to $U(1)_{E\&M}$, the size of its set of generators must decrease from four to one. Within the unbroken symmetry, A^{1-3}_μ and B_μ represented massless vector fields which carried associated forces. After breaking to U(1), however, three of the four basis vectors lose their symmetry and need not remain massless. The Higgs field also began with four real degrees of freedom (two in each complex component), and is now left with only one in the direction of v. The three broken symmetry dimensions become massive vector fields by absorbing degrees of freedom previously held by the Higgs field.

We see that the previously massless vector fields A^{1-3}_μ and B_μ have been grouped into two quadratic terms as a result of multiplication by the VEV. The factor v^2 multiplying them creates mass terms from previously massless vectors. The quadratic terms show the linear combinations which result in three massive particles, W^+, W^-, and Z. The state orthogonal to these is a massless particle. It is a linear combination of the original basis vectors, which remains massless and becomes the vector field associated with the the residual $\mathrm{U(1)_{E\&M}}$ gauge symmetry. This is defined to be the photon, A_μ. We also define $\cos\theta_W$ and $\sin\theta_W$ for notational simplicity,

$$\cos\theta_W \equiv \frac{g}{\sqrt{g^2+g'^2}}\ , \quad \sin\theta_W \equiv \frac{g'}{\sqrt{g^2+g'^2}}\ , \tag{1.26}$$

$$W^\pm_\mu = \frac{1}{\sqrt{2}}(A^1_\mu \mp iA^2_\mu)\ , \tag{1.27}$$

$$Z_\mu = \frac{1}{\sqrt{g^2+g'^2}}(g'B_\mu - gA^3_\mu)\ = \sin\theta_W B_\mu - \cos\theta_W A^3_\mu\ , \tag{1.28}$$

$$A_\mu = \frac{1}{\sqrt{g^2+g'^2}}(gB_\mu + g'A^3_\mu)\ = \cos\theta_W B_\mu + \sin\theta_W A^3_\mu\ . \tag{1.29}$$

These new particles define the mass basis, the basis in which particles will propagate energy through space and time. To see how the other fields will interact with these new particles, we rewrite the covariant derivative in the new basis. To do this we substitute the original generators for linear combinations of the new massive particles, using

$$A^1_\mu = \frac{1}{\sqrt{2}}\left(W^+_\mu + W^-_\mu\right)\ , \quad A^2_\mu = \frac{i}{\sqrt{2}}\left(W^+_\mu - W^-_\mu\right)\ , \tag{1.30}$$

$$A^3_\mu = \sin\theta_W A_\mu - \cos\theta_W Z_\mu\ , \tag{1.31}$$

$$B_\mu = \cos\theta_W A_\mu + \sin\theta_W Z_\mu\ . \tag{1.32}$$

Substituting these into the general covariant derivative (1.3), we see:

$$\begin{aligned}
D_\mu =&\partial_\mu - i\frac{g}{2}\sigma^a A^a_\mu - ig'Y\mathbb{I}B_\mu \\
=&\partial_\mu - \frac{ig}{2}(\sigma^1 A^1_\mu + \sigma^2 A^2_\mu + \sigma^3 A^3_\mu) - ig'Y\mathbb{I}B_\mu \\
=&\partial_\mu - \frac{ig}{2\sqrt{2}}\sigma^1\left(W^+_\mu + W^-_\mu\right) - \frac{i^2 g}{2\sqrt{2}}\sigma^2\left(W^+_\mu - W^-_\mu\right) \\
&- \frac{ig}{2}\sigma^3\left(\sin\theta_W A_\mu - \cos\theta_W \mathbb{I}Z_\mu\right) - ig'\left(\cos\theta_W A_\mu + \sin\theta_W Y\mathbb{I}Z_\mu\right) \\
=&\partial_\mu - \frac{ig}{2\sqrt{2}}(\sigma^1 + i\sigma^2)W^+_\mu - \frac{ig}{2\sqrt{2}}(\sigma^1 - i\sigma^2)W^-_\mu \\
&- \left(\frac{ig}{2}\sin\theta_W\sigma^3 + ig'\cos\theta_W Y\mathbb{I}\right)A_\mu - \left(-\frac{ig}{2}\cos\theta_W\sigma^3 + ig'\sin\theta_W Y\mathbb{I}\right)Z_\mu\ .
\end{aligned} \tag{1.33}$$

The coefficient of A_μ is useful because it relates hypercharge to electric charge. Naming the electric coupling $e \equiv g\sin\theta_W = g'\cos\theta_W = \frac{g'g}{\sqrt{g^2+g'^2}}$, the coefficient becomes

$$ie\left(\frac{1}{2}\sigma^3 + Y\mathbb{I}\right)A_\mu \ . \tag{1.34}$$

Thus the electric charge of an SU(2) doublet is

$$q = \frac{1}{2}\sigma^3 + Y\mathbb{I}\ . \tag{1.35}$$

The Higgs' kinetic term can now be expressed in terms of massive gauge bosons with the absence of $SU(2)_L \otimes U(1)_Y$ symmetry:

$$\left[\frac{1}{\sqrt{2}}D_\mu\begin{pmatrix}0\\ v\end{pmatrix}\right]^\dagger \frac{1}{\sqrt{2}}D_\mu\begin{pmatrix}0\\ v\end{pmatrix} = \frac{v^2g^2}{4}W_\mu^+W^{-\mu} + \frac{v^2}{8}\left(g^2+g'^2\right)Z_\mu Z^\mu\ . \tag{1.36}$$

And so spontaneous symmetry breaking has dismantled the $SU(2)_L \otimes U(1)_Y$ symmetry. We began with four massless vectors: $A^1_\mu, A^2_\mu, A^3_\mu$, and B_μ. Those have now been rearranged into three massive particles, $W^\pm$ and Z, a massless photon $A_\mu$associated with the remaining $U(1)_{\mathrm{E\&M}}$ symmetry.

1.3 Beyond the Standard Model

Although the Standard Model is successful in describing many natural phenomena, there are still unanswered questions. A complete model of the universe would therefore include more than is currently described in the SM. Beyond the Standard Model (BSM) research seeks to add to the SM by identifying possible model extensions and testing them against experimental data. In this discussion we focus on two of these unanswered questions: the nature of dark matter and the naturalness of the Higgs VEV.

Astronomical data points to the existence of Dark Matter (DM) which has very limited interaction with SM matter through any force except gravity. Several observations provide evidence for the existence of DM throughout the universe. These include gravitational lensing in galaxy clusters [3,4], rotation curves of spiral galaxies [5], and in the cosmic microwave background [6]. A Quantum Field Theory (QFT) model which describes DM would ideally provide fields which have little or no interaction with the gauge symmetries $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$, but which still provide some sort of detectable signals in order to check the model. There are unlimited possibilities for the structure of DM, from a single particle to hundreds of new fields. We therefore wish to include additional considerations which help motivate certain regions of this huge possibility space, such as Higgs physics.

The mass parameter μ of the Higgs potential is a free parameter in the SM. Measurements have determined the mass of the Higgs to be 125 GeV [7, 8], where $m_h^2 = 2\mu^2$. This is the only explicit scale in the SM, but has no apparent cause for its value. The only other known scale is the Planck scale, which is bigger by a factor of 10^{17}. While there is no inherent problem with the value of μ, physics associated with the Planck scale is generically expected to produce a Planck scale mass for the Higgs. This large, arbitrary discrepancy appears fine-tuned or “unnatural". It is therefore an appealing quality of a BSM model to provide an explanation or mechanism by which the value of μ is limited to its low scale, thus making its value “natural."

1.3.1 Twin Higgs Models

A promising BSM theory called the Twin Higgs [9] model presents some possibilities that we explore here. The Twin Higgs model supposes a new set of fields that are similar in structure to the Standard Model, but distinct from the SM gauge symmetries. This new sector provides excellent DM candidates, as it contains a variety of fields which are explicitly separated from the SM forces. Although there are many possibilities for the structure of DM, we choose to mirror the SM both because it is a known structure and because it appeals to the prevalence of symmetry in nature.

The twin sector is described by a contribution added to the SM Lagrangian. In the “Mirror Twin

Higgs" model, the additional Lagrangian contains identical fields describing mirror quarks, leptons, bosons, and gauge symmetries. The two sectors are connected through interactions between the Higgs field and its twin. The full scalar potential of the two sectors exhibits an approximate global U(4) symmetry. This can be seen by considering a four component scalar field:

$$H = \begin{pmatrix} H_A \\ H_B \end{pmatrix} , \tag{1.37}$$

where

$$H_A = \begin{pmatrix} H^+ \\ H_0 \end{pmatrix}_{\text{SM}} \quad H_B = \begin{pmatrix} H^+ \\ H_0 \end{pmatrix}_{\text{dark}} . \tag{1.38}$$

Here the Higgs potential takes the form

$$V_H = -M_H^2 |\mathbf{H}|^2 + \lambda_H |\mathbf{H}|^4 + \delta_H (|H_A|^4 + |H_B|^4) + m^2 \left(|H_A|^2 - |H_B|^2\right) , \tag{1.39}$$

where M_H and λ_H are constants similar to those in Eq. (1.21), δ_H and m determine the strength of the symmetry-breaking terms, and

$$|\mathbf{H}|^2 = |H_A|^2 + |H_B|^2 . \tag{1.40}$$

This approximate symmetry results in the creation of the observed Higgs boson as a pseudo-Nambu-Goldstone boson, which naturally has a low mass scale. In this way, the Twin Higgs model also provides naturalness to the Higgs mass parameter.

The Lagrangian for this model can be written as

$$\mathscr{L} = \mathscr{L}_{A,\text{MTH}} + \mathscr{L}_{B,\text{MTH}} - V_H \tag{1.41}$$

where $\mathscr{L}_{A,\text{MTH}}$ contains the Standard Model sector, $\mathscr{L}_{B,\text{MTH}}$ is a mirror of A, and the potential V_H contains the interaction terms between the A and B sectors. As the Mirror Twin Higgs model copies the exact Lagrangian from the SM, we have

$$\begin{aligned}\mathscr{L}_{A,\mathrm{MTH}} = &-\frac{1}{4}T^{a}_{\mu\nu A}T^{a\mu\nu}_{A} - \frac{1}{4}W^{a}_{\mu\nu A}W^{a\mu\nu}_{A} - \frac{1}{4}B_{\mu\nu A}B^{\mu\nu}_{A} \\ &+ i\overline{L}_A \not{D} L_A + i\overline{E}_A \not{D} E_A + i\overline{Q}_A \not{D} Q_A + i\overline{D}_A \not{D} D_A + i\overline{U}_A \not{D} U_A \\ &+ (\lambda_l)^i_j \overline{L}_{iA} H_A E^j_A + (\lambda_d)^i_j \overline{Q}_{iA} H_A D^j_A + (\lambda_u)^i_j \overline{Q}_{iA} \tilde{H}_A U^j_A + \mathrm{H.c.} \\ &+ (D_\mu H_A)^\dagger D_\mu H_A \ , \end{aligned} \tag{1.42}$$

and

$$\begin{aligned}\mathscr{L}_{B,\mathrm{MTH}} = &-\frac{1}{4}T^{a}_{\mu\nu B}T^{a\mu\nu}_{B} - \frac{1}{4}W^{a}_{\mu\nu B}W^{a\mu\nu}_{B} - \frac{1}{4}B_{\mu\nu B}B^{\mu\nu}_{B} \\ &+ i\overline{L}_B \not{D} L_B + i\overline{E}_B \not{D} E_B + i\overline{Q}_B \not{D} Q_B + i\overline{D}_B \not{D} D_B + i\overline{U}_B \not{D} U_B \\ &+ (\lambda_l)^i_j \overline{L}_{iB} H_B E^j_B + (\lambda_d)^i_j \overline{Q}_{iB} H_B D^j_B + (\lambda_u)^i_j \overline{Q}_{iB} \tilde{H}_B U^j_B + \mathrm{H.c.} \\ &+ (D_\mu H_B)^\dagger D_\mu H_B \ . \end{aligned} \tag{1.43}$$

There are many possible variations, however, which alter the twin sector in some way. For instance, in the so-called fraternal twin Higgs [10] the twin sector only includes the third generation and may not include a photon. Others have considered situations where the gauge symmetry of the twin sector is modified through spontaneous symmetry breaking [11–14]. A more comprehensive review of the spectrum of possibilities is given in [15].

The Mirror Twin Higgs model has some side effects which make it naively in tension with experiment. The cosmological measurement N_{eff} describes the number of relativistic degrees of freedom measured to contribute to the energy density of the Universe. For a BSM model, we require that ΔN_{eff}, the variation from the SM's value for N_{eff}, not exceed observational bounds. By doubling the Standard Model with no changes, we introduce a new set of three neutrinos and a photon. By adding these four new degrees of freedom, ΔN_{eff} approaches 5.5 [16–18]. This is well outside observational limits [6, 19], and so the Twin Higgs model requires some alteration from the Mirror formulation in order to match experiment. This has been successfully accomplished in several ways [12, 16–18, 20–23].

The Twin Higgs framework also accommodates many BSM cosmological successes. These include viable baryogenesis [14, 22, 24–29] and many dark matter possibilities [14, 16, 22–24, 29–45]. There have also been efforts to explain the tension between various measurements of the Hubble parameter within the Twin Higgs framework [35, 46, 47].

Chapter 2

A More Colorful Twin Higgs

In order to resolve some of the issues with the Mirror Twin Higgs, we begin by altering just one aspect of the model: SU(3)$_c$ is extended to become SU(4)$_c$. This expanded SU(4) color group with no Twin Higgs [48, 49], along with SU(5)$_c$ [50, 51], were explored by Foot *et. al.* A 4-dimensional color symmetry would alter the interactions between quarks and the way they form hadronic matter. Because experiment has clearly shown 3-color interactions, a fourth color must be found only at higher energy levels. The SU(4) symmetry would then be spontaneously broken into SU(3) and some residual pieces, just as the SM Higgs broke SU(2)$_L \otimes$U(1)$_Y$. For this to happen, we introduce a new Higgsing scalar field which behaves similarly to the SM Higgs. In this discussion, we name this new field ϕ. However, since we only have observations of the SM sector, there is no such requirement for ϕ to break SU(4) in the twin sector. This allows for unique interactions in the twin sector, which may provide interesting effects for dark matter and N_{eff}.

We add to our Lagrangian this ϕ field and its twin via kinetic terms in their respective sectors, as well as a potential which both mimics and interacts with that of the SM Higgs. Just as with the Mirror Twin Higgs, we separate the Lagrangian into three pieces:

$$\mathscr{L} = \mathscr{L}_A + \mathscr{L}_B + \mathscr{L}_{\text{inter}} \,, \tag{2.1}$$

where $\mathscr{L}_A$ and $\mathscr{L}_B$ add only one term to the Mirror Twin Higgs' Lagrangian. $\mathscr{L}_{\text{inter}}$ contains an expanded potential with terms for both H and ϕ:

$$\mathscr{L}_A = \mathscr{L}_{A,\text{MTH}} + (D_\mu \phi_A)^\dagger D_\mu \phi_A \ , \tag{2.2}$$

$$\mathscr{L}_B = \mathscr{L}_{B,\text{MTH}} + (D_\mu \phi_B)^\dagger D_\mu \phi_B \ , \tag{2.3}$$

$$\begin{aligned} \mathscr{L}_{\text{inter}} = & - M_H^2 |\mathbf{H}|^2 + \lambda_H |\mathbf{H}|^4 - M_\phi^2 |\Phi|^2 + \lambda_\phi |\Phi|^4 + \lambda_{H\phi} |\mathbf{H}|^2 |\Phi|^2 \\ & + \delta_H (|H_A|^4 + |H_B|^4) + \delta_\phi (|\phi_A|^4 + |\phi_B|^4) \\ & + \delta_{H\phi} (|H_A|^2 - |H_B|^2)(|\phi_A|^2 - |\phi_B|^2) \\ & + \mathscr{L}_\Phi \ . \end{aligned} \tag{2.4}$$

Where $|\Phi|^2$ is similar to $|\mathbf{H}|^2$, with its specific form being discussed further on. $\mathscr{L}_\Phi$ is a placeholder for the ϕ interaction terms, which we also determine later. Note that there is no explicit twin Z_2 breaking term in $\mathscr{L}_{\text{inter}}$, meaning that an exchange of A and B subscripts leaves the potential unchanged. This breaking is generated spontaneously when ϕ_A acquires a VEV, while ϕ_B does not. This differentiates the resulting forms of the two sectors, allowing for new, non-SM structures in the twin sector. Similar potentials with spontaneous twin symmetry breaking were explored previously in [11–13, 52].

The $\text{SU}(3)_c \times \text{SU}(2)_L \times \text{U}(1)_Y$ gauge symmetry of the SM has passed all experimental scrutiny. We ensure that this model produces the identical structure, including hypercharge, by including $\text{U}(1)_Y$ in the breaking pattern. To this end, our breaking pattern is $\text{SU}(4)_c \otimes \text{U}(1)_X \rightarrow \text{SU}(3)_c \otimes \text{U}(1)_Y$, where X is posited as an unbroken Abelian gauge symmetry. The breaking field ϕ must therefore interact with the $\text{SU}(4)_c \otimes \text{U}(1)_X$ symmetries and acquire a VEV.

Using group theory, we explore two different ways in which SU(4) can break into SU(3).

First, we represent the SU(4) generators as a 4×4 square matrix with 15 unique elements:

$$C_\mu^a T_a =$$
$$\begin{pmatrix} C_\mu^3+\frac{1}{\sqrt{3}}C_\mu^8+\frac{1}{\sqrt{6}}C_\mu^{15} & C_\mu^1 - iC_\mu^2 & C_\mu^4 - iC_\mu^5 & C_\mu^9 - iC_\mu^{10} \\ C_\mu^1 + iC_\mu^2 & -C_\mu^3 + \frac{1}{\sqrt{3}}C_\mu^8 + \frac{1}{\sqrt{6}}C_\mu^{15} & C_\mu^6 - iC_\mu^7 & C_\mu^{11} - iC_\mu^{12} \\ C_\mu^4 + iC_\mu^5 & C_\mu^6 + iC_\mu^7 & -\frac{2}{\sqrt{3}}C_\mu^8 + \frac{1}{\sqrt{6}}C_\mu^{15} & C_\mu^{13} - iC_\mu^{14} \\ C_\mu^9 + iC_\mu^{10} & C_\mu^{11} + iC_\mu^{12} & C_\mu^{13} + iC_\mu^{14} & -\sqrt{\frac{3}{2}}C_\mu^{15} \end{pmatrix}. \quad (2.5)$$

Notably, within the top left 3×3 section of this matrix we can see the equivalent SU(3) matrix using $C^1 - C^8$ as described in Sec. 1.1. On their own, the 8 generators of SU(3) have an identical structure.

Their presence within SU(4) indicates that a VEV which isolates them would also break SU(4) into SU(3) as desired. Just as the SM symmetries were broken by the Higgs acquiring a VEV, we do the same here with a VEV in the ϕ field. There are two forms the ϕ can take that break the symmetry correctly:

$$\langle\phi\rangle = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & w \end{pmatrix}, \qquad \langle\phi\rangle = \begin{pmatrix} 0 \\ 0 \\ 0 \\ w \end{pmatrix}. \quad (2.6)$$

For our initial symmetry breaking calculations, the choice of form has no effect on the result, as it does not change $|\phi|^2$ or $(D_\mu\phi)^\dagger D_\mu\phi$. In our exploration of additional terms in the Lagrangian, we refer to the 4×4 matrix form and the 4×1 vector form as models I and II respectively.

2.1 SU(4)$_c$⊗U(1)$_X$ Symmetry Breaking

To begin exploring the effects of the ϕ field's VEV on the SU(4)$_c$ symmetry, we first examine the kinetic term $(D_\mu\phi_A)^\dagger D_\mu\phi_A$. In the SM, the breaking of the Higgs kinetic term resulted in gauge bosons $W^\pm$ and Z. Here, we find a similar result.

The covariant derivative in Eq. (1.3), after replacing SU(3)$_c \to$ SU(4)$_c$ and U(1)$_Y \to$ U(1)$_X$, reads:

$$D_\mu = \partial_\mu - ig_X X X_\mu - i\frac{g}{2}\sigma^a A^a_\mu - i\frac{g_s}{2}T^a C^a_\mu \tag{2.7}$$

We begin by writing D_μ in matrix form, making changes to include SU(4)$_c\otimes$ U(1)$_X$:

$$D_\mu = \begin{pmatrix} \begin{matrix}\partial_\mu - \frac{ig_X}{2}X_\mu X_\phi \\ -\frac{ig_s}{2}(C^3_\mu + \frac{1}{\sqrt{3}}C^8_\mu + \frac{1}{\sqrt{6}}C^{15}_\mu)\end{matrix} & \frac{ig_s}{2}(C^1_\mu - iC^2_\mu) & \frac{ig_s}{2}(C^4_\mu - iC^5_\mu) & \frac{ig_s}{2}(C^9_\mu - iC^{10}_\mu) \\ \frac{ig_s}{2}(C^1_\mu + iC^2_\mu) & \begin{matrix}\partial_\mu - \frac{ig_X}{2}X_\mu X_\phi - \frac{ig_s}{2}(-C^3_\mu \\ + \frac{1}{\sqrt{3}}C^8_\mu + \frac{1}{\sqrt{6}}C^{15}_\mu)\end{matrix} & \frac{ig_s}{2}(C^6_\mu - iC^7_\mu) & \frac{ig_s}{2}(C^{11}_\mu - iC^{12}_\mu) \\ \frac{ig_s}{2}(C^4_\mu + iC^5_\mu) & \frac{ig_s}{2}(C^6_\mu + iC^7_\mu) & \begin{matrix}\partial_\mu - \frac{ig_X}{2}X_\mu X_\phi \\ -\frac{ig_s}{2}(-\frac{2}{\sqrt{3}}C^8_\mu + \frac{1}{\sqrt{6}}C^{15}_\mu)\end{matrix} & \frac{ig_s}{2}(C^{13}_\mu - iC^{14}_\mu) \\ \frac{ig_s}{2}(C^9_\mu + iC^{10}_\mu) & \frac{ig_s}{2}(C^{11}_\mu + iC^{12}_\mu) & \frac{ig_s}{2}(C^{13}_\mu + iC^{14}_\mu) & \begin{matrix}\partial_\mu - \frac{ig_X}{2}X_\mu X_\phi \\ + ig_s\sqrt{\frac{3}{8}}C^{15}_\mu\end{matrix} \end{pmatrix} . \tag{2.8}$$

Either form of $\langle\phi\rangle$ in Eq. 2.6 gives the result (sans zeroes):

$$D_\mu\phi = \begin{pmatrix} \frac{ig_s}{2}(C^9_\mu - iC^{10}_\mu) \\ \frac{ig_s}{2}(C^{11}_\mu - iC^{12}_\mu) \\ \frac{ig_s}{2}(C^{13}_\mu - iC^{14}_\mu) \\ -\frac{ig_X}{2}X_\mu X_\phi + ig_s\sqrt{\frac{3}{8}}C^{15}_\mu \end{pmatrix} . \tag{2.9}$$

This result indicates the usefulness of the definition:

$$\xi = \begin{pmatrix} \frac{i}{\sqrt{2}}(C^9_\mu - iC^{10}_\mu) \\ \frac{i}{\sqrt{2}}(C^{11}_\mu - iC^{12}_\mu) \\ \frac{i}{\sqrt{2}}(C^{13}_\mu - iC^{14}_\mu) \end{pmatrix} , \tag{2.10}$$

which allows us to write simply:

$$D_\mu\langle\phi\rangle = \begin{pmatrix} \frac{1}{\sqrt{2}}g_s\xi \\ -\frac{ig_X}{2}X_\mu X_\phi + ig_s\sqrt{\frac{3}{8}}C^{15}_\mu \end{pmatrix} w . \tag{2.11}$$

It is now clear to see the kinetic term result in:

$$\left(D_\mu\langle\phi\rangle\right)^\dagger D_\mu\langle\phi\rangle = \frac{1}{2}g_s^2 w^2 \xi^\dagger_\mu \xi^\mu + \left(\frac{1}{2}g_X X_\mu X_\phi - g_s\sqrt{\frac{3}{8}}C^{15}_\mu\right)^2 w^2 \tag{2.12}$$

As we did in the SM case, we introduce here a notational shorthand

$$\cos\theta_X = \frac{g_s\sqrt{\frac{3}{8}}}{\sqrt{g_X^2 X_\phi^2 + \frac{3}{8}g_s^2}} \ , \quad \sin\theta_X = \frac{g_X X_\phi}{\sqrt{g_X^2 X_\phi^2 + \frac{3}{8}g_s^2}} \tag{2.13}$$

which allows us to rewrite our result in a normalized form:

$$\left(D_\mu\langle\phi\rangle\right)^\dagger D_\mu\langle\phi\rangle = \frac{w^2 g_s^2}{2}\xi_\mu^\dagger\xi^\mu + w^2\left(g_X^2 X_\phi^2 + \frac{3}{8}g_s^2\right)\left(C_\mu^{15}\cos\theta_X - X_\mu\sin\theta_X\right)^2 \ . \tag{2.14}$$

We can now identify the massive state similar to the Z in the SM:

$$B'_\mu \equiv C_\mu^{15}\cos\theta_X - X_\mu\sin\theta_X \ , \quad m_{B'}^2 = 2w^2\left(g_X^2 X_\phi^2 + \frac{3}{8}g_s^2\right) \ , \tag{2.15}$$

and a massless, orthogonal state associated with SM hypercharge U(1)$_Y$:

$$B_\mu = X_\mu\cos\theta_X + C_\mu^{15}\sin\theta_X \ . \tag{2.16}$$

We invert this relationship and find

$$X_\mu = B_\mu\cos\theta_X - B'_\mu\sin\theta_X \ , \qquad C_\mu^{15} = B'_\mu\cos\theta_X + B_\mu\sin\theta_X \ , \tag{2.17}$$

With this, we can rewrite the SU(4) terms of our broken covariant derivative:

$$\frac{g_s}{2}T^a C_\mu^a = \frac{g_s}{2}\lambda^a G_\mu^a + \frac{g_s}{\sqrt{2}}\begin{pmatrix} 0 & \xi_\mu \\ \xi_\mu^\dagger & 0 \end{pmatrix} + g_s\sqrt{\frac{3}{8}}T_c\left(B'_\mu\cos\theta_X + B_\mu\sin\theta_X\right) \ , \tag{2.18}$$

where the λ^a matrices are the generators of SU(3)$_c$ and

$$T_c = \begin{pmatrix} \frac{1}{3} & 0 & 0 & 0 \\ 0 & \frac{1}{3} & 0 & 0 \\ 0 & 0 & \frac{1}{3} & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} . \tag{2.19}$$

By including the U(1)$_X$ part we find

$$D_\mu \supset -iB'_\mu\left[g_s\sqrt{\frac{3}{8}}\cos\theta_X T_c - g_X X_\psi\sin\theta_X\right] - ig'B_\mu\left[X_\psi + X_\phi T_c\right] \ , \tag{2.20}$$

where the hypercharge gauge coupling is given by

$$g' \equiv \frac{g_X g_s \sqrt{\frac{3}{8}}}{\sqrt{g_X^2 X_\phi^2 + \frac{3}{8} g_s^2}} \,. \tag{2.21}$$

The relationship in (2.21) can also be inverted, giving us the value of g_X in terms of known quantities:

$$g_X = \frac{\sqrt{\frac{3}{8}} g_s g'}{\sqrt{\frac{3}{8} g_s^2 - g'^2 X_\phi^2}} \,. \tag{2.22}$$

We have now found the SM hypercharge and coupling in terms of the SU(4) and X charges and couplings. The coefficient of B_μ allows us to translate between X charge and Y hypercharge, such that a state in the $(\mathbf{4}, X)$ representation of SU(4)$_c\times$U(1)$_X$ becomes

$$(\mathbf{4}, X) \to \left(\mathbf{3}, X + \frac{X_\phi}{3}\right) \oplus (\mathbf{1}, X - X_\phi) \,, \tag{2.23}$$

under SU(3)$_c\times$U(1)$_Y$.

In the SM, the quark fields transform under SU(3)$_c\times$SU(2)$_L\times$U(1)$_Y$ as

$$D \sim \left(\mathbf{3}, \mathbf{1}, -\frac{1}{3}\right) \,,\ U \sim \left(\mathbf{3}, \mathbf{1}, \frac{2}{3}\right) \,,\ Q \sim \left(\mathbf{3}, \mathbf{2}, \frac{1}{6}\right) \,. \tag{2.24}$$

Using the relation in Eq. (2.23) and the required Y hypercharge values of the SM, we find that under the original unbroken SU(4)$_c\times$SU(2)$_L\times$U(1)$_X$ symmetry the quarks must transform as

$$D \sim \left(\mathbf{4}, \mathbf{1}, -\frac{X_\phi + 1}{3}\right) \,,\ U \sim \left(\mathbf{4}, \mathbf{1}, \frac{2 - X_\phi}{3}\right) \,,\ Q \sim \left(\mathbf{4}, \mathbf{2}, \frac{1 - 2X_\phi}{6}\right) \,. \tag{2.25}$$

For completeness we record the lepton and Higgs fields, though their values do not change from the SM,

$$E \sim (\mathbf{1}, \mathbf{1}, -1) \,,\ L \sim \left(\mathbf{1}, \mathbf{2}, -\frac{1}{2}\right) \,,\ H \sim \left(\mathbf{1}, \mathbf{2}, \frac{1}{2}\right) \,. \tag{2.26}$$

The quark definitions imply that after symmetry breaking, the broken fourth color component of each quark becomes a new fermion with SM charges

$$D_4 \sim \left(\mathbf{1}, \mathbf{1}, -\frac{1 + 4X_\phi}{3}\right) \,,\ U_4 \sim \left(\mathbf{1}, \mathbf{1}, \frac{2 - 4X_\phi}{3}\right) \,,\ Q_4 \sim \left(\mathbf{1}, \mathbf{2}, \frac{1 - 8X_\phi}{6}\right) \,. \tag{2.27}$$

Using the relation in Eq. (1.35), we find the fermions have electric charges of

$$q_D = -\frac{1+4X_\phi}{3} \ , \quad q_U = \frac{2-4X_\phi}{3} \ . \tag{2.28}$$

As there is no choice of X_ϕ such that these both vanish, it must be that this construction predicts at least one new electrically charged state. Experimental bounds on new charged states are at least as high as about 100 GeV [53]. If these charged fermions were on the same mass scale as SM fermions, they would certainly have been discovered. Therefore, they must have additional mass terms provided by the ϕ VEV within $\mathscr{L}_{\text{inter}}$ in Eq. (2.2). The form of these terms varies depending on the representation chosen for ϕ, so we now divide our discussion between models I and II.

2.1.1 Model I

In model I, we introduce the ϕ field in the **10** representation of SU(4)$_c$. A 10-dimensional object that can be multiplied by the 4×4 SU(4) matrices is most easily represented as a symmetric matrix, as shown in Equation (2.6). Labelling this field by its representations under each of the gauge symmetries SU(4)$_c$⊗SU(2)$_L$⊗U(1)$_X$ represents it as

$$\phi \sim \left(\mathbf{10}, \mathbf{1}, X_\phi\right) \ , \tag{2.29}$$

where we can also see that it is a singlet under SU(2)$_L$ and has a U(1)$_X$ charge of X_ϕ.

With ϕ in the **10** representation of SU(4)$_c$, we are ready to determine the terms of $\mathscr{L}_\phi$ from Eq. (2.2). Coupling between ϕ and quarks is allowed in combinations which produce gauge-invariant multiplets of all gauge groups. For the **10** representation, we can include the Lagrangian terms:

$$\mathscr{L}_\phi = \lambda_{A1}\overline{u}_R\phi(d_R)^c + \lambda_{A2}\overline{Q}_L\phi(Q_L)^c + \text{H.c.} \ . \tag{2.30}$$

Let's examine the gauge invariance of these terms in SU(4):

$$\begin{aligned}\overline{u}_R\phi(d_R)^c &\rightarrow \overline{\mathbf{4}}\otimes\mathbf{10}\otimes\overline{\mathbf{4}}\\ &= (\mathbf{36}\oplus\mathbf{4})\otimes\overline{\mathbf{4}}\\ &= \left(\mathbf{36}\otimes\overline{\mathbf{4}}\right)\oplus\left(\mathbf{4}\otimes\overline{\mathbf{4}}\right)\\ &= \left(\mathbf{36}\otimes\overline{\mathbf{4}}\right)\oplus\mathbf{15}\oplus\mathbf{1}\ . \end{aligned} \tag{2.31}$$

Since this multiplication produces a gauge singlet,the **1** portion, it is an allowed coupling. In $\mathrm{U}(1)_Y$, hypercharges must sum to zero:

$$Y_\phi - 2Y_Q = 0\ . \tag{2.32}$$

When ϕ takes on its VEV, the terms in Eq. (2.30) become mass terms, giving the 4$^{\text{th}}$ component mass:

$$\lambda_{A1}w\overline{u}_4(d_4)^c + \lambda_{A2}w\overline{Q}_4(Q_4)^c + \text{H.c.}\ . \tag{2.33}$$

Unfortunately, it was shown in [49] for the case of three generations of quarks that the λ_{A2} mass term leads to a new state of about 20 GeV. As we shall see, these states have nonzero electric charge. Such a particles would have been discovered at LEP [53]. Thus, model I with ϕ in the **10** representation is ruled out, and we do not consider it further.

2.1.2 Model II

In model II, which we use throughout the rest of this discussion, we instead introduce the ϕ field in the fundamental **4** representation of $\mathrm{SU}(4)_c$. In this case, gauge invariant coupling between ϕ and the quarks requires the introduction of new fields with appropriate quantum numbers. To correctly form gauge invariant mass terms of that type, we must introduce new color singlet fields, which we name F^c, P^c, V^c. These allow us to describe Yukawa interactions between ϕ and the fourth-component quarks:

$$\lambda_F\overline{Q}\phi F^c + \lambda_P\overline{U}\phi P^c + \lambda_V\overline{D}\phi V^c + \text{H.c.}\ . \tag{2.34}$$

The charge structures dictate that the new fields have quantum numbers:

$$F^c \sim \left(\mathbf{1},\mathbf{2},\frac{1-8X_\phi}{6}\right)\ ,\ \ P^c \sim \left(\mathbf{1},\mathbf{1},\frac{2-4X_\phi}{3}\right)\ ,\ \ V^c \sim \left(\mathbf{1},\mathbf{1},-\frac{1+4X_\phi}{3}\right)\ . \tag{2.35}$$

so that each term is neutral under all charges. F is left-handed while P and V are right-handed. The charge conjugation has been used to keep the SM-like situation that SU$(2)_L$ couples to left-handed fields and not to right-handed ones.

The left- and right-handedness of these new particles has the potential to introduce anomalies into the theory. Anomalies are the result of a classically valid symmetry being disrupted in a quantum theory, introducing unexpected interactions. Here, we require new anomalies to vanish so as to not alter observed signals. The SU$(2)_L$ anomaly is governed by the sum over hypercharges of the fields which are in the **2** representation of SU$(2)_L$. As we have only introduced one such field, F, we require its charge X_F to vanish. From Eq. (2.35), this implies that

$$X_\phi = \frac{1}{8}\ . \tag{2.36}$$

The remaining anomalies cancel if

$$X_P^3 + X_V^3 = 0 \ \text{ and } \ X_P + X_V = 0\ . \tag{2.37}$$

This is indeed satisfied for this same choice of X_ϕ, which also provides

$$X_P = -X_V = \frac{1}{2}\ . \tag{2.38}$$

We note that this also implies the fourth component quark fields have electric charges of $\pm\frac{1}{2}$.

The additional fermion fields gain mass by the same Higgs mechanism as SM particles, through the Higgs couplings

$$\lambda_{FV}\overline{F}^c H V^c + \lambda_{FP}\overline{F}^c \widetilde{H} P^c + \text{ H.c.}\ , \tag{2.39}$$

where $\widetilde{H}$ is defined as in the up-type quark Higgs Yukawa coupling in the SM. These provide another source of mass for these fermions. More importantly, in the twin sector, where ϕ_B does not get a VEV, these mass terms prevent the introduction of new light states.

2.2 Symmetry Breaking and Gauge Bosons

Now that model II is described in terms of the SM symmetries plus residual pieces, we are ready to once again break SU(2)$_L$⊗U(1)$_Y$ →U(1)$_{\text{E\&M}}$ using the Higgs field. As the Higgs field is now charged under X charge rather than Y hypercharge, there are slight differences to account for. The applicable covariant derivative matrix for the Higgs under SU(2)$_L$⊗U(1)$_X$ is

$$\begin{aligned}D_\mu &= \partial_\mu - i\frac{g}{2}\sigma^a A^a_\mu - i\frac{g_X}{2}\mathbb{I}X_\mu \\ &= \frac{1}{\sqrt{2}}\begin{pmatrix}\partial_\mu - \frac{i}{2}(g_X X_\mu + gA^3_\mu) & \frac{ig}{2}(A^1_\mu - iA^2_\mu) \\ \frac{ig}{2}(A^1_\mu + iA^2_\mu) & \partial_\mu - \frac{i}{2}(g_X X_\mu - gA^3_\mu)\end{pmatrix}\end{aligned} \tag{2.40}$$

The $W^\pm$ bosons form in the same way as in the SM case, so we define them here and omit them from future calculations:

$$W^\pm_\mu = \frac{1}{\sqrt{2}}\left(A^1_\mu \mp iA^2_\mu\right) \ . \tag{2.41}$$

The remaining terms correspond to the bottom right element of the derivative matrix. We adjust the covariant derivative to show the broken symmetry and group the massless vectors

$$\begin{aligned}D_\mu\begin{pmatrix}0\\ v\end{pmatrix} =& \partial_\mu v - i\frac{g_X}{2}X_\mu v + i\frac{g}{2}A^3_\mu v \\ =& i\frac{g}{2}A^3_\mu v - i\frac{g_X}{2}\left(B_\mu\cos\theta_X - B'_\mu\sin\theta_X\right)v \\ =& i\frac{vg_X}{2}\sin\theta_X B'_\mu + \frac{i}{2}\left(gA^3_\mu - g'B_\mu\right)v \ .\end{aligned} \tag{2.42}$$

Note that from Eqs. (2.13) and (2.21), we see that $g_X\cos\theta_X = g'$.

We can now define a massless photon and a massive boson in anticipation of expanding the kinetic term:

$$\overline{Z}_\mu = \frac{1}{\sqrt{g^2+g'^2}}\left(gA^3_\mu - g'B_\mu\right) \ , \qquad A_\mu = \frac{1}{\sqrt{g^2+g'^2}}\left(g'A^3_\mu + gB_\mu\right) \ . \tag{2.43}$$

Then the mass term from $|D_\mu\phi|^2$ is easily identified as:

$$\frac{1}{2}\left[D_\mu\begin{pmatrix}0\\v\end{pmatrix}\right]^\dagger D_\mu\begin{pmatrix}0\\v\end{pmatrix}\supset\frac{v^2}{8}\left(g_X\sin\theta_X B'_\mu+\sqrt{g^2+g'^2}\overline{Z}_\mu\right)\left(g_X\sin\theta_X B'^\mu+\sqrt{g^2+g'^2}\overline{Z}^\mu\right)\ . \tag{2.44}$$

We now see that B' and $\overline{Z}$ both gain mass from the Higgs VEV. Due to the cross terms in multiplying the above mass term, B' and $\overline{Z}$ are not actually in their mass basis yet. By diagonalizing their mass matrix, we can see two distinct massive neutral bosons. First we substitute the SM Z mass, m_{Z_0}, and write the terms in a matrix. As a shorthand, $s_X\equiv\sin\theta_X$,

$$m_{Z_0}=\frac{v}{2}\sqrt{g^2+g'^2}\ , \tag{2.45}$$

$$\frac{1}{2}\left(B'_\mu,\overline{Z}_\mu\right)\begin{pmatrix}m_{B'}^2+\frac{v^2}{4}g_X^2s_X^2 & m_{Z_0}\frac{v}{2}g_Xs_X\\ m_{Z_0}\frac{v}{2}g_Xs_X & m_{Z_0}^2\end{pmatrix}\begin{pmatrix}B'^\mu\\ \overline{Z}^\mu\end{pmatrix}\ . \tag{2.46}$$

The diagonal masses are

$$m_\pm^2= \tag{2.47}$$
$$\frac{1}{2}\left(m_{B'}^2+m_{Z_0}^2+\frac{v^2}{4}g_X^2s_X^2\pm\sqrt{(m_{B'}^2-m_{Z_0}^2)^2+(m_{B'}^2+m_{Z_0}^2)\frac{v^2}{2}g_X^2s_X^2+\frac{v^4}{16}g_X^4s_X^4}\right)\ .$$

The mass eigenstates are defined by

$$Z'_\mu=B'_\mu c_M-\overline{Z}_\mu s_M\ ,\qquad Z_\mu=B'_\mu s_M+\overline{Z}_\mu c_M\ , \tag{2.48}$$

where $m_+=m_{Z'}$ and $m_-=m_Z$. The mixing angles, $\sin\theta_M\equiv s_M$ and $\cos\theta_M\equiv c_M$, are defined by

$$\cos 2\theta_M=\frac{m_{B'}^2+\frac{v^2}{4}g_X^2s_X^2-m_{Z_0}^2}{m_{Z'}^2-m_Z^2}\ ,\ \ \sin 2\theta_M=-\frac{m_{Z_0}vg_Xs_X}{m_{Z'}^2-m_Z^2}\ . \tag{2.49}$$

We note that as the energy scale of color breaking by ϕ must be much greater than that of electroweak breaking by H, $v\ll w$. Using this, we can expand the mass eigenstates to find, to leading order in v/w, that

$$m_{Z'}^2\approx m_{B'}^2+\frac{v^2}{4}g_X^2s_X^2\ ,\ \ m_Z^2\approx m_{Z_0}^2-\frac{m_{Z_0}^2v^2g_X^2s_X^2}{4m_{B'}^2}\ . \tag{2.50}$$

The mixing angle to the same order in v/w, is

$$\theta_M \sim -\frac{m_{Z_0} v g'^2}{3w^2 g_s^3 \sqrt{6}} \approx -2\times 10^{-4}\left(\frac{1\text{ TeV}}{w}\right)^2 . \tag{2.51}$$

We rewrite the unbroken gauge fields in terms of the mass eigenstate fields, in order to rewrite the broken covariant derivative:

$$X_\mu = c_X c_W A_\mu - (s_X s_M + c_X s_W c_M) Z_\mu + (c_X s_W s_M - s_X c_M) Z'_\mu \tag{2.52}$$

$$A^3_\mu = s_W A_\mu + c_W c_M Z_\mu - c_W s_M Z'_\mu \tag{2.53}$$

$$C^{15}_\mu = s_X c_W A_\mu + (c_X s_M - s_X s_W c_M) Z_\mu + (s_X s_W s_M + c_X c_M) Z'_\mu . \tag{2.54}$$

From the covariant derivative we can use these relations to determine the couplings of the various fields. We find that

$$\begin{aligned}
&g_X X_\psi X_\mu + g T^3 A^3_\mu + g_s \sqrt{\frac{3}{8}} T_c C^{15}_\mu = \\
&A_\mu \left[g_X c_X c_W X_\psi + g s_W T^3 + g_s \sqrt{\frac{3}{8}} s_X c_W T_c \right] \\
&+ Z_\mu \left[-g_X (s_X s_M + c_X s_W c_M) X_\psi + g c_W c_M T^3 + g_s \sqrt{\frac{3}{8}} (c_X s_M - s_X s_W c_M) T_c \right] \\
&+ Z'_\mu \left[g_X (c_X s_W s_M - s_X c_M) X_\psi - g c_W s_M T^3 + g_s \sqrt{\frac{3}{8}} (c_X c_M + s_X s_W s_M) T_c \right] .
\end{aligned} \tag{2.55}$$

We note that many constants in the A_μ term can be simplified, since

$$e = g s_W = g' c_W , \quad g' = g_X c_X = 8\sqrt{\frac{3}{8}} g_s s_X . \tag{2.56}$$

This lets us define the electric charge from the coefficient of A_μ as

$$Q_\psi = X_\psi + T^3 + \frac{1}{8} T_c . \tag{2.57}$$

So, we find

$$\begin{aligned}
&g_X X_\psi X_\mu + gT^3 A^3_\mu + g_s\sqrt{\frac{3}{8}}T_c C^{15}_\mu = \\
&eQ_\psi A_\mu + Z_\mu \frac{g}{c_W}\left[c_M(T^3 - s_W^2 Q_\psi) + \frac{s_M s_W c_X}{s_X}\left(Q_\psi - T^3 - \frac{X_\psi}{c_X^2}\right)\right] \\
&+ Z'_\mu \frac{g}{c_W}\left[\frac{c_M s_W c_X}{s_X}\left(Q_\psi - T^3 - \frac{X_\psi}{c_X^2}\right) - s_M(T^3 - s_W^2 Q_\psi)\right] \ .
\end{aligned} \tag{2.58}$$

The coefficients of these terms tell us all of the interactions between charged particles and gauge bosons. The photon couplings are as expected. Because the photon is the carrier of the electromagnetic force, its coupling to any particle is simply e times that particle's electric charge. The SM Z couples primarily through the weak force. Here, the Z has small modifications from the SM couplings; as $s_M \to 0$, it matches the SM exactly. Z', which does not appear in the SM, is mainly the result of color breaking. Intuitively, then, it couples most strongly to quarks due to the first T_c term depending on color.

2.3 Charged Gauge Bosons

The remaining gauge bosons in the model are the $W^\pm$ and the ξ. These formed as the result of symmetry breaking, when the off-diagonal terms of the covariant derivative grouped together with their Hermitian conjugates. Their mass terms in Eqs. (1.36) and (2.12) show this, being $\sim m_W^2 W^+ W^-$ and $\sim m_\xi^2 \xi^\dagger \xi$. The $W^\pm$ boson shows no new interactions from the SM, as $SU(2)_L$ was unaltered. The ξ boson, however, has new interactions with the quarks.

In the kinetic terms of the quark fields, they are acted on by the covariant derivative. We have now seen that this covariant derivative carries gauge bosons after symmetry breaking. These kinetic terms are:

$$i\overline{D}\not{D}D + i\overline{U}\not{D}U + i\overline{Q}\not{D}Q \ . \tag{2.59}$$

Because the ξ appears in the off-diagonal quadrants of $\not{D}$, we will focus only on those.

For these terms, we denote the quarks in two components: the SM color triplet and the fourth color component, as follows:

$$U = \begin{pmatrix} U_i \\ U_4 \end{pmatrix} , \qquad D = \begin{pmatrix} D_i \\ D_4 \end{pmatrix} , \qquad Q = \begin{pmatrix} Q_i \\ Q_4 \end{pmatrix} . \tag{2.60}$$

Expanding the kinetic terms like this, we find interactions in which the ξ interacts with one SM quark and one fourth-color quark:

$$\frac{g_s}{\sqrt{2}} \left[\overline{U}_i \not{\xi} U_4 + \overline{D}_i \not{\xi} D_4 + \overline{Q}_i \not{\xi} Q_4 \right] + \text{ H.c. } . \tag{2.61}$$

Gauge invariance, as always, holds for these terms. ξ and $\overline{U}_i$ are in the fundamental and anti-fundamental representations of SU(3)$_c$, so they are color-neutral. Because the total electric charge of any term must be zero, we can quickly see that ξ^μ has an electric charge of $Q_\xi = \frac{1}{6}$. This combination of color and electric charge is unique. Therefore, ξ^μ can only be pair-produced in collider experiments through $\xi^\mu \xi^\dagger_\mu$ couplings to electrically neutral states, such as gluons and other gauge bosons.

2.4 New Fermion Masses

In this section we determine the mass eigenstates of the new BSM fermions: F^c, P^c, V^c, and the fourth-color quarks. We express the SU(2)$_L$ doublet fields as

$$F^c = \begin{pmatrix} F^c_u \\ F^c_d \end{pmatrix} , \quad Q_4 = \begin{pmatrix} U_{L4} \\ D_{L4} \end{pmatrix} . \tag{2.62}$$

From the interactions in Eqs. (2.34) and (2.39), we put together the fermion mass matrices

$$(U_4, F^c_u)\, \mathscr{M}_u \begin{pmatrix} P^c \\ U_{L4} \end{pmatrix} , \quad (D_4, F^c_d)\, \mathscr{M}_d \begin{pmatrix} V^c \\ D_{L4} \end{pmatrix} , \tag{2.63}$$

with coefficients (from the highest generations where available)

$$\mathscr{M}_u = \begin{pmatrix} \lambda_P w & m_{tA} \\ \lambda_{FP}\frac{v_A}{\sqrt{2}} & \lambda_F w \end{pmatrix} , \quad \mathscr{M}_d = \begin{pmatrix} \lambda_V w & m_{bA} \\ \lambda_{FV}\frac{v_A}{\sqrt{2}} & \lambda_F w \end{pmatrix} , \tag{2.64}$$

where we denote the Higgs VEV v_A and fermion masses of the visible sector with an A. By diagonalizing these mass matrices, we find the physical masses are

$$m_{u\pm}^2 = \frac{1}{2}\left[w^2(\lambda_P^2+\lambda_F^2) + m_{tA}^2 + \lambda_{FP}^2\frac{v_A^2}{2} \right. \\ \left. \pm\sqrt{\left(\lambda_P^2 w^2 + \lambda_F^2 w^2 + m_{tA}^2 + \lambda_{FP}^2\frac{v_A^2}{2}\right)^2 - 4\left(\lambda_F\lambda_P w^2 - \lambda_{FP} m_{tA}\frac{v_A}{\sqrt{2}}\right)^2}\, \right] . \tag{2.65}$$

To leading order in v_A^2/w^2 these become

$$m_{u+}^2 = w^2\lambda_P^2 + \mathscr{O}\left(\frac{v_A^2}{w^2}\right) , \quad m_{u-}^2 = w^2\lambda_F^2 + \mathscr{O}\left(\frac{v_A^2}{w^2}\right) . \tag{2.66}$$

This shows that these fermions have masses on the order of the ϕ VEV w, as long as the Yukawa couplings λ_P and λ_F are not too small. This is in contrast to the low-mass charged fermions in model I.

We use the physical states of these fermions for naturalness considerations. They are given by

$$U_{L+} = \cos\theta_{UL} P^c - \sin\theta_{UL} U_{L4} , \qquad U_{L-} = \sin\theta_{UL} P^c + \cos\theta_{UL} U_{L4} , \tag{2.67}$$

$$U_{R+} = \cos\theta_{UR} U_4 - \sin\theta_{UR} F_u^c , \qquad U_{R-} = \sin\theta_{UR} U_4 + \cos\theta_{UR} F_u^c , \tag{2.68}$$

where

$$\cos 2\theta_{UL} = \frac{w^2(\lambda_P^2-\lambda_F^2) + m_{tA}^2 - \lambda_{FP}^2 v_A^2/2}{m_{u+}^2 - m_{u-}^2} , \quad \sin 2\theta_{UL} = -2w\frac{m_{tA}\lambda_F + \lambda_P\lambda_F v_A/\sqrt{2}}{m_{u+}^2 - m_{u-}^2} , \tag{2.69}$$

$$\cos 2\theta_{UR} = \frac{w^2(\lambda_P^2-\lambda_F^2) - m_{tA}^2 + \lambda_{FP}^2 v_A^2/2}{m_{u+}^2 - m_{u-}^2} , \quad \sin 2\theta_{UR} = -2w\frac{m_{tA}\lambda_P + \lambda_F\lambda_F v_A/\sqrt{2}}{m_{u+}^2 - m_{u-}^2} . \tag{2.70}$$

A similar set of equations holds for $\mathscr{M}_d$ with $U \to D$, $u \to d$, $m_t \to m_b$, and $P \to V$.

In the twin sector the $SU(4)_c$ color is unbroken, so there are no w contributions to the mass. The fourth-component of the quark does not separate from the rest of the multiplet, and gets the same mass. The F^c, P^c, and V^c states also get masses from the normal Higgs symmetry breaking mechanism. These are given by

$$m_{FP} \equiv \lambda_{FP} \frac{v_B}{\sqrt{2}} \ , \quad m_{FV} \equiv \lambda_{FV} \frac{v_B}{\sqrt{2}} \ . \tag{2.71}$$

2.5 Vacuum Expectation Value

Having verified that this model is not excluded in the same way as model I, we now include calculations to find its VEV. The scalar potential in $\mathscr{L}_{\text{inter}}$ can have non-zero local minima, to which the fields H and ϕ can settle at low energies. Minimizing the potential in its four fields simultaneously yields the values of these minima.

Having chosen the fundamental **4** representation for ϕ, the VEVs take the forms:

$$\langle H \rangle = \begin{pmatrix} 0 \\ v \end{pmatrix} \quad \text{and} \quad \langle \phi \rangle = \begin{pmatrix} 0 \\ 0 \\ 0 \\ w \end{pmatrix} . \tag{2.72}$$

Using subscripts to differentiate A and B sectors, and inserting these into the scalar potential given in Eq. (2.2) yields:

$$\begin{aligned} V_{\text{scalar}}(v_a, v_b, w_a, w_b) = & - M_H^2 (v_a^2 + v_b^2) + \lambda_H (v_a^2 + v_b^2)^2 \\ & - M_\phi^2 (w_a^2 + w_b^2) + \lambda_\phi (w_a^2 + w_b^2)^2 + \lambda_{H\phi} (v_a^2 + v_b^2)(w_a^2 + w_b^2) \\ & + \delta_H (v_a^4 + v_a^4) + \delta_\phi (w_a^4 + w_b^4) + \delta_{H\phi} (v_a^2 - v_b^2)(w_a^2 - w_b^2) \ . \end{aligned} \tag{2.73}$$

Next we differentiate in terms of each variable, finding a system of equations which gives the simultaneous minimum for all fields.

$$\partial_{H_A} : 0 = v_a \left[-M_H^2 + 2\lambda_H \left(v_a^2 + v_b^2\right) + \lambda_{H\phi}\left(w_a^2 + w_b^2\right) + 2\delta_H v_a^2 + \delta_{H\phi}\left(w_a^2 - w_b^2\right)\right] \ , \tag{2.74}$$

$$\partial_{H_B} : 0 = v_b \left[-M_H^2 + 2\lambda_H \left(v_a^2 + v_a^2\right) + \lambda_{H\phi}\left(w_a^2 + w_b^2\right) + 2\delta_H v_b^2 - \delta_{H\phi}\left(w_a^2 - w_b^2\right)\right] \ , \tag{2.75}$$

$$\partial_{\phi_A} : 0 = w_a \left[-M_\phi^2 + 2\lambda_\phi \left(w_a^2 + w_b^2\right) + \lambda_{H\phi}\left(v_a^2 + v_b^2\right) + 2\delta_\phi w_a^2 + \delta_{H\phi}\left(v_a^2 - v_b^2\right)\right] \ , \tag{2.76}$$

$$\partial_{\phi_B} : 0 = w_b \left[-M_\phi^2 + 2\lambda_\phi \left(w_a^2 + w_b^2\right) + \lambda_{H\phi}\left(v_a^2 + v_b^2\right) + 2\delta_\phi w_b^2 - \delta_{H\phi}\left(v_a^2 - v_b^2\right)\right] \ . \tag{2.77}$$

To restrict the set of possible solutions to those that do not contradict the SM but still provide new phenomena, we are interested in only cases in which $v_a, v_b, w_a \neq 0$ and $w_b = 0$. This simplifies the equations to:

$$\partial_{H_A} : 0 = -M_H^2 + 2\lambda_H \left(v_a^2 + v_b^2\right) + \lambda_{H\phi} w_a^2 + 2\delta_H v_a^2 + \delta_{H\phi} w_a^2 \ , \tag{2.78}$$

$$\partial_{H_B} : 0 = -M_H^2 + 2\lambda_H \left(v_a^2 + v_b^2\right) + \lambda_{H\phi} w_a^2 + 2\delta_H v_b^2 - \delta_{H\phi} w_a^2 \ , \tag{2.79}$$

$$\partial_{\phi_A} : 0 = -M_\phi^2 + 2\lambda_\phi w_a^2 + \lambda_{H\phi}\left(v_a^2 + v_b^2\right) + 2\delta_\phi w_a^2 + \delta_{H\phi}\left(v_a^2 - v_b^2\right) \ , \tag{2.80}$$

$$\partial_{\phi_B} : 0 = w_b \ . \tag{2.81}$$

Adding and subtracting the equations gives the relations:

$$\partial_{H_A} - \partial_{H_B} : 0 = \delta_H \left(v_a^2 - v_b^2\right) + \delta_{H\phi} w_a^2 \ , \tag{2.82}$$

$$\partial_{H_A} + \partial_{H_B} : 0 = -M_H^2 + \left(2\lambda_H + \delta_H\right)\left(v_a^2 + v_b^2\right) + \lambda_{H\phi} w_a^2 \ , \tag{2.83}$$

which yields

$$v_a^2 - v_b^2 = -\frac{\delta_{H\phi}}{\delta_H} w_a^2 \ , \tag{2.84}$$

$$v_a^2 + v_b^2 = \frac{M_H^2 - \lambda_{H\phi} w_a^2}{2\lambda_H + \delta_H} \ . \tag{2.85}$$

Using these relations as we solve Eq. (2.76) now gives:

$$w_a^2 = \frac{1}{2(\lambda_\phi + \delta_\phi)}\left[M_\phi^2 - \lambda_{H\phi}(v_a^2 + v_b^2) + \delta_{H\phi}(v_a^2 - v_b^2)\right]$$
$$= \frac{M_\phi^2(\delta_H + 2\lambda_H) - M_H^2\lambda_{H\phi}}{\delta_{H\phi}^2(1 + 2\frac{\lambda_H}{\delta_H}) - \lambda_{H\phi}^2 + 2(\delta_H + 2\lambda_H)(\lambda_\phi + \delta_\phi)} \,. \tag{2.86}$$

We can now use w_a to solve for H VEVs:

$$v_a^2 = \frac{M_H^2}{4\lambda_H + 2\delta_H} - \frac{\delta_{H\phi}M_\phi^2}{4\delta_H(\lambda_\phi + \delta_\phi) - 2\delta_{H\phi}^2} \,, \tag{2.87}$$

$$v_b^2 = \frac{M_H^2}{4\lambda_H + 2\delta_H} + \frac{\delta_{H\phi}M_\phi^2}{4\delta_H(\lambda_\phi + \delta_\phi) - 2\delta_{H\phi}^2} \,. \tag{2.88}$$

2.6 Exclusion Criteria

Now that all of the features of the model have been determined, it is useful to determine the current experimental bounds on these phenomena. As a simple first example of the criteria, the model predicts a change to the mass of the Z boson from the SM value. From Eq. (2.50), we find that

$$m_Z^2 - m_{Z_0}^2 \approx \frac{m_{Z_0}^2 v^2 g_X^2 \sin\theta_X^2}{4m_{B'}^2} \,. \tag{2.89}$$

The most current estimate of the Z mass, m_{Z_0}, from the Particle Data Group (PDG) is 91.1876 ± 0.0021 GeV [54]. This, using substitutions from Eqs. (2.13), (2.15), and (2.22), shows that our predicted Z mass is within 2σ of the experimental value when

$$w \gtrsim 275 \text{ GeV} \,. \tag{2.90}$$

This provides a lower limit on the energy scale of color breaking. This is well below the TeV scale we expect, so the Z mass does not meaningfully limit the theory.

The couplings of the Z' boson shown in Eq. (2.58) give insight to the production and detection of this new boson. The couplings show that Z' can be produced through Drell-Yan processes or similar, in which a quark and anti-quark annihilate and produce a neutral boson. It can then decay into either a pair of SM fermions, new BSM fermions, Zh, or $W^{\pm}$. Of these signals, the cleanest channel to observe is a pair of SM leptons. However, as noted following Eq. (2.58), the coupling to leptons is much smaller than for the the coupling to quarks and other bosons. A search done for similar neutral bosons for other theories determined that the mass of such a boson is unlikely to be less than ≈ 3 TeV [55]. In conjunction with Eq. (2.50), this sets a lower limit on the ϕ VEV w in the TeV range.

The ξ^{μ} bosons, due to their unusual color charge, can only be pair-produced, which requires double the energy as producing a single particle like the Z'. Therefore, the signals detectable from ξ^{μ} production are expected to be much weaker than those from the Z', giving it a less significant role in establishing bounds.

The BSM fermions F^c, V^c, and P^c can be analyzed similarly. These have electric charge $\pm\frac{1}{2}$. From Eq. (2.58) we see these fermions couple to A_{μ}, Z_{μ}, and Z'_{μ} with significant strength. They can also be pair produced through Drell-Yan processes at the LHC. Initial estimates of the bounds on states with exotic charges, from review of [56, 57], indicate that they are absent up to $\sim$500 GeV.

Taken together, these varying criteria seem to point to TeV range energies for the w VEV and the newly introduced particles. As all of these bounds are tied to w in some way, any experimental bound can be evaded by making the VEV larger. However, this increase tends to make the model less natural, as discussed below.

2.7 Naturalness Considerations

As a final consideration, we determine the potential of the observed Higgs boson by way of the Coleman-Weinberg [58] potential. This potential aggregates all one-loop corrections to the Higgs' potential up to a UV cutoff Λ from our Lagrangian:

$$V_{\rm CW} = -\frac{N_c}{8\pi^2}\Lambda^2 {\rm Tr}\mathcal{M}^2 - \frac{N_c}{16\pi^2}{\rm Tr}\left[\mathcal{M}^4\left(\ln\frac{\mathcal{M}^2}{\Lambda^2} - \frac{1}{2}\right)\right] , \tag{2.91}$$

where N_c is the number of colors of the included fermions and $\mathcal{M}$ is the matrix of scalar field dependent fermion masses.

As a warm-up, we consider the generated potential in the standard Mirror Twin Higgs framework. We use a nonlinear parameterization of the Higgs boson (see for instance [59]) such that

$$H_A = \frac{f}{\sqrt{\boldsymbol{h}^\dagger\boldsymbol{h}}}\boldsymbol{h}\sin\left(\frac{\sqrt{\boldsymbol{h}^\dagger\boldsymbol{h}}}{f}\right) , \quad H_B = \begin{pmatrix} 0 \\ f\cos\left(\frac{\sqrt{\boldsymbol{h}^\dagger\boldsymbol{h}}}{f}\right)\end{pmatrix} , \tag{2.92}$$

where f is the VEV of the four-component H given in Eq. (1.39). The fermion mass matrix in the top-quark sector is given by

$$(t_{RA}, t_{RB})\begin{pmatrix}\lambda_t f\widetilde{H}_A & 0 \\ 0 & \lambda_t f\widetilde{H}_B\end{pmatrix}\begin{pmatrix} Q_A \\ Q_B\end{pmatrix} . \tag{2.93}$$

Note that the quark masses are given by

$$m_{tA} = \lambda_t f\sin\vartheta , \quad m_{tB} = \lambda_t f\cos\vartheta , \tag{2.94}$$

where $\vartheta \equiv v/(f\sqrt{2})$.

The Λ^2 term of the potential is independent of $\boldsymbol{h}$, because the trace combines $\sin^2(\sqrt{\mathbf{h}^\dagger\mathbf{h}}/f) + \cos^2(\sqrt{\mathbf{h}^\dagger\mathbf{h}}/f)$ from the parameterized values of H_A and H_B:

$${\rm Tr}\mathcal{M}^\dagger\mathcal{M} = \lambda_t^2 f^2 {\rm Tr}\begin{pmatrix}\sin^2\left(\frac{\sqrt{\boldsymbol{h}^\dagger\boldsymbol{h}}}{f}\right) & 0 \\ 0 & \cos^2\left(\frac{\sqrt{\boldsymbol{h}^\dagger\boldsymbol{h}}}{f}\right)\end{pmatrix} = \lambda_t^2 f^2 = m_{tA}^2 + m_{tB}^2 . \tag{2.95}$$

Because this value is a constant, it does not contribute to the $\boldsymbol{h}^\dagger\boldsymbol{h}$ part of the Higgs potential, and in particular does not lead to a mass proportional to Λ^2. We next evaluate

$$\mathrm{Tr}\left[\mathscr{M}^4\left(\ln\frac{\mathscr{M}^2}{\Lambda^2}-\frac{1}{2}\right)\right]=$$

$$\lambda_t^4 f^4\left\{\sin^4\left(\frac{\sqrt{\boldsymbol{h}^\dagger\boldsymbol{h}}}{f}\right)\left[\ln\frac{\lambda_t^2 f^2\sin^2\left(\frac{\sqrt{\boldsymbol{h}^\dagger\boldsymbol{h}}}{f}\right)}{\Lambda^2}-\frac{1}{2}\right]+\cos^4\left(\frac{\sqrt{\boldsymbol{h}^\dagger\boldsymbol{h}}}{f}\right)\left[\ln\frac{\lambda_t^2 f^2\cos^2\left(\frac{\sqrt{\boldsymbol{h}^\dagger\boldsymbol{h}}}{f}\right)}{\Lambda^2}-\frac{1}{2}\right]\right\}$$

$$\approx \text{ constant } +2\boldsymbol{h}^\dagger\boldsymbol{h}\,\lambda_t^2(m_{tA}^2+m_{tB}^2)\ln\frac{\Lambda^2}{m_{tA}^2+m_{tB}^2}+\mathscr{O}\left(\boldsymbol{h}^\dagger\boldsymbol{h}\right)^2 \,. \tag{2.96}$$

Therefore, we find only a logarithmic contribution to the Higgs mass parameter. In other words, the corrections to the Higgs mass parameter μ^2 from all additional one-loop interactions is

$$\delta\mu^2=-\frac{3\lambda_t^2}{8\pi^2}(m_{tA}^2+m_{tB}^2)\ln\frac{\Lambda^2}{m_{tA}^2+m_{tB}^2}\ , \tag{2.97}$$

where the 3 in the numerator comes from the number of SM colors. We then define the tuning in the Higgs mass [60] as

$$\Delta=\left|\frac{2\delta\mu^2}{m_h^2}\right|^{-1}\ , \tag{2.98}$$

where m_h is the mass of the Higgs. In Fig. 2.1 this tuning is plotted for cutoffs of 2 TeV and 5 TeV as a function of m_{tB}. We see that as the cutoff gets higher the tuning becomes more stringent, as indicated by a lower value of percent tuning for the blue line than for the yellow line. As the twin top mass gets larger, moving along the x-axis, the tuning also becomes more stringent as expected. This means that the lower the mass of the top partner, the more natural and less fine-tuned the model is.

In our expanded SU(4)$_c$ model this analysis is changed in a few significant ways. First, in the twin sector the SU(4)$_c$ color group is unbroken, so N_c is 4 rather than 3 as it is in the SM quark sector. This mismatch is compensated at still higher scales by the missing components of the SU(4)$_c$ quark multiplets.

We first consider the $N_c\mathrm{Tr}\mathscr{M}^2$ part of the potential. In the twin section this is fairly straightforward. We have that $N_c=4$. Also, the trace of the squared matrix is independent of the Higgs, while

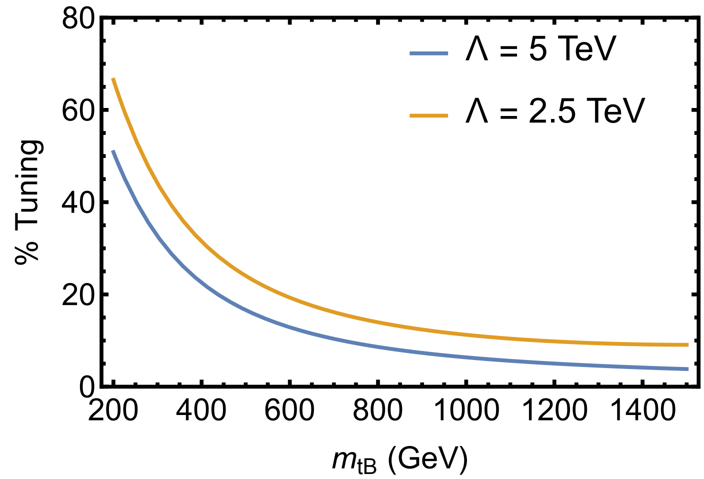

Figure 2.1 Plot of percent tuning in twin Higgs models for a range of cutoffs as a function of the twin top mass. A higher value on the y-axis indicates a more natural result.

the same is not true for the matrix to the fourth contribution. Isolating the twin sector matrix $\mathscr{M}_B$, we find

$$N_c \text{Tr}\mathscr{M}^2\big|_B = (4\lambda_t^2 + \lambda_{FP}^2 + \lambda_{FV}^2) f^2 \cos^2\left(\frac{\sqrt{\boldsymbol{h}^\dagger \boldsymbol{h}}}{f}\right) , \tag{2.99}$$

where we have dropped SM Yukawas smaller than λ_t. In the visible sector we find

$$N_c \text{Tr}\mathscr{M}^2\big|_A = (3\lambda_t^2 + \lambda_t^2 + \lambda_{FP}^2 + \lambda_{FV}^2) f^2 \sin^2\left(\frac{\sqrt{\boldsymbol{h}^\dagger \boldsymbol{h}}}{f}\right) + w^2(\lambda_P^2 + \lambda_V^2 + 2\lambda_F^2) . \tag{2.100}$$

While somewhat more complicated, the sum of the contributions from each sector does lead to a $\boldsymbol{h}$ independent contribution to the potential through the addition of $\sin^2(\sqrt{\mathbf{h}^\dagger \mathbf{h}}/f) + \cos^2(\sqrt{\mathbf{h}^\dagger \mathbf{h}}/f)$.

The $\mathscr{M}^4$ contribution from the hidden sector is simply

$$N_c \mathrm{Tr}\left[\mathscr{M}^4\left(\ln\frac{\mathscr{M}^2}{\Lambda^2}-\frac{1}{2}\right)\right]\Bigg|_B = 4\lambda_t^4 f^4\cos^4\left(\frac{\sqrt{\boldsymbol{h}^\dagger\boldsymbol{h}}}{f}\right)\left(\ln\frac{\lambda_t^2 f^2\cos^2\left(\frac{\sqrt{\boldsymbol{h}^\dagger\boldsymbol{h}}}{f}\right)}{\Lambda^2}-\frac{1}{2}\right)$$
$$+\lambda_{FP}^4 f^4\cos^4\left(\frac{\sqrt{\boldsymbol{h}^\dagger\boldsymbol{h}}}{f}\right)\left(\ln\frac{\lambda_{FP}^2 f^2\cos^2\left(\frac{\sqrt{\boldsymbol{h}^\dagger\boldsymbol{h}}}{f}\right)}{\Lambda^2}-\frac{1}{2}\right)$$
$$+\lambda_{FV}^4 f^4\cos^4\left(\frac{\sqrt{\boldsymbol{h}^\dagger\boldsymbol{h}}}{f}\right)\left(\ln\frac{\lambda_{FV}^2 f^2\cos^2\left(\frac{\sqrt{\boldsymbol{h}^\dagger\boldsymbol{h}}}{f}\right)}{\Lambda^2}-\frac{1}{2}\right)\,. \qquad (2.101)$$

The $\mathscr{M}^4$ terms are more complicated in the visible sector, where we must use the mass eigenstates given in Eq. (2.66) with $v\to\sqrt{2}f\sin(\sqrt{\boldsymbol{h}^\dagger\boldsymbol{h}}/f)$. Employing this substitution we have

$$N_c \mathrm{Tr}\left[\mathscr{M}^4\left(\ln\frac{\mathscr{M}^2}{\Lambda^2}-\frac{1}{2}\right)\right]\Bigg|_A = 3\lambda_t^4 f^4\sin^4\left(\frac{\sqrt{\boldsymbol{h}^\dagger\boldsymbol{h}}}{f}\right)\left(\ln\frac{\lambda_t^2 f^2\sin^2\left(\frac{\sqrt{\boldsymbol{h}^\dagger\boldsymbol{h}}}{f}\right)}{\Lambda^2}-\frac{1}{2}\right)$$
$$+m_{u+}^4\left(\ln\frac{m_{u+}^2}{\Lambda^2}-\frac{1}{2}\right)+m_{u-}^4\left(\ln\frac{m_{u-}^2}{\Lambda^2}-\frac{1}{2}\right)+m_{d+}^4\left(\ln\frac{m_{d+}^2}{\Lambda^2}-\frac{1}{2}\right)+m_{d-}^4\left(\ln\frac{m_{d-}^2}{\Lambda^2}-\frac{1}{2}\right)\,. \qquad (2.102)$$

Putting the two sectors together we do find logarithmic contributions to the Higgs mass parameter

$$\begin{aligned}\delta\mu^2 = -\frac{1}{8\pi^2}\Bigg\{&4\lambda_t^2(m_{tA}^2+m_{tB}^2)\ln\frac{\Lambda^2}{m_{tA}^2+m_{tB}^2}+\lambda_{FP}^4 f^2\ln\frac{\Lambda^2}{\lambda_{FP}^2 f^2}+\lambda_{FV}^4 f^2\ln\frac{\Lambda^2}{\lambda_{FV}^2 f^2}\\ &+w^2\lambda_P\frac{\lambda_P(\lambda_{FP}^2+\lambda_t^2)+2\lambda_F\lambda_{FP}\lambda_t}{\lambda_F^2-\lambda_P^2}\ln\frac{\Lambda^2}{w^2\lambda_P^2}+w^2\frac{\lambda_{FV}^2\lambda_V^4}{\lambda_F^2-\lambda_V^2}\ln\frac{\Lambda^2}{w^2\lambda_V^2}\\ &-w^2\lambda_F\frac{\lambda_F(\lambda_{FP}^2+\lambda_t^2)+2\lambda_P\lambda_{FP}\lambda_t}{\lambda_F^2-\lambda_P^2}\ln\frac{\Lambda^2}{w^2\lambda_F^2}-w^2\frac{\lambda_{FV}^2\lambda_F^4}{\lambda_F^2-\lambda_V^2}\ln\frac{\Lambda^2}{w^2\lambda_F^2}\Bigg\}\,. \qquad (2.103)\end{aligned}$$

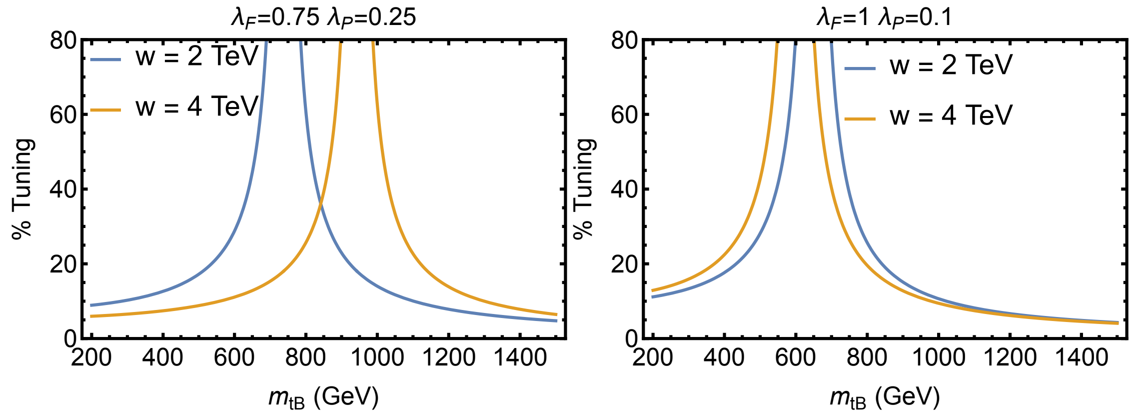

Figure 2.2 Percent tuning as a function of m_{tB} for a cutoff of 5 TeV for a range of w VEVs. We see that tuning is reduced in a central region where the different contributions cancel. The twin top mass with mild tuning depends on the Yukawa couplings.

The λ_{FV} and λ_{FP} couplings primarily serve to ensure there are not more light degrees of freedom in the twin sector. Hence, we take them to be about the size of λ_b and neglect their effects, finding:

$$\begin{aligned}\delta\mu^2 &= -\frac{\lambda_t^2}{8\pi^2}\left[4(m_{tA}^2+m_{tB}^2)\ln\frac{\Lambda^2}{m_{tA}^2+m_{tB}^2}+\frac{w^2\lambda_P^2}{\lambda_F^2-\lambda_P^2}\ln\frac{\Lambda^2}{w^2\lambda_P^2}-\frac{w^2\lambda_F^2}{\lambda_F^2-\lambda_P^2}\ln\frac{\Lambda^2}{w^2\lambda_F^2}\right]\\ &= -\frac{\lambda_t^2}{8\pi^2}\left[4(m_{tA}^2+m_{tB}^2)\ln\frac{\Lambda^2}{m_{tA}^2+m_{tB}^2}-w^2\ln\frac{\Lambda^2}{w^2\lambda_F^2}+\frac{w^2\lambda_P^2}{\lambda_F^2-\lambda_P^2}\ln\frac{\lambda_F^2}{\lambda_P^2}\right]\,. \end{aligned} \tag{2.104}$$

Without loss of generality, suppose that $\lambda_F > \lambda_P$. We see that there is at least a partial cancellation between the top-sector contributions. In Fig. 2.2 we plot the tuning as a function of m_{tB} for a few values of w. The cutoff is taken to be 5 TeV. One sees that the tuning at low m_{tB} is about 10%, as the fourth component of the top field has a w scale mass. At larger masses, however, the tuning lessens dramatically, before returning to percent level values. In the plot on the left both λ_F and λ_P are taken order 1 and with order one difference. In this case the region of reduced tuning depends on w. In the plot on the right there is a mild hierarchy between the Yukawa couplings and so the region of reduced tuning does not change much with the color breaking VEV.

We note also that regions of exact cancellation may seem fine tuned themselves. However, the regions of reduced tuning are more generic and do not depend on exact cancellation of independent parameters. Therefore, we feel that this reduction in tuning for higher m_{tB} is an accurate characterization, at least within the low energy theory. It is also the case that the color breaking VEV w produces both of the new mass scales. It directly gives mass to the new fermions (modulated by a Yukawa coupling) and provides the Z_2 breaking mass term in the Higgs potential, being multiplied by the small quartic coupling $\delta_{H\phi}$.

Chapter 3

Conclusion

The potential signals and tuning measurements of this model have proven interesting. Our exploration of this model has resulted in updated phenomenology of Twin Higgs models. We discovered that model I using SU(4)$_c$ is excluded due to its low-mass states with electric charge. We have also improved the calculation of bounds for model II, finding lower limits for the new charged fermions. Most interestingly, there are new, less-studied signals which can be tested at the LHC in the form of fractionally charged particles.

The most qualitatively novel result from this model lies in the tuning measurements assessment found from the calculations of the Coleman-Weinberg potential. In most models that introduce a symmetry partner for the top quark, the tuning of the model becomes more severe as the partner mass increases. This lowers the motivation of the models as higher energies fail to discover a new partner. Using SU(4) model II, however, we see in Fig. 2.2 a region of increasing top partner mass in which the tuning improves. Current experimental methods aim to test to high precision the deviations of Higgs parameters from the SM. In the case that little to no deviation is found, many models be pushed into fine-tuned regions of parameters space. With the new pattern of improved tuning at higher masses, however, a lack of deviation would not make the model unnatural.

Our future work includes more precise testing of the limits of this model. One aspect of that will be through improved calculations on bounds from collider experiments. We expect that di-lepton resonance searches with provide the most powerful probe of the color breaking scale w through the Z'. We also anticipate that the bounds on fractionally charged particles can be strengthened significantly. Additionally, we will explore in more detail the limits provided by cosmology through Dark Matter and ΔN_{eff}. Further analysis of the Dark Matter candidates of this model, especially the baryons of the unbroken twin color, will likely provide additional interesting phenomena.

Appendix A

Representations of SU(4)

This appendix provides a more detailed description of the Lie Group SU(4) and its representations. The principles and some of the matrices included are also valuable in understanding other Lie Groups used in the Standard Model.

The generator matrices of SU(4) are:

$$\lambda_1 = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad \lambda_2 = \begin{pmatrix} 0 & -i & 0 & 0 \\ i & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \lambda_3 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{A.1}$$

$$\lambda_4 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad \lambda_5 = \begin{pmatrix} 0 & 0 & -i & 0 \\ 0 & 0 & 0 & 0 \\ i & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \lambda_6 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{A.2}$$

$$\lambda_7 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & -i & 0 \\ 0 & i & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad \lambda_8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -2 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \lambda_9 = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix} \tag{A.3}$$

$$\lambda_{10} = \begin{pmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ i & 0 & 0 & 0 \end{pmatrix} \quad \lambda_{11} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \lambda_{12} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -i \\ 0 & 0 & 0 & 0 \\ 0 & i & 0 & 0 \end{pmatrix} \tag{A.4}$$

$$\lambda_{13} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \quad \lambda_{14} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \end{pmatrix} \lambda_{15} = \frac{1}{\sqrt{6}} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -3 \end{pmatrix} \tag{A.5}$$

These matrices are all orthogonal to each other, and $Tr(\lambda_i^2) = 2$. To normalize all 15 generators in Killing form, we define:

$$T_i = \frac{1}{2}\lambda_i \tag{A.6}$$

Note also that 3 of these are simultaneously diagonalizable: T_3, T_8, and T_{15}. These 3 form the Cartan subalgebra of the group:

$$T_3 = \frac{1}{2}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad T_8 = \frac{1}{2\sqrt{3}}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -2 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad T_{15} = \frac{1}{2\sqrt{6}}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -3 \end{pmatrix} \tag{A.7}$$

The structure constants of the 15 generator matrices are defined by:

$$[T_i, T_j] = if^{ijk}T_k$$

Many of these f^{ijk} values are zero. For the commutators which obtain a diagonal generator matrix, the sum over $f^{ijk}T_k$ gives us multiple non-zero terms, such that

$$[T_i, T_j] = if^{ij3}T_3 + if^{ij8}T_8 + if^{ij15}T_{15}$$

.

With the non-zero structure constants being:

i	j	k	f^{ijk}
1	2	3	1
1	4	7	$\frac{1}{2}$
1	5	6	$-\frac{1}{2}$
1	9	12	$\frac{1}{2}$
1	10	11	$-\frac{1}{2}$
2	4	6	$\frac{1}{2}$
2	5	7	$\frac{1}{2}$
2	9	11	$\frac{1}{2}$
2	10	12	$\frac{1}{2}$
3	4	5	$\frac{1}{2}$
3	6	7	$-\frac{1}{2}$
3	9	10	$\frac{1}{2}$
3	11	12	$-\frac{1}{2}$
4	5	8	$\frac{\sqrt{3}}{2}$
4	9	14	$\frac{1}{2}$

i	j	k	f^{ijk}
4	10	13	$-\frac{1}{2}$
5	9	13	$\frac{1}{2}$
5	10	14	$\frac{1}{2}$
6	7	8	$\frac{\sqrt{3}}{2}$
6	11	15	$\frac{1}{2}$
6	12	14	$-\frac{1}{2}$
7	11	13	$\frac{1}{2}$
7	12	14	$\frac{1}{2}$
8	9	10	$\frac{1}{2\sqrt{3}}$
8	11	12	$\frac{1}{2\sqrt{3}}$
8	13	14	$-\frac{1}{\sqrt{3}}$
9	10	15	$\sqrt{\frac{2}{3}}$
11	12	15	$\sqrt{\frac{2}{3}}$
13	14	15	$\sqrt{\frac{2}{3}}$

(A.8)

The Cartan elements can help us define important representations of the group. Starting with the fundamental representation known as **4**, we define its basis vectors simply as:

$$f_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \qquad f_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} \qquad f_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} \qquad f_4 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \tag{A.9}$$

To find the weights of this representation, we find the eigenvalues of each of these vectors when acted on by each of the Cartan generators T_3, T_8, and T_{15}.

$$T_3 f_1 = \frac{1}{2} f_1 \qquad T_8 f_1 = \frac{1}{2\sqrt{3}} f_1 \qquad T_{15} f_1 = \frac{1}{2\sqrt{6}} f_1 \tag{A.10}$$

$$T_3 f_2 = -\frac{1}{2} f_2 \qquad T_8 f_2 = \frac{1}{2\sqrt{3}} f_2 \qquad T_{15} f_2 = \frac{1}{2\sqrt{6}} f_2 \tag{A.11}$$

$$T_3 f_3 = 0 f_3 \qquad T_8 f_3 = -\frac{1}{\sqrt{3}} f_3 \qquad T_{15} f_3 = \frac{1}{2\sqrt{6}} f_3 \tag{A.12}$$

$$T_3 f_4 = 0 f_4 \qquad T_8 f_4 = 0 f_4 \qquad T_{15} f_4 = -\frac{3}{2\sqrt{6}} f_4 \tag{A.13}$$

We then assign each f_i an associated vector μ_i, containing its 3 eigenvalues. This μ vector is called the weight vector, which we write in the order T_3, T_8, T_{15}.

$$\mu_1 = \left(\frac{1}{2}, \frac{1}{2\sqrt{3}}, \frac{1}{2\sqrt{6}}\right) \tag{A.14}$$

$$\mu_2 = \left(-\frac{1}{2}, \frac{1}{2\sqrt{3}}, \frac{1}{2\sqrt{6}}\right) \tag{A.15}$$

$$\mu_3 = \left(0, -\frac{1}{\sqrt{3}}, \frac{1}{2\sqrt{6}}\right) \tag{A.16}$$

$$\mu_4 = \left(0, 0, -\frac{3}{2\sqrt{6}}\right) \tag{A.17}$$

We now define the complex conjugate representation of the fundamental. The Lie algebra is defined as:

$$\overline{T}_i = -(T_i)^*$$

The basis vectors f remain the same, because the dimension of the representation has not changed. Acting on these basis vectors with the Cartan subalgebra of $\overline{T}$ yields the weights of the conjugate representation of the fundamental.

$$\overline{\mu}_1 = \left(-\frac{1}{2}, -\frac{1}{2\sqrt{3}}, --\frac{1}{2\sqrt{6}}\right) \tag{A.18}$$

$$\overline{\mu}_2 = \left(\frac{1}{2}, -\frac{1}{2\sqrt{3}}, -\frac{1}{2\sqrt{6}}\right) \tag{A.19}$$

$$\overline{\mu}_3 = \left(0, \frac{1}{\sqrt{3}}, -\frac{1}{2\sqrt{6}}\right) \tag{A.20}$$

$$\overline{\mu}_4 = \left(0, 0, \frac{3}{2\sqrt{6}}\right) \tag{A.21}$$

The highest of these weights is $\overline{\mu}_2$.

Next, we want to find the adjoint representation. The adjoint is a representation in which the generators are also the basis states. Their action on each other is defined as:

$$[X, Y] = X|Y\rangle$$

To find the adjoint representation, we change our basis such that each of our generators are eigenvectors of our Cartan subalgebra. This is done by taking the nonzero structure constants involving any Cartan subalgebra element and combining its associated pair of generators into a new normalized eigenstate. For example:

$$f^{123} = 1 \tag{A.22}$$

$$[T_3, T_1] = iT_2 \tag{A.23}$$

$$[T_3, T_2] = -iT_1 \tag{A.24}$$

$$[T_3, \frac{1}{\sqrt{2}}(T_1 \pm iT_2)] = \pm\frac{1}{\sqrt{2}}(T_1 \pm iT_2) \tag{A.25}$$

Following the same pattern shown above, we find:

$$[T_3, \frac{1}{\sqrt{2}}(T_1 \pm iT_2)] = \pm\frac{1}{\sqrt{2}}(T_1 \pm iT_2) \quad \text{(A.26)}$$

$$[T_8, \frac{1}{\sqrt{2}}(T_1 \pm iT_2)] = 0 \quad \text{(A.27)}$$

$$[T_{15}, \frac{1}{\sqrt{2}}(T_1 \pm iT_2)] = 0 \quad \text{(A.28)}$$

$$[T_3, \frac{1}{\sqrt{2}}(T_4 \pm iT_5)] = \pm\frac{1}{2}\frac{1}{\sqrt{2}}(T_4 \pm iT_5) \quad \text{(A.29)}$$

$$[T_8, \frac{1}{\sqrt{2}}(T_4 \pm iT_5)] = \pm\frac{\sqrt{3}}{2}\frac{1}{\sqrt{2}}(T_4 \pm iT_5) \quad \text{(A.30)}$$

$$[T_{15}, \frac{1}{\sqrt{2}}(T_4 \pm iT_5)] = 0 \quad \text{(A.31)}$$

$$[T_3, \frac{1}{\sqrt{2}}(T_6 \pm iT_7)] = \mp\frac{1}{2}\frac{1}{\sqrt{2}}(T_6 \pm iT_7) \quad \text{(A.32)}$$

$$[T_8, \frac{1}{\sqrt{2}}(T_6 \pm iT_7)] = \pm\frac{\sqrt{3}}{2}\frac{1}{\sqrt{2}}(T_6 \pm iT_7) \quad \text{(A.33)}$$

$$[T_{15}, \frac{1}{\sqrt{2}}(T_6 \pm iT_7)] = 0 \quad \text{(A.34)}$$

$$[T_3, \frac{1}{\sqrt{2}}(T_9 \pm iT_{10})] = \pm\frac{1}{2}\frac{1}{\sqrt{2}}(T_9 \pm iT_{10}) \quad \text{(A.35)}$$

$$[T_8, \frac{1}{\sqrt{2}}(T_9 \pm iT_{10})] = \pm\frac{1}{2\sqrt{3}}\frac{1}{\sqrt{2}}(T_9 \pm iT_{10}) \quad \text{(A.36)}$$

$$[T_{15}, \frac{1}{\sqrt{2}}(T_9 \pm iT_{10})] = \pm\sqrt{\frac{2}{3}}\frac{1}{\sqrt{2}}(T_9 \pm iT_{10}) \quad \text{(A.37)}$$

$$[T_3, \frac{1}{\sqrt{2}}(T_{11} \pm iT_{12})] = \mp\frac{1}{2}\frac{1}{\sqrt{2}}(T_{11} \pm iT_{12}) \quad \text{(A.38)}$$

$$[T_8, \frac{1}{\sqrt{2}}(T_{11} \pm iT_{12})] = \pm\frac{1}{2\sqrt{3}}\frac{1}{\sqrt{2}}(T_{11} \pm iT_{12}) \quad \text{(A.39)}$$

$$[T_{15}, \frac{1}{\sqrt{2}}(T_{11} \pm iT_{12})] = \pm\sqrt{\frac{2}{3}}\frac{1}{\sqrt{2}}(T_{11} \pm iT_{12}) \quad \text{(A.40)}$$

$$[T_3, \frac{1}{\sqrt{2}}(T_{13} \pm iT_{14})] = 0 \quad \text{(A.41)}$$

$$[T_8, \frac{1}{\sqrt{2}}(T_{13} \pm iT_{14})] = \mp\frac{1}{\sqrt{3}}\frac{1}{\sqrt{2}}(T_{13} \pm iT_{14}) \quad \text{(A.42)}$$

$$[T_{15}, \frac{1}{\sqrt{2}}(T_{13} \pm iT_{14})] = \pm\sqrt{\frac{2}{3}}\frac{1}{\sqrt{2}}(T_{13} \pm iT_{14}) \quad \text{(A.43)}$$

This identifies 12 eigenvectors of the Cartan subalgebra. We can relabel these states for simplicity, in an order which will become clear momentarily, as follows:

$$X_1^{\pm} = \frac{1}{\sqrt{2}}(T_4 \pm iT_5) \tag{A.44}$$

$$X_2^{\pm} = \frac{1}{\sqrt{2}}(T_{13} \pm iT_{14}) \tag{A.45}$$

$$X_3^{\pm} = \frac{1}{\sqrt{2}}(T_{11} \pm iT_{12}) \tag{A.46}$$

$$X_4^{\pm} = \frac{1}{\sqrt{2}}(T_9 \pm iT_{10}) \tag{A.47}$$

$$X_5^{\pm} = \frac{1}{\sqrt{2}}(T_6 \pm iT_7) \tag{A.48}$$

$$X_6^{\pm} = \frac{1}{\sqrt{2}}(T_1 \pm iT_2) \tag{A.49}$$

These 12 vectors, as well as the Cartan subalgebra T_3, T_8, and T_{15}, we define to be the basis states of our adjoint representation. The weights of this basis are important to all representations, and are called the roots of the algebra. The root vectors are found in the same way as the weights of the fundamental were, using the eigenvalues determined above. They are:

$$\alpha_{\pm 1} = \left(\pm\frac{1}{2}, \pm\frac{\sqrt{3}}{2}, 0\right) \qquad \alpha_{\pm 2} = \left(0, \mp\frac{1}{\sqrt{3}}, \pm\sqrt{\frac{2}{3}}\right) \tag{A.50}$$

$$\alpha_{\pm 3} = \left(\pm\frac{1}{2}, \mp\frac{1}{2\sqrt{3}}, \mp\sqrt{\frac{2}{3}}\right) \qquad \alpha_{\pm 4} = \left(\pm\frac{1}{2}, \pm\frac{1}{2\sqrt{3}}, \pm\sqrt{\frac{2}{3}}\right) \tag{A.51}$$

$$\alpha_{\pm 5} = \left(\pm\frac{1}{2}, \mp\frac{\sqrt{3}}{2}, 0\right) \qquad \alpha_{\pm 6} = (\pm 1, 0, 0) \tag{A.52}$$

We now want to select a subset of these as our positive roots. Positive roots are a subset of the roots that obey the following rules:

1) multiplying a positive root by -1 does not yield another positive root

2) if the sum of any two positive roots is also a root, that root is positive

Displaying the roots graphically as vectors, this is equivalent to choosing one half of the unit sphere and setting its contained roots as positive. In our case, we will choose our set of positive roots to be:

$$\alpha_{+1} = \left(\frac{1}{2}, \frac{\sqrt{3}}{2}, 0\right) \qquad \alpha_{+2} = \left(0, -\frac{1}{\sqrt{3}}, \sqrt{\frac{2}{3}}\right) \tag{A.53}$$

$$\alpha_{+3} = \left(\frac{1}{2}, -\frac{1}{2\sqrt{3}}, -\sqrt{\frac{2}{3}}\right) \qquad \alpha_{+4} = \left(\frac{1}{2}, \frac{1}{2\sqrt{3}}, \sqrt{\frac{2}{3}}\right) \tag{A.54}$$

$$\alpha_{+5} = \left(\frac{1}{2}, -\frac{\sqrt{3}}{2}, 0\right) \qquad \alpha_{+6} = (1, 0, 0) \tag{A.55}$$

Having chosen the positive roots, we now take a subset of them to be the positive simple roots. Simple roots are positive roots that cannot be written as the sum of two other positive roots. By checking which of these positive roots sum to give another positive root, we find that:

$$\alpha_{+4} = \alpha_{+1} + \alpha_{+2} \qquad \alpha_{+5} = \alpha_{+2} + \alpha_{+3} \tag{A.56}$$

$$\alpha_{+6} = \alpha_{+3} + \alpha_{+4} \qquad \alpha_{+6} = \alpha_{+1} + \alpha_{+2} + \alpha_{+3} \tag{A.57}$$

This means that we can exclude α_{+1}, α_{+3}, and α_{+4} from the positive simple roots, and we are left with the set of α_{+2}, α_{+5}, and α_{+6}. We can now write all roots as linear combinations of the positive simple roots.

$$\pm\alpha_1 \qquad \pm\alpha_2 \qquad \pm\alpha_3 \tag{A.58}$$

$$\pm(\alpha_1 + \alpha_2) \qquad \pm(\alpha_2 + \alpha_3) \qquad \pm(\alpha_1 + \alpha_2 + \alpha_3) \tag{A.59}$$

Using these roots, the eigenvectors of the Cartan subalgebra, defined above as X_α, are identified as the raising and lowering operators. Relabeling them using our linear combinations of simple roots, they are:

$$X_1^{\pm} = \frac{1}{\sqrt{2}}(T_4 \pm iT_5) \tag{A.60}$$

$$X_2^{\pm} = \frac{1}{\sqrt{2}}(T_{13} \pm iT_{14}) \tag{A.61}$$

$$X_3^{\pm} = \frac{1}{\sqrt{2}}(T_{11} \pm iT_{12}) \tag{A.62}$$

$$X_{12}^{\pm} = \frac{1}{\sqrt{2}}(T_9 \pm iT_{10}) \tag{A.63}$$

$$X_{23}^{\pm} = \frac{1}{\sqrt{2}}(T_6 \pm iT_7) \tag{A.64}$$

$$X_{123}^{\pm} = \frac{1}{\sqrt{2}}(T_1 \pm iT_2) \tag{A.65}$$

These operators relate vectors with weight μ to vectors with weight $\mu \pm \alpha$.

It is important to note that the highest weight of the adjoint representation, the root $(1,0,0)$, is the sum of the highest weights of the fundamental and conjugate representations. This is an example of a general result. A representation can be defined by a pair of integers (m,n) which indicate its highest weight as a linear combination of the highest weights of the fundamental and its conjugate. In other words, the highest weight of the (m,n) representation of SU(4) equals $m\mu_1 + n\overline{\mu}_2$. Written this way, the fundamental is $(1,0)$, its conjugate is $(0,1)$, and the adjoint representation is the $(1,1)$ representation. We will use this fact to identify further representations.

J Classifying Vector

In addition, each of the raising and lowering operators $X^{\pm}$ has an associated X_α^3 operator analogous to the σ^3 angular momentum operator, defined by:

$$X_\alpha^3 = \frac{1}{\alpha^2}\left(\alpha_1 T_3 + \alpha_2 T_8 + \alpha_3 T_{15}\right) \tag{A.66}$$

$$X_\alpha^3|\mu\rangle = \frac{\alpha \cdot \mu}{\alpha^2}|\mu\rangle \tag{A.67}$$

We can also use the simple positive roots to identify and label all representations of SU(4), by defining a list j which classifies the highest weight of the representation. Using Notes 9.201.

We take a representation, find its state with the highest weight μ_1, and act on that state with the X^3_α operator for each simple root.

$$j^{(1)} = \frac{\alpha^{(1)} \cdot \mu_1}{|\alpha^{(1)}|^2} \qquad j^{(2)} = \frac{\alpha^{(2)} \cdot \mu_1}{|\alpha^{(2)}|^2} \qquad j^{(3)} = \frac{\alpha^{(3)} \cdot \mu_1}{|\alpha^{(3)}|^2} \tag{A.68}$$

These eigenvalues then go into a list $j = \left(2j^{(1)}, 2j^{(2)}, 2j^{(3)}\right)$ to label the representation.

Using this method on the fundamental representation, we find:

$$\mu_1 = \left(\frac{1}{2}, \frac{1}{2\sqrt{3}}, \frac{1}{2\sqrt{6}}\right) \tag{A.69}$$

$$j^{(1)} = \frac{\left(\frac{1}{2}, \frac{\sqrt{3}}{2}, 0\right) \cdot \left(\frac{1}{2}, \frac{1}{2\sqrt{3}}, \frac{1}{2\sqrt{6}}\right)}{1^2} = \frac{1}{2} \tag{A.70}$$

$$j^{(2)} = \frac{\left(0, -\frac{1}{\sqrt{3}}, \sqrt{\frac{2}{3}}\right) \cdot \left(\frac{1}{2}, \frac{1}{2\sqrt{3}}, \frac{1}{2\sqrt{6}}\right)}{1^2} = 0 \tag{A.71}$$

$$j^{(3)} = \frac{\left(\frac{1}{2}, -\frac{1}{2\sqrt{3}}, -\sqrt{\frac{2}{3}}\right) \cdot \left(\frac{1}{2}, \frac{1}{2\sqrt{3}}, \frac{1}{2\sqrt{6}}\right)}{1^2} = 0 \tag{A.72}$$

$$\tag{A.73}$$

Thus the fundamental representation gives us $j = (1,0,0)$. Repeating this process with the adjoint:

$$\mu_1 = (1,0,0) \tag{A.74}$$

$$j^{(1)} = \frac{\left(\frac{1}{2}, \frac{\sqrt{3}}{2}, 0\right) \cdot (1,0,0)}{1^2} = \frac{1}{2} \tag{A.75}$$

$$j^{(2)} = \frac{\left(0, -\frac{1}{\sqrt{3}}, \sqrt{\frac{2}{3}}\right) \cdot (1,0,0)}{1^2} = 0 \tag{A.76}$$

$$j^{(3)} = \frac{\left(\frac{1}{2}, -\frac{1}{2\sqrt{3}}, -\sqrt{\frac{2}{3}}\right) \cdot (1,0,0)}{1^2} = \frac{1}{2} \tag{A.77}$$

We find $j = (1,0,1)$.

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