An Approximation for Early Universe Neutron Abundance Until the Onset of Neutron Decay

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ABSTRACT

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[An approximation for the fractional neutron abundance in the 1 MeV to .1 MeV range in the early universe is derived. This is then computed and plotted using the Mathematica software. It is compared with and found in agreement with similar plots and acts as an upper limit on ^{4}He during Big bang nucleosynthesis. The approximation falls short at around 0.1 MeV due the onset of neutron decay and D production, changing effective degrees of relativistic freedom, and the increasing consequence of the mass of the electron as the universe cooled.]

Keywords: [Big bang nucleosynthesis, Cosmic microwave background, Neutron abundance]

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Chapter 1

Introduction

1.1 General Overview of BBN

There are light nuclei that are commonly found throughout the universe such as ^{1}H (written as just H), ^{2}H (written as D and called Deuterium), ^{3}H (written as T and called Tritium) ^{4}He, and to a much lesser extent ^{7}Li. To an observer, the question naturally arises as to how these nuclei were formed, and why do we have the particular mix that we have; that is to say, why is the ratio of D to H what it is, etc. The best agreement with observed abundances of these light elements is in the predictions that result from the hot big bang model with the process for their formation known as Big Bang Nucleosynthesis (BBN) [3]. In BBN, these nuclei formed as the result of a very dense, high temperature plasma that rapidly expanded [4]. As it expanded, it cooled enough such that electrons, protons, and neutrons could form. Further expansion led to sufficient cooling that allowed for protons and neutrons to begin lumping together, with more and more interactions resulting in the formation of heavier and heavier elements. All the while the universe continued to expand and this lowering of density and temperature reduced the number of interactions which effectively put a cap on how heavy of elements, to an appreciable amount, were formed in the early universe. Nuclei

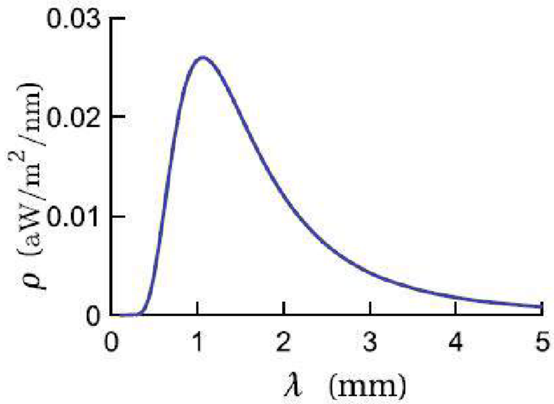

Figure 1.1 Black Body radiation of the CMB is plotted, which has a temperature of 2.725 K. The energy density is plotted as a function of wavelength as a function of wavelength. Graph from Peatross and Ware (2015) [1].

up to ^{7}Be had formed to an abundance worth noting within the window given by the expanding universe. ^{7}Be though, being unstable, mostly decays into ^{7}Li and a positron, and thus ^{7}Li is the heaviest of the light elements that is of interest in looking at as far as model matching for early universe nucleosynthesis goes classically.

1.2 CMB and BBN

As the early universe can be modeled in a way to expect black body radiation, it also follows that there should be a characteristic spectrum of intensities of wavelengths being given off along with a principle wavelengths associated with the temperature [1]. This associated spectrum of wavelengths is indeed found throughout the universe. The name for this is the Cosmic Microwave Background (CMB). The associated temperature to the CMB is 2.725 K [2] [5].

With the temperature constrained through the CMB, BBN is able to predict what abundances we should see of light elements that are a product from the early Universe. Standard practice is to express these abundances as ratios with respect to the observed ^{1}H. BBN's predictions agree well with observed abundances with the exception of ^{7}Li which has historically not agreed. In recent years, the disagreement has come into question and the so called Lithium-7 problem is no longer strongly motivated. [6]. This general agreement builds confidence both in the Standard Model and our understanding of basic nuclear interactions.

1.3 Neutron Abundance in the Early Universe's Part in BBN

In this paper, it will be convenient to roll the Boltzmann's constant into the temperature and express temperatures as energies, mostly as MeV. It will also be convenient to choose units such that atomic masses normally written as MeV/c^2 can also be expressed as energies in MeV. At temperatures much higher than 1 MeV, the ratio of neutrons to protons was 1 due to the ratio [2]

$$\frac{n_p^{(0)}}{n_n^{(0)}} = e^{\mathscr{Q}/T}. \tag{1.1}$$

n_i is the particle number density, in this case for protons and neutrons respectively. The super script (0) denotes that the number densities are equilibrium number densities. $\mathscr{Q}$ is the mass difference between neutrons and protons which is 1.293 MeV and T is the temperature. Thus as the Universe cools and draws closer to the MeV energy scale, the ratio in (1.1) increases which skews it in favor of the protons.

This ratio works well at high temperatures, and gives an accurate big picture that as the temperature goes down the number of neutrons goes down, but it also misses the real ratio substantially as it assumes perfect efficiency in weak interactions which would convert neutrons into protons. This is referring to interactions discussed in Chapter 2 other than neutron decay, as neutrons are unstable while protons are stable. The time scale for neutron decay is longer than the time scale of interactions at this point in the early Universe and is a smaller contributing factor. Perfection in conversion through the weak interactions doesn't happen for similar reasons as to why later in the BBN process the baryons don't instantaneously clump into iron nuclei, which has the highest binding energy [2].

The baryon densities aren't high enough in the time scale given by the expanding early universe for every neutron to be converted. A critical aspect of modeling BBN and light element production is to be able to accurately approximate how the neutron-proton ratio as a function of temperature.

1.4 Approximating Neutron Abundance Prior to Neutron Decay

In this paper, I recreate an approximation for the neutron abundance done in Dodelson's Modern Cosmology. This approximation is aimed at being accessible yet somewhat accurate prior to neutron decay, and later D production. This approximation serves the purpose of setting an upper bound on the expected ^{4}He production in BBN, providing a good starting point for adding to when calculating the neutron abundance during light element production, and showing that the neutron-proton ratio falls out of equilibrium as the temperature drops.

In the following chapter I discuss the theoretical methods for calculating the neutron abundance as well as plotting it. Then in chapters 3 and 4, I compare this plot to the plot found in Dodelson's Modern Cosmology and find it in agreement. In chapter 5 I discuss where the approximation falls short as neutron decay and D production become more dominant in their contributions. In chapter 5 I also discuss future work that can use the result of this approximation of the neutron abundance.

Chapter 2

Methods

In this chapter, I go over the methods used for deriving the theoretical neutron abundance as a function of temperature as well as plotting it. In section 2.1, I begin with the Boltzmann equation for annihilation and end with an ODE for the number density. In section 2.2 I derive an ODE for neutron abundance using the result from 2.1. In section 2.3 I go through my methods of solving the ODE from section 2.2 using the method of variation of parameters. In section 2.4 I discuss how the result found in 2.3 was used to compute and plot the neutron abundance as a function of temperature.

2.1 Boltzmann Equation for Annihilation to ODE for Number Density

$$
\begin{aligned}
a^{-3}\frac{d(n_1a^3)}{dt} = &\int \frac{d^3p_1}{(2\pi)^3 2E_1}\int \frac{d^3p_2}{(2\pi)^3 2E_2}\int \frac{d^3p_3}{(2\pi)^3 2E_3}\int \frac{d^3p_4}{(2\pi)^3 2E_4} \\
&\times (2\pi)^4\delta^3(p_1+p_2-p_3-p_4)\delta(E_1+E_2-E_3-E_4)|\mathscr{M}|^2 \\
&\times (f_3f_4(1\pm f_1)(1\pm f_2)-f_1f_2(1\pm f_3)(f\pm f_4))
\end{aligned}
\tag{2.1}
$$

$$1 + 2 \leftrightarrow 3 + 4 \tag{2.2}$$

The Boltzmann equation for annihilation, (2.1), operates under the principle that the rate that an abundance changes is simply the net result of processes that create or annihilate said species. In (2.1) a is the scale factor of the universe, n_i is the species number density for species involved in the interaction in (2.2). The notation for p_i and E_i is the same, which are the momenta and energies respectively. The δ's are Dirac deltas and serve to conserve momentum and energy. The $\mathscr{M}$ is the amplitude of the interaction and is determined based on the physics involved with the interaction between particular particles. The f_i's are the occupation numbers of the particular species.

There is a lot to unpack here in general, but for the purposes of this paper we make note of a few things. The quantity μ is the chemical potential when we are dealing with things in equilibrium. Also of note is that we will be working with species not in equilibrium and at temperatures smaller than $E - \mu$. This simplifies distributions significantly. Secondly, we choose to define the thermally averaged cross section as (2.3), where $n_i^{(0)}$ denotes the equilibrium number density. The superscript (0) denotes that we are referring specifically to equilibrium number densities.

$$\begin{aligned} \langle \sigma v \rangle \equiv & \frac{1}{n_1^{(0)} n_2^{(0)}} \int \frac{d^3 p_1}{(2\pi)^3 2E_1} \int \frac{d^3 p_2}{(2\pi)^3 2E_2} \int \frac{d^3 p_3}{(2\pi)^3 2E_3} \int \frac{d^3 p_4}{(2\pi)^3 2E_4} E^{-(E_1+E_2)/T} \\ & \times (2\pi)^4 \delta^3(p_1 + p_2 - p_3 - p_4) \delta(E_1 + E_2 - E_3 - E_4) |M|^2 \end{aligned} \tag{2.3}$$

Applying these definitions to (2.1) allows us to arrive at (2.4) which is what we want to work with. For a more thorough treatment and derivation, see Dodelson (2003) [2].

$$a^{-3} \frac{d(n_1 a^3)}{dt} = n_1^{(0)} n_2^{(0)} \langle \sigma v \rangle \left(\frac{n_3 n_4}{n_3^{(0)} n_4^{(0)}} - \frac{n_1 n_2}{n_1^{(0)} n_2^{(0)}} \right) \tag{2.4}$$

2.2 ODE for Neutron Abundance

Going from the general case in Eq. (2.4) to the case for neutrons, we consider these two interactions shown in (2.5) and (2.6), that can contribute to neutron production or annihilation.

$$p + \bar{\nu} \leftrightarrow n + e^{+} \tag{2.5}$$

$$p + e^{-} \leftrightarrow n + \nu \tag{2.6}$$

Here p is for proton, e is for electrons and positrons and ν is for neutrinos. The leptons (electrons, positrons, and neutrinos) at the MeV temperature scale are no longer coupled, but are still relativistic, share the same temperature, and are close to the same abundance as one another [2]. As a result, their densities are the same, and we let any number density by a lepton be denoted as n_l. For our situation, we will apply (2.4) to our situation of interest by letting 1 be neutrons, 2 and 4 be leptons, and 3 be protons, which then gives us

$$a^{-3}\frac{d(n_n a^3)}{dt} = n_n^{(0)} n_l^{(0)} \langle \sigma v \rangle \left(\frac{n_p n_l}{n_p^{(0)} n_l^{(0)}} - \frac{n_n n_l}{n_n^{(0)} n_l^{(0)}} \right). \tag{2.7}$$

Rearranging (2.7) and recognizing that $n_l^{(0)} = n_l$ as the leptons are in equilibrium, we get

$$a^{-3}\frac{d(n_n a^3)}{dt} = n_l^{(0)} \langle \sigma v \rangle \left(n_p \frac{n_n^{(0)}}{n_p^{(0)}} - n_n \right). \tag{2.8}$$

It is also useful to define the ratio of neutrons to baryons in general, which is denoted by X_n and is given as

$$X_n \equiv \frac{n_n}{n_n + n_p}. \tag{2.9}$$

We then find

$$n_n = X_n(n_n + n_p) \;, \tag{2.10}$$

and

$$n_p = (n_n + n_p)(1 - X_n) \;. \tag{2.11}$$

Substituting (2.10) and (2.11) into (2.8) and also the equilibrium proton-neutron ratio, (1.1) gives us

$$a^{-3}\frac{d(X_n(n_n+n_p)a^3)}{dt} = n_l^{(0)}\langle\sigma v\rangle\left((n_n+n_p)(1-X_n)e^{-\mathcal{Q}/T} - X_n(n_n+n_p)\right) . \tag{2.12}$$

On the left hand side of the equation, we make note of the fact that in the case where we are concerned with protons being converted into neutrons or vice versa, their sum density, n_n+n_p, times the space, a^3, does not change with time. We can then pull the two together out of the derivative, giving

$$(n_n+n_p)a^3a^{-3}\frac{dX_n}{dt} = n_l^{(0)}\langle\sigma v\rangle\left((n_n+n_p)(1-X_n)e^{-\mathcal{Q}/T} - X_n(n_n+n_p)\right) . \tag{2.13}$$

We can then simplify the equation by canceling the n_n+n_p and the a terms out, leaving us with

$$\frac{dX_n}{dt} = n_l^{(0)}\langle\sigma v\rangle\left((1-X_n)e^{-\mathcal{Q}/T} - X_n\right) . \tag{2.14}$$

At this point we should focus on the term outside the parenthesis on the right hand side, $n_l^{(0)}\langle\sigma v\rangle$. This is the product of the number density of a lepton and the thermally averaged cross section, which is the reaction rate of the reactions we are considering. This is then the conversion rate of neutrons and protons. This rate we'll denote as $\lambda_n p$. Applying this to to 2.14 gives us

$$\frac{dX_n}{dt} = \lambda_{np}\left((1-X_n)e^{-\mathcal{Q}/T} - X_n\right) . \tag{2.15}$$

Finally, we make a change of variable where we let $x \equiv \mathcal{Q}/T$ in order to simplify the work that we will have to do later on due to the time dependence that both T and λ_{np} have. In doing so, we are seeking to let x be our independent variable. In order to do so, we rewrite the left hand side of (2.15) as $\frac{dx}{dt}\frac{dX_n}{dx}$. From how we've defined x, $dx/dt = -\mathcal{Q}(dT/dt)/T^2 = -(x/T)(dT/dt)$.

Now, there are a couple of important relations to draw in order to make the next substitution. First is that $T \propto 1/a$, which is to say that as the universe expands, it also cools at an inverse rate. Next is that the Hubble rate H, which is a measure of how rapidly the scale factor of the universe changes, is defined as $H(t) \equiv \frac{da/dt}{a}$. Lastly, a relation that comes as a result from analysis done with the Einstein equations is that $H^2(t) = 8\pi G\rho/3$ [2]. Where G is the gravitational constant, and ρ is the energy density of the Universe. Stringing this all together gives

$$\frac{dT/dt}{T} = -\frac{da/dt}{a} = -H = -\left(\frac{8\pi G\rho}{3}\right)^{1/2} . \tag{2.16}$$

This then makes $dx/dt = xH$, and the left hand side of 2.15 $xH\frac{dX_n}{dx}$. Isolating the derivative leaves us with

$$\frac{dX_n}{dx} = \frac{\lambda_{np}}{xH}\left((1-X_n)e^{-x} - X_n\right) . \tag{2.17}$$

Now, before we move onto solving our ODE, it is important that we figure out how H and $\lambda_n p$ depend on our independent variable x. For H's part, ρ is the only thing that is a function of x. In order to simplify our calculation, we'll treat ρ as roughly constant with x through our area of interest. At the 1 MeV temperature scale, the relativistic particles are the main contributors to the energy density, whose contributions are quantified as [2]

$$\rho = \frac{\pi^2}{30}T^4\left[\sum_{i=\text{bosons}} g_i + \frac{7}{8}\sum_{i=\text{fermions}} g_i\right] . \tag{2.18}$$

Where g_i denotes the degrees of freedom of the contributing species. Photons are the relativistic bosons that contribute with 2 degrees of freedom. From the fermions neutrinos contribute 6, and electrons and positrons each contribute 2. This makes the effective degrees of freedom denoted as g_* equal to 10.75. In this regime then, with we defined x so that $T^4 = Q^4/x^4$, we end up with 2.19.

$$\frac{dX_n}{dx} = \frac{\lambda_{np}}{x}\left(\frac{3*30x^4}{8\pi^3 G\mathscr{Q}^4 g_*}\right)^{1/2}\left[(1-X_n)e^{-x} - X_n\right] \tag{2.19}$$

Which when cleaned up gives 2.20.

$$\frac{dX_n}{dx} = x\lambda_{np}\left(\frac{45}{4\pi^3 G\mathscr{Q}^4 g_*}\right)^{1/2}\left[(1-X_n)e^{-x} - X_n\right] \tag{2.20}$$

For $\lambda_n p$'s part it has its own rather extensive calculation [2] and for the purpose of this paper we'll just use the result 2.21.

$$\lambda_{np} = \frac{255}{\tau_n x^5}(12 + 6x + x^2) \tag{2.21}$$

Where τ_n = 886.7 sec, and is the half life of the neutron. This sets us up to now solve the first order nonhomogeneous ODE, 2.20, for X_n, the neutron fractional abundance.

2.3 Solving for the Neutron Fractional Abundance as a Function of Temperature

We'll now go about solving the ODE we just found in the previous subsection. Rearranging 2.20 into a more familiar form, and making some definitions allows the application of first order ODE theory to be straight forward. First we rearrange the right hand-side so that we separate the nonhomogeneous part of the equation from the bits that are multiplying X_n, giving us 2.22.

$$\frac{dX_n}{dx} = x\lambda_{np}\left(\frac{45}{4\pi^3 G\mathscr{Q}^4 g_*}\right)^{1/2} e^{-x} - x\lambda_{np}\left(\frac{45}{4\pi^3 G\mathscr{Q}^4 g_*}\right)^{1/2}(1-e^{-x})X_n \tag{2.22}$$

Next we define

$$C \equiv \left(\frac{45}{4\pi^3 G\mathscr{Q}^4 g_*}\right)^{1/2} = 1.13\text{sec}^{-1}, \tag{2.23}$$

as well as

$$\xi(x) \equiv Cx\lambda_{np}e^{-x}, \tag{2.24}$$

and

$$\chi(x) \equiv Cx\lambda_{np}(1+e^{-x}). \tag{2.25}$$

We then isolate the nonhomogeneous term on the right hand side leaving us with

$$\frac{dX_n}{dx} + \chi(x)X_n = \xi(x). \tag{2.26}$$

Now, in the case that $\xi(x)$ is 0, 2.26 is homogeneous and the solution is simply given by 2.27.

$$X_n = Ae^{-P} \tag{2.27}$$

Where A is a constant and $P(x)$ is the anti-derivative of $\chi(x)$. As $\xi(x)$ is not 0, we can use the method of variation of parameters to reach a solution. This entails taking the homogeneous solution and altering it so that it takes the form of 2.28.

$$X_n = \eta(x)e^{-P(x)} \tag{2.28}$$

P doesn't change, so if we solve for $\eta(x)$ we will have solved for X_n. Plugging 2.28 into 2.26 we get 2.29.

$$\frac{d}{dx}(\eta(x)e^{-P(x)}) + \chi(x)\eta(x)e^{-P(x)} = \xi(x) \tag{2.29}$$

Carrying through the derivative by applying the product rule in the first term of 2.29 gives us 2.30.

$$\frac{d\eta(x)}{dx}e^{-P(x)} - \chi(x)\eta(x)e^{-P(x)} + \chi(x)\eta(x)e^{-P(x)} = \xi(x) \tag{2.30}$$

The second and third terms on the left hand side cancel out and if we isolate the derivative term, we have 2.31.

$$\frac{d\eta(x)}{dx} = \xi(x)e^{-P(x)} \tag{2.31}$$

Taking the anti-derivative of both sides gives us $\eta(x)$ and so, mathematically speaking, we've solved for X_n, the neutron fractional abundance in terms of x. To get it in terms of T we substitute back in for x using $x = \mathcal{Q}/T$. The only other bits that have to be considered are the bounds of the integrals and also substituting dx for something with dT. This relation is shown in 2.32 and 2.33.

$$\frac{dx}{dT} = \frac{d}{dT}\left(\frac{\mathcal{Q}}{T}\right) = -\frac{\mathcal{Q}}{T^2} \tag{2.32}$$

$$dx = -\frac{\mathcal{Q}}{T^2}dT \tag{2.33}$$

2.4 Computing and Graphing the Solution with Mathematica

While the solution stands solved mathematically speaking, actually computing the integrals is its own problem. The integrals turned out to be quite messy and so were numerically evaluated using the Mathematica software. As a complicated integral had to be evaluated for every value of T being plotted, just using the Plot function proved to be too time consuming. Instead, a table of values was produced from plugging in values of T across the range of interest. These points were then plotted. To be particular, the range was from 1 MeV to 0.01 MeV, with the step size between points being 0.03 MeV. The high temperature used in the integrals as the lower bound was 100 MeV. Additionally, $2X_n$ was plotted instead of X_n to follow suite as was done by Dodelson [2], and to show the upper bound for ^{4}He as it uses 2 neutrons, and so the asymptotic behavior of $2X_n$ is the limit as to how abundant ^{4}He can be.

Chapter 3

Results

3.1 Graph of Neutron Fractional Abundance Approximation

Figure 3.1 is the resulting plot for both the approximation and the equilibrium neutron fractional abundances as a function of temperature. Included below is figure 3.2 for comparison which is the plot from Dodelson's Modern Cosmology. It includes plots for ^{4}He and D in addition to the two plots I recreated.

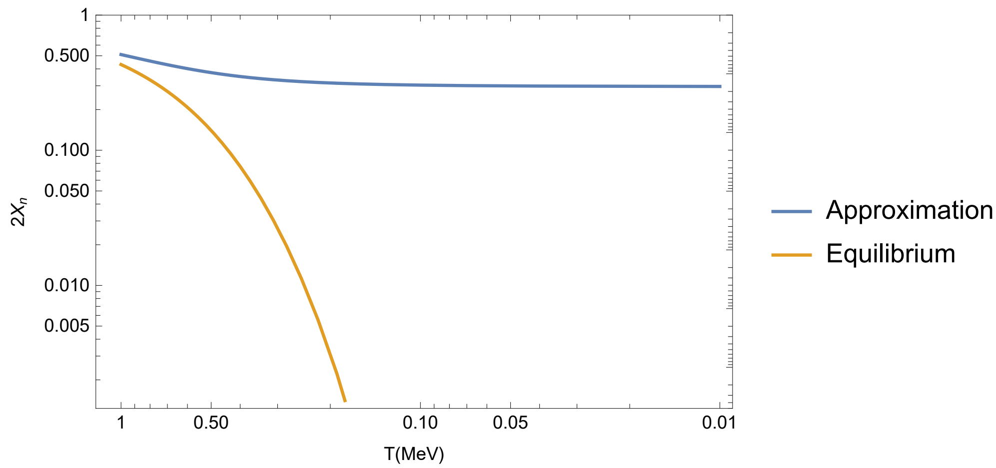

Figure 3.1 Twice the fractional neutron abundance (X_n) plotted as a function of decreasing temperature. The approximation done in this paper is plotted in blue and the equilibrium abundance is plotted in orange

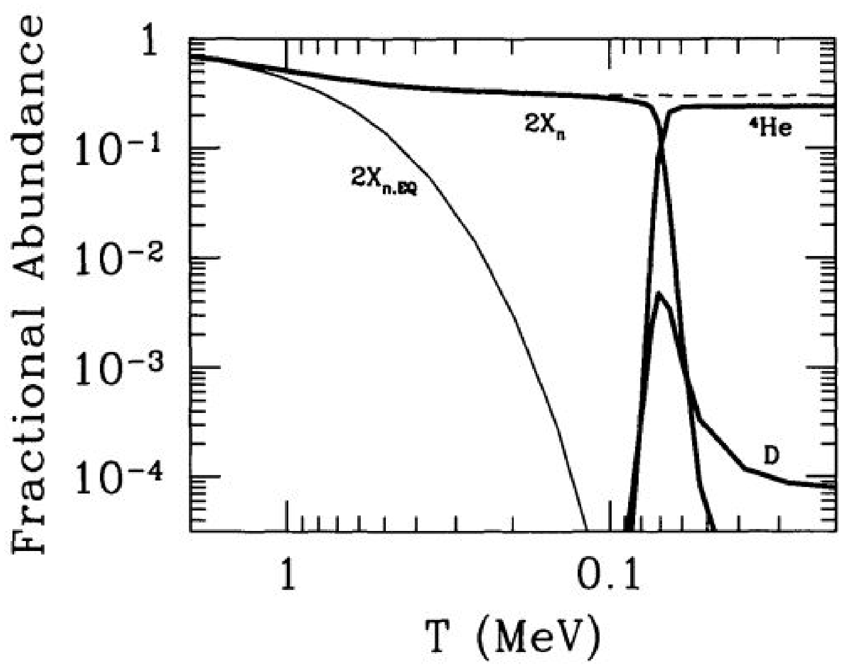

Figure 3.2 Fractional abundances for neutrons (denoted by X_n), ^{4}He, and D. $X_{n,EQ}$ is what the neutron fractional abundance would've been if equilibrium had been maintained. The dashed line is the approximation for $2X_n$ done in this paper and is only visibly different from the more complete treatment at around 0.1 MeV. Figure from Dodelson (2003) [2]

Chapter 4

Discussion

4.1 Graph Comparison

The two figures, 3.1 and 3.2 agree quite well on the approximation and equilibrium plots. A few things do need to be taken into account otherwise there might appear to be discrepancies. First is that in 3.1 the plots do not go all the way to the edges of the graph. Second is that the vertical axes, while similar, are not the same on the two graphs. If 3.1 were cropped so that the plot went to the edges, and if we cut off both graphs at 10^{-3} on the vertical axis, the agreement between the graphs would become even clearer.

Beyond agreement, there are a few things that are worth noting. First is that the equilibrium fractional abundance would have been extremely low before Deuterium (D) production had began, effectively cutting off the process of BBN before it even began. It is also worth noting that around and above 1 MeV, there is significant agreement between the approximations and the equilibrium prediction. With regards to the approximation and the full treatment, they agree up until about 0.1 MeV. Finally, we see that the ^{4}He fractional abundance does not surpass the upper limit set by the $2X_n$ approximation done in this paper as expected.

Chapter 5

Conclusion

5.1 Approximation Works until Neutron Decay and D production

The approximation made in this paper works well for the fractional neutron abundance until the onset of two interactions we didn't take into account, neutron decay 5.2

$$n \rightarrow p + e^- + \bar{\nu} \ , \tag{5.1}$$

and later D production,

$$n + p \rightarrow D + \gamma \ . \tag{5.2}$$

In addition, there are a few approximations that were made that do not hold as the temperature cools. The first is the number of relativistic degrees of freedom. This goes down significantly from 10.75 at 1 MeV to 3.36 [2] around 0.1 MeV. The second is that we've been able to get away with treating the electron as if it were massless due to the incredible amount of energy it had from it's momentum at relativistic speeds. Again, as the universe cooled, this approximation became less and less viable. Regardless, the approximations made in this paper work in the regime hoped for and is set to act as a great basis to build off of in future work.

5.2 Future Work

To keep going forward with this work, it would be good to take neutron decay and D production into account as well as the changing effective degrees of relativistic freedom and electron mass. This should give a good approximation of the neutron abundance going into BBN.

It would then be good to take that neutron abundance and use it along with the principle interactions to get some predictions for BBN with regards to D and ^{4}He and compare those with observed quantities. Going further, it would be good to use a recent BBN program such as PRyMordial [7] to get light element abundance predictions and compare that with the basic calculations done and observation.

Having this basis for BBN opens doors to exploring the current situation of the ^{7}Li problem as well as baryogenesis and possibly dark matter production in the early universe.

Appendix A

Mathematica Code for Neutron Abundance

Below is the code that was used to plot 3.1. A few things to note is that Mathematica threw a lot of precision errors when calculating the integrals and that in order to get the temperature value to decrease as it went to the right and be logarithmic on both axes, I used the ScalingFunction, as can be seen on A.2

```
In[18]:= τn = 886.7; λnp = 255 / (τn * x^5) * (12 + 6 * x + x^2); c = 1.13;

In[39]:= γ = x * λnp / c; (*to simplify ξ and χ*)
         ξ = γ * Exp[-x];
         χ = γ * (1 + Exp[-x]);
         P = -Integrate[χ, x];
         Xhn = Exp[P]; (*Homogeneous solution with A = 1*)
         Q = 1.293;
         η = Integrate[ξ / Xhn * -Q / T^2 /. x → Q / T, {T, 100, Tn}];
         Xnhn = η * Xhn /. x → Q / Tn;
         Tmin = .01;
         Tmax = 1;
         dT = .03;
         Tvalues = Table[Tvalue, {Tvalue, Tmin, Tmax, dT}];
         Xnvalues = Table[2 * Xnhn, {Tn, Tvalues}];
         equilibrat = Exp[-Q / T];
         Xnequilib = equilibrat / (1 + equilibrat);
         Xnequilibvalues = Table[2 * Xnequilib, {T, Tvalues}];
```


Figure A.1 First page from the Mathematica code used to create 3.1

In[61]:=
```
Xnpoints = {Tvalues, Xnvalues} // Transpose;
Xnequilibpoints = {Tvalues, Xnequilibvalues} // Transpose;
plot = ListLinePlot[{Xnpoints, Xnequilibpoints},
  ScalingFunctions → {{-Log10[#] &, 10^-# &}, "Log"},
  Frame → {True, True, True, True}, PlotRange → {0, 1},
  FrameLabel → {"T(MeV)", "2Xn"}, PlotLegends → {"Approximation", "Equilibrium"}]
```

NIntegrate: The precision of the argument function $\left(-0.117731\, e^{-\frac{1.293}{T}-0.117731\, e^{-1.293/T}\left(4+e^{\text{Times}[\ll 2\gg]}\left(4+\text{Times}[\ll 2\gg]+\text{Times}[\ll 2\gg]\right)+\frac{1.293}{T}\right)T^3}\left(12+\frac{1.67185}{T^2}+\frac{7.758}{T}\right)T^2\right)$ is less than WorkingPrecision (26.`).

NIntegrate: The precision of the argument function $\left(-0.117731\, e^{-\frac{1.293}{T}-0.117731\, e^{-1.293/T}\left(4+e^{\text{Times}[\ll 2\gg]}\left(4+\text{Times}[\ll 2\gg]+\text{Times}[\ll 2\gg]\right)+\frac{1.293}{T}\right)T^3}\left(12+\frac{1.67185}{T^2}+\frac{7.758}{T}\right)T^2\right)$ is less than WorkingPrecision (26.`).

NIntegrate: The precision of the argument function $\left(-0.117731\, e^{-\frac{1.293}{T}-0.117731\, e^{-1.293/T}\left(4+e^{\text{Times}[\ll 2\gg]}\left(4+\text{Times}[\ll 2\gg]+\text{Times}[\ll 2\gg]\right)+\frac{1.293}{T}\right)T^3}\left(12+\frac{1.67185}{T^2}+\frac{7.758}{T}\right)T^2\right)$ is less than WorkingPrecision (26.`).

General: Further output of NIntegrate::precw will be suppressed during this calculation.

Out[63]=

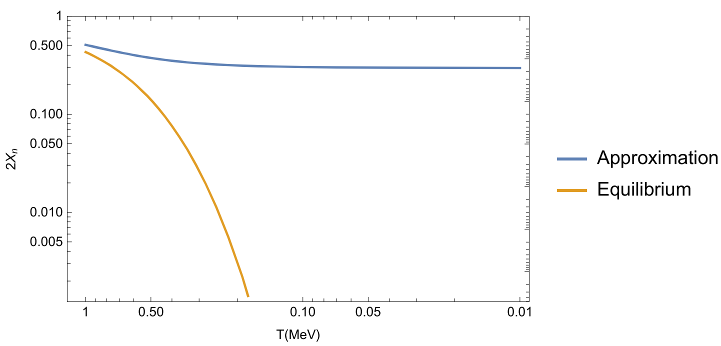

Figure A.2 Second page from the Mathematica code used to create 3.1

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