Honors Thesis

Implementing a Problem-Solving Framework in Physics Tutorials for Upper-Division Electricity and Magnetism Students

by
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ABSTRACT

Implementing a Problem-Solving Framework in Physics Tutorials for Upper-Division Electricity and Magnetism Students

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We study the effects of implementing a problem-solving framework in tutorials intended for use in upper-division electricity and magnetism courses. These tutorials were integrated into an electricity and magnetism course in Fall 2024, and were compared alongside a set of tutorials on the same topics which did not include the problem-solving framework. We performed a mixed-methods analysis on exam scores, interviews, and survey responses from students who were in the class. Students seem to prefer the tutorials that implement the problem-solving framework, and these tutorials seem to assist students with conceptual understanding somewhat; however, the qualitative data on conceptual understanding is not statistically significant given the small class size. We also provide here an analysis on upper-division physics students' problem-solving practices, and insights on how our data can inform tutorial design and physics instruction.

Keywords: Physics Education Research, Problem Solving, Tutorials, Electricity and Magnetism, Upper-division

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Chapter 1

Introduction and Background

Early physics education research (PER) initially studied student conceptual understanding, largely in introductory college physics, but in recent decades the field has expanded to address a variety of issues at many different stages of education [1]. Here, we study the implementation of a set of physics tutorials intended for upper-division electricity and magnetism (E&M) courses, informed by recent PER on problem solving and problem-solving frameworks [2], use of engagement tools such as tutorials [3, 4], expert-like versus novice-like practices [2, 5], and physics education interventions for levels other than the introductory level [6]. This introduction chapter will give an overview to each of these topics and how they apply to the context of E&M courses. In the following section, we will introduce the tutorials we used, how we tested them in Physics 441, Brigham Young University's (BYU) first-semester upper-division E&M course, and what data we collected from the course. Finally, we will report the effects that the tutorials had on students' understanding and describe the problem-solving practices of students who participated in this study. We primarily aim to answer the research question, *What are the impacts of tutorials structured around problem-solving frameworks on student understanding in upper-division E&M, both in terms of their conceptual understanding and ability to accurately* *solve physics problems in E&M?* In addition to presenting results, we will also offer practical insights on how instructors can consider these results for implementation in their classroom.

1.1 Background

1.1.1 Problem Solving and Problem-Solving Frameworks

Solving physics problems can help students gain familiarity with certain topics or procedures, such as how a student can become acquainted with the motion of charges in electromagnetic fields by performing calculations using the Lorentz force law. However, apart from the actual theory or mathematical techniques associated with a physics problem, problems are also an important setting for students to demonstrate their understanding of problem solving itself and improve their overall ability to solve problems. Therefore, students also need to be taught effective practices for problem solving, as it is a skill they will need throughout their education and career.

A common method for teaching this is through problem-solving frameworks, which break problem solving into a series of smaller steps applicable to most problems one encounters in physics. Problem solving frameworks have been used for a long time in math and in physics education. For example, an early framework is proposed by George Polya in his book *How to Solve It* in 1945, focused on solving math problems [7]. The framework here breaks problem solving into four distinct steps, which include understanding the problem, making a plan, carrying out the plan, and looking back. Other frameworks have been proposed since then in physics education, such as the seven-step framework proposed by Gaigher et al. [8], or the five-step framework used in the University of Minnesota's physics program [9, 10]. Though new problem-solving frameworks have been introduced, they still often use or base themselves around the core steps in Polya's described procedure, although they may also be framed more specifically toward physics problem solving, or may include extra steps specific to physics problems. For example, Gaigher et al. include drawing a diagram as the first step of their framework [8]. Though this might be part of "understanding the problem" in Polya's framework, this is focused more specifically toward the spatial reasoning or visual representations often needed in physics problems.

Here, we have followed the Minnesota framework [9]. The five steps are given as

Describe the problem
Describe knowns and unknowns, facts about the problem, or draw diagrams to provide an effective description of the situation
Identify correct physics principles
List physical laws or equations that may be relevant to the problem
Apply principles correctly
Synthesize problem description and physics principles to derive new information or rewrite equations into mathematically solvable forms
Use proper mathematical procedures
Solve equations or use mathematical logic to arrive at numerical answers, new mathematical expressions, or proofs
Check that solution is logical
Interpret physical meaning of final results and reflect on the reasoning used to arrive at the final result

Figure 1.1 The problem-solving framework used throughout this study, following what is used in the University of Minnesota's physics program [9].

describing the problem, identifying correct physics principles, applying principles correctly, using proper mathematical procedures, and checking to ensure that the solution is logical, as shown in **Figure 1.1**. This framework is intended to capture the key aspects of physics problem solving, based both on instructor needs and existing problem-solving literature, and has been used as a means of assessing student problem solving [10]. These aspects make it appropriate for our study where we are investigating interventions that may further assist students with problem solving.

1.1.2 The Role of Tutorials in Physics Education

Tutorials are the educational setting we are studying for implementing the problem-solving framework. They might be considered "in-between" lecture and homework instruction: traditional lecture might be at one end of instructor versus student participation, where the instructor does most of the speaking while students ask questions or passively listen, and homework is at the other, where students are engaged in problem solving but without much involvement of the instructor. Tutorials typically consist of problems or problem sets given in class, so students are actively engaged in learning while an instructor and/or teaching assistant is still present. In our instance, students were given worksheets containing problem sets in class in place of lecture time.

Though many colleges and universities tend to use to lecture-style instruction, tutorials as a learning environment have existed for a relatively long time in physics education. The *Tutorials in Introductory Physics* are an early instance of research-based tutorials that have been present in physics education for over twenty years [3]. Tutorials are commonly studied, and generally accepted in physics education research literature as more effective than traditional lecture at developing conceptual understanding among students [11, 12]. This effectiveness may be because tutorials employ active learning techniques, which tend to be more effective in developing conceptual understanding and problem-solving ability than passive learning techniques [13, 14]. Additionally, in our own previous research, we have found that tutorials have brought several benefits to physics classrooms: guiding students in a step-by-step structure through new problems has led to less time feeling lost or unproductive on problems, and students have responded positively to being in an environment where they can receive support from instructors and their fellow classmates [15].

Given the positive findings regarding tutorials at the introductory level [11], there have been efforts to create tutorials for the upper-division level, such as in quantum mechanics [16], electrodynamics [4, 12], and other advanced topics [17]. Upper-division level resources may focus on specific concepts or problem-solving steps, but have yet to serve as a comprehensive resource for the course [18, 19]. Structuring tutorials around the entire problem-solving framework can result in tutorials that teach more comprehensively: some of the steps, such as "identifying concepts" and "applying concepts" provide ways for students to consider physics topics conceptually, and steps such as "using proper mathematical procedures" provide a place for students to learn the mathematical methods that can help them better apply these topics. In addition, utilizing a problem-solving framework in tutorials can more explicitly teach students about problem solving itself, which upper-division students need to continue to learn in the context of their advanced problems, just as they did as introductory students [20].

1.1.3 Cognitive Apprenticeship Theory

The idea of cognitive apprenticeship theory is to structure educational practices in ways analogous to a traditional apprenticeship, as described in [21]. When learning a skill in a traditional apprenticeship, experts in the skill will first show a novice how it is performed, assist the novice as they attempt the skill themselves, and then gradually give less assistance as the novice becomes capable of performing the skill independently. These practices are known as modeling, coaching, and fading, respectively. Cognitive apprenticeship theory employs these practices while considering that in certain spaces, the skills we are attempting to teach students may require cognitive and metacognitive processes, rather than solely physical skills.

Problem solving is an example of a skill where we can apply cognitive apprenticeship theory. Both cognitive and metacognitive tasks are required for problem solving to be performed expertly: cognitive processes are used in the creation of the solution, and metacognitive processes are used in evaluating one's own problem-solving procedure. In fact, the problem-solving frameworks we discussed in **1.1.1** are an effective scaffold for these processes (a scaffold is a temporary educational support for students as they are learning new concepts or skills [22]). A framework contains the cognitive skills of problem solving, such as with drawing descriptive pictures, applying physical principles, and executing mathematical procedures. Additionally, a step such as checking the reasonableness of a solution requires students to evaluate their process metacognitively. Therefore, problem-solving frameworks support the practice of scaffolding expert-like practices into instruction, a common part of cognitive apprenticeship theory. The Minnesota framework we follow here is effective at scaffolding expert-like practices into instruction by its design, since it is structured on instructor expectations as well as expert-novice research [10].

Tutorials can also serve important roles in cognitive apprenticeship models. When an instructor shows students how to solve an example problem in class, this is an example of modeling. On the other hand, when students approach problems in their homework, this is part of the fading process, where students are expected to solve these problems with little assistance from the instructor. Tutorials can be effective for the intermediate step of coaching when tutorials are structured to guide students step-by-step through problem solving (especially in our case where most or all steps of the framework are present in each tutorial). Additionally, they will likely occur in class, where an instructor is present to aid students as needed. These aspects of tutorials apply cognitive apprenticeship theory in different ways from what lecture or homework allow.

1.1.4 Considerations for Upper-Division Electricity and Magnetism

Application of physics problem-solving strategies, tutorials, and cognitive apprenticeship theory in education may follow consistent principles, but can vary depending on the course or the needs of the students within the course. In this section, we describe Physics 441 as taught at BYU, or "first-semester E&M," which refers to the common practice of dividing upper-division E&M instruction across two courses. First-semester E&M differs from "introductory E&M," physics students' first formal course in E&M, which is taken in the freshman or sophomore year of university physics and is only one semester in length. Physics 441 follows the structure of many first-semester E&M courses taught at universities, and what is discussed here will likely apply to other universities' versions of first-semester E&M.

Physics 441 primarily covers electrostatics and magnetostatics, theories that describe electric and magnetic fields constant in time. The textbook used in this course, and often used for upper-division E&M, is Griffith's *Introduction to Electrodynamics*, where the first half of the text is dedicated to introducing these theories, the methods of describing electromagnetic fields by using them, and explaining the properties of these fields in matter [23]. By the end of the course, Maxwell's four equations are introduced, which begins to transition into electrodynamics where the electromagnetic fields vary in time.

A primary difference that separates upper-division from introductory E&M is the mathematical rigor. In fact, students have noted at times that much of Physics 441 differed simply in its level of mathematical involvement. When comparing the course to introductory E&M on the Fall 2024 end-of-semester survey (see **2.2.3**), a student noted that "most of it felt like just more rigorous versions of that class." The first chapter of Griffith's *Introduction to Electrodynamics* is consistent with this, exclusively devoting its first chapter to advanced mathematical concepts, such as vector analysis, in preparation for a rigorous treatment of electromagnetic fields [23]. This is necessary since electromagnetic fields are represented mathematically as vector fields, a function of spatial coordinates where each coordinate in 3-dimensional space is associated with a 3-dimensional vector. However, students generally do not encounter vector fields in a math course until after students they have taken their introductory E&M course, so it is not feasible to treat introductory E&M with same rigor as it is done at the upper-division level.

Despite the difference in mathematical rigor, many E&M concepts taught are shared between the two courses. In each, we introduce Gauss's law in its integral form as

$$\oint \mathbf{E} \cdot d\mathbf{a} = \frac{q_{\text{enc}}}{\varepsilon_0}, \tag{1.1}$$

where $\mathbf{E}$ is the electric field, $d\mathbf{a}$ is an area vector representing an infinitesimally small portion of a closed surface, q_{enc} is the charge within the closed surface, and ε_0 is a physical constant, the permittivity of free space. Using Gauss's law to compute the electric field is of interest in both courses. However, while we may expect upper-division students to have a thorough understanding of the integral and dot product operations present in the equation, with introductory students we may just teach which shape, or "Gaussian surface," to draw around the charges of interest. In this case, the above equation simplifies to the expression

$$E = \frac{q_{\text{enc}}}{\varepsilon_0 A}, \tag{1.2}$$

where A is the total surface area of the shape through which the electric field passes, and E now just represents the magnitude of this electric field (distinct from $\mathbf{E}$ which is a vector with direction and magnitude). Introductory courses follow an approach more like this [24]. Therefore we can teach effective methods for computing electric fields at either level, which allows an effective discussion of the behavior of fields in electrostatics at either course's degree of mathematical rigor.

Cases like these are important to keep in mind when studying interventions for upper-division physics. E&M is unlike other upper-division courses, such as quantum mechanics, where students have much less intuition about the subject, let alone a full previous course. Students in our case begin the course with existing experience studying the subject, which may assist them in some ways—such as familiarity with the reoccurring topics—or impede them in others—such as misconceptions or biases.

Despite student perceptions about the similarities between the courses, about half of the topics treated in first-semester E&M are new. Students' deeper mathematical background at the upper-division level also has an important role here, as many of the new topics will require concepts from their most recent math courses. Potentials for the electromagnetic fields are an example of this. Students will use vectors (and tensors, a mathematical generalization of vectors) to study the multipole expansion and how to approximate the electric potential V using dipoles and quadrupoles. Another method students learn to compute V is by solving Laplace's equation,

$$\nabla^2 V = 0\,, \tag{1.3}$$

a partial differential equation. Finally, we can introduce the magnetic vector potential $\mathbf{A}$ to upper-division students, which is defined as

$$\nabla \times \mathbf{A} = \mathbf{B}\,. \tag{1.4}$$

We say that the magnetic field $\mathbf{B}$ is the curl of the vector potential $\mathbf{A}$. For many upper-division students, only their most recent math courses may have covered vector calculus or partial differential equations. Because of this, students may still be making sense of these advanced mathematical concepts, and E&M may be one of their first courses where they are consistently applying them.

When problems are highly mathematical, it can become increasingly difficult to remain connected to the physical meaning of these problems. However, introductory E&M often teaches tools to connect new equations to a student's physical intuition. The Lorentz force law,

$$\mathbf{F}_{\text{mag}} = q\mathbf{v} \times \mathbf{B}\,, \tag{1.5}$$

taught in introductory E&M, describes the magnetic force $\mathbf{F}_{\text{mag}}$ applied to a charge of magnitude q, traveling at a velocity $\mathbf{v}$ in a magnetic field $\mathbf{B}$. Even without previous physical experience or intuition (since students tend not to watch the deflection of microscopic charges by magnetic fields in their everyday life), students can develop a simple visualization of the physical effect that the magnetic field has on charges by using the "right-hand rule." But for $\mathbf{A}$ in upper-division E&M, its relationship to the magnetic field is through the curl, a more complicated mathematical operation that what we saw in the Lorentz force law, and comes at a higher level of abstraction (though this is not to say that there are not visual representations of the curl). A student said of this field (and of another field like it) that they "feel too much like mathematical constructs and not physical realities so it's hard to really understand how they fit into things," on a past year's survey. Other students have made similar comments during the study about the course's new fields [15]. From these comments, it seems that as E&M employs increasingly abstract mathematical concepts, students struggle more with understanding the physical meaning behind them.

The challenge of upper-division E&M can come from the increased mathematical rigor being applied to familiar topics, as well as attempting to understand new concepts that emerge alongside abstract mathematical equations. For this, a comprehensive tutorial that addresses all aspects of the problem-solving framework becomes particularly beneficial. Students that struggle more with the conceptual side of problem solving may find more benefit from parts of a tutorial that ask them to consider explicitly what principles they will use and why it applies to that situation. On the other hand, students that struggle more with mathematics may benefit from the tutorial walking them step-by-step how to apply previously learned mathematical techniques to electrostatics, and preventing the "actual physics" from being obscured. Though upper-division E&M may present unique difficulties, a tutorial that comprehensively introduces problem-solving can help meet students' challenges at this stage of their education.

Chapter 2

Methods

2.1 Design and Distribution of Problem-Solving Framework-Based Tutorials

Here we provide a description of the design of this study's tutorials and how the framework was integrated into them. For those used during the Fall 2024 semester, they were based on a set of tutorials already used during the same course in Fall 2023. These tutorials were then rewritten in two different versions: Version A, which explicitly integrated the problem-solving framework given by [9], and Version B, which was the same tutorial except without any explicit references or problems based on the framework. As described in **1.1.1**, this framework has five steps, which include describing the problem, identifying correct physics principles, applying principles correctly, using correct mathematical procedures, and checking to ensure that the solution is logical. For example, if the topic of a tutorial is on Ampere's Law, we might organize a solution based on the framework as follows:

1. **Describe:** Based on the problem prompt, describe any currents or current densities present, possibly by drawing diagrams or describing the them using vectors.

2. **Identify:** Recognize that Ampere's law can be used if the currents exhibit certain symmetries, such as a cylindrical symmetry along the z-axis.

3. **Apply:** Write down Ampere's Law in integral form and draw Amperian loops on the figure that will allow for proper integration to obtain the magnetic field.

4. **Mathematical Procedures:** Perform integration and use symmetries present in the scenario to determine direction of the resulting magnetic field.

5. **Check:** Verify that the field is consistent with information we previously described; ensure that units are correct; perform a calculation showing that the field gives the currents when using differential form of Ampere's Law; perform some other check to verify that the solution is reasonable.

Version A of the tutorial integrated questions based on these steps. Examples of questions from Version A and Version B from Fall 2024's Ampere's Law tutorial are given below:

A: What information are you given about the current? Use this information to find the current density in the regions $r < a$ and $b < r < c$.

B: Find the current density in the regions $r < a$ and $b < r < c$.

Finding the current density is an essential step to correctly using Ampere's Law, so students receiving either tutorial should be given a prompt to find current density in order to be taught a successful approach to solving problems with Ampere's Law. However, Version A instructs students to more consciously describe the current before immediately moving to steps that involve calculations. While both progress the student in solving the problem, only one prompt explicitly asks students to consider something from the framework. As seen in this example, structuring a tutorial around a problem-solving framework leads to significant differences between the two versions of the tutorial beyond simply rewording questions. For further examples of differences in tutorial versions, both versions of each tutorial used during Fall 2024 are contained in **Appendix A**.

On days where a tutorial was given, the class was organized into two groups, Group 1 and Group 2. Group 1 was given Version A for the first tutorial, then Version B for the second, whereas Group 2 did the reverse, as shown in **Table 2.1**. The topics of the seven tutorials given during the study are also shown.

Tutorial #	Tutorial Topic	Group 1	Group 2
1	Point Charges and the Electric Field	A	B
2	Gauss's Law	B	A
3	Solving Laplace's Equation	A	B
4	The Quadrupole Electric Field	B	A
5	Dielectrics	A	B
6	Magnetostatics and Ampere's Law	B	A
7	Displacement Current	A	B

Table 2.1 Tutorial topics and versions given to each group during the Fall 2024 semester. Version A explicitly implements the problem solving framework described in this section.

Though giving the tutorials in the manner described above does not give a true control group, this avoids the issue of offering some students a possibly helpful intervention while not offering it to others. In addition, because we were working with a small class size, this allowed us in our analysis to more accurately identify whether exam performance was due to the tutorial version that the student used rather than other factors, such as the student's typical exam performance and study habits.

2.2 Data Collection

The following sections will give a brief overview of the data collected and its purposes. In total, 24 students consented to this study at varying levels of participation, which are described in section **2.2.5**. Because of this, we will be clear in each section about how many students we have data from of that type. All data was collected with approval from the Institutional Review Board (IRB).

2.2.1 Student Coursework

The minimum participation option was a consent to use of coursework. This includes post-tutorial self-assessments, students' work on the tutorials, and exams. Here, we limit our discussion to the exams, though other coursework has been used in previous years [15], and we plan to carry out future analyses with non-exam coursework. The exams were analyzed quantitatively to determine whether students performed better on exams as a result of one version of the tutorial or the other, an important part of responding to the primary research question we initially outlined in the **Introduction**. In total, we collected three different exams from all 24 students who participated, but only graded the questions on the exams that related to the topics given on the tutorials.

2.2.2 Interviews with Students

We also conducted interviews in two rounds—one in the middle of the semester and one at the end of the semester. Three interviews were conducted each round for a total of six interviews, administered among four students. The interviews were aimed at obtaining qualitative data regarding student perceptions of tutorials, as well as how they view problem solving and problem-solving frameworks.

In addition to the questions posed during each interview, students were also given a brief exercise at the end of each interview that consisted of two questions, each relevant to the topics they had worked through on tutorials. One of the problems would correspond to a topic for which they had received a framework-based tutorial on, and the other would correspond to a topic for which they had received the non-framework-based tutorial on. These exercises were administered using a think-aloud protocol, meaning that we asked students to verbally explain each step that they took in their problem solving as they were doing it.

The interview questions, as well as the exercise prompts for each round, are contained in **Appendix B**.

2.2.3 End-of-Semester Survey

A brief survey was also given at the conclusion of the semester for participants to respond to anonymously. In total, ten students completed this survey. The survey consisted of questions asking students about difficult topics they encountered during the semester and their perceptions of the two tutorials, particularly in regard to the two formats they were exposed to during the semester. They were asked to compare two versions of a tutorial side by side, without being told which version used the problem-solving framework.

The survey questions are also included as a part of **Appendix B**.

2.2.4 Classroom Observations

We collected general classroom observations as a part of this study. This included more general trends, rather than monitoring specific students. For example, a general observation might be "students tended to express frustration with early parts of the tutorial," but we would not make a statement such as "student *X* gained more confidence on math problems on this tutorial." General observations allowed us to view overall patterns in student interaction of the tutorials while still interfering minimally with the class.

2.2.5 Participant Consent

This study involved three levels of consent:

1. **Full participation:** Students consented to participation in any part of the study, including interviews, surveys, and us using their coursework. Six students chose to participate fully.

2. **Reduced participation:** Students consented to surveys and us studying their coursework, but did not opt to participate in interviews. Fifteen students chose reduced participation.

3. **Consent to use of coursework:** Students would not do anything beyond what the course required, but consented to us using their coursework. Three students chose this level of participation.

Students who participated in the study were kept anonymous, and any specific student referred to in data analyses or reporting of results was simply referred to as "Student X," where X is a number assigned to that student for the purposes of the study. No data was collected from students who didn't consent to the study; however, general classroom observations may implicitly include students who did not sign a consent form.

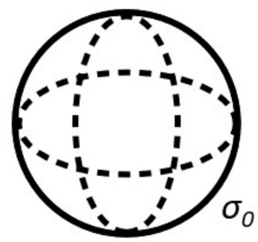
2.3 Analysis

2.3.1 Quantitative Analysis of Exam Scores

We performed a quantitative analysis of exam scores by grading students' exams according to a rubric specific to the study, then comparing the scores of students who had a framework-based tutorial on the topic for that problem versus those who did not.

The rubric scored students on two parts of problem-solving: invoking and applying. Invoking describes a student's ability to identify the correct concepts associated with a problem. Applying refers to a student's ability to then use those principles to properly arrive at the correct solution. Below is an example problem and the rubric that would be used for that problem:

Find the electric field outside a spherical shell of constant surface charge density σ_0.



Invoking
Recognizes that this physical scenario exhibits spherical symmetry (0.5/1.0)
Recognizes that Gauss's Law can determine the electric field with spherical symmetry (0.5/1.0)

Applying
Integrates or uses the surface area times σ_0 to find the correct total enclosed charge (0.33/1.0)
Relates this correctly to the integral of an electric field which points radially outward (0.33/1.0)
Obtains correct **E** field after integral has been set up and enclosed charge has been found (0.33/1.0)

Figure 2.1 An example problem with the corresponding rubric used to grade that problem.

Each part, invoking and applying, is scored out on a scale from 0 to 1 for every problem. As shown above, if there are more concepts to be recognized and/or steps to be taken to solve the problem, then the rubric simply gives smaller parts out of 1 to each concept or step being scored.

Each participant was scored as described above, and classified as being a part of either Group 1 or Group 2 (see **Table 2.1**). In our analysis, we observed that Group 2 outperformed Group 1 significantly regardless of tutorial. To account for this, the average difference in scores between the two groups was subtracted from Group 2, which was intended to account for them consistently scoring higher than Group 1. After having included the subtraction as a means of controlling for Group 2's overall better performance on both tutorial types, we then compared the scores for Version A to scores for Version B for the topic related to a given problem. We specifically compared the invoking and applying parts of each problem, and also compared the averages for these across all exams. These averages for invoking, applying, and total scores across all exams were compared using a t-test for statistical significance and Cohen's d to measure effect size.

2.3.2 Qualitative Analysis of Other Data

All other data was analyzed qualitatively. We may report qualitative data in the following ways:

Quotations: Across surveys and interviews, we will observe trends and themes in remarks from students. Additionally, providing a quotation from a student can offer detailed explanations on their sentiments or experiences with the tutorials.

Survey Results: We will describe how many students selected certain options on multiple choice questions given in the survey, which may show students' preferences or perceptions about the tutorials.

Coding: Coding is the practice of organizing or categorizing qualitative data. For example, when coding interviews, we might identify the types of responses students gave to a question; something like "when students were asked about ___ in an interview, students X, Y, and Z referenced 'mathematical procedures' in the problem-solving framework." Coding was also used beyond the analysis of quotations, such as when we coded students' problem-solving practices in order to better understand how students approach this task. In this case, we identified when they used parts of the problem-solving framework and how successful they were in using them.

2.3.3 Overall Features of Analysis

Because we are performing both quantitative and qualitative analyses, we are using a mixed-methods approach. Quantitative analyses may help in answering the research questions in a way that the qualitative analysis cannot, and the same is true for the qualitative analysis with respect to our quantitative results. The research questions may be answered using patterns seen in both analyses, such as an idea expressed by a student in an interview being further supported by trends in the exam scores.

Chapter 3

Results and Discussion

3.1 Impact of Tutorials on Exam Scores

Here, we report the exam scores, scored as described in **2.3.1**, using an invoking and applying rubric. The results of this scoring are given as a chart in **Figure 3.1**. Because the scores of individual problems vary considerably, and are taken from small collections of scores, we provide a summary of scores across all exams, which includes a much larger set of scores. The score summaries are separated into invoking, applying, and total categories, and for each tutorial type. In addition, the average score differences, p-values indicating the likelihood of getting our result randomly, and Cohen's d measuring the effect size are given in **Table 3.1**.

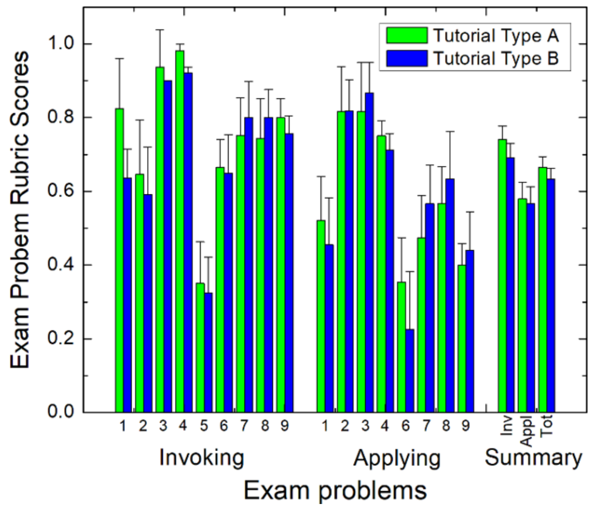


Figure 3.1 Scores are given for each of the tutorials on various exam problems. Invoking refers to the ability to identify the correct physics principles, while applying refers to the ability to apply those principles to get the correct result of that problem (as described in **2.3.1**). The summary reports the overall trends for each category across all exams. In the legend, "Tutorial Type A" refers to the framework-based tutorial. Error bars are given by the standard error.

	Invoking	**Applying**	**Total**
Score Difference	4.9%	1.3%	3.2%
***p*-value**	0.36	0.84	0.45
Cohen's *d*	0.14	0.034	0.088

Table 3.1 Score difference in students using Tutorial A over Tutorial B on exams. In each category, students using Tutorial A outperform those using Version B; however, a t-test and a calculation of Cohen's d indicates that the effect on exam scores is neither large or statistically significant.

First, we note that the p-values are high for each category, so if we are to consider only the exam scores as the indicator of problem-solving performance, these scores are not sufficient to claim that the students' performance is meaningfully increased by using one version of the tutorial or the other. A null result like this is not surprising with an upper-division study, as we are facing the limitations of small class sizes. However, we will still describe trends we see in this data, as they still offer possible insights, especially when considered in the context of our qualitative results.

One trend is that students completing problems for which they received Version A, the framework-based version, of the tutorials outperformed Version B in both the invoking and applying (and therefore also in the total scores) categories. We see this difference stronger in the invoking scores, with a 4.5% difference, than in the applying scores, a 1.3% difference. Cohen's d is low for both, indicating only a small impact on scores. However, the exam results begin to suggest that the tutorials assist students more with conceptual parts of problem solving than they do with "getting the right answer." This perhaps should not be terribly surprising given the changes in the structure of the tutorials. Both Version A and Version B guide students through solving the problem, but the structure of the framework present in Version A offers questions such as "what principles are you going to apply here?" or "how can you know that this is the correct principle to use?" As students seem to be scoring higher in their ability on exams to identify the correct physics principles associated with a problem, this seems to be consistent with the types of questions one would introduce with the framework.

In addition, the greater difference in invoking scores over applying scores also seems to be reflect the trends of previous tutorial research, where tutorials tend to impact conceptual understanding more [11,12]. In particular, the results of [12] were that scores on a conceptual exam were improved among students who used tutorials, but that there seemed to be little effect on the regular exams of the course when comparing students who used or did not use tutorials. Though we have tested a modification to tutorials, rather than testing the effects of classes with or without tutorials, it is interesting to note that our result is similar to results of previous tutorial research.

3.2 Qualitative Impact of Tutorials on Student Understanding

To further explore the possibility of conceptual understanding being strengthened by the framework-based version of the tutorials, we examine qualitative trends in student comments from surveys and interviews. For example, comparing comments between Student 3's two interviews suggests that they grew in their conceptual understanding through their experience with the tutorials. In their first interview, following the first mid-term, they said, "I feel like when an example is provided or when I'm guided through how to get a quantitative result, I feel pretty confident with those. But the questions that [have] tripped me up, more have been like, 'what principles are you applying?' or 'what concepts are you applying to this problem?' And I don't think it's because the question is intrinsically hard. But it's like I'm trying to figure out what kind of response is expected." While at this point in the semester, they note difficulty with these types of questions, their confidence with these types of questions change in a later interview, where they say, "I think I've started to understand better some of the more qualitative questions like 'what principles are you applying?' and 'how would you apply this to different scenarios?' I know...there was a bunch of hypothetical questions of like, 'what if this were changed?' 'what if this were different?' I think over the semester, I'm starting to become more comfortable with those questions, whereas at the beginning of semester, I felt like I really just wanted to do the quantitative stuff, like I just wanted to get the right answer." These comments indicate a shift in confidence in these types of conceptual questions and an improved understanding of expectations when responding to them.

Understandably, as every student in the study was using both versions of the tutorial, alternating each time, some might consider it difficult to justify that this shift in conceptual understanding or problem-solving is due to one version of the tutorial or the other. However, we note that these types of questions, such as "what principles are you applying?" are exclusive to the framework-based format of tutorials. Questions like these would not appear on the other version, since considering principles explicitly is not strictly necessary to perform a calculation and arrive at a correct expression or numerical answer to the problem (consider our explanation in **2.1** of how students can progress towards a solution of a problem without explicitly using all problem-solving steps). Only the framework-based versions of tutorials directly ask students to consider the principles like this. As students regularly encounter these types of questions, they seem to improve in their ability to answer them.

This may seem an obvious result, that students with regular practice answering questions about the principles they are applying would remark that they have improved at responding to those types of questions. On the other hand, considering previous research that affirms that upper-division students do not always apply expert-like practices in solving their upper-division problems [20], it may be incorrect to assume that students are proficient in this aspect of problem solving if they are not encountering these questions after their introductory courses. Whether or not these aspects of problem solving are directly assessed as part of upper-division courses, they remain good practice when solving a physics problem, which the framework we use was designed in mind with [10]. From Student 3's interviews, continuing to integrate these questions into an upper-division course may assist students in continuing to learn these effective problem-solving principles in upper-division courses.

Other student comments indicate that students felt the tutorials helped their conceptual understanding, although these comments are not as straightforward to connect to the differences between tutorials and the effects of implementing the framework. Some survey responses include statements such as "[tutorial practice] helped my conceptual understanding and my application of the things we learned" and "I think they were useful for understanding the concepts better." In particular, one student mentions specific features of a tutorial that helped their conceptual understanding: "I do feel like [the displacement current tutorial] helped me understand the principle better. I liked how we solved the problem two different ways and compared the methods to produce the same result. This made it very clear how the principle is applied." While these all indicate that these are helping with conceptual understanding rather than being simply a math exercise, none of these comments reference the framework or compare versions of the tutorials. However, they also are consistent with the trends of previous tutorial research [11, 12].

We note that even with the assistance of the tutorials, some students remain in novice-like problem-solving patterns. Student 5 in their final interview described their problem-solving process: "I approach the problem and I'll think, 'What am I looking at? Where are the things I need to pay attention to?' And then what? Look at my formula sheets, go 'what formula can I apply here to get some answer?"' While starting the problem-solving process in a way that aligns with the framework (describing the problem), from there they seem to immediately jump to trying to find formulas and calculating an answer. While many student statements note the increased conceptual understanding that comes through tutorials, tutorials will not have the same impact on conceptual understanding for every student.

To summarize these qualitative findings, we have found positive results from the tutorials in increasing conceptual understanding. In particular, Student 3's interview comments and the exam scores both seem to indicate that this could be due to the implementation of the framework. Much ambiguity surrounds this however, and with other students, it is unclear whether their sentiments about conceptual help from the tutorials are related to the implementation of the framework. Additionally, with Student 5, despite the fact that they were exposed to both formats of tutorials, from their description it does not seem that they use or consider concepts all that heavily when solving a problem. Though the qualitative data strengthens the claim that the framework-based tutorials are assisting students in conceptual parts of problem-solving, it does not seem to make that result conclusive.

3.3 Student Perceptions of Tutorials

Some qualitative data taken in this study helps us to understand other ways tutorials have impacted students apart from problem-solving ability or conceptual understanding. This includes whether the students enjoy using the tutorials or not, as well as common successes or difficulties that students seem to encounter.

Students seem to overall respond positively and appreciate the tutorials in general. On the survey, we received comments such as "I think they were helpful because some of them did help break down those long and tedious problems and made them easier to understand," and "They were all helpful. I think there ought to have been a tutorial every week."

Looking at perceived benefits more specifically, students find that the tutorials help them comprehensively with problem solving. As discussed throughout **3.2**, students feel that the tutorials help their conceptual understanding. But for students that struggle more with quantitative parts of problem solving, or the "math," they feel that the tutorials are effective in helping with this as well: "I think looking at things qualitatively is helpful, but I often have a very incomplete understanding of the actual quantitative methods (more incomplete than I think even when I may already have a qualitative understanding). Getting help with the quantitative methods in a classroom setting is helpful for me." Students tend to find themselves more confident in concepts than the they do the math, or vice versa (the above survey response is an example of a student who likely finds themselves more confident in concepts than in math, a student from a previous year is an example of the other case: "I know math very well ... but it often takes me a while to identify correct principles"). Seeing remarks regarding both the conceptual and mathematical parts of problem solving, we can see that students seem to be able to find assistance from the tutorials on whichever aspect they feel more deficient.

Results indicate that though either version of the tutorials is beneficial to students, there seems to be a preference from students towards Version A of the tutorials, or the version that uses the problem-solving framework. On the survey, students were asked whether they preferred Version A or Version B of the tutorials, and students tend to slightly prefer Version A, as shown in **Table 3.2**. The benefits they perceive about Version A become more clear when asked about why they chose that option. Students made comments such as "The wording in Tutorial A is easier to follow," "It provides a bit deeper of thinking and

Tutorial Version Preference	**Count**
Version A	5
Version B	1
No preference between formats	2
Cannot identify a meaningful difference	1
Preferred format would depend on topic	1

Table 3.2 Student responses to a survey question where they are asked to compare two versions of the tutorial side by side. They were labeled "Tutorial A" and "Tutorial B," but they were not told which version implemented the problem-solving framework.

forces you to look into the details of a problem rather than just scraping the surface," and "I think the questions in tutorial A were less ambiguous. I also appreciate that it asked me to draw images to illustrate the problem/solution, which helps me visualize the concepts I am trying to learn better than tutorial B would." In response to an earlier question, before students were asked to compare the tutorials side by side, a student said "I liked the problem solving structure. It helped break things down into steps that were easier to understand." Students note various benefits of using the version of the tutorials that include the framework, including more clarity, deeper thinking, and being guided step by step through a problem. Students did not note any particular benefits regarding Version B. The one student that selected Version B when comparing the two tutorials side by side remarked earlier in the survey that they preferred tutorials that used the problem-solving framework, indicating that they were likely unaware of which version implemented the framework.

This has been another example where identifying the effects of implementing the framework has been difficult. From both the interviews and survey, it seems that students do not know which version includes the problem-solving framework, at least from their in-class experience. Student 3 remarks in his end-of-semester interview, "to me it seems like ... each of these tutorials walked us through the problem in some problem-solving framework. I mean, they were each like three pages long with steps walking us through how to do the problem." Student 6 also notes, "I don't remember noticing any specific differences with the tutorials," and Student 5 even says "I didn't even realize there were two formats." When asked about preferences between the two formats before asking them to compare tutorial versions side by side, many students also gave similar comments, as shown in **Table 3.3**.

Looking at both survey results together, along with interview comments, it seems that many students do not notice the difference between tutorials or have a strong preference, but those who do have a preference tend to prefer the problem-solving framework as part

Tutorial Version Preference (before comparing)	Count
Preferred problem-solving framework	4
Did not notice a difference	4
Felt tutorial versions were about the same	1
Responded "N/A"	1

Table 3.3 Categorized, typed student responses to a survey question where they were asked "You have been given tutorials in multiple formats this semester, some being structured with a problem solving framework and others not having this structure. Do you feel that one format was more helpful than the other? Please explain." They were not yet asked to compare tutorial versions side by side.

of the tutorials. Overall, this seems favorable to the implementation of problem-solving frameworks in tutorials, as students tend to respond positively to this, as well as being able to describe specific reasons for their preference.

This is not to say that the using the problem-solving framework in tutorials comes without difficulties for students. As Student 3 mentioned in their first interview, they initially struggled with the questions added due to the implementation of the problem-solving framework, such as "what concepts are you applying to this problem?" They describe this in detail throughout their interview: "I don't think it's because the question is intrinsically hard. But it's like I'm trying to figure out what kind of response is expected." He brings out Version A of a tutorial to talk about a specific question: "None of it was too hard. I think it was overall written, really well. ... Like this question, 'list some of the mathematical procedures you can think of needing to use to solve Laplace's equation.' So, it's like, 'what constitutes as being a mathematical procedure?' That was a question that went through my head. And maybe that's just me as a STEM major not knowing how to qualitatively answer questions as well as I can just spit out a number. ... Some of these first questions took me a second." The questions added due to the framework seem to feel different to this student than many of the questions they are responding to as a STEM major, which leads to difficulty interpreting what the question means and what is expected of them. Other students on the survey noted ambiguity about some of the tutorials, which may have also been due to unfamiliarity with how to respond to conceptual or problem-solving focused questions introduced because of the framework. If students are not used to responding to these types of questions, they may struggle, but Student 3's improved confidence on these questions throughout the semester also shows promise that students can gain the skills necessary to understand the concepts and problem-solving considerations for their problems on a deeper level.

Student 3's comments also suggests that as it is, there could be improvements made to how courses focus on problem-solving in upper-division courses, as they experienced difficulty with talking about this at first. As a design principle for future tutorials, if students would not likely be familiar with a term like "mathematical procedures" (though this term is more commonplace in problem-solving literature), it is worth using language or a description that would be more familiar with students: for example, including a statement such as "what mathematical procedures, *such as integration, matrix multiplication, techniques to solve differential equations, etc.*, would you use here?" Otherwise, the tutorials will tend to expect students to answer questions with no example or explanation, which can be frustrating to students, and fails to incorporate any "modeling" from the cognitive apprenticeship model.

3.4 Trends in Student Problem Solving

Apart from providing insights about the tutorials themselves, qualitative data from these students also provides insights about typical problem-solving processes and thoughts of

Problem-solving Step	Frequency	Correctness
Describing	12/12	12/12
Identifying	12/12	11/12
Applying	11/12	8/11
Mathematical Procedures	10/12	6/10
Checking Work	4/12	2/4

Table 3.4 Results of analyzing how students used the problem-solving framework during exercises. This analyzes twelve problem-solving attempts from various students in various interviews. “Frequency” indicates if the problem-solving step was attempted in the problem, and “Correctness” describes whether that step was done successfully.

students at this level. These insights are useful for further assisting students with problem solving in future tutorial design, as well as better informing upper-division instruction surrounding problem-solving in general.

At the conclusion of interviews, students were asked to complete a problem-solving exercise based on concepts and problems they were taught through the tutorials. We analyzed how often students demonstrated the use of a step of the framework, consciously or unconsciously, and how correctly that step was performed, with the results of this analysis shown in **Table 3.4**. The analysis focuses on twelve problem-solving attempts, which is any attempt made by any student to solve a problem during an interview. However, the problem-solving exercise administered contained two problems on two different tutorial topics, so the problem -solving attempts are only taken from six interviews. In addition, two students were interviewed both at the middle of the semester and at the end of the semester, so the problem-solving attempts only come from four students. While we are analyzing twelve total attempts, this comes from a small sample of interviews and students, which should be considered when we reference these attempts in this discussion.

Still, from these results, it seems clear that students are competent in initial steps of problem set-up. In the problem-solving exercises, they always attempted to describe the problem and identify some relevant principle to it, and were successful in this in almost all of those attempts. In fact, some students recognize that they would be unsuccessful in their problem-solving attempt overall if they were not to address these initial steps. In their interview, Student 1 provides a visual problem-description by drawing point charges in a 3-D space, saying "Does anyone do this without drawing a picture? I just want to have a conversation with them. How does their brain work?" Two students responding to the survey also remarked that being asked to draw pictures was a feature they liked about Version A of the tutorials, one of these students saying, "[it] helps me visualize the concepts I am trying to learn better than tutorial B would." While visualization is not the only method of describing a problem, students seem to reference this in particular as a helpful problem-solving strategy for them. Existing problem-solving literature seems to indicate the importance of visualization as a problem description strategy as well [25, 26].

Unsurprisingly, as students continue in the problem-solving exercises, fewer attempts are made at the later problem-solving steps. Student 5 in one exercise feels that they know which principles are relevant to the problem, but unsure of what to do from here in applying those principles or performing any calculations, goes no further on the problem. This is also interesting to note, given that in this interview, the problem-solving strategy they described seemed to be consist of understanding the situation, then looking for formulas to get an answer—a strategy that seems to involve very few conceptual steps. However, when asked to explain their problem-solving aloud with an interviewer, they seem to reference the conceptual parts of the problem with no extra prompting, then find themselves unable to recognize or use the correct formulas in order to complete a solution.

With other students, even as they make attempts to complete the problem, they also unsurprisingly tend to make more mistakes with later framework steps. For example, students attempted ten times to use mathematical procedures or calculations of some kind to complete the problem, but were only successful in this six of the ten times: only half of the problem-solving attempts overall completed correct calculations.

The final and sharpest drop we see in use of problem-solving steps is with checking work. This is only attempted four times, and was only done correctly two of those times. From the problem-solving exercises, it appears that this is a particular struggle for students to know to do and know how to do. This can also be seen in the survey comments where one student relates their experience with using the tutorials: "I also struggled with the lack of solutions for the tutorials, as this meant I had absolutely no way to check my work if I wanted to use them as study materials for the exams." They make this comment, despite the fact that at least the A versions of the tutorial would have explicitly included some question asking them to check their solution to the tutorial and guided them through how to do so.

From this data, students seem unfamiliar with the methods that they might use to check the reasonability of an answer without relying on an answer key. It may also not occur to students what role that this step plays in problem-solving, both in helping to assess whether the answer obtained is correct, and in making sense of the answer and the process followed to obtain it. However, if a student has already obtained an answer and has few doubts about it, they may choose not to take any extra steps to perform a check. Student 6 seemed confident in their exercise solution, even saying "I could be more rigorous about that but that's the right answer." Student 1 on the other hand, seeming less confident in their solution to the Coulomb's Law exercise, performed a check to ensure that they had the right directions on their fields. A student working on the quadrupole moment tutorial in class also asked if the fact that the quadrupole moment matrix is traceless could be used to check if they had done their calculations correctly, despite them working on Version B of the tutorial and not being prompted to do this. Students looking for more confidence in their work may more naturally look for ways to check the reasonability of their answer. However, it may also be particularly difficult for students still gaining conceptual understanding of a problem to know how to perform checks on their work. This may also prove to be an area for instructors to focus on, as performing this step can play such a crucial role in developing conceptual understanding of a problem. The exact reasoning for when students tend to check (or not check) their work is worth looking into in future work.

3.5 Discussion

Here we give a summary of the major results. Statements of these results, as well corresponding quotes are organized in **Table 3.5**.

Overall, the tutorials seem to have positive effects on the students who use them, with the assistance students seem to find both in conceptual and mathematical parts of problem solving. However, the results show a stronger increase in how implementing the problem-solving framework assists students with conceptual learning. Statistically, the quantitative data is not strong enough to show this conclusively, but the effects on conceptual learning can be seen in quantitative and qualitative data. Students also seem to have a stronger preference toward the tutorials structured using the problem-solving framework, feeling like they are effective at breaking problems into steps, less ambiguous, and give you a deeper look at the problems and concepts taught in the tutorials.

Though overall, the framework-based tutorials seem to be positive for students, their most notable drawback is that students seem to struggle with being able to answer conceptual questions and questions about problem solving, not being as sure of the expectations and

Benefits	**Evidence**
Tutorials appear to assist with conceptual understanding	Exam results, Student 3: “I’ve started to understand better some of the more qualitative questions ...”
Break new problems into steps	Survey: “I liked the problem-solving structure. It helped break things down ...”
Easier wording to follow	Survey: “Questions in tutorial A were less ambiguous.”
Deeper look at problems	Survey: “It provides a bit deeper of thinking and forces you to look into the details ...”

Drawbacks	**Evidence**
Tutorial assistance with understanding is minor and may not help every student	*p*-values above 0.05, Student 5: “And then what? [I] look at my formula sheets ...”
Many students do not notice benefits of one tutorial type versus other	Student 6: “I don’t remember noticing any specific differences with the tutorials ...”
Students may have difficulty with knowing how to answer conceptual and problem-solving questions	Student 3: “I’m trying to figure out what kind of response is expected.”

Table 3.5 Summary of benefits and drawbacks of framework-based tutorials, as discussed in the data. Quotes are abbreviated; if viewing electronically, clicking on the quote can take you to the full quote where it is discussed in the text.

possibly feeling less confident in their ability to discuss concepts as they are with calculations. Students seem to be able to improve at these questions, and this may have also have led to why we see a stronger effect on conceptual understanding than on completing problems overall.

The conclusions of this study could be stronger. The tutorials may not help every student in the way that Student 3 saw themselves benefited. Though students tend to prefer Version A and seem not to prefer Version B, many students do not have a preference at all toward one version or the other, or cannot tell a difference. Finally, many statements we have seen come from only one or a few students. In this we note that this study is limited due to the small sample size, and that it is possible that if this study was replicated with larger classes or multiple classes across universities, similar trends would appear, and would strengthen the results we have seen in this part of the study.

3.6 Application of Research in Designing and Using Tutorials

To conclude our discussion, we consider how the research done here can be applied to designing future tutorials and using tutorials in classrooms.

3.6.1 Structuring Materials Using a Problem-Solving Framework

We provide all of the tutorials we used in this study in **Appendix A**. However, instructors may be interested in having a tutorial for a topic other than the ones their are existing tutorials for here, or may want to adapt an existing tutorial (such as the University of Colorado Boulder tutorials [17]) to more fully address problem solving with the assistance of the framework at this level. This section will provide considerations for structuring course materials so that they assist with problem solving.

As discussed in **2.1**, focusing on problem solving in a tutorial can include reframing existing questions about physics in the context of problem solving and problem-solving frameworks, changing existing questions to new questions so that they focus more explicitly on problem-solving itself, or adding new questions to tutorials. We provide examples of these changes below.

When reframing questions, this can involve wording a problem to focus more on problem solving. This was done for Question 6 of Tutorial 1B. On that version, the question was

6. *Here we will look deeper at our result from the previous problem and compare to a well-known problem. Simplify your answer to the case where P is on the z-axis. Compare your result to the result of Example 2.2.*

The same question was given as Question 8 on Tutorial 1A as

8. *Check your answer by showing that it reduces to the result in Example 2.2 when P is on the z-axis. (This is an example of performing a check by comparing to an easier problem, because Griffiths used symmetry to simplify the problem, and the result holds only on the z-axis.)*

Students are doing the same task in each of these questions. However, on Version B, students are asked to do the task and make conclusions for themselves about the significance of the task. On Version A, students are told more explicitly the role that this calculation plays in problem solving and reasoning that their earlier result was correct, by instructing them to do the task to check.

Question 1 on Tutorial 1 is an example of changing the task to focus on a problem-solving step. On Version B, for a point charge q and an observer at point P, students are asked to express $\mathbf{r}$, $\mathbf{r}'$ and $\boldsymbol{\mathscr{r}}$ (vectors used in E&M to describe the position of the charge, observer, and their position relative to each other) as vectors for this configuration. Version A instead gives students the task of explaining what each of these vectors describe in the scenario, focusing more explicitly on the practice of description of physical scenarios. However, by changing the task, students will not have practice with the the task it replaced. Instructors who feel that both describing and practicing writing out the vectors are important to include may want to add both instead of just having a framework task replace calculations. But replacing questions may be effective for situations where instructors may want to prioritize focusing on problem solving while still balancing practical limitations, such as time constraints in class.

Question 1. (a) of Tutorial 2A on Gauss's Law is an additional question that Version B of the tutorial does not have:

1. (a) *How are the charges arranged in this problem? What about this makes our coordinate system a convenient choice?*

Responding to this question may not be strictly required to eventually use Gauss's Law, but adding this question provides guidance in considering how charges are arranged (describing) and how that will impact symmetries and the coordinate system they will use (applying), rather than simply carrying out a procedure for certain situations.

Adding such questions might make parts of the tutorial more conceptual in nature. Even a question asking students about what mathematical procedures they are using in a problem and why that would be a useful mathematical tool, though asking about math, will involve connections to the concepts associated with that problem. As we saw in **3.3**, students had some difficulty with these types of questions if they were less confident in their ability to respond to conceptual questions. However, we note that that student comments indicated that the difficulty was partially due to not knowing the expectations of the question. As mentioned at the end of **3.3**, tutorials could provide examples to students of what effectively describing a problem looks like, or what constitutes a mathematical procedure. While students may improve with practice, providing examples or setting clear expectations of what response is expected may help students as they are responding to these questions.

3.6.2 Using Tutorials and Supporting Students as they Complete Tutorials

We used seven tutorials throughout the course of a fourteen week semester. Though the time between tutorials was sometimes shorter or longer, tutorials were typically every other week, around the time the concept was being taught. Some students expressed a desire for more frequent tutorials, such as the student who remarked on the survey that thought there should be a tutorial every week. However, even despite attempts to shorten tutorials to fit within class time, to give students proper time to complete tutorials will use most, sometimes all, of a 50-minute lecture period. While tutorials can provide a thorough exploration of a concept or problem, as well as being a form of active learning rather than passive, they lose the advantage that lectures can have of being able to present a lot of information in a relatively short time. Given the amount of content in a typical upper-division E&M course, though students may respond positively to frequent tutorials, due to time considerations, instructors may want to prioritize the most important or most difficult concepts to be addressed in the tutorial format.

When tutorials were given in class, students completed the tutorial on their own or with a small group of other students, with the instructor and teaching assistants available to provide guidance or answer questions. We did not study or consider other styles of administering tutorials, but we received a comment from one student that particularly disliked this format:

> "The only thing not helpful about the tutorials was how they were executed. It was not really a tutorial, but rather another homework assignment we were told to do. Yes it broke things down into steps, but sometimes we didn't know how or what to do still. In class it was sort of a free for all, and if you were lucky you'd get the professor or the [teaching assistant] to answer your question. The problem was everyone had different questions and they all worked at different paces, and so the professor was always with someone else. It would have been better if we were given a section to do, given five minutes to do it, and then the professor went over how to do it together as a class."

While it may be an advantage in some aspects to provide an environment where students are allowed to work at their own pace, this student seemed to find difficulty from how little guidance they were given in this style. After their comment above, the student notes that they found a time in class where their suggestion of being given a little time on their own, followed by time working on it as a class, was helpful for them. In any case, this student's comment suggests that the tutorials, even with the guidance of a framework, may not be effective as a self-study resource, and should still be accompanied by the guidance of an instructor or teaching assistant.

3.6.3 Grading and Assessment

Tutorials, especially those implementing a problem-solving framework, guide students through a thorough process of solving a problem, in contrast to many homework problems where students are expected to obtain results with little guidance. One advantage of the framework-based tutorial format is that it can clarify where students are experiencing difficulty in problem solving. Since they tend to ask questions asking students specifically about what principles they are applying and how they are applying them, if students are responding incorrectly to such questions or choosing to provide little to no response at all, then this may clarify that a student's difficulties with a certain problem do not reside in their ability to perform the calculations, but in their understanding of and effectiveness in applying the concepts. In fact, it may highlight scenarios where students are able to obtain a correct numerical answer while still needing more work on understanding the concepts. The structure of framework-based tutorials provides such opportunities for formative assessment: instructors may better understand what aspects of problem-solving to focus on in their instruction, and students receiving feedback on the tutorials may better understand which areas of problem-solving they would benefit from further practice on.

Appendix A

Tutorials Used in Physics 441

Provided here are the tutorials used in this study, as given to students in the class. Version A implements the problem-solving framework while Version B does not. All tutorials for Fall 2024 were modified from existing tutorials, created and/or modified by David Neilsen and John Colton for upper-division E&M as taught at Brigham Young University during Fall 2023.

Point Charges and the Electric Field

A point charge q is at a location given by the position vector $\mathbf{r}' = (x', y', z')$. So the electric field observed at $\mathbf{r} = (x, y, z)$ is given by

$$\mathbf{E}(\mathbf{r}) = \frac{q}{4\pi\epsilon_0}\frac{\hat{\boldsymbol{\mathscr{r}}}}{\mathscr{r}^2}, \qquad \text{where} \qquad \boldsymbol{\mathscr{r}} = \mathbf{r} - \mathbf{r}'.$$

When computing the field from multiple point charges, the principle of superposition states that the fields add together linearly, i.e., for charges q_i at positions $\mathbf{r}_i'$, $\mathbf{E}(\mathbf{r}) = \mathbf{E}_1(\mathbf{r}, \mathbf{r}_1') + \mathbf{E}_2(\mathbf{r}, \mathbf{r}_2') + \cdots$. When adding multiple vectors together, it may be convenient to write the field from a single charge q_i as

$$\mathbf{E}_i(\mathbf{r}, \mathbf{r}_i) = \frac{q_i}{4\pi\epsilon_0}\frac{\boldsymbol{\mathscr{r}}_i}{\mathscr{r}_i^3} = \frac{q_i}{4\pi\epsilon_0}\frac{(\mathbf{r} - \mathbf{r}_i')}{|\mathbf{r} - \mathbf{r}_i'|^3},$$

where then the total field is

$$\mathbf{E}(\mathbf{r}) = \sum_i \mathbf{E}_i(\mathbf{r}, \mathbf{r}_i) = \frac{1}{4\pi\epsilon_0}\sum_i \frac{q_i}{|\mathbf{r} - \mathbf{r}_i'|^3}(\mathbf{r} - \mathbf{r}_i').$$

Situation 1. A point charge q is located at position $(6, -2, 0)$ m. You observe the charge at a point P at $(-2, 5, 0)$ m.

1. Briefly explain what each of these vectors is describing in this situation:

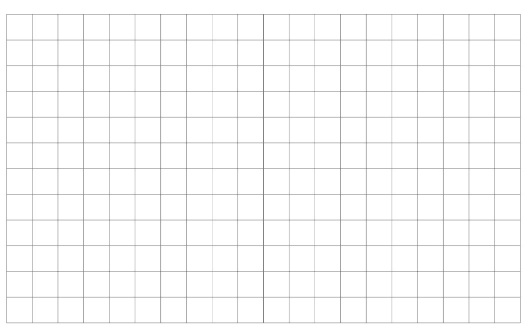
   (a) $\mathbf{r}$

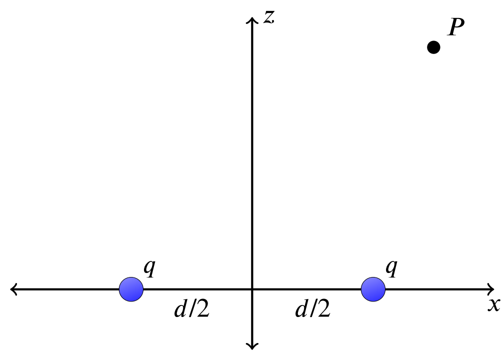
   (b) $\mathbf{r}'$

   (c) $\boldsymbol{\mathscr{r}}$

2. While Griffiths chooses to use the notation of $\boldsymbol{\mathscr{r}}$, often it is still written just as $\mathbf{r} - \mathbf{r}'$. What might be the advantages of each use of notation? Does one describe the situation differently than the other?

3. On the grid below, draw x- and y-axes and indicate the locations of the origin O, point P, and the charge q. Draw and label the vectors $\mathbf{r}$, $\mathbf{r}'$, and $\boldsymbol{\mathscr{r}}$.





Situation 2. In Example 2.2 of Griffiths, two point charges with charge q are located on the x-axis, as shown in the figure above. In the example problem, point P is on the z-axis, but we will generalize this so that P can be anywhere in space.

4. Draw the vectors $\mathbf{r}$, $\mathbf{r}'_1$, $\mathbf{r}'_2$, $\boldsymbol{\mathscr{r}}_1$, and $\boldsymbol{\mathscr{r}}_2$ on the figure.

5. What will $\mathbf{r}$ represent in this situation? How is this different from situation 1?

6. Find the Cartesian components for

   (a) $\mathbf{r}'_1$

   (b) $\mathbf{r}'_2$

7. Briefly explain how you will use the *principle of superposition* in this problem to find $\mathbf{E}$. Then calculate the total electric field at point P from both charges. Express your answer in terms of the Cartesian components of the vectors.

An important skill for physicists to develop is learning how to check the results of difficult computations. One way to do this is to check if our answer reduces to a known result to an easier problem. A second way is to look at special limits, such as letting some quantity, such as the charge or the position, go to zero or infinity.

8. Check your answer by showing that it reduces to the result in Example 2.2 when P is on the z-axis. (This is an example of performing a check by comparing to an easier problem, because Griffiths used symmetry to simplify the problem, and the result holds only on the z-axis.)

$$\left[\textit{Result of Ex. 2.2:}\quad \mathbf{E} = \frac{1}{4\pi\epsilon_0}\frac{2qz}{\left[z^2+(d/2)^2\right]^{3/2}}\hat{\mathbf{z}}\right]$$

9. In example 2.2, Griffiths checked his result by taking the limit as $z \gg d$, and showed that his result reduced to the field of a point charge at the origin with a total charge $2q$. Perform a similar check by calculating $\mathbf{E}$ on the x-axis in the limit that $x \gg d$.

10. Later in this class, you will be given a ring of positive charge distribution with radius R and will find the electric field on the z-axis. Think through some problem solving steps you have seen in this tutorial and how they may apply or change on that problem.

   (a) How will your description of the problem change? (For example, how are the $\mathbf{r}$ vectors different? What other ways is that problem different from a situation with point charges?)

   (b) How will the equations be different in that situation? Even with those differences, do any principles still apply from this tutorial?

   (c) Do you think you could produce a similar check where $z \gg R$ like what was done in this tutorial? How can this help you know if your answer made sense?

11. Answer the additional self-assessment questions on Max.

P441 Tutorial 1B Name: ______________________________

Point Charges and the Electric Field

A point charge q is at a location given by the position vector $\mathbf{r}' = (x', y', z')$. So the electric field observed at $\mathbf{r} = (x, y, z)$ is given by

$$\mathbf{E}(\mathbf{r}) = \frac{q}{4\pi\epsilon_0}\frac{\hat{\boldsymbol{\mathscr{r}}}}{\mathscr{r}^2}, \qquad \text{where} \qquad \boldsymbol{\mathscr{r}} = \mathbf{r} - \mathbf{r}'.$$

When computing the field from multiple point charges, the principle of superposition states that the fields add together linearly, i.e., for charges q_i at positions $\mathbf{r}_i'$, $\mathbf{E}(\mathbf{r}) = \mathbf{E}_1(\mathbf{r}, \mathbf{r}_1') + \mathbf{E}_2(\mathbf{r}, \mathbf{r}_2') + \cdots$. When adding multiple vectors together, it may be convenient to write the field from a single charge q_i as

$$\mathbf{E}_i(\mathbf{r}, \mathbf{r}_i) = \frac{q_i}{4\pi\epsilon_0}\frac{\boldsymbol{\mathscr{r}}_i}{\mathscr{r}_i^3} = \frac{q_i}{4\pi\epsilon_0}\frac{(\mathbf{r} - \mathbf{r}_i')}{|\mathbf{r} - \mathbf{r}_i'|^3},$$

where then the total field is

$$\mathbf{E}(\mathbf{r}) = \sum_i \mathbf{E}_i(\mathbf{r}, \mathbf{r}_i) = \frac{1}{4\pi\epsilon_0}\sum_i \frac{q_i}{|\mathbf{r} - \mathbf{r}_i'|^3}(\mathbf{r} - \mathbf{r}_i').$$

Situation 1. A point charge q is located at position $(6, -2, 0)$ m. You observe the charge at a point P at $(-2, 5, 0)$ m.

1. Write each of the unit vectors in terms of $\hat{\mathbf{x}}$, $\hat{\mathbf{y}}$, and $\hat{\mathbf{z}}$:

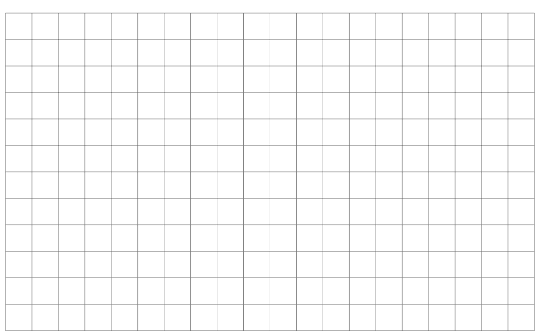
   (a) $\mathbf{r}$

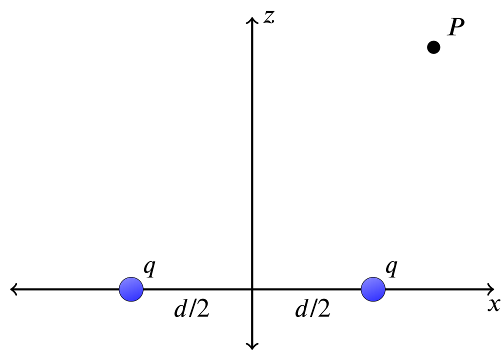
   (b) $\mathbf{r}'$

   (c) $\boldsymbol{\mathscr{r}}$

2. While Griffiths chooses to use the notation of $\boldsymbol{\mathscr{r}}$, often it is still written just as $\mathbf{r} - \mathbf{r}'$. What might be the advantages of each use of notation?

3. On the grid below, draw x- and y-axes and indicate the locations of the origin O, point P, and the charge q. Draw and label the vectors $\mathbf{r}$, $\mathbf{r}'$, and $\boldsymbol{\mathscr{r}}$.





Situation 2. In Example 2.2 of Griffiths, two point charges with charge q are located on the x-axis, as shown in the figure above. In the example problem, point P is on the z-axis, but we will generalize this so that P can be anywhere in space.

4. Start by finding the Cartesian components for

   (a) $\mathbf{r}$

   (b) $\mathbf{r}'_1$

   (c) $\mathbf{r}'_2$

5. Use your expressions of $\mathbf{r}$ vectors to then calculate the total electric field at point P from both charges. Express your answer in terms of the Cartesian components.

6. Here we will look deeper at our result from the previous problem and compare to a well-known problem. Simplify your answer to the case to where P is on the z-axis. Compare your result to the result of Example 2.2.

$$\left[\textit{Result of Ex. 2.2:} \quad \mathbf{E} = \frac{1}{4\pi\epsilon_0} \frac{2qz}{\left[z^2 + (d/2)^2\right]^{3/2}} \hat{\mathbf{z}} \right]$$

7. Calculate $\mathbf{E}$ on the x-axis in the limit that $x \gg d$. What does your equation reduce to? Why is this significant?

8. Later in this class, you will be given a ring of positive charge with radius R and will find the electric field on the z-axis. Think through what you have seen in this tutorial and how that may apply or change on that problem.

   (a) Summarize how you have solved these problems in the case of point charges, and then summarize how you think you will find the electric field with a continuous charge distribution. Compare the two approaches.

   (b) Given that all the charge on the ring is positive, what do you think would happen in that scenario if you were to take a limit where $z \gg R$ (similar to problem 6)?

9. Answer the additional self-assessment questions on Max.

P441 Tutorial 2A Name: ______________________

Gauss' Law

A hollow sphere has an inner radius a_1, and an outer radius a_2. In addition, there is a constant surface charge density σ_0 on the boundary, a_1. Thus, we might write the charge density as

$$\rho(r) = \begin{cases} \frac{\sigma_0}{4\pi a_1^2}\delta(r - a_1) & r = a_1 \\ \rho_1(r) & a_1 < r < a_2, \end{cases}$$

where ρ_1 is a function of the radius r.

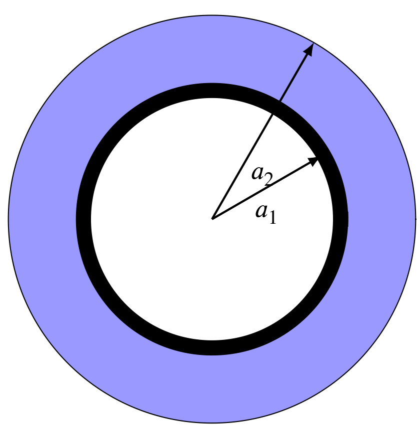
1. We will write the electric field in spherical coordinates $\mathbf{E} = E_r\,\hat{\mathbf{r}} + E_\theta\,\hat{\boldsymbol{\theta}} + E_\phi\,\hat{\boldsymbol{\phi}}$.

   (a) How are the charges arranged in this problem? What about this makes our coordinate system a convenient choice?

   (b) Based on the description of the charge distribution in this problem, which of the coordinates $\{r, \theta, \phi\}$ will $\mathbf{E}$ depend on, and which components of $\mathbf{E}$ will be zero? Explain.

   (c) How does this allow you to find $\mathbf{E}$ using Gauss' law in integral form $\oint \mathbf{E} \cdot \mathrm{d}\mathbf{a}$? Would you be able to to use Gauss's law if the charge distribution had some θ dependence, $\rho_1(r, \theta)$?

2. The figure below shows the sphere. Draw the electric field for each region of the sphere. Then draw on the diagram the Gaussian surfaces, and the vector $\mathbf{n}$ for those surfaces, that you will use to find the electric field $\mathbf{E}$ for each region of this sphere. Label each region.



3. What is the total charge Q on the surface $r = a_1$? One method is to multiply the charge distribution by the total area of the surface. Why does this work here and in which situations might it not work? What mathematical methods are appropriate for those situations?

4. Now find a general expression for the total charge Q in the region between a_1 and a_2, that is, the enclosed charge for a sphere of radius $a_1 < r < a_2$.

5. Assume that $\rho_1(r) = Ar$ for a constant A. Use Gauss's law to relate the charge expressions you found to the flux through the surfaces you drew, then find $\mathbf{E}$ (a) inside the shell and (b) for the region $a_1 < r < a_2$.

6. Check whether your answer makes sense. Is there any discontinuity in your expression for $\mathbf{E}$ for the sphere? Would you expect there to be, and where if so? Explain.

7. Answer the additional self-assessment questions on Max.

P441 Tutorial 2B   Name: ______________________

Gauss' Law

A hollow sphere has an inner radius a_1, and an outer radius a_2. In addition, there is a constant surface charge density σ_0 on the boundary, a_1. Thus, we might write the charge density as

$$\rho(r) = \begin{cases} \dfrac{\sigma_0}{4\pi a_1^2}\delta(r - a_1) & r = a_1 \\ \rho_1(r) & a_1 < r < a_2, \end{cases}$$

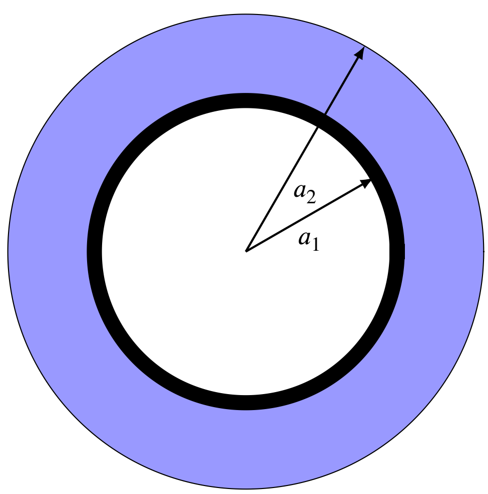
where ρ_1 is a function of the radius r.

1. We will write the electric field in spherical coordinates $\mathbf{E} = E_r\,\hat{\mathbf{r}} + E_\theta\,\hat{\boldsymbol{\theta}} + E_\phi\,\hat{\boldsymbol{\phi}}$.

   (a) Which of the coordinates $\{r, \theta, \phi\}$ will $\mathbf{E}$ depend on, and which components of $\mathbf{E}$ will be zero? Explain.

   (b) Would you be able to to use Gauss's law if the charge distribution had some θ dependence, $\rho_1(r, \theta)$?

2. The figure below shows the sphere. Draw the electric field for each region of the sphere. You may also use the diagram to aid you in other parts of this tutorial as necessary.



3. What is the total charge Q on the surface $r = a_1$? One method is to multiply the charge distribution by the total area of the surface. What is the other method you could use? (You only need to calculate using one method).

4. Now find a general expression for the total charge Q in the region between a_1 and a_2, that is, the enclosed charge for a sphere of radius $a_1 < r < a_2$.

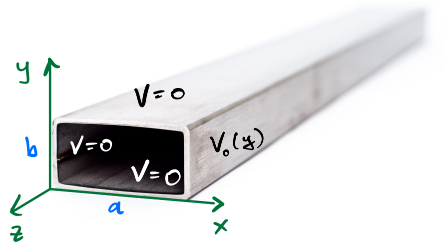
5. Assume that $\rho_1(r) = Ar$ for a constant A. Use Gauss's law to find $\mathbf{E}$ (a) inside the shell and (b) for the region $a_1 < r < a_2$.

6. Where is there a discontinuity in the $\mathbf{E}$ field? Why is this discontinuity there?

7. Answer the additional self-assessment questions on Max.

P441 Tutorial 3A Name: ____________________

Separation of Variables



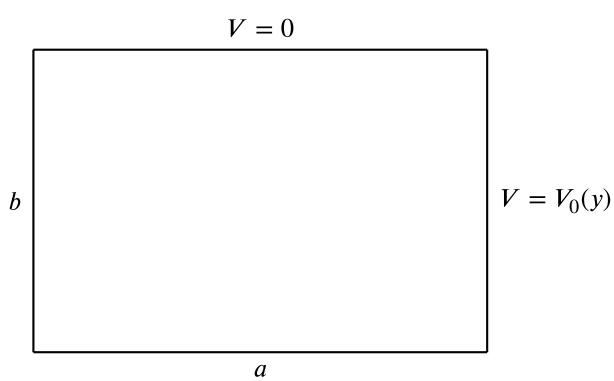
A very long hollow pipe has a rectangular cross section, with lengths a and b. The walls of the pipe are held at a fixed potential. Three of the walls are grounded, and the fourth wall has a potential that varies with position, $V(a, y, z) = V_0(y)$. To find the potential everywhere inside the pipe, solve Laplace's equation for the potential $V(x, y, z)$

$$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0.$$

1. Describe the scenario in the following ways: what do we know so far about the voltage? Why are Cartesian coordinates an ideal choice for this problem? Does this system have any symmetries that could simplify solving for the potential?

2. Outside of this being a tutorial about Laplace's equation, what about this situation would help you to know that Laplace's equation could help you solve for the potential as opposed to other methods you have learned?

3. Assume that the boundary condition at $x = a$ is $V_0(y) = \sin(\pi y/b)$ on $0 < y < b$. Guess a solution for $V(x, y, z)$ by sketching contours of constant V on the cross section of the pipe below.



As much of problems having to do with Laplace's equation require heavy use of mathematical procedures and techniques, the next few questions will focus on using these mathematical procedures and how to check results.

4. Briefly list some of the mathematical procedures you can think of needing to use to solve Laplace's equation.

5. What does it mean to "separate variables" for $V(x, y, z)$?

6. Plug the separated form of $V(x, y, z)$ into Laplace's equation. You may make any simplifications that the symmetries of this problem allows.

7. What are the boundary conditions for this problem (a) at $x = 0$ and $x = a$, (b) at $y = 0$ and $y = b$, and (c) in the z direction? How will you use the boundary conditions when finding your solution in this scenario?

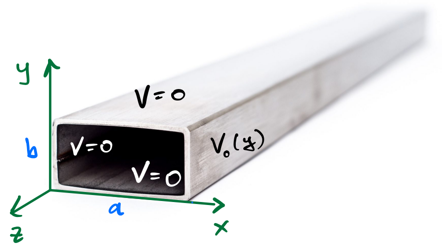
8. To solve Laplace's equation, you will expand V in terms of eigenfunctions, such as $\sin\xi$, $\cos\xi$, $\sinh\xi$, or $\cosh\xi$.

   (a) Which eigenfunctions will you choose for expansion in the x-direction? Check that these eigenfunctions make sense by showing that they can satisfy your boundary conditions.

   (b) Which eigenfunctions will you choose for expansion in the y-direction? Once again check that these eigenfunctions make sense by showing that they can satisfy your boundary conditions.

9. Using the eigenfunctions you found, write $V(x, y, z)$ in general form.

P441 Tutorial 3B Name: ____________________

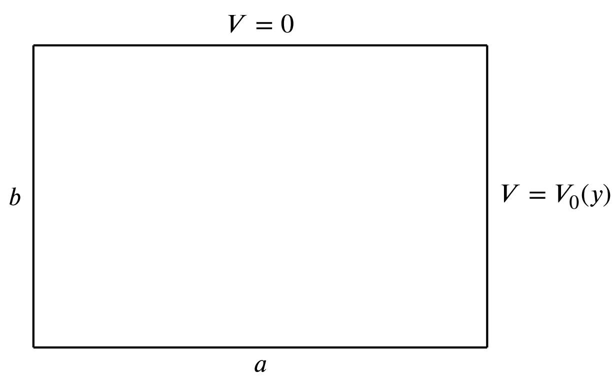


A very long hollow pipe has a rectangular cross section, with lengths a and b. The walls of the pipe are held at a fixed potential. Three of the walls are grounded, and the fourth wall has a potential that varies with position, $V(a, y, z) = V_0(y)$. To find the potential everywhere inside the pipe, solve Laplace's equation for the potential $V(x, y, z)$

$$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0.$$

1. How can the symmetries in this system help simplify solving for the potential? Explain.

2. Assume that the boundary condition at $x = a$ is $V_0(y) = \sin(\pi y/b)$ on $0 < y < b$. Guess a solution for $V(x, y, z)$ by sketching contours of constant V on the cross section of the pipe below.



3. What does it mean to "separate variables" for $V(x, y, z)$?

4. Plug the separated form of $V(x, y, z)$ into Laplace's equation. You may make any simplifications that the symmetries of this problem allows.

5. What are the boundary conditions for this problem (a) at $x = 0$ and $x = a$, (b) at $y = 0$ and $y = b$, and (c) in the z direction?

6. To solve Laplace's equation, you will expand V in terms of eigenfunctions, such as $\sin\xi$, $\cos\xi$, $\sinh\xi$, or $\cosh\xi$.

   (a) Use your boundary conditions described in the previous question to choose eigenfunctions for expansion in the x-direction.

   (b) Use your boundary conditions described in the previous question to choose eigenfunctions for expansion in the y-direction.

7. Using the eigenfunctions you found, write $V(x, y, z)$ in general form.

P441 Tutorial 4A Name: ______________________________

The Quadrupole Electric Field

The quadrupole potential and the quadrupole moment are

$$V(\mathbf{r}) = \frac{1}{4\pi\epsilon_0}\frac{Q_{ij}x_ix_j}{r^5}, \qquad Q_{ij} = \frac{1}{2}\int \left(3x'_ix'_j - r'^2\delta_{ij}\right)\rho(\mathbf{r}')\,d\tau',$$

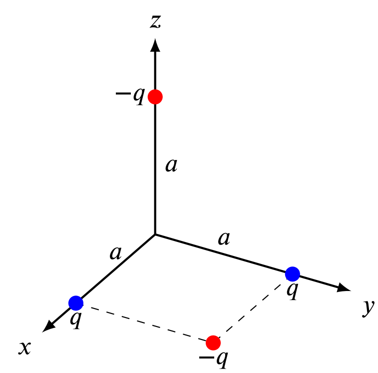
where r is the magnitude of the radial vector, $\mathbf{r} = x_i\mathbf{e}_i$, and $\rho(\mathbf{r})$ is the charge density. The quadrupole moment is symmetric and traceless

$$Q_{ij} = Q_{ji}, \qquad \mathrm{Tr}\,Q_{ij} = Q_{ii} = 0.$$

The Cartesian components of Q_{ij} are often written in matrix form

$$Q_{ij} = \begin{pmatrix} Q_{xx} & Q_{xy} & Q_{xz} \\ Q_{xy} & Q_{yy} & Q_{yz} \\ Q_{xz} & Q_{yz} & Q_{zz} \end{pmatrix}.$$

Consider the charge distribution shown to the right with four point charges. Positive charges with magnitude q are located on the x- and y-axes, a distance a from the origin. Negative charges $-q$ are located on the z-axis and in the x-y plane.



1. What is the total charge present? Without calculating the monopole field, does the monopole, Q, vanish for this charge distribution?

2. Draw the vectors $\mathbf{r}'$ that describe the location of each of the charges on the diagram. Without doing a calculation, does the total dipole moment, $\mathbf{p}$, vanish for this charge distribution? If $\mathbf{p} \neq 0$ for this distribution, does it depend on the location of the origin? Explain why or why not.

3. Now consider the elements of the quadrupole moment matrix, Q_{ij}. What information from our problem will we use to calculate them? Do they depend on the location of the origin? Explain why or why not.

4. When we have individual charges instead of a distribution, we can write the formula for Q_{ij} without the integral. Write the formula for Q_{ij} in the case of a single charge, q.

5. Calculate the quadrupole moment Q_{ij} about the origin for the four charges in the figure. To do this, calculate the moment for each charge individually, $Q_{ij}^{(\ell)}$, and then sum them together,

$$Q_{ij} = Q_{ij}^{(1)} + Q_{ij}^{(2)} + Q_{ij}^{(3)} + Q_{ij}^{(4)}.$$

   (a) Find $Q_{ij}^{(1)}$ for the charge on the x-axis. Find each component and then check that the moment you got makes sense by showing that it is symmetric and traceless.

   (b) Find $Q_{ij}^{(2)}$ for the charge on the y-axis. From here, now that you have checked, you may use that $\mathrm{Tr}\, Q_{ij} = 0$, and that $Q_{ij} = Q_{ji}$ to simplify calculations.

   (c) Find $Q_{ij}^{(3)}$ for the charge on the z-axis.

   (d) Find $Q_{ij}^{(4)}$ for the charge in the x-y plane.

(e) Find the total Q_{ij}. Is it still symmetric and traceless after you have summed the individual quadrupole moments?

6. Looking at your computations, what information is encoded in the different components of Q_{ij}? For example, for the single charge calculations where the charge was on the axis, how did the diagonal components of Q_{ij} compare to the off-diagonal components? How about for the charge off axis? Finally, how did this impact the total Q_{ij}?

7. Answer the additional self-assessment questions on Max.

P441 Tutorial 4B Name: ______________________________

The Quadrupole Electric Field

The quadrupole potential and the quadrupole moment are

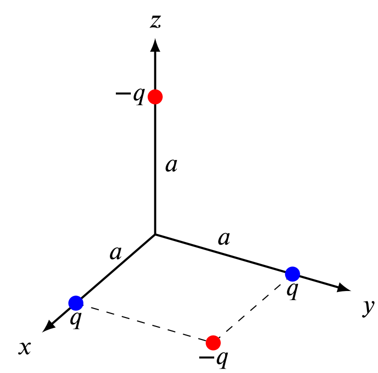
$$V(\mathbf{r}) = \frac{1}{4\pi\epsilon_0}\frac{Q_{ij}x_i x_j}{r^5}, \qquad Q_{ij} = \frac{1}{2}\int \left(3x'_i x'_j - r'^2\delta_{ij}\right)\rho(\mathbf{r}')\,d\tau',$$

where r is the magnitude of the radial vector, $\mathbf{r} = x_i\mathbf{e}_i$, and $\rho(\mathbf{r})$ is the charge density. The quadrupole moment is symmetric and traceless

$$Q_{ij} = Q_{ji}, \qquad \mathrm{Tr}\, Q_{ij} = Q_{ii} = 0.$$

The Cartesian components of Q_{ij} are often written in matrix form

$$Q_{ij} = \begin{pmatrix} Q_{xx} & Q_{xy} & Q_{xz} \\ Q_{xy} & Q_{yy} & Q_{yz} \\ Q_{xz} & Q_{yz} & Q_{zz} \end{pmatrix}.$$



Consider the charge distribution shown to the right with four point charges. Positive charges with magnitude q are located on the x- and y-axes, a distance a from the origin. Negative charges $-q$ are located on the z-axis and in the x-y plane.

1. Without doing a calculation, does the monopole charge, Q, vanish for this charge distribution?

2. Without doing a calculation, does the total dipole moment, $\mathbf{p}$, vanish for this charge distribution? If $\mathbf{p} \neq 0$ for this distribution, does it depend on the location of the origin? Explain why or why not.

3. Now consider the elements of the quadrupole moment matrix, Q_{ij}. Do they depend on the location of the origin? Explain why or why not.

4. When we have individual charges instead of a distribution, we can write the formula for Q_{ij} without the integral. Write the formula for Q_{ij} in the case of a single charge, q.

5. Calculate the quadrupole moment Q_{ij} about the origin for the four charges in the figure. To do this, calculate the moment for each charge individually, $Q_{ij}^{(\ell)}$, and then sum them together,

$$Q_{ij} = Q_{ij}^{(1)} + Q_{ij}^{(2)} + Q_{ij}^{(3)} + Q_{ij}^{(4)}.$$

You may use that $\mathrm{Tr}\, Q_{ij} = 0$, and that $Q_{ij} = Q_{ji}$ to simplify calculations.

(a) Find $Q_{ij}^{(1)}$ for the charge on the x-axis.

(b) Find $Q_{ij}^{(2)}$ for the charge on the y-axis.

(c) Find $Q_{ij}^{(3)}$ for the charge on the z-axis.

(d) Find $Q_{ij}^{(4)}$ for the charge in the x-y plane.

(e) Find the total Q_{ij}.

6. Looking at your computations, what information is encoded in the different components of Q_{ij}? For example, for the single charge calculations where the charge was on the axis, how did the diagonal components of Q_{ij} compare to the off-diagonal components? How about for the charge off axis? Finally, how did this impact the total Q_{ij}?

7. Answer the additional self-assessment questions on Max.

P441 Tutorial 5A Name: ______________________________

Dielectrics

A uniform linear dielectric medium with dielectric constant ϵ fills a large region of space. A small spherical cavity of radius a is hollowed out from the material, and a point dipole $\mathbf{p}$ is placed at the center of the cavity. In this problem, we will find the potential V everywhere (inside and outside of the cavity).

1. Draw the situation described in this problem. Label everything you know on the diagram. Indicate on you drawing or diagram the coordinate system you will use.

2. We will look at using Laplace's Equation to find the potential V in this problem.

   (a) What other physical principles or techniques will you need to take into account while solving this problem?

   (b) What are some other techniques you have learned to find potentials? Why would we emphasize using Laplace's Equation in this situation over some other techniques you have learned in this class?

3. What symmetries are present in this problem? Write the general form for the solution of Laplace's Equation in the coordinate system you chose, taking these symmetries into account.

4. Consider the two regions.

   (a) Let the potential within the cavity be V_{in}. How does the dipole act as a "condition" for what form V_{in} will take? Write the form of V_{in}.

   (b) Let the potential outside the cavity be V_{out}. What conditions affect the form of V_{out}? Write the form of V_{out}.

   (c) What boundary conditions need to be applied at $r = a$ to relate V_{in} and V_{out}?

5. Apply your boundary conditions and find the solution for V everywhere. When you have found your solution, what ways can you check that your answer is correct or makes sense?

The following questions are primarily thought questions. You do not need to find complete solutions if a new scenario is given, but you should consider (a) any physical principles involved beyond what was needed for the above problem, (b) how you might apply them, and (c) any changes to mathematical techniques from the problem you have just done.

6. Why did we split the solution into two parts, V_{in} and V_{out}, with matching conditions, instead of writing one expression for the global solution?

7. How would you find the bound polarization charge σ_b on the wall of the cavity at $r = a$?

8. How would the solution change if the dipole were replaced by a point charge?

9. How would the solution change if a constant, external field, aligned to the dipole, was introduced?

10. How would the solution change if the cavity in the dielectric were replaced with a conducting sphere of the same size with charge Q?

11. How would the solution change if $a \to 0$?

P441 Tutorial 5B Name: ____________________

Dielectrics

A uniform linear dielectric medium with dielectric constant ϵ fills a large region of space. A small spherical cavity of radius a is hollowed out from the material, and a point dipole $\mathbf{p}$ is placed at the center of the cavity. In this problem, we will find the potential V everywhere (inside and outside of the cavity).

1. We will look at using Laplace's Equation to find the potential V in this problem. Describe briefly your approach to solving this problem. What coordinate system will you use?

2. Write the general form for the solution of Laplace's Equation in the coordinate system you chose. Are there any coordinates that the solution does not depend on?

3. Consider the two regions.

   (a) Let the potential within the cavity be V_{in}. Write the form of V_{in}. Which terms can be set to zero?

   (b) Let the potential outside the cavity be V_{out}. Write the form of V_{out}. Which terms can be set to zero?

   (c) What boundary conditions need to be applied at $r = a$ to relate V_{in} and V_{out}?

4. Now find the solution for V everywhere.

The following questions are primarily thought questions. You do not need to find complete solutions for them, but you may write a couple of equations if needed to explain your answers.

5. Why did we split the solution into two parts, V_{in} and V_{out}, with matching conditions, instead of writing one expression for the global solution?

6. How would you find the bound polarization charge σ_b on the wall of the cavity at $r = a$?

7. How would the solution change if the dipole were replaced by a point charge?

8. How would the solution change if a constant, external field, aligned to the dipole, was introduced?

9. How would the solution change if the cavity in the dielectric were replaced with a conducting sphere of the same size with charge Q?

10. How would the solution change if $a \to 0$?

P441 Tutorial 6A Name: ______________________

Magnetostatics and Ampere's Law

Maxwell's Equations for Static Magnetic Fields.

1. A magnetic field has the form $\mathbf{B} = 3xy\,a(x, y)\,\hat{\mathbf{x}} + 4y^2\,\hat{\mathbf{y}}$, where $a(x, y)$ is an unknown function.

   (a) We will first find $a(x, y)$. Identify any physical law(s) about the magnetic field that might help you find $a(x, y)$.

   (b) Apply any physical laws to the $\mathbf{B}$ field described in the problem. What kind of equation do you get, and what mathematical techniques will you use to solve it?

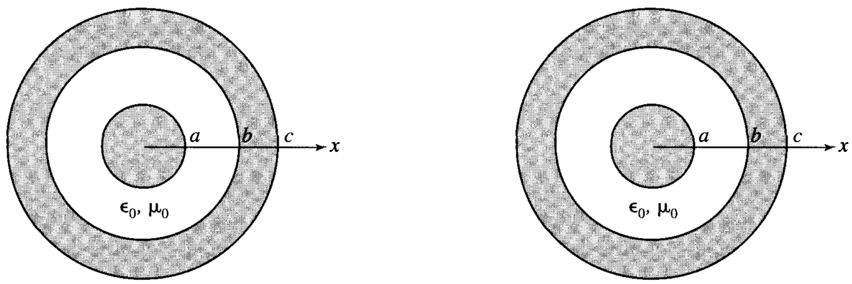
   (c) Find $a(x, y)$ for this magnetic field. Then check your work by inserting $a(x, y)$ into the expression for $\mathbf{B}$ and show that it does satisfy the physical law(s) that apply to magnetic fields.

   (d) What physical law relates $\mathbf{B}$ to $\mathbf{J}$? Write an equation for the current $\mathbf{J}$ that produces this field.

Ampere's Law. Ampere's law in integral form is written

$$\oint_C \mathbf{B} \cdot d\mathbf{l} = \mu_0 \int_S \mathbf{J} \cdot d\mathbf{a}.$$

2. The figure below shows the cross section of a long coaxial cable. The center conductor ($r < a$) carries a current I_0 in the direction out of the page. The outer conductor ($b < r < c$) carries the return current I_0 in to the page. The current density in the conductors is constant.



   (a) Visualize the current by sketching the direction of current on each region of the cable. Use what you know about the direction of the current to sketch the direction of magnetic field lines on the figure in all regions. Two identical figures are provided if you prefer not to overlay your sketches, but label each figure.

   (b) Why can you use Ampere's law in this scenario? What changes to this scenario would make it so that Ampere's law couldn't be used?

   (c) What information are you given about the current? Use this information to find the current density in the regions $r < a$ and $b < r < c$.

   (d) Sketch a curve C and surface S in the region $r < a$ to apply Ampere's law. Apply Ampere's law on this curve to find $\mathbf{B}(\mathbf{r})$.

(e) Sketch a curve C and surface S in the region $a < r < b$ to apply Ampere's law. Apply Ampere's law on this curve to find $\mathbf{B}(\mathbf{r})$.

(f) (Skip this if short on time) Sketch a curve C and surface S in the region $b < r < c$ to apply Ampere's law. Apply Ampere's law on this curve to find $\mathbf{B}(\mathbf{r})$.

(g) Without doing any calculations, what will $\mathbf{B}(\mathbf{r})$ be in the region $r > c$? Explain by applying Ampere's law.

(h) Sketch the magnitude of B as a function of r. Label each region on your plot (make a reasonable prediction for $b < r < c$ if you did not compute $\mathbf{B}(\mathbf{r})$ for that region).

P441 Tutorial 6B Name: ______________________

Magnetostatics I

Maxwell's Equations for Static Magnetic Fields.

1. A magnetic field has the form $\mathbf{B} = 3xy\, a(x, y)\, \hat{\mathbf{x}} + 4y^2\, \hat{\mathbf{y}}$, where $a(x, y)$ is an unknown function.

   (a) We will first find $a(x, y)$. Use conditions from Maxwell's equations to find a differential equation for $a(x, y)$.

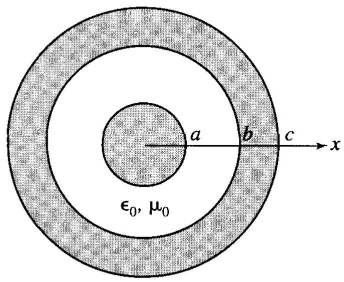
   (b) Using your equation, find a solution for $a(x, y)$ for this magnetic field.

   (c) Write an equation for the current $\mathbf{J}$ that produces this field.

Ampere's Law. Ampere's law in integral form is written

$$\oint_C \mathbf{B} \cdot d\mathbf{l} = \mu_0 \int_S \mathbf{J} \cdot d\mathbf{a}.$$

2. The figure below shows the cross section of a long coaxial cable. The center conductor ($r < a$) carries a current I_0 in the direction out of the page. The outer conductor ($b < r < c$) carries the return current I_0 in to the page. The current density in the conductors is constant.



(a) Sketch the direction of magnetic field lines on the figure in all regions.

(b) Find the current density in the regions $r < a$ and $b < r < c$.

(c) Sketch a curve C and surface S in the region $r < a$ to apply Ampere's law. Apply Ampere's law on this curve to find $\mathbf{B}(\mathbf{r})$.

(d) Sketch a curve C and surface S in the region $a < r < b$ to apply Ampere's law. Apply Ampere's law on this curve to find $\mathbf{B}(\mathbf{r})$.

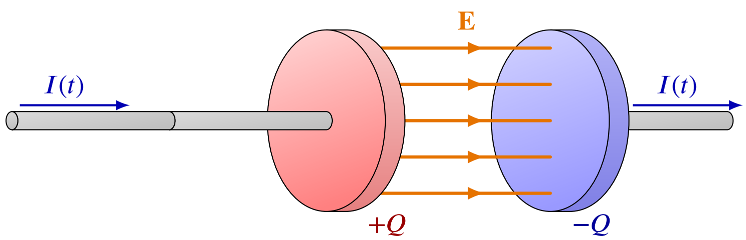
(e) (Skip this if short on time) Sketch a curve C and surface S in the region $b < r < c$ to apply Ampere's law. Apply Ampere's law on this curve to find $\mathbf{B}(\mathbf{r})$.

(f) Without doing any calculations, what will $\mathbf{B}(\mathbf{r})$ be in the region $r > c$? Explain.

(g) Sketch the magnitude of B as a function of r. Label each region on your plot (make a reasonable prediction for $b < r < c$ if you did not compute $\mathbf{B}(\mathbf{r})$ for that region).

P441 Tutorial 7A Name: ______________________________

Displacement Current



A capacitor with parallel circular plates is charged by a current $I(t)$, such that the charge on the capacitor as a function of time t is $Q = Q_0(1 - e^{-t/\tau})$, where Q_0 and τ are positive constants. The plates have a radius a and separation d. You can assume that $\mathbf{E}(t)$ is constant between the plates.

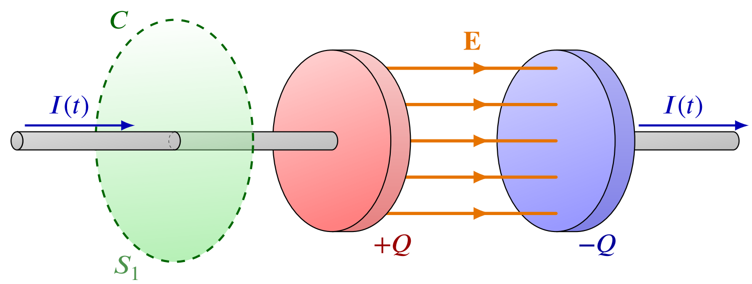
1. Assume that charge on the capacitor plates is redistributed much faster than the time scale, τ, for charging the capacitor. Thus, we can assume that $\sigma(t)$ is approximately constant over the capacitor plates.

   (a) There are many different formulas you have learned in this class relating to the current. What information are you given in this problem, that can help you find the current $I(t)$? What formula will you use? Use this to calculate the current $I(t)$ that charges the capacitor.

   (b) Again, you have learned many methods of calculating an electric field in this class. Which method or principles will you use to calculate $\mathbf{E}(t)$ here? Calculate the electric field, $\mathbf{E}(t)$, between the capacitor plates.

   (c) Calculate $\dfrac{\partial \mathbf{E}}{\partial t}$.

   (d) What is the displacement current (a short conceptual or mathematical explanation is fine)? Given what you have found, is there a displacement current between the capacitor plates? Why or why not?

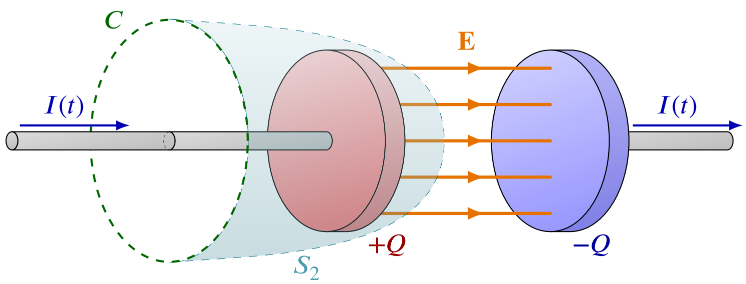


2. The figure above shows a loop C with radius s that encloses the shaded surface S_1 in the plane of the loop. You will find the magnetic field **B** around the wire.

   (a) What physical information will be relevant with the curve and surface given?

   (b) What physical principles or laws will help you determine the magnetic field induce about the wire? How will you apply them?

   (c) Use these physical principles to find **B** (magnitude and direction). Indicate the direction of **B** on the figure above.



3. Consider now the surface S_2 enclosed by the Amperian loop C, as shown in the figure above.

   (a) What physical information will be relevant with this curve and surface?

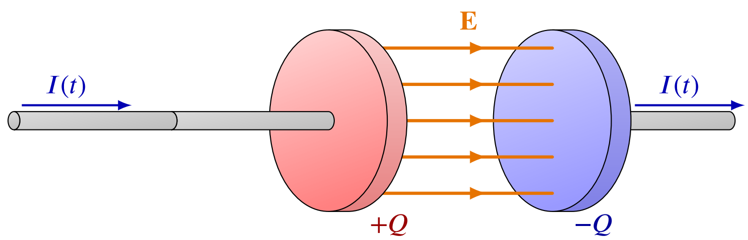
   (b) What physical principles or laws will help you determine the magnetic field induce about the wire? How will you apply them?

   (c) Use these physical principles to find $\mathbf{B}$ (magnitude and direction). Indicate the direction of $\mathbf{B}$ on the figure above.

   (d) Compare your result in question 3 to the result of question 2. Does changing the enclosed surface from S_1 to S_2 change $\mathbf{B}$ at C? How can this be used to check if your answer is reasonable in these kinds of scenarios?

P441 Tutorial 7B Name: ______________________________

Displacement Current



A capacitor with parallel circular plates is charged by a current $I(t)$, such that the charge on the capacitor as a function of time t is $Q = Q_0(1 - e^{-t/\tau})$, where Q_0 and τ are positive constants. The plates have a radius a and separation d. You can assume that $\mathbf{E}(t)$ is constant between the plates.

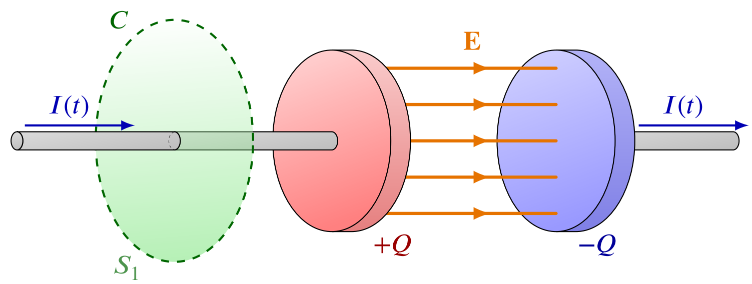
1. Assume that charge on the capacitor plates is redistributed much faster than the time scale, τ, for charging the capacitor. Thus, we can assume that $\sigma(t)$ is approximately constant over the capacitor plates.

   (a) Calculate the current $I(t)$ that charges the capacitor.

   (b) Calculate the electric field, $\mathbf{E}(t)$, between the capacitor plates.

   (c) Finally, calculate $\dfrac{\partial \mathbf{E}}{\partial t}$.

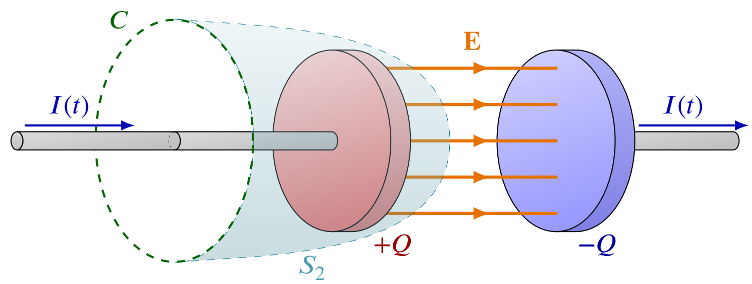
   (d) Given what you have found, is there a displacement current between the capacitor plates? If there is, what is the displacement current for this situation?



2. The figure above shows a loop C with radius s that encloses the shaded surface S_1 in the plane of the loop. You will find the magnetic field $\mathbf{B}$ around the wire.

   (a) What will be the physical source of $\mathbf{B}$ with this curve and this surface?

   (b) Use Ampere's Law to find $\mathbf{B}$ (magnitude and direction). Indicate the direction of $\mathbf{B}$ on the figure above.



3. Consider now the surface S_2 enclosed by the Amperian loop C, as shown in the figure above.

   (a) What will be the physical source of $\mathbf{B}$ with this curve and this surface?

   (b) Use Ampere's Law to find $\mathbf{B}$ (magnitude and direction). Indicate the direction of $\mathbf{B}$ on the figure above.

   (c) Based on your results in question 2 and question 3, does changing the enclosed surface from S_1 to S_2 change $\mathbf{B}$ at C? What is the significance of this for Ampere's Law and Maxwell's correction?

Appendix B

Interview and Survey Questions

Provided here are the questions used in interviews with students and in the end-of-semester survey.

B.1 Interview Questions

B.1.1 Mid-Semester Interview

Here is an initial question to get you thinking about things.

1. In your view, what role do homework and tutorial problems play in helping you learn physics?

   Consider the problems you've worked on in this class over the past month. You have been familiar, in some way, with the notion of a problem-solving framework in physics:

   - Describe what is going on: draw a picture, identify knowns and unknowns, identify the target variable and other variables you might need

- Identify the principle or set of principles about which the problem is concerned
- Apply the principles correctly
- Use proper mathematical procedures
- Check your work and make sure your progression is logical

Consider specifically the tutorials that you have been doing in the past month of the course or so. Take a minute to review these tutorials, if you need to recall them.

2. Some tutorials you have been given have explicitly included a problem-solving framework while others have not. Do you feel one format is more helpful than the other?
3. Considering the problem-solving framework you have just looked at, do you see any parts for which you think you individually did well on any of the tutorials from this past month?
4. Do you see any parts of the problem-solving framework that apply to what you individually struggled with on any of the tutorials from this past month?
5. Do you find yourself using the problem-solving framework on tutorials or homework where you are not explicitly asked to use it?
   (a) (If yes) Describe how you use the framework when you are working on a problem that does not ask you to use it.
   (b) (If no) Why do you not use the framework when you do choose not to use it?
6. Do you think any of the specific topics you covered in this past month's tutorial worked better or worse when being given a problem solving framework? Do you think any topics worked better or worse without the framework?

Note: the problem-solving exercise given to students at the end of the interview is on the following page.

The following exercises are based on topics you have seen in tutorials. For both problems given, solve the problem and explain your thinking as you complete each problem. You may express your solution verbally, or through writing or drawing on the paper, but please explain verbally what you are thinking as you complete the problem. You may request more paper if you need more space than what is given on this sheet.

1. Given a large plate of constant surface charge density $\sigma(x, y) = \sigma_0$, what is the electric field experienced by a charge a distance z above the plate? You may assume that this distance z is much less than the length of the plate.

2. A charge $+q$ is placed at a position $(1, 0, 0)$, and a charge $-q$ is placed at a position $(0, 1, 0)$. What is the electric field at the origin?

B.1.2 End-of-Semester Interview

Here is an initial question to get you thinking about things.

1. In your view, what role do homework and tutorial problems play in helping you learn physics?

   Consider the problems you've worked on in this class over the past month. You have been familiar, in some way, with the notion of a problem-solving framework in physics:

   - Describe what is going on: draw a picture, identify knowns and unknowns, identify the target variable and other variables you might need
   - Identify the principle or set of principles about which the problem is concerned
   - Apply the principles correctly
   - Use proper mathematical procedures
   - Check your work and make sure your progression is logical

   Consider specifically the tutorials that you have been doing in the past month of the course or so. Take a minute to review these tutorials, if you need to recall them.

2. Some tutorials you have been given have explicitly included a problem-solving framework while others have not. Do you feel one format is more helpful than the other?

3. As you are completing the tutorials in class, do you notice differences between the two tutorial formats?

   (a) (If yes) What differences do you notice?

   (b) (If no, no follow-up question)

4. Considering the problem-solving framework you have just looked at, do you see any parts for which you think you individually did well on any of the tutorials from this past month?

5. Do you see any parts of the problem-solving framework that apply to what you individually struggled with on any of the tutorials from this past month?

6. Do you find yourself using the problem-solving framework on tutorials or homework where you are not explicitly asked to use it?

   (a) (If yes) Describe how you use the framework when you are working on a problem that does not ask you to use it.

   (b) (If no) Why do you not use the framework when you do choose not to use it?

7. Do you think any of the specific topics you covered in this past month's tutorial worked better or worse when being given a problem solving framework? Do you think any topics worked better or worse without the framework?

Note: the mid-semester and end-of-semester interview questions are identical, except the end-of-semester interview asks if students notice differences between the tutorial formats at Question 3. Like for the mid-semester interview, the exercise given to students is on the following page.

The following exercises are based on topics you have seen in tutorials. For both problems given, solve the problem and explain your thinking as you complete each problem. You may express your solution verbally, or through writing or drawing on the paper, but please explain verbally what you are thinking as you complete the problem. You may request more paper if you need more space than what is given on this sheet.

1. A large plate has a constant surface current density with magnitude K_0 in the $+x$ direction. What is the magnetic field just above the plate?

2. A square capacitor plate at $z = d$ is held at a potential V_0 while the opposite plate at $z = 0$ is grounded. You may assume that the distance between the plates, d, is much smaller than the area of the plates. What is the electric potential between the plates?

B.2 End-of-Semester Survey Questions

1. The following is an extensive list of topics which we covered in the class. Please rate each one in terms of a 5 point scale where 1 = you struggled a lot with the topic and 5 = you had little to no difficulty with the topic. (Likert scale)

    - How to do integrals and derivatives of scalar and vector functions
    - How to use and/or prove gradient theorem, divergence theorem, curl theorem
    - How to draw electric field lines or make other field-related plots
    - How to use Coulomb's law to find E from charges or charge densities ("script r method")
    - How to use Gauss's law to find E for high symmetry situations of spherical, cylindrical, and planar charge densities
    - How to find E from V (negative gradient); and V from E (negative line integral)
    - How to find V from charges or charge densities ("script r")
    - How to calculate energy stored in field/work done to assemble charges
    - How to conceptually or numerically solve Laplace's equation using relaxation
    - How to solve boundary value problems using separation of variables: for both rectangular and spherical coordinates, esp. using Fourier's trick
    - How to solve image problems
    - How to find electric dipole moments, dipole potentials, and dipole fields
    - How to find electric quadrupole moements and quadrupole potentials
    - How to use Gauss's law for D to find D for high symmetry situations
    - How to find field given P (via bound charge densities or Gauss's law for D)

- How to find capacitance for a given geometry (including with possible dielectrics)
- How to use Biot-Savart law to calculate magnetic field from currents or current densities ("script r")
- How to use Ampere's law to find B for high symmetry situations of cylindrical, solenoidal, planar, and toroidal current densities
- How to find B from A (curl)
- How to find A from currents or current densities ("script r")
- How to calculate energy stored in field/work done to assemble currents
- How to find magnetic dipole moments, dipole potentials, and dipole fields
- How to use Ampere's law for H to find H for high symmetry situations
- How to find field given M (via bound current densities or Ampere's law for H)
- How to find EMF (including motional EMF, and Faraday's flux rule)
- How to find E for a changing B (the "Faraday current")
- How to find B for a changing E (the "displacement current")
- How to use boundary conditions for all fields and potentials (E, V, D, B, A, H) to relate quantities in one region of space to another; including distinctions between parallel and perpendicular when applicable
- How to find properties of isolated charges or dipoles (electric or magnetic) in both types of fields: force, torque, energy

2. Are there any topics you struggled with which are not listed here? (text entry)
3. For some of the topics which you struggled with, are there specific aspects about those topics you can point to which caused you to struggle? (text entry)

The following is a list of tutorials which you did this semester:

1 Point Charges and the Electric Field

2 Gauss's Law

3 Laplace's Equation and Separation of Variables

4 The Quadrupole Electric Field

5 Dielectrics

6 Magnetostatics and Ampere's Law

7 Displacement Current

4. Do you think any of the tutorials used this semester were particularly helpful? Please describe how or why. (text entry)

5. Do you think any of the tutorials you used this semester were not so helpful? Please describe how or why. (text entry)

6. You have been given tutorials in multiple formats this semester, some being structured with a problem solving framework and others not having this structure. Do you feel that one format was more helpful than the other? Please explain. (text entry)

   Below are two tutorials on the same topic, but in different formats. You do not need to solve any of the problems or answer any questions given on the tutorials, and you can take as much or as little time as you would like viewing them. *Note: The tutorials mentioned in this question were given to students as links to view at this point in the survey, and are shown following the remaining questions. Differences between the two versions of the tutorial were highlighted in yellow to assist students in comparing the two versions.*

7. Between these two tutorial formats, if you had the choice to use one format instead of the other, which format would you choose? (multiple choice)

   (a) Tutorial A

   (b) Tutorial B

   (c) No preference between formats

   (d) Cannot identify a meaningful difference between formats

   (e) Preferred format would depend on topic

8. Explain why you chose the option above. (text entry)

9. Consider the course you had for freshman-level electromagnetism. Some of this course may have come across as more mathematically rigorous versions of that course, while other topics may have been completely new to you. Please describe below some examples of both cases that you experienced in this course (overall, specific examples, etc.). (text entry)

Tutorial A Name: ______________________________

Gauss' Law

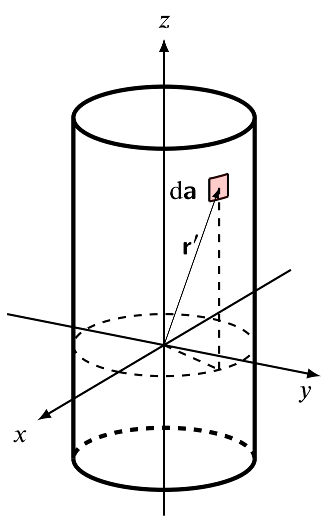
(Part 1). The main accelerator at the SLAC National Accelerator Laboratory has a 3.2 km (2 mi) clyindrical beam line with a radius of $a = 6$ cm. Electrons in the beam are accelerated to 50 GeV. In a strange accident that occurs while the accelerator is running, a lightning strike gives the beam line a constant charge density per unit length, σ, on the outside surface of the beam line.

1. In what direction does the electric field $\mathbf{E}$ point outside the outer surface of the beam line? Use what you know about the arrangement of charges to explain why it points in this direction. Draw the electric field lines on the figure.

2. Now based on the direction and symmetry of the electric field, draw a Gaussian surface on the figure. What shape will it take and why did you choose this shape? What is the total area on this surface? (You can leave variables general).

3. How much total charge is enclosed in this surface? How does Gauss's Law relate this and the previous problem?

4. With this information, determine $\mathbf{E}$ outside the beam line.



5. Consider what happens inside the tube. Does the surface charge on the beam line affect the charged particles (electrons) inside the tube? How do we know this from physical laws or principles?

6. A cylinder has surface charge density σ (much like the SLAC Accelerator situation described above), except now there is a volume charge density $\rho(\mathbf{r}) = \rho_0 s$, where s is the radius from the center of the cylinder. You do not need to find the electric field for this situation, but do respond to the following to prepare yourself for future problems like this:

   (a) How does the additional charge distribution affect the electric field, inside and outside the cylinder? Will it change any of the symmetries?

   (b) Will this change how you use Gauss's Law in finding the electric field? Are there any other physical principles that might apply when you have multiple charge distributions?

(Part 2). In the next three situations, draw a picture of the situation and explain whether you can or cannot use Gauss's Law:

7. A long glass rod has a constant surface charge σ on only half of the rod, $0 \leq \phi < \pi$. You want to find the electric field outside the rod.

8. A cube has a uniform charge density and is centered on the z-axis. You want to find the electric field on the z-axis.

9. A point charge is at the origin. You want to find the electric flux through the upper hemisphere of a sphere also centered at the origin.

10. Answer the additional self-assessment questions on Max.

Tutorial B Name: ______________________

Gauss' Law

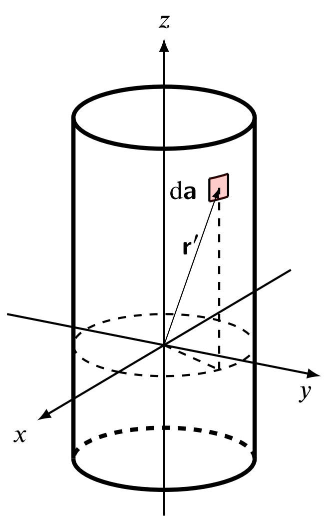
(Part 1). The main accelerator at the SLAC National Accelerator Laboratory has a 3.2 km (2 mi) clyindrical beam line with a radius of $a = 6$ cm. Electrons in the beam are accelerated to 50 GeV. In a strange accident that occurs while the accelerator is running, a lightning strike gives the beam line a constant charge density per unit length, σ, on the outside surface of the beam line.

1. In what direction does the electric field $\mathbf{E}$ point outside the outer surface of the beam line? How do you know this?

2. A cylinder encloses the section of the beam line depicted below with some height h and some radius $s > a$. What is the total area on this surface? (You can leave variables in terms of h and s). Express the electric flux for a general electric field $\mathbf{E}$ using the area you found (remember which parts of the cylinder through which the flux is 0).

3. How much total charge is enclosed in this surface?

4. Finally, use the flux and the enclosed charge in combination with Gauss' law to determine $\mathbf{E}$ outside the beam line.



5. Consider $\mathbf{E}$ inside the tube. Does the surface charge on the beam line affect the charged particles (electrons) inside the tube?

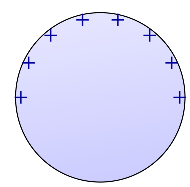
6. A cylinder has surface charge density σ (much like the SLAC Accelerator situation described above), except now there is a volume charge density $\rho(\mathbf{r}) = \rho_0 s$, where s is the radius from the center of the cylinder. You do not need to find the electric field for this situation, but do respond to the following to prepare yourself for future problems like this:

   (a) How will this affect the steps you took above to solve the problem?

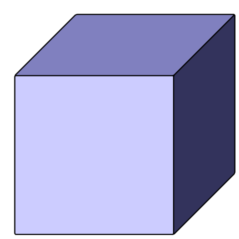
   (b) Will you need to take any additional steps to solve the problem when there is additional charge distribution?

(Part 2). In the next three situations, determine whether you can or cannot use Gauss's Law:

7. A long glass rod has a constant surface charge σ on only half of the rod, $0 \leq \phi < \pi$. You want to find the electric field outside the rod.



8. A cube has a uniform charge density and is centered on the z-axis. You want to find the electric field on the z-axis.



9. A point charge is at the origin. You want to find the electric flux through the upper hemisphere of a sphere also centered at the origin.

10. Answer the additional self-assessment questions on Max.

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