Expanding the Gauge Symmetry of Quantum Chromodynamics in a Twin Higgs Model

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ABSTRACT

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An extension of the Standard Model of particle physics is proposed that is motivated primarily by the hierarchy problem. The model expands the gauge group of quantum chromodynamics in an attempt to provide a viable model of dark matter and a reason for the unexpected nature of the Higgs vacuum expectation value. These changes introduce new fermion and boson fields to the Standard Model, as well as interesting new collider phenomenology.

Keywords: Higgs, Higgs Mechanism, Vacuum Expectation Value, Hierarchy Problem, Gauge Symmetries, Twin Higgs Model, Symmetry Breaking

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Chapter 1

Introduction

Particle physics is the study of the universe's most fundamental interactions. By understanding how nature behaves at the shortest length scales, we broaden our scope of knowledge and capacity for discovery. Particle physicists probe the dynamics of nature's most fundamental particles by employing two primary techniques: theory and experiment. Theoretical particle physicists use mathematical methods to assign structure and dynamics to elementary particles. Experimental particle physicists employ various experimental techniques to physically observe these particles. The two classes of physicists work hand in hand; theorists expand on the existing theoretical framework and predict new particles, while experimentalists look for these new particles to validate or rule out corresponding theoretical models.

This partnership is prevalent in nearly all areas of modern physics; experimentalists and theorists work together to understand natural phenomena to the greatest extent possible. Experiment attempts to explain *what* nature is doing, while theory attempts to cover *why* that is the case. Both are incredibly important to the scientific process.

To illustrate this, we consider Brownian motion. Brownian motion is the random motion of particles suspended in a fluid. It has been observed for centuries in many fields; it is named after a Robert Brown, who observed it in pollen suspended in water. Brown ruled out the possibility that the random motion was driven by living things, but was unable to fully explain the phenomenon [1]. So by the early 1900s, Brownian motion was well-observed, but not *well-understood*. That was the case until Einstein proposed his model of Brownian motion in 1905. He was able to provide a physical and mathematical explanation for Brownian motion, which is now used in countless fields. The development of a theoretical model would not have been possible without the experimental evidence, and the experimental evidence would have gone unexplained without a theoretical model.

Both experiment and theory are critically important to fully understanding the dynamics of natural phenomena. And while these fields are often distinct, they are rarely working independently. Theorists work to explain phenomena already observed, but also to predict new phenomena. Experimentalists search for new phenomena, but also seek to validate theoretically predicted models. This is very true of particle physics.

Modern particle physics is centered on what is called the Standard Model (SM). The SM represents our current understanding of fundamental particles, and it does an excellent job predicting experiment. However, it is not a product of one flash of genius; countless physicists have contributed to our understanding of fundamental particles, in an ongoing process that has taken over a century. These physicists have been both experimentalists and theorists, expanding our knowledge of fundamental phenomena. And their efforts have not been in vain —the Standard Model matches experimental data incredibly well. However, there is still work to be done.

While it is incredibly accurate to experiment, the SM lacks explanations for some specific phenomena. A large portion of particle theory is devoted to providing these explanations. Theoretical particle physicists attempt to expand the scope of the SM by building models of what might exist. Generally, the goals of these projects are not to improve SM accuracy but to improve understanding of the underlying principles; the SM predicts most phenomena very accurately, but is occasionally lacking descriptions of why physical phenomena are what they are. This is where our work will begin; we present a theoretical model that modifies the properties of the SM. Our model is motivated by both theory and experiment, and we seek to provide explanations not yet provided by the Standard Model. By extending the SM in specific ways, our model not only provides potential descriptions previously lacking, but also predicts new particles. These particles are particularly important when considering our model in an experimental framework, as they provide a concrete way for the model to be experimentally verified.

In this thesis, we discuss the Standard Model in Chapter 2 in order to cultivate an understanding of its structure and dynamics. This discussion will include areas the SM is lacking explanation, including those our model addresses. Having built an understanding of the current model, Chapter 3 will provide the mathematical framework required to build our extension of the SM that alleviates some of its limitations. After our model is built, we discuss its potential ramifications in Chapter 4, including ways to observe new predicted particles experimentally.

Chapter 2

Background

Much of this thesis is taken from our not yet published paper [2], which builds an extension of the Standard Model that addresses some of the limitations of the Standard Model. In order to understand where the Standard Model is lacking, a working understanding of its structure is required.

2.1 Standard Model Lagrangian

Before extending the Standard Model, we first consider its current structure. This will be a basis from which we move to consider its current limitations, followed by our new model's extensions. The Standard Model describes the interactions of three fundamental forces. It governs interaction under the electromagnetic, weak, and strong forces. Fundamental particles can be divided into two groups —fermions and bosons. Fermions are particles with half-odd-integer spin, spin-1/2 particles for example, where bosons have integer spin. The fermions of the SM include familiar particles such as electrons, but also include quarks and other leptons.

Quarks, in addition to having electric charge, are charged under the strong force. The strong force is driven by the the color charge, which has three charges; these analogous to thes to the electric charge, but with three rather than one. These charges have been labeled as red, green, and blue, though they have no intrinsic connection to the visible light spectrum. Just like electric charge can be negative, these charges can be as well; so the strong force also has charges of antired, antigreen, and antiblue, which are the negative counterparts of the standard color charges. This color charge is what binds the baryons like the proton together, as well as the mesons. Observable kinematically allowed particles must be colorless, which is possible via two methods: 1) having an equal number of red, green, and blue charge; 2) having equal parts color and corresponding anticolor [3].

Quarks are particles bound under all SM forces, including color; thus, they are not allowed to be independent and must be bound into larger colorless particles. Leptons are not charged under color and are rarely bound into larger states. The lightest leptons are the electron and the neutrinos, which are colorless with spin-1/2. The remaining leptons have similar properties but greater mass.

Most bosons in the Standard Model are gauge bosons with spin-1 that correspond to different forces. In the theoretical framework, these bosons are modeled to mediate fundamental forces by moving between other particles. The Z and W bosons are massive particles tied to the weak force. The photon is massless and responsible for carrying the electromagnetic force. Gluons are also massless and mediate the nuclear force [4].

The SM bosons have the notable outlier in the Higgs boson, which does not carry any particular force. The Higgs is a scalar boson, which means it is spin-0. It is related to the weak and electric forces, but does not carry them in the same way the gauge bosons do. The Higgs plays a unique and important role in the dynamics of the Standard Model, which is explained in later sections.

A Lagrangian simply put is the quantity that governs the dynamics of a given system. In the case of the SM, the Lagrangian must include all known particles, as well as their dynamics. We will consider sections of the SM Lagrangian separately in this chapter in order to better understand the implications of each term. The gauge symmetries of the SM are given as

$$-\frac{1}{4}G^a_{\mu\nu}G^{a\mu\nu}-\frac{1}{4}A^a_{\mu\nu}A^{a\mu\nu}-\frac{1}{4}B_{\mu\nu}B^{\mu\nu}\ . \tag{2.1}$$

These terms outline the fundamental forces as their underlying symmetries predict. G^a, A^a, and B each represent the generators of the symmetry corresponding to the force; more information regarding gauge symmetries is provided in Section 2.2. In the SM, G^a represents the gluon fields, as their symmetry is preserved at all energy scales. However, A^a and B are related to the remaining gauge bosons, but do not represent the photon or the W and Z; this phenomenon is explained further in Section 2.3.

Another important section of the SM Lagrangian are often called the Yukawa interactions. They outline how the Higgs boson interacts with the SM fermions. The terms are given as

$$\lambda_l \overline{L} H E + (\lambda_d)^i_j \overline{Q}_i H D^j + (\lambda_u)^i_j \overline{Q}_i \tilde{H} U^j + \text{h.c.}. \tag{2.2}$$

H represents the Higgs field and E represents the various lepton fields. Q, U, and D all represent the quark fields. Any terms shared by fields outline their interactions. Here h.c. stands for Hermitian Conjugate, which means the Hermitian Conjugate of each term is also included, but shortened for brevity. It is simply a way of covering all possible interactions. These terms in the Lagrangian are what determine fermion mass—interaction with the Higgs is what results in the masses of the fermions. The various λ are called Yukawa couplings, and they are a measure of how much the fermions couple to the Higgs. The Yukawa coupling corresponding to a given fermion field determines how massive its particle is.

The final term we consider is the called the kinetic term. It outlines how the Higgs field interacts with itself, given by

$$(D_\mu H)^\dagger D_\mu \ . \tag{2.3}$$

H still represents the Higgs field, and D_μ represents the covariant derivative corresponding to the Higgs. The covariant derivative depends on the field it is acting on, but in general can be thought of as representative of the intrinsic motion of a field. This term results in an important process called the Higgs Mechanisim, outlined in Section 2.3.

2.2 Gauge Symmetries

Gauge symmetries in the Standard Model are tied to the fundamental forces. The structure of a symmetry is determined by its Lie Algebra, which is a set of matrices. When these matrices operate on other matrices, specific patterns are observed that result in symmetries. Lie Algebras are different for each symmetry. In the SM, each generator corresponds to a massless boson that mediates the force corresponding to its gauge symmetry. The gauge symmetries of the Standard Model can be expressed as

$$\mathrm{SU}(3)_c \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y \tag{2.4}$$

$\mathrm{SU}(3)_c$ corresponds to the strong nuclear force, where color charges determine chromodynamic interactions. The Standard Model for QCD includes three color charges—red, green, and blue. The Lie Algebra for the SU(3) group has 8 generators, so there are 8 force-carrier bosons for SU(3) color (called gluons).

$\mathrm{SU}(2)_L$ is related to both electromagnetism and the weak force. The Lie Algebra for SU(2) has 3 generators. $\mathrm{U}(1)_Y$ is also related to the electroweak force, and the Lie Algebra for $U(1)$ has 1 generator. So, there are four massless gauge bosons collectively for the combined symmetry of $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$. This becomes important when discussing the breaking of this symmetry.

2.3 Higgs Mechanism

In the Standard Model, spontaneous symmetry breaking occurs as the Higgs boson field breaks

$$\mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y \to \mathrm{U}(1)_{\mathrm{E\&M}} \tag{2.5}$$

The symmetry breaking occurs because the Higgs has a nonzero vacuum expectation value (VEV). A VEV is the value that a field takes in the vacuum. Nature seeks to minimize energy, so the VEV is the value a field has when its potential energy is minimized.

The Higgs transforms under both the $\mathrm{SU}(2)_L$ and $\mathrm{U}(1)_Y$ gauge symmetries. Thus, when its nonzero VEV breaks symmetry, it results in a change of basis for all other fields that couple to $\mathrm{SU}(2)_L$ and $\mathrm{U}(1)_Y$. The symmetry that remains after symmetry is broken is $\mathrm{U}(1)_{E\&M}$.

The kinetic term of the Higgs field in the SM Lagrangian is

$$\mathcal{L} = (D_\mu H)^\dagger D_\mu H \tag{2.6}$$

This potential has a minimum at $\langle H \rangle = \frac{\mu}{\sqrt{2\lambda}}$ which is the VEV. Writing H as a two component object that transforms under $\mathrm{SU}(2)_L$, and using a convenient gauge transformation yields

$$\langle H \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix} \tag{2.7}$$

As the Higgs field couples only to $\mathrm{SU}(2)_L$ and $\mathrm{U}(1)_Y$, its covariant derivative can be written as

$$D_\mu = \partial_\mu + i\frac{g}{2}\sigma^a A^a_\mu + ig' Y_h B_\mu \ , \tag{2.8}$$

where A^a_μ and $B_\mu$represents the massless gauge bosons, g and g' represent the couplings of their corresponding forces. ∂_μ is the derivative of the field, which will disappear when discussing the VEV. σ and Y are the generators of $\mathrm{SU}(2)_L$ and $\mathrm{U}(1)_Y$ respectively. Using these terms, we expand the Higgs' kinetic term.

$$\left[\frac{1}{\sqrt{2}} D_\mu \begin{pmatrix} 0 \\ v \end{pmatrix}\right]^\dagger \frac{1}{\sqrt{2}} D_\mu \begin{pmatrix} 0 \\ v \end{pmatrix} = \frac{v^2 g^2}{8}(A^1_\mu + iA^2_\mu)(A^{1\mu} - iA^{2\mu}) + \frac{v^2}{2}(g' Y_h B_\mu - \frac{g}{2} A^3_\mu)^2 \tag{2.9}$$

We see sets of these massless bosons grouped together, which we can rewrite as new fields. These new states have masses, which are determined by the magnitude of their term in Equation 2.9. We now find the mass eigenstates of gauge generators after symmetry breaking:

$$W_\mu^\pm = \frac{1}{\sqrt{2}}(A_\mu^1 \mp iA_\mu^2) \tag{2.10}$$

$$Z_\mu = \frac{1}{\sqrt{g^2 + 4Y_h^2 g'^2}}(2g' Y_h B_\mu - gA_\mu^3) \tag{2.11}$$

$$A_\mu = \frac{1}{\sqrt{g^2 + 4Y_h^2 g'^2}}(gB_\mu + 2g' Y_h A_\mu^3) \tag{2.12}$$

So the mass eigenstates after symmetry breaking yield the $W_\mu^\pm$ and Z_μ bosons. This diagonalization in the mass basis also yields a definition A_μ of the photon field, which is defined by its orthogonality to the Z_μ boson, which requires it to be massless. The Higgs Mechanism results in massive gauge bosons, which we can use to rewrite the SM Lagrangian in the mass basis. This basis is particularly useful because it puts our model in terms of observable particles. Once symmetry breaking has taken place, observing the massless bosons corresponding to the broken symmetry is impossible, as it is a part of a new massive particle. In the mass basis, actual physical particles make up the basis, which makes determining the physical dynamics much easier.

2.4 Hierarchy Problem

The previous section discussed the nonzero Higgs VEV resulting in spontaneous symmetry breaking. The Hierarchy Problem rises from the unexpected magnitude of the Higgs VEV. Experimentally, the Higgs VEV is approximately 246 GeV. This poses no specific problem, but it is not what is expected. Naive dimensional analysis predicts a VEV on the order of 10^{19} GeV [5].

This disparity is 17 orders of magnitude in scale, which is significant. And while this is not mathematically inconsistent, it is puzzling. The Standard Model provides no explanation for why the Higgs VEV is so low. And the magnitude of difference implies there are physical principles not yet understood regarding the Higgs field.

That is the nature of the Hierarchy Problem—explaining why the Higgs VEV is so much lower than its predicted value. The Hierarchy Problem is something our new model attempts to address.

2.5 Dark Matter and Twin Higgs Models

Dark matter makes up approximately 26% of the universe's energy density. Matter accounted for by the Standard Model makes up only about 5% [6]. Considering the magnitude of dark matter compared to SM matter, it stands to reason that the dark matter could be just as complex as normal matter. Considering the complicated nature of the Standard Model, this school of thinking allows for potentially very complex dark matter structures.

Mirror Twin Higgs models are a class of Beyond the Standard Model (BSM) models that provide a potential structure for dark matter. These models propose a sector of matter that is symmetric to visible matter; this can alleviate the Hierarchy Problem in addition to the dark matter potential. In a Twin Higgs Model, each SM particle has a twin sector counterpart with identical properties, but transforms under separate twin sector gauge groups. Twin sector matter has a symmetric structure to SM matter but has its own gauge groups, so there is little interaction with standard matter. This complex structure with little interaction with visible matter creates a compelling area for potential dark matter. Dark matter may be composed of one or many of the fields that make up the twin sector.

The Lagrangian for a Twin Higgs model has the form

$$\mathcal{L}_{\text{visible}} + \mathcal{L}_{\text{dark}} + \mathcal{L}_{\text{scalar}} \,. \tag{2.13}$$

The Lagrangian for the visible and dark sectors are distinct, so the fields from one do not directly interact with one another. However, both dark and visible sectors interact with the scalar sector. So the indirect interactions between the sectors happen via scalar bosons. In the classic Twin Higgs model, the only scalar boson at low energies is the Higgs, so it carries all interactions. The Higgs field in a Twin Higgs model splits its VEV with the twin sector Higgs and the Standard Model Higgs. This splitting of the Higgs VEV somewhat alleviates the disparity between the observed and expected magnitude of the Higgs VEV [7]. This alleviation of the hierarchy problem is present in the model proposed here as well. However, with the addition of color-charged scalar boson field, the interactions become more complicated.

Chapter 3

Model

In the Standard Model, color-charged particles remain invariant under SU(3) transformations. In this BSM model, we replace the SU(3) gauge symmetry of chromodynamics with an SU(4) symmetry. This symmetry adds a fourth color charge to QCD, which allows us to introduce interesting dynamics that help expand the scope of the Standard Model. However, experimental indicates there are three color charges. To match experimental data, we break the $\mathrm{SU}(4)$ symmetry we have introduced down to the $\mathrm{SU}(3)$ symmetry that matches experiment. This breaking means that at current experimental energy levels, our model matches with three color charges. But at higher energy levels above current experiments color gains a fourth color charge.

To yield this qualitative behavior mathematically, we introduce a scalar boson field similar to the Higgs that breaks this SU(4) symmetry such that the remaining symmetry is the SU(3) found in the Standard Model; this field we will call ϕ. We are interested in breaking the $\mathrm{SU}(4)_C$ symmetry, so ϕ will be charged under both $\mathrm{SU}(4)_C$; in order to avoid issues with hypercharge later on, ϕ will also couple to $\mathrm{U}(1)_X$, which is related to SM $\mathrm{U}(1)_Y$. The representation of SU(4) that this field is in determines its specific properties. We move forward with ϕ in the **4** representation, but a discussion of an alternative is located in Appendix B. The generators of the SU(4) Lie algebra, which result in the SU(4) symmetry structure, are given in Appendix A. The SU(4) Lie Algebra has 15 generators;

we note that generators T^{1-8} are the Gell-Mann matrices - the generators for the SU(3) Lie Algebra. Thus, SU(4) color can be broken such that the preserved symmetry is the SU(3) symmetry structure of the Standard Model.

3.1 Breaking SU(4) Color

ϕ is charged under $\mathrm{SU}(4)_C$ and $\mathrm{U}(1)_X$.While ϕ is in the fundamental representation of SU(4), it has a VEV of the form

$$\langle\phi\rangle = \begin{pmatrix} 0 \\ 0 \\ 0 \\ w \end{pmatrix} . \tag{3.1}$$

This leads to a symmetry breaking pattern of

$$\mathrm{SU}(4)_C \times \mathrm{U}(1)_X \to \mathrm{SU}(3)_C \times \mathrm{U}(1)_Y \tag{3.2}$$

Before symmetry breaking, this color group has a total of 15 Lie algebra generators; after symmetry breaking the preserved symmetry SU(3) has only 8. This results in 7 Nambu-Goldstone bosons (NGBs) being produced. These bosons are "eaten" by the new massive vector bosons that are produced when diagonalizing into the mass basis. Eight of the SU(4) gauge bosons become the eight gluons of the $\mathrm{SU}(3)_C$ subgroup which will be called G^a_μ. Six of the seven NGBs are eaten by the vector boson we will call ξ_μ:

$$\xi_\mu = \frac{1}{\sqrt{2}} \begin{pmatrix} C^9_\mu - iC^{10}_\mu \\ C^{11}_\mu - iC^{12}_\mu \\ C^{13}_\mu - iC^{14}_\mu \end{pmatrix} \tag{3.3}$$

This vector boson is analagous to the W_μ bosons. Similar to the W_μ bosons after the Higgs symmetry breaking, ξ_μ is composed of the unpreserved SU(4) generators left after color symmetry breaking.

The seventh NGB is eaten by a linear combination of X_μ and C_μ^{15}. With $\langle\phi\rangle$ defined for our symmetry breaking pattern, we consider the covariant derivative acting on it:

$$D_\mu\langle\phi\rangle = -i \begin{pmatrix} \frac{g_s}{\sqrt{2}}\xi_\mu \\ \frac{g_X}{8}X_\mu - g_s\sqrt{\frac{3}{8}}C_\mu^{15} \end{pmatrix} w \,. \tag{3.4}$$

Covariant derivatives show up in the Lagrangian when the term concerns the self-interaction of a given field. In this case, we look at the term $|D_\mu\langle\phi\rangle|^2$, called the kinetic term of the ϕ field. This outlines the self-interaction of ϕ. After expanding the kinetic term with the new VEV of ϕ, we find that the generators not preserved under SU(3) group together to form the ξ_μ as discussed, but also a new linear combination. The kinetic term becomes

$$\frac{w^2 g_s^2}{2}\xi_\mu^\dagger\xi^\mu + w^2\left(\frac{g_X^2}{64}+\frac{3}{8}g_s^2\right)\left(C^{15\mu}c_X - X^\mu s_X\right)\left(C^{15\mu}c_X - X^\mu s_X\right) \,, \tag{3.5}$$

where

$$\cos\theta_X \equiv c_X = \frac{g_s\sqrt{24}}{\sqrt{24g_s^2+g_X^2}} \,, \quad \sin\theta_X \equiv s_X = \frac{g_X}{\sqrt{24g_s^2+g_X^2}} \,. \tag{3.6}$$

Equation 3.5 tells us what combinations of generators produce massive bosons, as well as their masses. From the first term, we identify the mass of ξ_μ as

$$m_\xi = \frac{g_s w}{\sqrt{2}} \,. \tag{3.7}$$

From the second term of Eq. 3.5, we can define the massive linear combination of C_μ^{15} and X_μ in the mass basis as

$$B'_\mu = C_\mu^{15}c_X - X_\mu s_X, \tag{3.8}$$

which has a resulting mass of

$$m_{B'} = \frac{w}{4\sqrt{2}}\sqrt{24g_s^2 + g_X^2}\ . \tag{3.9}$$

We have found massive linear combinations of generators, however the parameters that determine their specific values and masses are not in terms of SM values. Specifically, we have values in terms of X_μ and its coupling, rather than SM hypercharge. To do so we rewrite these relations as

$$X_\mu = B_\mu c_X - B'_\mu s_X\ , \qquad C^{15}_\mu = B'_\mu c_X + B_\mu s_X \tag{3.10}$$

For a given field ψ this means that the covariant derivative attached to the $\mathrm{SU}(4)_C$ gauge couplings becomes

$$\frac{g_s}{2}T^a C^a_\mu = \frac{g_s}{2}\lambda^a G^a_\mu + \frac{g_s}{\sqrt{2}}\begin{pmatrix} 0 & \xi_\mu \\ \xi^\dagger_\mu & 0 \end{pmatrix} + g_s\sqrt{\frac{3}{8}}T_c\left(B'_\mu c_X + B_\mu s_X\right)\ , \tag{3.11}$$

where the λ^a matrices are the Gell-Mann matrices and

$$T_c = \begin{pmatrix} \frac{1}{3} & 0 & 0 & 0 \\ 0 & \frac{1}{3} & 0 & 0 \\ 0 & 0 & \frac{1}{3} & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}\ . \tag{3.12}$$

Now adding the covariant derivative attached to the $\mathrm{U}(1)_X$ symmetry yields

$$D_\mu\psi \supset -iB'_\mu\left[g_s\sqrt{\frac{3}{8}}c_X T_c - g_X X_\psi s_X\right]\psi - ig' B_\mu\left[X_\psi + X_\phi T_c\right]\psi\ , \tag{3.13}$$

where the hypercharge gauge coupling is given by

$$g' = \frac{g_X g_s\sqrt{\frac{3}{8}}}{\sqrt{\frac{3}{8}g_s^2 + g_X^2 X_\phi^2}} \tag{3.14}$$

$$g_X = \frac{g' g_s\sqrt{\frac{3}{8}}}{\sqrt{\frac{3}{8}g_s^2 + g'^2 X_\phi^2}} \tag{3.15}$$

This puts the coupling associated with $\mathrm{U}(1)_X$ in terms of SM values and X_ϕ, which will be determined later in the chapter. This in turn allows us to find the SM hypercharge of any field charged under $\mathrm{SU}(4)_C \times \mathrm{U}(1)_X$:

$$(\mathbf{4}, X) \rightarrow \left(\mathbf{3}, X + \frac{X_\phi}{3}\right) \oplus (\mathbf{1}, X - X_\phi) \tag{3.16}$$

Thus we have found Standard Model hypercharge and its associated coupling in terms of the BSM parameters of $\mathrm{U}(1)_X$. We find ϕ to be in a specific representation of $\mathrm{U}(1)_X$ which corresponds to its charge under this gauge.

We are also able to find the SM values for quark fields with the values in our model.

$$\mathcal{D} \sim \left(\mathbf{4}, \mathbf{1}, -\frac{X_\phi + 1}{3}\right) \ , \ \mathcal{U} \sim \left(\mathbf{4}, \mathbf{1}, \frac{2 - X_\phi}{3}\right) \ , \ \mathcal{Q} \sim \left(\mathbf{4}, \mathbf{2}, \frac{1 - 2X_\phi}{6}\right) \ . \tag{3.17}$$

And the lepton and Higgs fields remain unchanged, as does the $\widetilde{H}$ field from the quark Yukawa terms:

$$E \sim (\mathbf{1}, \mathbf{1}, -1) \ , \ L \sim \left(\mathbf{1}, \mathbf{2}, -\frac{1}{2}\right) \ , \ H \sim \left(\mathbf{1}, \mathbf{2}, \frac{1}{2}\right) \ , \ \widetilde{H} \sim \left(\mathbf{1}, \overline{\mathbf{2}}, -\frac{1}{2}\right) \ . \tag{3.18}$$

After breaking color symmetry, the quarks are bound under SM $\mathrm{SU}(3)_C$. But what happens to the quarks carrying the fourth color? These appear as three colorless BSM fermions of SM charges $(\mathrm{SU}(3)_c \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y)$

$$D_4 \sim \left(\mathbf{1}, \mathbf{1}, -\frac{4X_\phi + 1}{3}\right) \ , \ U_4 \sim \left(\mathbf{1}, \mathbf{1}, \frac{2 - 4X_\phi}{3}\right) \ , \ Q_4 \sim \left(\mathbf{1}, \mathbf{2}, \frac{1 - 8X_\phi}{6}\right) \ . \tag{3.19}$$

There is no X_ϕ value that allows both electric charges to vanish, so these BSM fermions predicted by our model are electrically charged. The experimental data regarding charged particles such as these will put experimental constraints on our model. The **10** representation of ϕ produced charged particles that were too light; to avoid repeating this issue, we introduce several color singlet fields to push the masses of these fourth color quarks above experimental limits. These fields will provide mass terms in the Yukawa interactions:

$$\lambda_F \overline{\mathcal{Q}} \phi F^c + \lambda_P \overline{\mathcal{U}} \phi P^c + \lambda_V \overline{\mathcal{D}} \phi V^c + \mathrm{H.c.} \ , \tag{3.20}$$

F, P, and V are new BSM fermions providing mass to the fourth color quarks. To preserve Lorentz invariance in the Lagrangian, these fermion fields will have quantum numbers

$$V^c \sim \left(\mathbf{1},\mathbf{1},-\frac{4X_\phi+1}{3}\right) \ , \ P^c \sim \left(\mathbf{1},\mathbf{1},\frac{2-4X_\phi}{3}\right) \ , \ F^c \sim \left(\mathbf{1},\mathbf{2},\frac{1-8X_\phi}{6}\right) \ . \tag{3.21}$$

In order to avoid introducing additional anomalies into the model, we require any of these left-handed fields to have vanishing hypercharge. The only left-handed fermion of these three is F. Requiring the hypercharge of F to be zero requires

$$X_\phi = \frac{1}{8} \ . \tag{3.22}$$

To avoid anomalies from our right-handed fermions, we require

$$X_P^3 + X_V^3 = 0 \ \text{ and } \ X_P + X_V = 0 \ . \tag{3.23}$$

These requirements, assuming the same choice of X_ϕ, yields

$$X_P = -X_V = \frac{1}{2} \ . \tag{3.24}$$

These are the values required by our model to avoid introducing anomalies, so from these we are able to determine properties of the fourth color quarks.

3.1.1 Massive Fermionic States

We begin by determining physical constraints on the BSM particles produced by the model. After the mass mixing caused by color breaking, the fourth color quarks and the F, P, and V fermions are combined to create a set of fermions capable of being observed. Expressing F^c and Q_4 as $\mathrm{SU}(2)_L$ doublets we find

$$F^c = \begin{pmatrix} F_u^c \\ F_d^c \end{pmatrix} \ , \ Q_4 = \begin{pmatrix} U_{L4} \\ D_{L4} \end{pmatrix} \ . \tag{3.25}$$

These doublets mix using the mass matrices

$$\mathcal{M}_u = \begin{pmatrix} \lambda_P w & \lambda_U \frac{v_A}{\sqrt{2}} \\ \lambda_{FP} \frac{v_A}{\sqrt{2}} & \lambda_F w \end{pmatrix} , \quad \mathcal{M}_d = \begin{pmatrix} \lambda_V w & \lambda_D \frac{v_A}{\sqrt{2}} \\ \lambda_{FV} \frac{v_A}{\sqrt{2}} & \lambda_F w \end{pmatrix} . \tag{3.26}$$

This mixing results in four fermions that we will call $U_\pm$ and $D_\pm$, which linear combinations of the previous fields, given by

$$U_{L+} = c_{UL} P^c - s_{UL} U_{L4} , \qquad U_{L-} = s_{UL} P^c + c_{UL} U_{L4} , \tag{3.27}$$

$$U_{R+} = c_{UR} U_4 - s_{UR} F_u^c , \qquad U_{R-} = s_{UR} U_4 + c_{UR} F_u^c , \tag{3.28}$$

where

$$\cos 2\theta_{UL} = \frac{w^2(\lambda_P^2 - \lambda_F^2) + v_A^2/2\left(\lambda_{FP}^2 - \lambda_U^2\right)}{m_{u+}^2 - m_{u-}^2} , \quad \sin 2\theta_{UL} = -\sqrt{2} w v_A \frac{\lambda_U \lambda_P + \lambda_F \lambda_{FP}}{m_{u+}^2 - m_{u-}^2} , \tag{3.29}$$

$$\cos 2\theta_{UR} = \frac{w^2(\lambda_P^2 - \lambda_F^2) + v_A^2/2\left(\lambda_U^2 - \lambda_{FP}^2\right)}{m_{u+}^2 - m_{u-}^2} , \quad \sin 2\theta_{UR} = -\sqrt{2} w v_A \frac{\lambda_U \lambda_F + \lambda_P \lambda_{FP}}{m_{u+}^2 - m_{u-}^2} . \tag{3.30}$$

These characterize the $U_\pm$ fermions, and a similar definition is true for $D_\pm$ where $U \to D$, $u \to d$ and $P \to V$. The masses of these fermions to leading order in $\frac{v_A^2}{w^2}$ are

$$m_{u+}^2 = w^2 \lambda_P^2 + \mathcal{O}\left(\frac{v_A^2}{w^2}\right) , \quad m_{u-}^2 = w^2 \lambda_F^2 + \mathcal{O}\left(\frac{v_A^2}{w^2}\right) . \tag{3.31}$$

These leading order masses are not dependent on any fourth color quark mass parameters, only w and the Yukawa couplings of P, F, and V. As such, the masses do not change with generation. So while SM fermions have three generations of particles, these BSM fermions are generation-blind, resulting only four new observable fermions. Remembering our choice of X_ϕ we find that the fermions $u_\pm$ and $d_\pm$ produced by color breaking have electric charge of $\pm\frac{1}{2}$.

3.1.2 Higgs Mechanism with Broken Color

The Higgs Mechanism with this broken color model does not occur exactly like the SM Higgs Mechanism. The introduction of the ϕ field also introduced the $\mathrm{U}(1)_X$ symmetry, which is distinct from the SM $\mathrm{U}(1)_Y$ symmetry. The Higgs field ultimately couples to X_μ, not just B_μ as in the SM. In the visible sector, the Higgs gets a VEV

$$\langle H \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix} \tag{3.32}$$

Because our model doesn't make any changes to the $\mathrm{SU}(2)_L$ symmetry, the W bosons remain unchanged, and can be defined as

$$W_\mu^\pm = \frac{1}{\sqrt{2}} \left(A_\mu^1 \mp i A_\mu^2 \right) , \tag{3.33}$$

where they receive the SM mass. But the masses of the neutral bosons are more complicated. In a process similar to the Higgs Mechanism, we look at the $|D_\mu \langle H \rangle|^2$ of the scalar Lagrangian. We ignore teh $W_\mu^\pm$ terms, as they remain unchanged from the SM. We find the term

$$\frac{v^2}{8} \left(g_X X_\mu - g A_\mu^3 \right) \left(g_X X^\mu - g A^{3\mu} \right) \tag{3.34}$$

where we define

$$\cos\theta_W \equiv c_W = \frac{g}{\sqrt{g^2 + g'^2}} , \quad \sin\theta_W \equiv s_W = \frac{g'}{\sqrt{g^2 + g'^2}} . \tag{3.35}$$

Using the SM definitions of the photon and the corresponding massive boson, we define

$$A_\mu = B_\mu c_W + A_\mu^3 s_W , \qquad \overline{Z}_\mu = - B_\mu s_W + A_\mu^3 c_W , \tag{3.36}$$

as the SM photon and Z. To determine how our model changes them, we substitute these terms into (2.31), which becomes

$$\frac{v^2}{8} \left(B'_\mu g_X s_X + \sqrt{g^2 + g'^2} \overline{Z}_\mu \right) \left(B'_\mu g_X s_X + \sqrt{g^2 + g'^2} \overline{Z}^\mu \right) . \tag{3.37}$$

These are no longer diagonalized, so there is some mass mixing between the B'_μ and $\overline{Z}_\mu$ fields. We then diagonalize the mixing matrix to find the mass states in our model.

$$\frac{1}{2} \left(B'_\mu, \overline{Z}_\mu \right) \begin{pmatrix} m_{B'}^2 + \frac{v^2}{4} g_X^2 s_X^2 & m_{Z_0} \frac{v}{2} g_X s_X \\ m_{Z_0} \frac{v}{2} g_X s_X & m_{Z_0}^2 \end{pmatrix} \begin{pmatrix} B'^\mu \\ \overline{Z}^\mu \end{pmatrix} \tag{3.38}$$

where

$$m_{Z_0} = \frac{v}{2}\sqrt{g^2 + g'^2} \tag{3.39}$$

is the SM mass of the Z boson. The mass eigenstates of this mass matrix are

$$Z'_\mu = B'_\mu c_M - \overline{Z}_\mu s_M \ , \qquad Z_\mu = B'_\mu s_M + \overline{Z}_\mu c_M \tag{3.40}$$

where the mixing angles are defined as

$$c_M = \cos 2\theta_M = \frac{m_{B'}^2 + \frac{v^2}{4} g_X^2 s_X^2 - m_{Z_0}^2}{m_{Z'}^2 - m_Z^2} \tag{3.41}$$

$$s_M = \sin 2\theta_M = -\frac{m_{Z_0} v g_X s_X}{m_{Z'}^2 - m_Z^2} \ . \tag{3.42}$$

The mass eigenvalues corresponding to these eigenstates are

$$m_\pm^2 = \tag{3.43}$$
$$\frac{1}{2}\left(m_{B'}^2 + m_{Z_0}^2 + \frac{v^2}{4} g_X^2 s_X^2 \pm \sqrt{(m_{B'}^2 - m_{Z_0}^2)^2 + (m_{B'}^2 + m_{Z_0}^2)\frac{v^2}{2} g_X^2 s_X^2 + \frac{v^4}{16} g_X^4 s_X^4} \right) \ .$$

where $m_+ = m_{Z'}$ and $m_- = m_Z$. Because $v << w$ we can expand the mass eigenvalues to leading order in $\frac{v}{w}$. This yields

$$m_{Z'}^2 \approx m_{B'}^2 + \frac{v^2}{4} g_X^2 s_X^2 \ , \ \ m_Z^2 \approx m_{Z_0}^2 - \frac{m_{Z_0}^2 v^2 g_X^2 s_X^2}{4 m_{B'}^2} \ . \tag{3.44}$$

Thus, our model predicts three electrically neutral bosons; in addition to the photon and Z_μ boson, we predict a Z'_μ boson that is a leftover of broken color symmetry. These fields can be written as linear combinations of the gauge generators as such:

$$A_\mu = c_W c_X X_\mu + s_W A_\mu^3 + c_W s_X C_\mu^{15} \tag{3.45}$$

$$Z_\mu = c_M c_W A_\mu^3 - (c_M s_W c_X + s_M s_X) X_\mu + (s_M c_X - c_M s_W s_X) C_\mu^{15} \tag{3.46}$$

$$Z'_\mu = (c_M c_X + s_M s_W s_X) C_\mu^{15} - s_M c_W A_\mu^3 - (c_M s_X + s_M s_W c_X) X_\mu \ . \tag{3.47}$$

We note that

$$e = g s_W = g' c_W \ , \quad g' = g_X c_X = 8\sqrt{\frac{3}{8}} g_s s_X \ , \tag{3.48}$$

and define the electric charge as

$$Q_\psi = X_\psi + T^3 + \frac{1}{8} T_c \ . \tag{3.49}$$

The kinetic term of the scalar Lagrangian can then be written in terms of these massive particles.

$$\begin{aligned} & g_X X_\psi X_\mu + g T^3 A^3_\mu + g_s \sqrt{\frac{3}{8}} T_c C^{15}_\mu = \\ & e Q_\psi A_\mu + Z_\mu \frac{g}{c_W} \left[c_M (T^3 - s_W^2 Q_\psi) + \frac{s_M s_W c_X}{8 s_X} \left(T_c - 8 \frac{s_X^2}{c_X^2} X_\psi \right) \right] \\ & + Z'_\mu \frac{g}{c_W} \left[\frac{c_M s_W c_X}{8 s_X} \left(T_c - 8 \frac{s_X^2}{c_X^2} X_\psi \right) - s_M (T^3 - s_W^2 Q_\psi) \right] \ . \end{aligned} \tag{3.50}$$

This equation shows how the fermion fields couple to the photon, the Z, and the Z'. We can see that the coupling to the photon is dependent only on electric charge, just as in the Standard Model. The couplings to the Z are similar to the Standard Model, but with an added dependence on T^c. Assuming s_M is small, the change from the Standard Model is small. The Z' couples very little to leptons because for them $T^c = 0$, but couples strongly to quark and fourth color quarks.

3.2 Dark Matter

Having defined our model in the visible sector, we are able to do the same for the dark sector. The dark sector is defined as being identical to the visible sector, with two differences. In order to avoid being $\mathbb{Z}_2$ symmetric between sectors, we set the VEV of the ϕ field in the dark sector to be zero

$$\langle \phi_B \rangle = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \ . \tag{3.51}$$

This makes the dark sector dynamically different from the visible sector, breaking the mirror symmetry between them. Having a VEV of zero means that color symmetry is not broken in the dark sector — there are four unbroken color charges regardless of energy scale. This results in colorless hadrons in dark the sector being bosons; our model produces a novel and interesting structure of dark matter that needs further exploration.

The dark sector also alleviates the hierarchy problem with the introduction of another Higgs field. We use the nonlinear parametrization of these fields

$$H_A = \boldsymbol{h}\frac{f}{\sqrt{\boldsymbol{h}^\dagger \boldsymbol{h}}}\sin\left(\frac{\sqrt{\boldsymbol{h}^\dagger \boldsymbol{h}}}{f}\right) \tag{3.52}$$

$$H_B = \begin{pmatrix} 0 \\ f\cos\left(\frac{\sqrt{\boldsymbol{h}^\dagger \boldsymbol{h}}}{f}\right) \end{pmatrix} \tag{3.53}$$

where f is the scale at which the Higgs and the twin Higgs become different. We define $\boldsymbol{h}$ as a global Higgs doublet that has the VEV

$$\langle \boldsymbol{h} \rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v \end{pmatrix} . \tag{3.54}$$

The resulting VEVs for these fields can be written as

$$\langle H_A \rangle = \begin{pmatrix} 0 \\ f\sin\frac{v}{f\sqrt{2}} \end{pmatrix} , \quad \langle H_B \rangle = \begin{pmatrix} 0 \\ f\cos\frac{v}{f\sqrt{2}} \end{pmatrix} \tag{3.55}$$

Here, the global Higgs VEV is shared by the visible and dark sector Higgs fields. The visible Higgs VEV is then only a portion of the global VEV, which is considerably larger. This alleviates the hierarchy problem by providing a reason for the low value of the Higgs VEV — the global VEV is higher than the observed value, but much of it is hidden in the dark sector Higgs VEV. This provides a limited explanation for why the Higgs VEV is so low, which alleviates the hierarchy problem.

3.3 New Model Lagrangian

Our model maintains the core structure of the Standard Model while adding a dark matter sector, as well as several new fields and symmetries. These changes are reflected in the Lagrangian for this model. We define the individual components of the Lagrangian for clarity; the full-model Lagrangian is given by their sum.

$$\mathcal{L}_{model} = \mathcal{L}_{visible} + \mathcal{L}_{dark} + \mathcal{L}_{scalar} \tag{3.56}$$

In our model, $\mathcal{L}_{visible}$ is very similar to that of the SM, but modified to reflect the extensions. The gauge symmetries from Eq. 2.1 becomes

$$\frac{1}{4}T^a_{\mu\nu}T^{a\mu\nu} - \frac{1}{4}W^a_{\mu\nu}W^{a\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} \ , \tag{3.57}$$

where T^a are the generators of SU(4) rather than SU(3). This change reflects the extended color of our model. The introduction of this extension also changes the interactions of the Higgs, so the Yukawa interactions in Eq. 2.2 becomes

$$\lambda_l \overline{L} HE + \lambda_d \overline{\mathcal{Q}} H \mathcal{D} + \lambda_u \overline{\mathcal{Q}} \widetilde{H} \mathcal{U} + \text{h.c.} \tag{3.58}$$

$$+\lambda_{A1}\overline{Q}\phi_A(F_L)^c + \lambda_{A2}\overline{u}\phi_A(E_R)^c + \lambda_{A3}\overline{d}\phi_A(V_R)^c + \text{h.c.} \tag{3.59}$$

Thus, the BSM fermions couple to the Higgs field like the SM fermions, which allows us to predict their masses. These changes are reflected in the dark sector, so $\mathcal{L}_{dark} = \mathcal{L}_{visible}$, with one change: the color breaking VEV w that is nonzero in the visible sector is zero in the dark sector. And finally, $\mathcal{L}_{scalar}$ includes the kinetic terms of all scalar fields, which is defined as

$$\mathcal{L}_{scalar} = (D_\mu H_A)^\dagger D_\mu H_A + (D_\mu \phi_A)^\dagger D_\mu \phi_A + (D_\mu H_B)^\dagger D_\mu H_B + (D_\mu \phi_B)^\dagger D_\mu \phi_B + V_{scalar} \ . \tag{3.60}$$

Here V_{scalar} is the scalar potential of our model, which dictate how the scalar fields H_A, H_B, ϕ_A, and ϕ_B interact with each other.

This Lagrangian defines all interactions our model predicts. It can be split into the visible and dark sectors, with any interactions between the two occurring via scalar fields. Having finished the model Lagrangian, all that remains is to determine BSM parameter constraints and potential collider signals. Our model introduces new fields and dynamics, which are partially defined by known parameters. But they are also defined by new free parameters that are not defined by the Standard Model. We use experimental data to determine constraints on these BSM parameters, and then determine potential collider signals after these parameters are determined.

Chapter 4

Results

In this thesis we have outlined a mathematical extension of the Standard Model that extends the gauge symmetry governing color interactions. This change introduced new fermion and boson fields which are outside the scope of existing models. As these fields are not predicted by the Standard Model, in most cases their properties are not fixed values. Their characteristics are determined by the parameters introduced in our model; these parameters do not have fixed values, but are affected by other factors that can be experimentally evaluated. In order to determine the experimental characteristics of our model and its fields, we seek to constrain the BSM parameters on which our field properties depend.

4.1 Fermion Masses

We first look at the $u_\pm$ and $d_\pm$ BSM fermions, which have electric charge $\pm\frac{1}{2}$. There are experimental bounds on such charged particles — it is likely their masses must be at least 1.5 TeV. Fixing their masses to 1.5 TeV, we plot the Yukawa couplings of the F, P, and V fermion fields for various w values in Figure 4.1. We are now able to characterize the masses of the $u_\pm$ and $d_\pm$ for a variety of physical parameters. In general, as w is decreased, the Yukawa couplings must increase to give the fermions enough mass to be experimentally viable. However, for a given color breaking VEV w, F, P, and V can couple to the Higgs to varying degrees. The experimental constraints on the value of w found in Figure 4.2 further specify the Yukawa couplings for our BSM fermions. The values of our BSM Yukawa couplings are not fixed, but we have constrained them to a smaller range of potential values. This constraining allows our model to be as specific as possible, which makes potential experimental observation more straightforward.

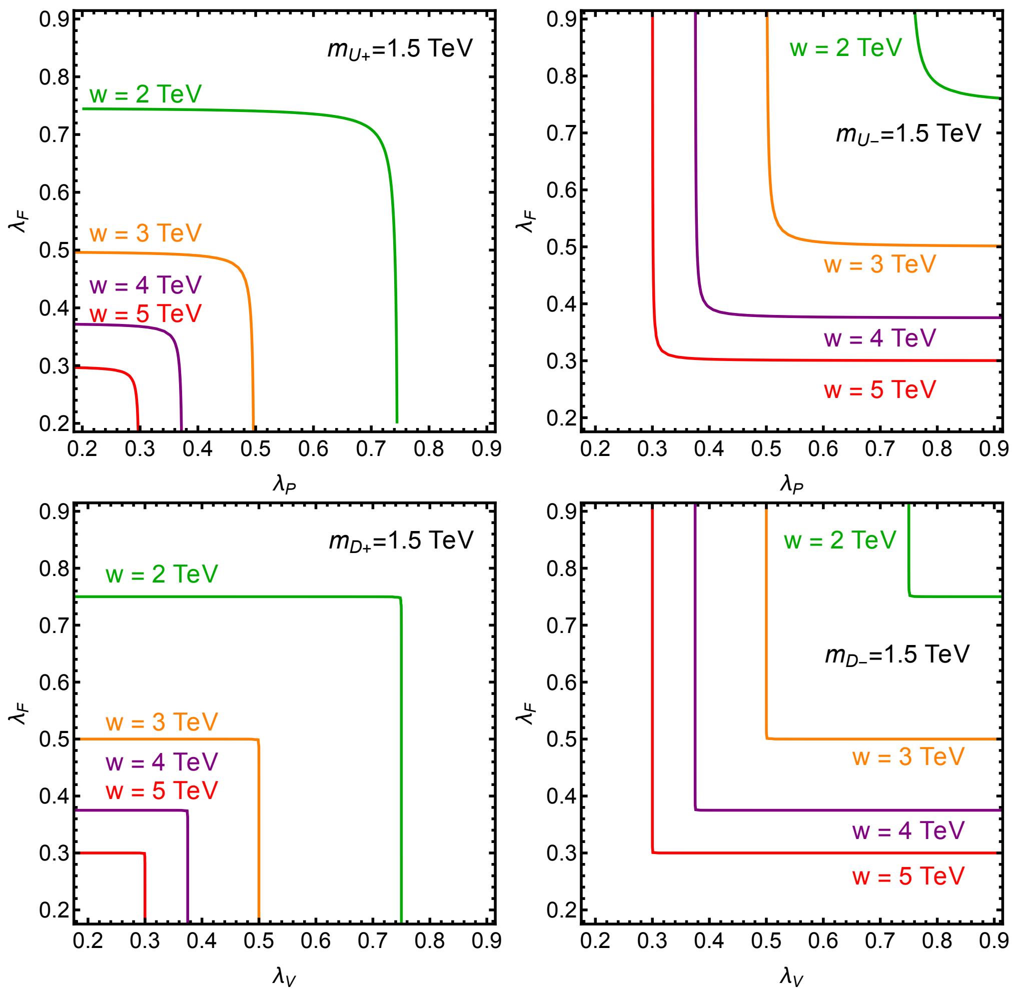


Figure 4.1 Contour plots of the BSM fermions $u_\pm$ (top) and $d_\pm$ (bottom) with masses of 1.5 TeV as a function of the Yukawa couplings and for various values of the color breaking VEV w. The coupling $\lambda_{FP} = \lambda_{FV}$ is taken to be 0.02 and has little effect on the outcome for values of this magnitude or smaller [2].

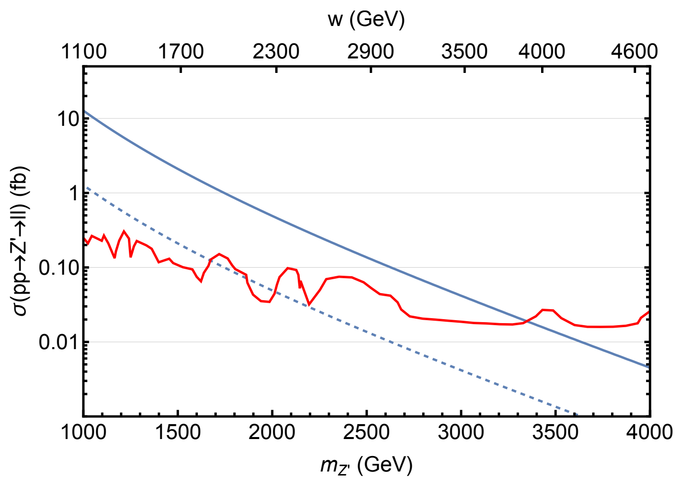
4.2 Collider Signals

This gives us some constraints on the masses of the fermions predicted by our model; the masses of the bosons must be characterized as well. The Z' boson can be produced in a collider through quark-antiquark interaction. These processes have been searched through dilepton [8, 9] and dijet [10] resonances for bosons like our Z'.

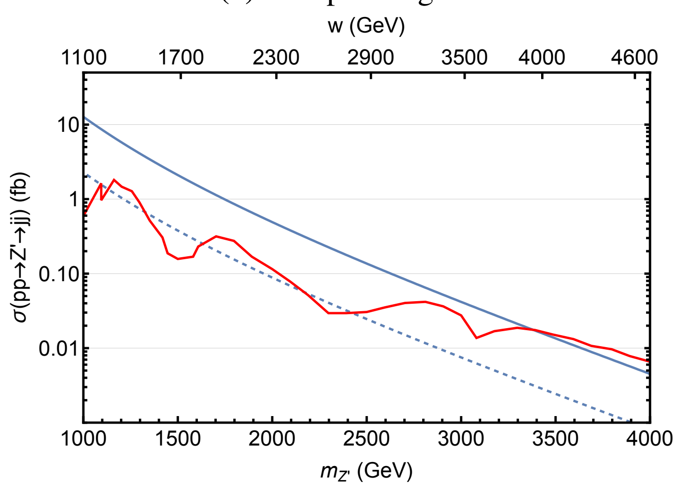
Because the Z' boson predicted in this model is a product of color breaking, it has comparatively large interactions with quarks, and comparatively small couplings to leptons. However, our Z' is color neutral, but carries that leftover fourth charge after symmetry breaking, just like the fourth color quarks $u_\pm$ and $d_\pm$. So, the Z' couples most strongly to the fourth color quarks if they are available. As found in Equations 3.31 and 3.44, the masses of both the massive fourth color states and the Z' both depend on the color breaking VEV w. However, it possible for the post-decay sum of masses of $u_\pm$ and d_pm to be greater than the mass of the Z'; in this case, the Z' cannot decay into the BSM fermions because of conservation of mass. If the masses of the fourth color quarks are lower, $u_\pm$ and $d_\pm$ are kinematically available for the Z' to decay into.

When the fourth color quarks are kinematically available to the Z', the branching fraction for our model's Z' to SM quarks is approximately 18%, while the branching fraction to leptons is only about 0.01%. But when the BSM fermions are too heavy for the Z' to decay into them, these branching fractions change to 99.5% into SM quarks and 0.1% into leptons [2]. In the case where BSM fermions are allowed kinematically, the Z' prefers to decay into $u_\pm$ and $d_\pm$. However, our model can be constructed with this allowance or without. This gives us two specific ways to check our model's Z' experimentally.

The red lines of Figure 4.2 are the experimental signal for bosons like the Z' as a function of the boson mass. In order for our model to not be experimentally ruled out, it must predict cross sections below the experimental signal, otherwise they would have been observed already. So, we



(a) Dilepton Signal



(b) Dijet Signal

Figure 4.2 (a) We plot the bounds, in red, on the Z' from an ATLAS dilepton search [8]. **(b)** We plot the bounds, in red, on the Z' from an CMS dijet search [10].
Both: Solid blue lines denote the cross section when Z' decays to BSM fermions **are** allowed. Dashed blue lines denote the cross section when Z' decays to BSM fermions **are not** allowed. [2]

are interested in regions of the graphs where our predicted Z' branching fractions are *below* the red lines.

The solid blue lines of Figure 4.2 corresponds to the case where Z' *can* decay into BSM fermions; this makes the branching fraction for SM fermions significantly lower. In this case, the Z' mass must have a mass of nearly 3.5 TeV to remain undetected. This Z' mass corresponds to a color breaking VEV of roughly 4 TeV. A search for our Z' which decays into BSM fermions would need to probe energies in that region.

The dashed blue lines of Figure 4.2 corresponds to the case where Z' *cannot* decay into BSM fermions; in this case Z' decays primarily into SM quarks. For this case, there are regions of Z' mass as low as nearly 1 TeV where our model's Z' would remain undetected. However, only with a Z' mass above roughly 2.5 TeV do both dijet and dilepton branching fraction predictions fall below experimental bounds. This mass corresponds to a color breaking VEV of roughly 2.9 TeV. A search for our Z' decaying only into SM fermions would probe masses above this energy level.

We have analyzed our model's Z' boson potential collider signals to determine its experimentally allowed masses. The Z' remains undetected and thus experimentally viable in regions of Figure 4.2 where the blue lines are below the red. Depending on the availability of BSM decays, the allowed Z' mass changes along with the color breaking VEV w. As discussed in Section 4.1, the Yukawa couplings of the F, P, and V fermion fields depend on allowed values for w. Our model's two cases of Z'-to-fermion decay require w values greater than 2.9 TeV and 4 TeV respectively. Thus, our analysis has allowed us to further constrain the Yukawa couplings of the BSM fermions and characterize our Z' boson more specifically.

4.3 Final Remarks

Our mathematical model predicts several BSM particles with the potential to be physically observed; the properties of these particles are dependent on variable parameters of the model which are constrained by experiment. The model remains experimentally feasible and mathematically rigorous while adding BSM fields and dynamics. The introduction of a dark sector alleviates the hierarchy problem and posits a novel structure for dark matter. The structure of this dark matter is complex and interesting, and the coupling through the Higgs field predicts potentially observable collider signals. However, further study of its theoretical properties is required — specifically, how the integer spin hadrons in the dark sector behave phenomenologically. The corresponding gauge symmetry that creates these hadrons is broken in the visible sector, which breaks the $\mathbb{Z}_2$ mirror symmetry of the model. This breaking of color symmetry in the visible sector leads to interesting new dynamics while maintaining the information of the Standard Model. At high energy scales, color in the visible sector has the same $\mathrm{SU}(4)_C$ symmetry as the dark sector. But at scales currently observable, quarks transform under the $\mathrm{SU}(3)_C$ symmetry that is identical to the Standard Model. Our model introduces BSM fields that are potentially observable in collider experiment, and resolves problems for which the Standard Model does not have solutions.

Appendix A

Generators of SU(4)

We use the following representation of the SU(4) Lie Algebra

$$T^1 = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad T^2 = \begin{pmatrix} 0 & -i & 0 & 0 \\ i & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad T^3 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{A.1}$$

$$T^4 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad T^5 = \begin{pmatrix} 0 & 0 & -i & 0 \\ 0 & 0 & 0 & 0 \\ i & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad T^6 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{A.2}$$

$$T^7 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & -i & 0 \\ 0 & i & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad T^8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -2 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad T^9 = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix} \tag{A.3}$$

$$T^{10} = \begin{pmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ i & 0 & 0 & 0 \end{pmatrix} \quad T^{11} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \quad T^{12} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -i \\ 0 & 0 & 0 & 0 \\ 0 & i & 0 & 0 \end{pmatrix} \tag{A.4}$$

$$T^{13} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \quad T^{14} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \end{pmatrix} \quad T^{15} = \frac{1}{\sqrt{6}} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -3 \end{pmatrix} \tag{A.5}$$

which satisfy

$$\mathrm{Tr}\left[\frac{1}{2}T^a \frac{1}{2}T^b\right] = \frac{1}{2}\delta^{ab} \,. \tag{A.6}$$

With the non-zero structure constants being:

$$\begin{aligned}
&f^{123} = 1\,, \ f^{458} = f^{678} = \frac{\sqrt{3}}{2}\,, \ f^{8\,9\,10} = f^{8\,11\,12} = \frac{1}{2\sqrt{3}}\,, \ f^{8\,13\,14} = -\frac{1}{\sqrt{3}} \\
&f^{147} = f^{165} = f^{1\,9\,12} = f^{1\,11\,10} = f^{246} = f^{257} = f^{2\,9\,11} = f^{2\,10\,12} = f^{345} = f^{376} = f^{3\,9\,10} = \frac{1}{2}\,, \\
&f^{3\,12\,11} = f^{4\,9\,14} = f^{4\,13\,10} = f^{5\,9\,13} = f^{5\,10\,14} = f^{6\,11\,15} = f^{6\,14\,12} = f^{7\,11\,13} = f^{7\,12\,14} = \frac{1}{2}\,, \\
&f^{9\,10\,15} = f^{11\,12\,15} = f^{13\,14\,15} = \sqrt{\frac{2}{3}}
\end{aligned} \tag{A.7}$$

Appendix B

Representations of SU(4)

10 Representation Model

In this model, the ϕ field is in the **10** representation of SU(4), with ten dimensions. In this representation, it gets a VEV of the form

$$\langle\phi\rangle = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & w \end{pmatrix} \tag{B.1}$$

Having ϕ take this form is attractive because it does not require the addition of more BSM fermions to maintain Lorentz invariance [11]. However, the subsequent model proposed by foot introduces charge $\pm\frac{1}{2}$ fermionic particles [12], which we found have since been experimentally excluded. Having been experimentally excluded, we move ϕ to the **4** representation of SU(4).

4 Representation Model

The **4** representation of SU(4) is the fundamental representation of this group, which means it is has the lowest dimensions possible while maintaining this symmetry pattern. While ϕ is in the fundamental representation of SU(4), it has a VEV of the form

$$\langle\phi\rangle = \begin{pmatrix} 0 \\ 0 \\ 0 \\ w \end{pmatrix} \tag{B.2}$$

Having a VEV of this form preserves the $\mathrm{SU}(3)_C$ subgroup after symmetry breaking

$$\frac{1}{2}T^{1,\ldots,8}\langle\phi\rangle = 0 \tag{B.3}$$

This VEV is also an eigenvector of the T^{15} generator because

$$\frac{1}{2}T^{15}\langle\phi\rangle = -\frac{1}{2}\sqrt{\frac{3}{2}}\langle\phi\rangle. \tag{B.4}$$

Using this model of ϕ we define the gauge boson of $\mathrm{U}(1)_X$ as X_μ and the gauge coupling g_X, We will define the gauge coupling of $\mathrm{SU}(4)_C$ as g_s because it is inherited by the residual $\mathrm{SU}(3)_C$ color group of the Standard Model. The gauge bosons of this color group will be denoted by C^a_μ. The complete gauge covariant derivative is given by

$$D_\mu\psi = \left(\partial_\mu - ig_X X_\psi X_\mu - i\frac{g}{2}\sigma^a A^a_\mu - i\frac{g_s}{2}T^a C^a_\mu\right)\psi, \tag{B.5}$$

where X_ψ is the $U(1)_X$ charge of ψ and g is the usual $\mathrm{SU}(2)_L$ gauge coupling. This $\mathrm{U}(1)_X$ charge is different from standard hypercharge, as it is attached to the U(1) gauge group broken by ϕ that results in $\mathrm{U}(1)_Y$ after symmetry breaking. The generators and gauge bosons of $\mathrm{SU}(2)_L$ are the same as the Standard Model – $\frac{1}{2}\sigma^a$ and A^a_μ respectively. In this thesis we model ϕ in the fundamental **4** representation of SU(4).

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