The Electro-Magnetism Inventory for Conceptual Evaluation

Preliminary Results and Refinement

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ABSTRACT

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Upper-division electricity and magnetism (E&M) courses can be conceptually difficult for students, yet few assessment tools exist to measure students' conceptual understanding at this advanced level. While qualitative methods such as interviews and written work analysis provide insight into student reasoning, they are difficult to apply across large cohorts or institutions, and existing quantitative assessments are either limited to introductory E&M or narrowly focused on specific upper-division topics, such as Gauss' Law, Ampère's Law, or vector potential. In this thesis, we developed and evaluated the Electro-Magnetism Inventory for Conceptual Evaluation (E-MICE), a concept inventory that emphasizes conceptual reasoning over mathematical problem-solving and was designed specifically for upper-division E&M courses to provide comprehensive coverage aligned with the first 7 chapters of Griffith's Introduction to Electrodynamics. Through analysis of existing concept inventories for lower-division E/M such as the Brief Electricity and Magnetism Assessment and the Electricity and Magnetism Concept Inventory, student and faculty interviews, and course materials, we constructed the instrument with distractors grounded in documented student misconceptions. The instrument was administered to Brigham Young University's Physics 441 course in Fall 2025, a group of 20 students. Preliminary item analysis indicated that 20 out of 33 questions effectively discriminate, or have a Kelley Discrimination index D such that $D \geq 0.4$, between varying levels of student understanding, while also identifying questions requiring further refinement. This work therefore establishes an initial instrument, though more validation is required before future multi-institutional deployment.

Keywords: Physics Education, Concept Inventory, Electricity and Magnetism

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Chapter 1

Introduction

A key concern in physics education is ensuring that students develop both good problem solving skills as well as a proper conceptual understanding of physical topics. Therefore, much physics education research (PER) focuses on those same two areas; McDermott and Redish outline dozens of papers for each of those focuses in their 1999 review of PER as a whole [1].

As part of that focus, researchers seek to evaluate how students perform in those areas. This is most easily accomplished through testing, as it serves as a way to measure understanding and gauge performance for both students and teachers; assessment can also lead to significant learning gains [2], which is another benefit. Thus, testing plays a large role in physics education research. This work focuses on a specific type of test, known as a concept inventory, that seeks to evaluate students' conceptual understanding.

1.1 Background on Concept Inventories

Concept inventories are exams targeting persistent misconceptions and patterns of reasoning rather than computational proficiency. They made their first appearance in physics education research with the introduction of the Force Concept Inventory (FCI) in the early 1990s [3]. The FCI was designed to diagnose students' conceptual understanding of Newtonian mechanics by targeting well-documented misconceptions rather than assessing mathematical problem-solving ability. Since the FCI was a multiple-choice exam, its widespread adoption demonstrated that carefully constructed multiple-choice instruments could reveal conceptual difficulties even among students who successfully completed traditional coursework. Its success served as a proof of concept for the effectiveness of conceptual inventories as a research tool in physics. Since its inception, a number of similar concept inventories, focused on different topics, have joined it in the physics education zeitgeist.

Following the FCI, numerous concept inventories were developed across a variety of physics disciplines, including electricity and magnetism. At the introductory level, instruments such as the Brief Electricity and Magnetism Assessment (BEMA) [4], the Conceptual Survey of Electricity and Magnetism (CSEM) [5], and the Electricity and Magnetism Concept Inventory (EMCI) [6] focus on qualitative reasoning about electric and magnetic fields, forces, potentials, and basic electromagnetic phenomena. These assessments have been widely used to compare traditional and transformed curricula and to identify persistent student difficulties in introductory E&M courses. Studies using these instruments have consistently demonstrated that traditional instruction yields only modest gains in conceptual understanding, and that many fundamental misconceptions about fields, superposition, and symmetry persist after instruction [7].

In contrast, fewer assessment tools exist for upper-division E&M. Notable examples include the Colorado Upper-Division Electrostatics Assessment (CUE), which emphasizes electrostatics solution strategies, boundary conditions, and the interpretation of mathematical expressions [8], and the Colorado UppeR-division ElectrodyNamics Test (CURrENT), which focuses on conceptual understanding of Maxwell's equations, time-varying fields, and electromagnetic waves [9]. While these instruments represent important advances in upper-division assessment, they are intentionally limited in scope and do not function as comprehensive concept inventories spanning the full range of advanced E&M topics. Additionally, both of these instruments are free-response assessments, which provide a great deal of insight into student thought but require much more time for scoring than multiple-choice exams. We note that multiple-response (select more than one answer) versions of the CUE exist, however, it retains the limited scope of its free-response counterpart [10].

Taken together, the existing literature reveals a clear gap. Most E&M concept inventories are either restricted to introductory material or narrowly focused on specific upper-division content areas. While there exists a multiple-response version of the CUE, we hope to be able to simplify that format into a simpler multiple-choice (select the best option) format for ease of use while retaining validity. The instrument developed in this thesis is therefore intended to be both simpler to administer and broader in scope than these alternatives while retaining usefulness as a valid tool.

1.2 Purpose of the Instrument

Although qualitative interviews provide rich insight into student reasoning, they are labor-intensive and difficult to scale. A validated concept inventory enables large-scale data collection, longitudinal tracking, and cross-institutional comparison, thereby supporting both instructional reform and research into student learning trajectories.

To address the limitations discussed above, we developed a concept inventory designed specifically for upper-division E&M courses. The design of this instrument is informed by the assessment framework established by the Force Concept Inventory (FCI), which demonstrated that carefully constructed multiple-choice instruments can probe students' conceptual understanding independent of computational proficiency [3]. Our objective is to extend this methodology to advanced E&M topics that are not adequately captured by existing assessment tools.

1.3 Research Aims

This project sought to address the following research aims:

1. Create a comprehensive upper-division electricity and magnetism concept inventory
2. Translate documented student difficulties and misconceptions into a multiple-choice format
3. Gather preliminary evidence to validate the instrument's usefulness as a research tool

This work is exploratory and should be best used as a backdrop for future research. Its primary contribution is the development and initial implementation of a new assessment tool, along with early evidence supporting its validation as a tool for use in upper-division E&M instruction and research. However, the tool will need continued development and validation before widespread implementation is practical.

Chapter 2

Methods

The development of the upper-division electricity and magnetism concept inventory followed an iterative, research-informed design process consistent with established practices in physics education research (PER) [3, 11]. The instrument was constructed through multiple stages, including review of existing assessments, analysis of student work, item drafting, and refinement for content coverage and clarity.

2.1 Assessment Format

Several upper-division assessments, including the CUE and CURrENT, employ open-ended formats that allow detailed insight into student reasoning [8, 9]. While such formats provide rich qualitative information, they require substantial scorer training and validation to achieve acceptable inter-rater reliability. For the purposes of scalability, cross-institutional deployment, and rapid scoring, we adopted a multiple-choice format consistent with other widely used concept inventories, including the FCI, BEMA, CSEM, and EMCI [3–6]. As evidenced by those instruments, carefully constructed distractor answer choices when grounded in documented student reasoning can effectively capture conceptual understanding while maintaining practicality for large-scale use [3, 11]. The instrument was therefore designed to preserve the diagnostic advantages of research-based distractors while ensuring ease of scoring and reproducibility across instructors and institutions.

2.2 Item Generation

Question development drew on multiple complementary sources in order to ground the instrument in authentic student reasoning. First, we reviewed existing lower-division and upper-division E&M assessments to identify established conceptual domains and structural features. Particular attention was given to topics emphasized in introductory E&M instruments such as BEMA and CSEM [4, 5], as well as to upper-division conceptual difficulties documented in the literature, particularly in regards to Gauss's Law and Ampére's Law [7, 12, 13]. The list of major difficult topics by Mason et al. were also useful in determining where to focus our efforts [14].

Second, we analyzed student responses to free-response conceptual questions previously administered in the upper-division (Physics 441) course. These free-response questions were primarily based on standard upper-division content drawn from *Introduction to Electrodynamics* by Griffiths. Student solutions were reviewed to identify recurring patterns of reasoning, persistent misconceptions, and common mathematical-physical disconnects. Consistent with established concept inventory design principles, these observed reasoning patterns were explicitly incorporated into multiple-choice distractors [3].

Third, additional candidate items were generated based on faculty experience and instructional observation. We examined past exams, particularly final exams, from BYU's Physics 220, the lower-division E&M course, to identify core conceptual topics that students are expected to encounter prior to upper-division coursework. Faculty contributors drew on multi-year experience teaching upper-division E&M to identify topics that consistently presented conceptual challenges. Student researchers contributed perspectives based on peer discussions and recent course experience, helping ensure that distractors reflected authentic student language and reasoning. This process resulted in an initial pool of approximately 60 candidate items.

2.3 Topic Selection and Content Structure

To ensure comprehensive yet balanced coverage of upper-division E&M, the initial item pool was organized into seven major topics:

1. Mathematical Foundations: Vector reasoning, coordinate systems, and general forms of differential equation solutions.
2. Electrostatics: Electric fields, scalar potential, charge distributions, Coulomb's law, and Gauss's law in symmetric contexts.
3. Magnetostatics: Magnetic fields, vector potential, current density, Biot–Savart law, and Ampère's law.
4. Electric Potential Methods: Relaxation techniques, method of images, and multipole expansions.
5. Materials and Macroscopic Fields: **P**, **D**, **M**, and **H** fields, along with susceptibility, permittivity, and permeability.
6. Force, Energy, and Momentum in Electromagnetic Systems.
7. Time-Dependent Maxwell's Equations: Displacement current, Faraday's law, and electromagnetic wave solutions.

While no single domain was allowed to dominate the inventory, greater representation was intentionally given to topics overlapping with lower-division E&M. This design choice supports the long-term goal of using the instrument in both pre-instruction and post-instruction contexts, analogous to the FCI model [3]. Including foundational topics enables the instrument to probe retained conceptual understanding from prior coursework as well as growth during the upper-division sequence.

Following this content-balancing process, the inventory was reduced from approximately 60 candidate items to a final set of 33 questions. The ordering of questions broadly follows the topical sequence presented in Griffiths' *Introduction to Electrodynamics*, thereby aligning the instrument with common instructional progressions.

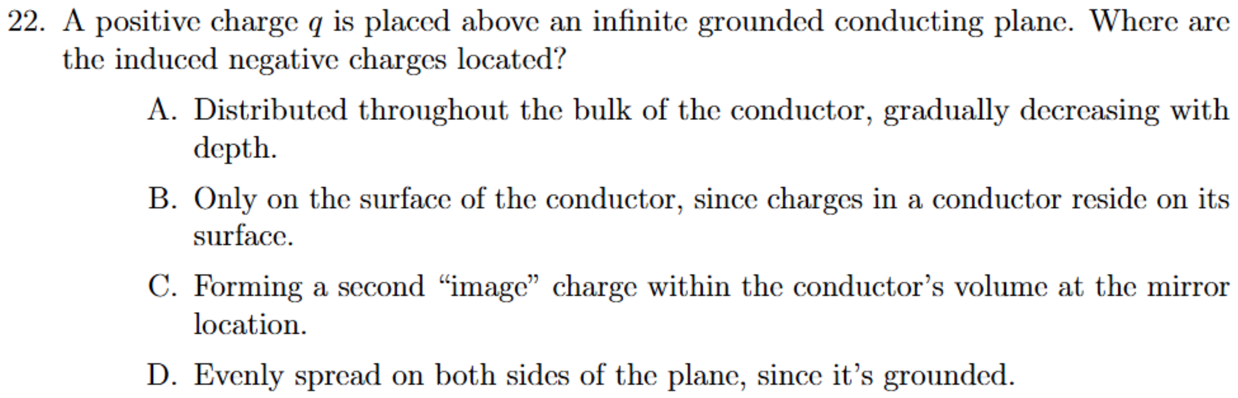
2.3.1 Example Question

As previously mentioned, each question was designed with common misconceptions in mind. An example of this, question 22 on the exam, is given in Figure 2.1. This question focuses on the negative charges that are induced when a charge is brought near a grounded conducting plane. The correct answer for this question is choice B, that the charge is induced only on the surface of the conductor. The other choices focus on several common misconceptions, as follows:

1. A, distributed throughout the bulk of the conductor, gradually decreasing with depth. Students often do not understand the properties of conductors compared to other materials, which this choice addresses.

2. C, forming a second "image" charge within the conductor's volume at the mirror location. The method of images is a useful model to think about this problem, but is itself only a mathematical tool - this option therefore focuses on those students that conflate model with the physical system.

3. D, evenly spread on both sides of the plane, since it's grounded. This focuses on a common misconception of grounded objects, namely that they cannot have charge. In this case, the



22. A positive charge q is placed above an infinite grounded conducting plane. Where are the induced negative charges located?
    - A. Distributed throughout the bulk of the conductor, gradually decreasing with depth.
    - B. Only on the surface of the conductor, since charges in a conductor reside on its surface.
    - C. Forming a second "image" charge within the conductor's volume at the mirror location.
    - D. Evenly spread on both sides of the plane, since it's grounded.

Figure 2.1 Question 22, focused on Electric Potential Methods

thought is that the charges on one side of the plane would cancel those on the other. However, in practice, grounding an object only requires that its potential equal zero, not the charge itself.

Chapter 3

Results

The preliminary version of the concept inventory was administered to a full section of Physics 441 with the cooperation of the course instructor. All enrolled students (N = 20) completed the assessment under standard testing conditions. Responses were analyzed using percent-correct statistics and the Kelley discrimination index [15], described in section 3.2. See appendix A for the instrument as it was given in this pilot.

3.1 Overall Performance

The mean score on the instrument was 72.42% (SD = 18.34%), with individual scores ranging from 39.40% to 96.97%. The distribution of total scores is shown in Figure 3.1. The absence of strong ceiling or floor effects suggests that the overall difficulty of the instrument was appropriate for this population.

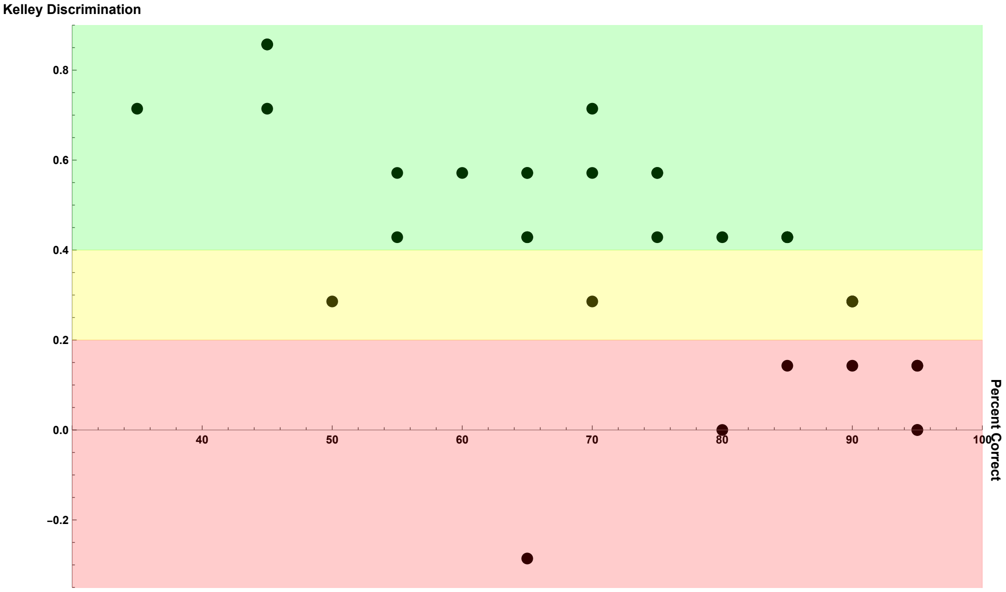


Figure 3.1 Distribution of total scores, based on percent correct (difficulty) and discrimination. Note that for validation purposes, discrimination is much more important than difficulty, hence the green shaded region represents strong discrimination, the yellow shaded region moderate discrimination, and the red shaded region low discrimination.

3.2 Item Difficulty and Discrimination

A major aspect of test validation is item discrimination, wherein individual questions are evaluated on how well they differentiate between high performers and low performers. This item discrimination was done via the Kelley Discrimination index [15], which involves taking the top and bottom 27% of scorers. The item discrimination index is calculated for each question by the following equation

$$D = \frac{H-L}{N},$$

where D is the discrimination, H is the number of people in the high-achieving group that correctly answer that question, L is the number of people in the low-achieving group that answered correctly, and N is the number of people in one of the groups and is the same for both the high-achieving and low-achieving groups. To fully encompass both the top and bottom 27%, our groups each had 7 people, so N=7.

This discrimination index gives a value from -1 to +1, in which -1 represents completely opposite discrimination, or that lower scorers did better on that questions. A discrimination of 0 then represents a question with no discrimination and +1 a question with perfect discrimination, in that only those that scored well on the whole performed well on that item.

We computed item difficulty (measured as fraction correct, hence values closer to 1 have lower difficulty) and discrimination indices for each question. Following conventional guidelines in educational measurement [16], items with discrimination values above 0.40 were considered strong discriminators, values between 0.20 and 0.39 were considered moderate, and values below 0.20 were flagged for review.

Of the 33 items:

- 20 exhibited strong discrimination ($D \geq 0.40$),
- 5 exhibited moderate discrimination ($0.20 \leq D < 0.40$),
- 8 exhibited low discrimination ($D < 0.20$).

We note that the Kelley Discrimination Index works best when item difficulty is moderate, $.25 \leq p \leq .75$ [17]. This was evident in our work, as all but 1 of the questions where $p \geq .85$ had poor discrimination, whereas of the questions where $.25 \leq p \leq .75$, only 1 had poor discrimination; that question had a typo which limits our ability to analyze that result. Table 3.1 gives a summary of these statistics as well as the mean difficulty and discrimination, and table 3.2 gives a breakdown of the mean difficulty and discrimination by topic and a full item-level breakdown is provided in Appendix C.

Several items with low discrimination were examined individually. In some cases, poor discrimination was attributable to correctable issues such as ambiguous wording or typographical errors (for an example of a typographical error, see Figure 3.2). These items were revised for deployment in the next iteration of the instrument. In other cases, low discrimination appeared

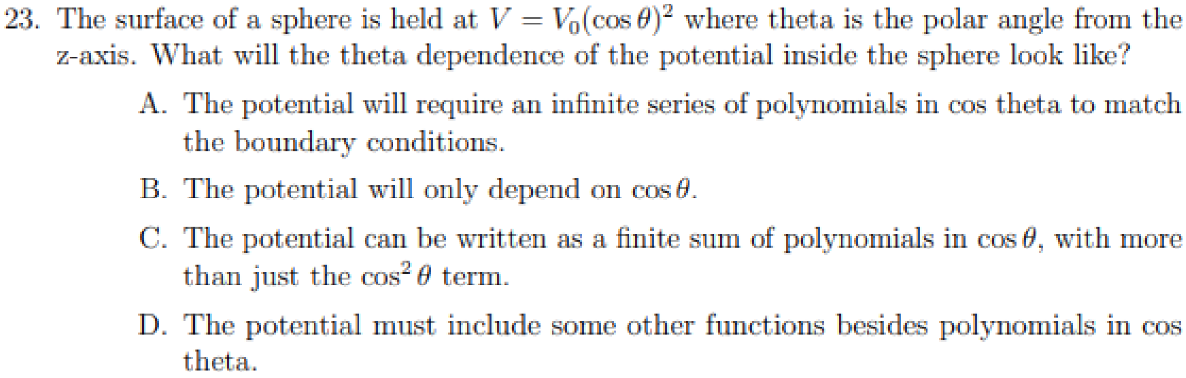
Statistic	Value
Number of Items	33
Mean Difficulty (p)	0.7242
Mean Discrimination (D)	0.3939
Items with $D \geq 0.40$	20
Items with $0.20 \leq D < 0.40$	5
Items with $D < 0.20$	8

Table 3.1 Summary of item-level statistics for the preliminary administration.

Topic	Mean Difficulty	Mean Discrimination
Mathematical Foundations	0.6800	0.4286
Electrostatics	0.7950	0.3144
Magnetostatics	0.6500	0.4608
Electric Potential Methods	0.7333	0.4608
Materials and Macroscopic Fields	0.5667	0.4297
Force, Energy, and Momentum	0.8333	0.2737
Time-Dependent Maxwell's Equations	0.7250	0.5355

Table 3.2 The average difficulty and discrimination by topic.

to reflect uniformly high or uniformly low performance across the cohort. Instructor experience suggests that certain concepts may be either broadly mastered or broadly misunderstood within a given course offering, naturally suppressing discrimination despite conceptual relevance. These items were retained pending further data collection across multiple semesters.



23. The surface of a sphere is held at $V = V_0(\cos\theta)^2$ where theta is the polar angle from the z-axis. What will the theta dependence of the potential inside the sphere look like?
    - A. The potential will require an infinite series of polynomials in cos theta to match the boundary conditions.
    - B. The potential will only depend on $\cos\theta$.
    - C. The potential can be written as a finite sum of polynomials in $\cos\theta$, with more than just the $\cos^2\theta$ term.
    - D. The potential must include some other functions besides polynomials in cos theta.

Figure 3.2 An example of a typographical error that resulted in poor discrimination. Note that answer choice B should have $\cos^2\theta$ rather than $\cos\theta$. Because of this error, both answer choices B and C were accepted.

3.3 Item Discrimination by Topic

It can be useful for instructors to examine both the item difficulty and discrimination by topic, given in Figure 3.3. The average difficulty does not appear to have any particular outliers, which is useful in ensuring that the Kelley Discrimination Index is a useful tool for validation.

It should also be noted that Electrostatics, which received the bulk of the coverage on the exam, had among the lowest item discrimination on average. Electrostatics also had the 2nd-highest difficulty value, implying that the average score was quite high. This makes sense, since electrostatics is also a major focus in lower-division E&M so there will likely be less discrepancy between various students' experience with that topic.

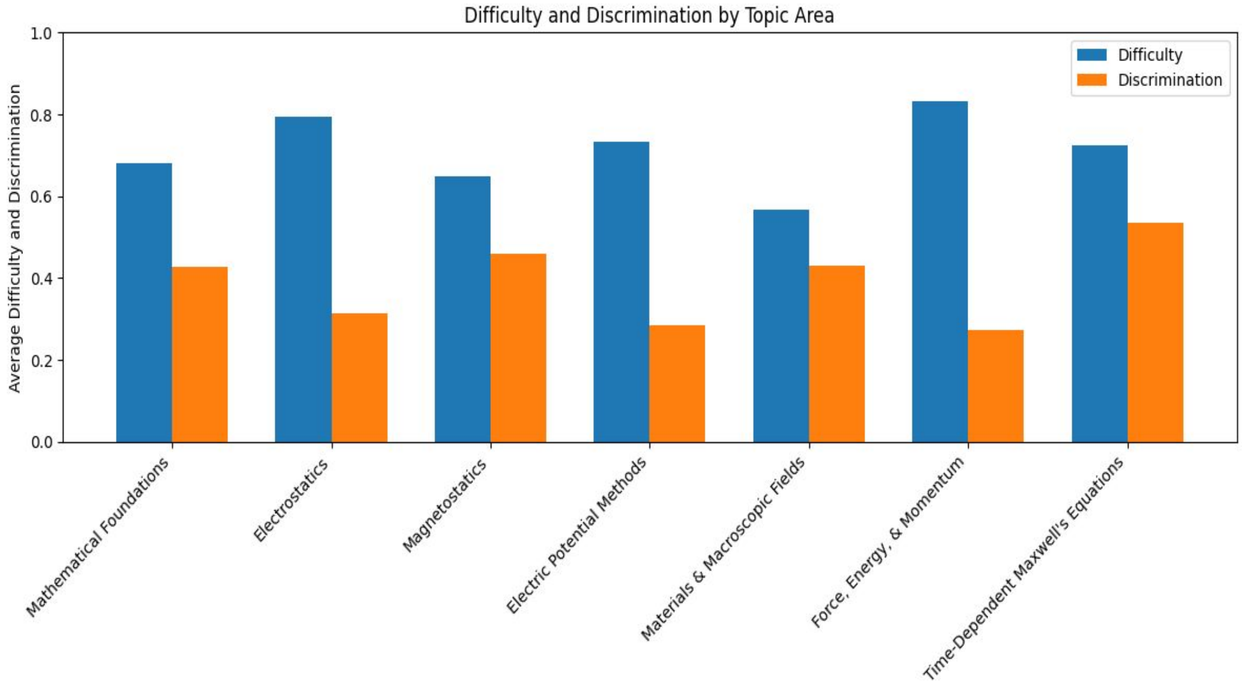
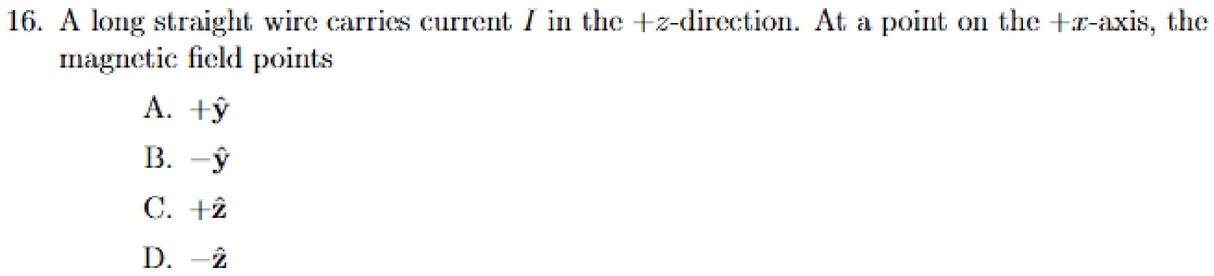


Figure 3.3 Difficulty and Discrimination by Topic. We recall that difficulty is defined as percent correct, so items with a greater difficulty value had the highest average score.

While comparing the average difficulty and discrimination by topic cannot definitively prove any patterns, it does provide interesting insight into how students understand the various topics of a comprehensive upper-division E&M course.

3.4 Example Item Revision

Figures 3.4 and 3.5 illustrate a representative revision made following discrimination analysis. The original version of the question exhibited low discrimination despite moderate difficulty. A minor modification was introduced to reduce cueing effects and eliminate an unintended shortcut. The revised version preserves the same conceptual target while reducing superficial cues that may have artificially inflated performance. Subsequent administrations will determine whether this modification improves item discrimination without altering conceptual difficulty.



16. A long straight wire carries current I in the $+z$-direction. At a point on the $+x$-axis, the magnetic field points

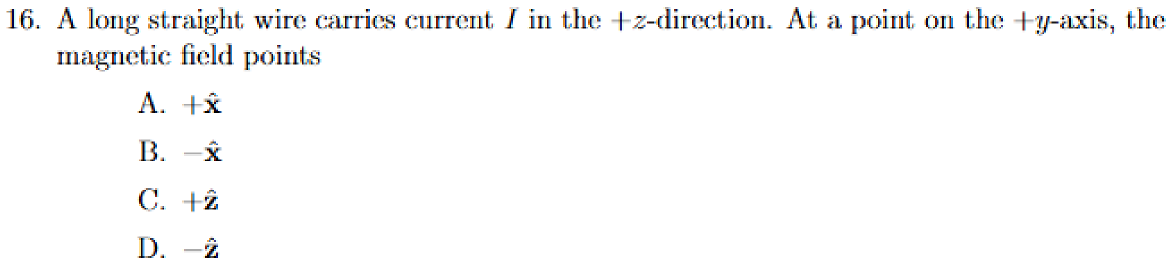
A. $+\hat{\mathbf{y}}$

B. $-\hat{\mathbf{y}}$

C. $+\hat{\mathbf{z}}$

D. $-\hat{\mathbf{z}}$

Figure 3.4 An example question from the exam given to students.



16. A long straight wire carries current I in the $+z$-direction. At a point on the $+y$-axis, the magnetic field points

A. $+\hat{\mathbf{x}}$

B. $-\hat{\mathbf{x}}$

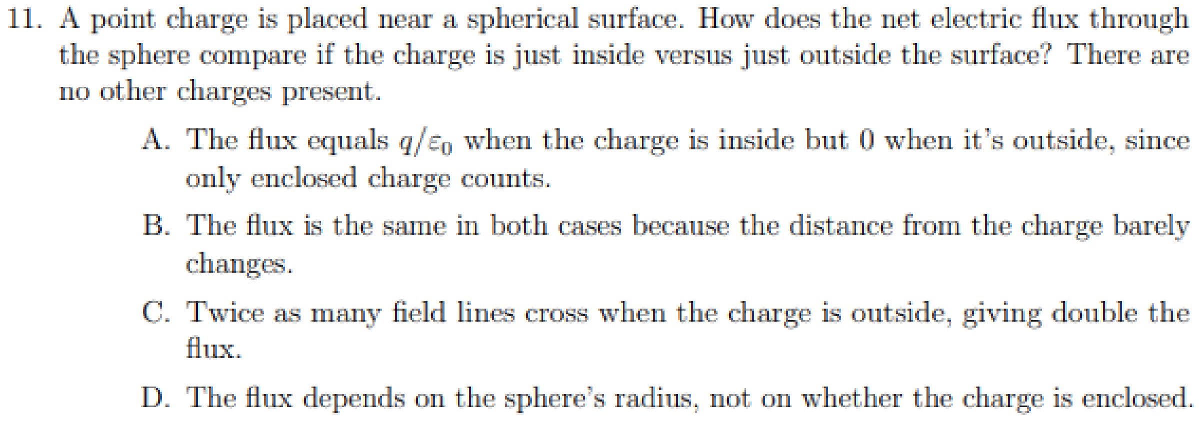
C. $+\hat{\mathbf{z}}$

D. $-\hat{\mathbf{z}}$

Figure 3.5 A slight revision after discrimination analysis. Note that while the question covers the same material, a simple change makes the result less immediately obvious.

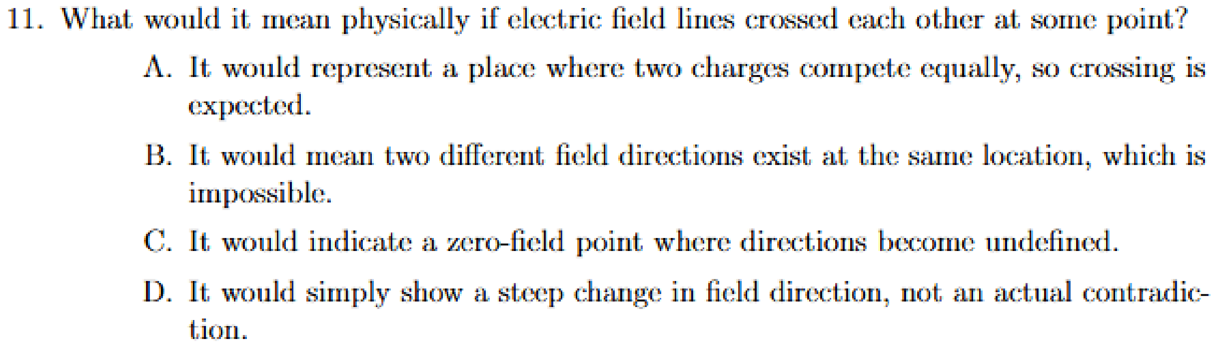
3.5 Item Removal

One item was identified as both exceptionally easy (difficulty = 0.95) and exhibiting negligible discrimination ($D \approx 0$). Because the item provided little information about variation in student understanding, it was removed from the instrument and replaced, see Figures 3.6 and 3.7. Aside from this removal and minor revisions, the majority of the inventory remained unchanged to preserve instrument stability for longitudinal data collection and future validation studies.



11. A point charge is placed near a spherical surface. How does the net electric flux through the sphere compare if the charge is just inside versus just outside the surface? There are no other charges present.
    - A. The flux equals q/ε_0 when the charge is inside but 0 when it's outside, since only enclosed charge counts.
    - B. The flux is the same in both cases because the distance from the charge barely changes.
    - C. Twice as many field lines cross when the charge is outside, giving double the flux.
    - D. The flux depends on the sphere's radius, not on whether the charge is enclosed.

Figure 3.6 Question 11, with both low difficulty and low discrimination.



11. What would it mean physically if electric field lines crossed each other at some point?
    - A. It would represent a place where two charges compete equally, so crossing is expected.
    - B. It would mean two different field directions exist at the same location, which is impossible.
    - C. It would indicate a zero-field point where directions become undefined.
    - D. It would simply show a steep change in field direction, not an actual contradiction.

Figure 3.7 Question 11, now completely rewritten.

3.6 Preliminary Interpretation

These preliminary results suggest that the majority of items function as intended, with acceptable levels of discrimination for an early-stage instrument. However, additional administrations across multiple semesters and institutions will be necessary to establish test reliability, ensure stability of item statistics, and gather evidence of test validity for institutions other than Brigham Young University.

Chapter 4

Discussion

This project sought to address a methodological gap in upper-division electricity and magnetism assessment by developing a scalable, research-informed concept inventory that complements qualitative approaches such as student interviews and written-response diagnostics. While qualitative methods provide rich insight into student reasoning [8, 12], they are labor-intensive and difficult to scale across multiple sections or institutions. The present instrument represents an attempt to extend the assessment framework established by the Force Concept Inventory (FCI) [3] to advanced E&M topics that are not comprehensively addressed by existing multiple-choice tools.

Our E-MICE was constructed using documented upper-division student difficulties, local free-response analysis, and established concept inventory design principles. Its scope spans a broad range of E&M domains, including electrostatics, magnetostatics, materials, and time-dependent Maxwell's equations. Unlike the CUE or CURrENT [8, 9], the E-MICE is designed as a generalized multiple-choice instrument suitable for pre-instruction and post-instruction deployment.

Preliminary results from a single administration suggest that the instrument functions appropriately for this population. The mean score of 72.4% indicates moderate overall difficulty without strong ceiling effects, and the mean Kelley discrimination index of 0.39 suggests that many items effectively differentiate between higher- and lower-performing students. Item-level analysis revealed that the majority of questions demonstrated acceptable or strong discrimination, while a small number required revision or removal. These findings are consistent with expectations for an early-stage instrument undergoing iterative refinement [11].

However, several limitations must be acknowledged. The instrument has thus far been administered to only one section at a single institution, limiting the generalizability of the current statistical findings. Additionally, reliability metrics such as Cronbach's α and item response theory modeling have not yet been performed.

It is also important to recognize the inherent limitations of multiple-choice assessments. While carefully constructed distractors can capture common reasoning patterns, they cannot fully represent the nuance and variability of student thought. Open-ended instruments remain essential for probing the structure of student reasoning in depth. The E-MICE is therefore best understood as a complementary tool that enables efficient large-scale measurement rather than as a replacement for qualitative diagnostic methods.

Future work should include repeated administrations across multiple semesters and institutions to establish stability of item statistics such that neither the difficulty nor discrimination of a given question fluctuate from semester to semester and to ensure test validity, or that the instrument accurately reflects students' conceptual understanding. In particular, pre-instruction and post-instruction administration would allow direct measurement of conceptual gains, analogous to the FCI model [3]. Additional analysis of performance by conceptual domain may also help identify persistent upper-division difficulties, particularly in areas such as materials and time-dependent Maxwell's equations, which prior research suggests are challenging for students [7]. While that information may be obvious to individual instructors, we hope to be able to track more nation-wide trends. Finally, expanding the instrument to subsequent courses in the upper-division sequence, such as BYU's Physics 442, may provide insight into longitudinal development of conceptual understanding across the E&*M* curriculum.

Appendix A

The E-MICE, as Administered to the Fall 2025 Physics 441 Class

Multiple Choice Exam

Each question has one correct answer. Select the best option.

1. Which of the following differential operators produces a scalar from a vector field?
   - A. gradient
   - B. curl
   - C. divergence
   - D. Laplacian of a scalar

2. A function $f(r, \theta, \phi)$ solves Laplace's equation inside a sphere. The general solution is typically written as a sum over
   - A. A combination of exponentials e^{kr} and e^{-kr}
   - B. Legendre polynomials to describe theta and regular polynomials to describe r
   - C. A combination of sines and cosines
   - D. A combination of Bessel functions and cosines functions

3. When solving boundary-value problems with the method of images, the image charge is chosen so that
   - A. Gauss's law is automatically satisfied.
   - B. energy is minimized for the true charge distribution.
   - C. the boundary condition on the conductor is exactly satisfied.
   - D. the total charge of the system remains zero.

4. In an eigenfunction expansion solution of Laplace's equation, the role of orthogonality is to
   - A. reduce the number of boundary conditions.
   - B. guarantee that all eigenfunctions have the same eigenvalue.
   - C. allow individual expansion coefficients to be solved independently.
   - D. ensure the potential inside is constant.

5. Which statement about the vector potential $\mathbf{A}$ is *always* true?
   - A. $\mathbf{A}$ is uniquely determined by the magnetic field $\mathbf{B}$.
   - B. $\mathbf{B} = \nabla \times \mathbf{A}$ regardless of gauge choice.
   - C. $\nabla \cdot \mathbf{A} = 0$ for all physical situations.
   - D. $\mathbf{A}$ must always point in the same direction as $\mathbf{B}$.

6. Two equal positive charges are fixed at $(1, 0, 0)$ and $(-1, 0, 0)$. As you move along the x-axis from $-\infty$ to $+\infty$, how does the direction of the electric field change?

   A. It points right, then left, then right again, since the charges repel in opposite directions.

   B. It points right everywhere, because both positive charges push a test charge in the same direction.

   C. It alternates randomly, because the fields from both charges interfere in unpredictable ways.

   D. It points left, then right, then left, then right, because the field points away from each positive charge.

7. For the same two positive charges, how does the electric potential vary along the x-axis?

   A. It increases steadily from $-\infty$ to $+\infty$ because you get closer to more positive charge overall.

   B. It is zero midway between the charges where their fields cancel and large but not infinite far to the left and right of the charges.

   C. It remains constant because the configuration is symmetric.

   D. It is small far to the left and the right of the charges, infinite near each charge, and finite but not zero between the charges.

8. Between two large parallel plates of a capacitor, the electric field is uniform and points from the positive plate to the negative plate. What does this imply about the electric potential between the plates?

   A. The potential is constant everywhere since the field strength does not vary.

   B. The potential alternates between high and low regions depending on proximity to the plates.

   C. The potential changes linearly with distance; a constant field means a constant rate of change of V.

   D. The potential is zero midway and highest near both plates.

9. A flat disc is divided into four quadrants: Q1 and Q3 carry + charge; Q2 and Q4 carry − charge. What is the electric field directly above the disc's center?

   A. It points upward because the positive regions dominate over the negative ones.

   B. It is zero because the horizontal and vertical components from all four quadrants cancel by symmetry.

   C. It points downward toward the negative quadrants since opposite charges attract.

   D. It fluctuates along z since the quadrants' fields reinforce in some directions.

10. For the same situation, at a point directly above the center of the same four-quadrant disc, what is the electric potential?
    - A. Nonzero, because the field lines curve and the charges are at finite distance.
    - B. Zero, since potential and field always vanish at the same points.
    - C. Positive, because the potential from the positive quadrants dominates near the axis.
    - D. Zero, because equal positive and negative charge regions contribute equal and opposite potentials.

11. A point charge is placed near a spherical surface. How does the net electric flux through the sphere compare if the charge is just inside versus just outside the surface? There are no other charges present.
    - A. The flux equals q/ε_0 when the charge is inside but 0 when it's outside, since only enclosed charge counts.
    - B. The flux is the same in both cases because the distance from the charge barely changes.
    - C. Twice as many field lines cross when the charge is outside, giving double the flux.
    - D. The flux depends on the sphere's radius, not on whether the charge is enclosed.

12. For a group of charges whose total (net) charge is zero, how does the electric potential behave far from the system?
    - A. It always decreases as $1/r^2$ because neutral systems act like dipoles at large distances.
    - B. It usually falls off as $1/r^2$ if a dipole moment is present, but faster ($1/r^3$, $1/r^4$, $\ldots$) if higher moments dominate.
    - C. It remains constant, since a neutral system produces no potential at a distance.
    - D. It decreases as $1/r$ like a single point charge because each charge contributes equally.

13. An electric dipole consists of a positive charge located at $(+1,-1)$ and a negative charge located at $(+1,+1)$ in the xy-plane. Which direction is the dipole moment directed toward within the xy-plane?

    A. $\uparrow$

    B. $\nearrow$

    C. $\rightarrow$

    D. $\searrow$

    E. $\downarrow$

    F. $\swarrow$

    G. $\leftarrow$

    H. $\nwarrow$

    I. some other direction

    J. no direction because the dipole moment is zero.

14. A negative point charge is placed to the left of the surface of an electrically neutral conducting solid sphere, inducing a non-uniform surface charge on the sphere. Let point A be on the left side of the sphere (closest to the charge) and point B be on the right side (furthest). The electric potential at point A will be ________ the potential at point B.

    A. greater than

    B. less than

    C. equal to

    D. cannot be determined

15. A uniform circular rod carries a steady current and has a constant potential difference between its ends, creating a uniform electric field inside the rod. The electric field in the rod is directed

    A. toward the end at lower potential

    B. toward the end at higher potential

    C. perpendicular to the rod

    D. in some other direction

16. A long straight wire carries current I in the $+z$-direction. At a point on the $+x$-axis, the magnetic field points

    A. $+\hat{\mathbf{y}}$

    B. $-\hat{\mathbf{y}}$

    C. $+\hat{\mathbf{z}}$

    D. $-\hat{\mathbf{z}}$

17. A square loop carries a steady current I. At the exact center of the loop, the magnetic field
    - A. is zero by symmetry.
    - B. points perpendicular to the plane of the loop.
    - C. lies in the plane of the loop but rotates depending on current direction.
    - D. depends on the loop's orientation but is always tangent to one side.

18. An infinite current sheet lies in the xy-plane and carries a surface current density $\mathbf{K} = K\,\hat{\mathbf{x}}$. What is the direction of the magnetic field just above the sheet?
    - A. $+\hat{\mathbf{x}}$
    - B. $-\hat{\mathbf{x}}$
    - C. $+\hat{\mathbf{y}}$
    - D. $-\hat{\mathbf{y}}$
    - E. $+\hat{\mathbf{z}}$
    - F. $-\hat{\mathbf{z}}$

19. A toroid with N turns carries a current I. Which statement about the magnetic field $\mathbf{B}$ is correct?
    - A. The field is uniform inside and outside the toroid.
    - B. The field exists only outside the toroid; inside it is zero.
    - C. The field inside is nonzero; outside it is approximately zero.
    - D. The field forms radial lines pointing outward from the center.

20. A magnetic dipole at the origin has its dipole moment along $+\hat{\mathbf{z}}$. At a point on the x-axis some distance away, the magnetic field from the dipole is oriented in which direction?
    - A. $+\hat{\mathbf{x}}$
    - B. $-\hat{\mathbf{x}}$
    - C. $+\hat{\mathbf{y}}$
    - D. $-\hat{\mathbf{y}}$
    - E. $+\hat{\mathbf{z}}$
    - F. $-\hat{\mathbf{z}}$

21. The electric field immediately above and below a charged sheet will:
    - A. Always be continuous if another external electric field is superimposed on it.
    - B. Show a discontinuity only if there are equal positive and negative charges on the sheet.
    - C. Be continuous whenever the sheet is part of a conducting plane in equilibrium.
    - D. Always have a discontinuity equal to σ/ε_0 caused by the surface charge itself, regardless of other fields.

22. A positive charge q is placed above an infinite grounded conducting plane. Where are the induced negative charges located?
    A. Distributed throughout the bulk of the conductor, gradually decreasing with depth.
    B. Only on the surface of the conductor, since charges in a conductor reside on its surface.
    C. Forming a second "image" charge within the conductor's volume at the mirror location.
    D. Evenly spread on both sides of the plane, since it's grounded.

23. The surface of a sphere is held at $V = V_0(\cos\theta)^2$ where theta is the polar angle from the z-axis. What will the theta dependence of the potential inside the sphere look like?
    A. The potential will require an infinite series of polynomials in cos theta to match the boundary conditions.
    B. The potential will only depend on $\cos\theta$.
    C. The potential can be written as a finite sum of polynomials in $\cos\theta$, with more than just the $\cos^2\theta$ term.
    D. The potential must include some other functions besides polynomials in cos theta.

24. Inside a dielectric, how does the total electric field compare with the external field applied from outside?
    A. It still points in the same direction as the external field but is weaker in magnitude because the induced field partially cancels it.
    B. It points in the opposite direction because the polarization field dominates inside the dielectric.
    C. It becomes zero because the induced field completely cancels the applied one.
    D. It alternates direction throughout the dielectric due to internal dipole alignment.

25. When a dielectric material is inserted between the plates of a charged capacitor (with Q held constant), what happens to the energy stored in the field?
    A. It increases, since the electric field strength becomes larger inside the dielectric.
    B. It stays the same, because the total charge Q has not changed.
    C. It decreases, because $U = \frac{1}{2}Q^2/C$ and the capacitance C increases when the dielectric is added.
    D. It first decreases and then increases as the dielectric fills the capacitor.

26. Which boundary condition on the magnetic field is always correct at an interface with no free surface current?
    - A. $B_\perp$ is discontinuous.
    - B. $B_\parallel$ is discontinuous.
    - C. $H_\perp$ is continuous.
    - D. $H_\parallel$ is continuous.
27. A charged particle moves through a region with only a magnetic field present. Which is true?
    - A. The magnetic field does work on the charge.
    - B. The speed of the particle changes but not its direction.
    - C. The magnetic force is always parallel to the velocity.
    - D. The magnetic force is always perpendicular to the velocity.
28. A charged particle enters a region where uniform $\mathbf{E}$ and $\mathbf{B}$ fields are both present. The particle's velocity is initially perpendicular to both fields. Which statement about the particle's kinetic energy is correct?
    - A. Only the electric field can change the particle's kinetic energy.
    - B. Only the magnetic field can change the particle's kinetic energy.
    - C. Both the electric and magnetic fields can change the particle's kinetic energy.
    - D. Neither field can change the particle's kinetic energy.
29. A particle with charge q moves with velocity $\mathbf{v}$ into a region with a uniform magnetic field $\mathbf{B}$, where $\mathbf{v}$ and $\mathbf{B}$ are almost (but not quite) parallel to each other. Which statement best describes the effect on the particle's momentum?
    - A. The magnetic field changes the magnitude of the particle's momentum but not its direction.
    - B. The magnetic field changes the direction of the particle's momentum but not its magnitude.
    - C. The particle's momentum remains completely unchanged.
    - D. The magnetic field reverses the particle's velocity after a short time.

30. A uniform electric field **E** points upward and a uniform magnetic field **B** points into the page. A narrow beam of positive charges enters this region moving to the left, so that **v** is in the direction of $\mathbf{E} \times \mathbf{B}$. The particles in the beam all move in the same direction, but they have different speeds. As the particles move through the region, what happens?

    A. All particles travel in straight lines through the region, regardless of their speed.

    B. Slower particles are deflected in the direction of **E**, faster particles are deflected opposite to **E**, and for some velocity in between those two speeds, a particle can move without deflection.

    C. Slower particles are deflected opposite to **E**, faster particles are deflected in the direction of **E**, and for some velocity in between those two speeds, a particle can be deflected exactly perpendicularly to **E**.

    D. All particles follow the same circular path around **B**, independent of their speed.

31. When the magnetic field through a conducting loop changes with time, what induces an emf in the loop?

    A. The electric field created by the time-varying magnetic field.

    B. The magnetic field lines themselves exerting a continuous Lorentz force on charges.

    C. The loop's resistance converting magnetic energy directly into voltage.

    D. The static electric field produced by stationary charges along the wire.

32. If magnetic field lines pass through a stationary conducting loop and remain constant in time, what emf is produced in the loop?

    A. A steady emf proportional to the field strength.

    B. An alternating emf that reverses as the field penetrates the loop.

    C. None, because the magnetic flux is constant and does not change with time.

    D. A transient emf which arises from non-uniformity of the field in space.

33. In a region of space with no free charges and no conduction currents, the electric field is changing with time. According to Maxwell's equations, what does this imply?
    - A. No magnetic field can exist because there is no current.
    - B. A changing electric field produces a magnetic field in the same way a conduction current would.
    - C. A magnetic field exists only if charges are accelerating nearby.
    - D. The magnetic field must be zero unless $\nabla \cdot \mathbf{E}$ is nonzero.

Appendix B

The E-MICE, as of February 14, 2026

Multiple Choice Exam

Each question has one correct answer. Select the best option.

1. Which of the following differential operators produces a scalar from a vector field?
    - A. gradient
    - B. curl
    - C. divergence
    - D. Laplacian of a scalar
2. A function $f(r, \theta, \phi)$ solves Laplace's equation inside a sphere. The general solution is typically written as a sum over
    - A. A combination of exponential functions e^{kr} and e^{-kr}, there is no θ dependence.
    - B. A combination of Legendre polynomials to describe θ and regular polynomials to describe r.
    - C. A combination of sinh and cosh functions to describe θ and exponential functions e^{kr} and e^{-kr}.
    - D. A combination of Bessel functions to describe r and cos functions to describe θ.
3. When solving boundary-value problems with the method of images, the image charge is chosen so that
    - A. Gauss's law is automatically satisfied.
    - B. energy is minimized for the true charge distribution.
    - C. the boundary condition on the conductor is exactly satisfied.
    - D. the total charge of the system remains zero.
4. In an eigenfunction expansion solution of Laplace's equation, the role of orthogonality is to
    - A. reduce the number of boundary conditions.
    - B. guarantee that all eigenfunctions have the same eigenvalue.
    - C. allow individual expansion coefficients to be solved independently.
    - D. ensure the potential inside is constant.
5. Which statement about the vector potential $\mathbf{A}$ is *always* true?
    - A. $\mathbf{A}$ is uniquely determined by the magnetic field $\mathbf{B}$.
    - B. $\mathbf{B} = \nabla \times \mathbf{A}$ regardless of gauge choice.
    - C. $\nabla \cdot \mathbf{A} = 0$ for all physical situations.
    - D. $\mathbf{A}$ must always point in the same direction as $\mathbf{B}$.

6. Two equal positive charges are fixed at $(1,0,0)$ and $(-1,0,0)$. As you move along the x-axis from $-\infty$ to $+\infty$, how does the direction of the electric field change?
   - A. It points right, then left, then right, then left, because the field points toward each positive charge.
   - B. It points right everywhere, because both positive charges push a test charge in the same direction.
   - C. It points left, is zero in between the charges, then right because the field from the charges cancel in the center.
   - D. It points left, then right, then left, then right, because the field points away from each positive charge.

7. For the same two positive charges, how does the electric potential vary along the x-axis?
   - A. It increases steadily from $-\infty$ to zero and then decreases from zero to $+\infty$ because you get closer to the charges and then far away again.
   - B. It is zero midway between the charges where their fields cancel and large but not infinite far to the left and right of the charges.
   - C. It remains constant because the configuration is symmetric so one charge cancels the potential from the other.
   - D. It is small far to the left and the right of the charges, infinite near each charge, and finite but not zero between the charges.

8. Between two large parallel plates of a capacitor, the electric field is uniform and points from the positive plate to the negative plate. What does this imply about the electric potential between the plates?
   - A. The potential is constant everywhere since the field strength does not vary.
   - B. The potential alternates between high and low regions depending on proximity to the plates.
   - C. The potential changes linearly with distance; a constant field means a constant rate of change of V.
   - D. The potential is zero midway and highest near both plates.

9. A flat disc is divided into four quadrants: Q1 and Q3 carry + charge; Q2 and Q4 carry − charge. What is the electric field directly above the disc's center?
   - A. It points upward because the positive regions dominate over the negative ones.
   - B. It is zero because the horizontal and vertical components from all four quadrants cancel by symmetry.
   - C. It points downward toward the negative quadrants since opposite charges attract.
   - D. It fluctuates along z as the quadrants' fields construcively and deconstructively interfere.

10. For the same situation, at a point directly above the center of the same four-quadrant disc, what is the electric potential?
    - A. Nonzero, because the field lines curve and the test charge is at finite distance above the center.
    - B. Zero, since potential and field always vanish at the same points.
    - C. Nonzero, because the potential from the positive quadrants dominates near the axis.
    - D. Zero, because equal positive and negative charge regions contribute equal and opposite potentials.

11. What would it mean physically if electric field lines crossed each other at some point?
    - A. It would represent a place where two charges compete equally, so crossing is expected.
    - B. It would mean two different field directions exist at the same location, which is impossible.
    - C. It would indicate a zero-field point where directions become undefined.
    - D. It would simply show a steep change in field direction, not an actual contradiction.

12. For a group of charges whose total (net) charge is zero, how does the electric potential behave far from the system?
    - A. It always decreases as $1/r^2$ because neutral systems act like dipoles at large distances.
    - B. It usually falls off as $1/r^2$ if a dipole moment is present, but faster ($1/r^3$, $1/r^4$, $\ldots$) if higher moments dominate.
    - C. It remains constant, since a neutral system produces no potential at a distance.
    - D. It decreases as $1/r$ like a single point charge because each charge contributes equally.

13. An electric dipole consists of a positive charge located at $(+1, -1)$ and a negative charge located at $(+1, +1)$ in the xy-plane. Which direction is the dipole moment directed toward within the xy-plane?

    A. $\uparrow$

    B. $\nearrow$

    C. $\rightarrow$

    D. $\searrow$

    E. $\downarrow$

    F. $\swarrow$

    G. $\leftarrow$

    H. $\nwarrow$

    I. some other direction

    J. no direction because the dipole moment is zero.

14. A negative point charge is placed to the left of the surface of an electrically neutral conducting solid sphere, inducing a non-uniform surface charge on the sphere. Let point A be on the left side of the sphere (closest to the charge) and point B be on the right side (furthest). The electric potential at point A will be ______ the potential at point B.

    A. greater than

    B. less than

    C. equal to

    D. cannot be determined

15. A uniform circular rod carries a steady current and has a constant potential difference between its ends, creating a uniform electric field inside the rod. The electric field in the rod is directed

    A. toward the end at lower potential

    B. toward the end at higher potential

    C. perpendicular to the rod

    D. in some other direction

16. A long straight wire carries current I in the $+z$-direction. At a point on the $+y$-axis, the magnetic field points

    A. $+\hat{\mathbf{x}}$

    B. $-\hat{\mathbf{x}}$

    C. $+\hat{\mathbf{z}}$

    D. $-\hat{\mathbf{z}}$

17. A square loop carries a steady current I. At the exact center of the loop, the magnetic field
    - A. is zero by symmetry.
    - B. points perpendicular to the plane of the loop.
    - C. lies in the plane of the loop but rotates depending on current direction.
    - D. depends on the loop's orientation but is always tangent to one of the four sides.

18. An infinite current sheet lies in the xy-plane and carries a surface current density $\mathbf{K} = K\,\hat{\mathbf{x}}$. What is the direction of the magnetic field just above the sheet?
    - A. $+\hat{\mathbf{x}}$
    - B. $-\hat{\mathbf{x}}$
    - C. $+\hat{\mathbf{y}}$
    - D. $-\hat{\mathbf{y}}$
    - E. $+\hat{\mathbf{z}}$
    - F. $-\hat{\mathbf{z}}$

19. A toroid with N turns carries a current I. Which statement about the magnetic field $\mathbf{B}$ is correct?
    - A. The field is uniform inside and outside the toroid.
    - B. The field exists only outside the toroid; inside it is zero.
    - C. The field inside is nonzero; outside it is approximately zero.
    - D. The field forms radial lines pointing outward from the center.

20. A magnetic dipole at the origin has its dipole moment along $+\hat{\mathbf{z}}$. At a point on the x-axis some distance away, the magnetic field from the dipole is oriented in which direction?
    - A. $+\hat{\mathbf{x}}$
    - B. $-\hat{\mathbf{x}}$
    - C. $+\hat{\mathbf{y}}$
    - D. $-\hat{\mathbf{y}}$
    - E. $+\hat{\mathbf{z}}$
    - F. $-\hat{\mathbf{z}}$

21. The electric field immediately above and below a charged sheet will:
    - A. Always be continuous if another external electric field is superimposed on it.
    - B. Show a discontinuity only if there are equal positive and negative charges on the sheet.
    - C. Be continuous whenever the sheet is part of a conducting plane in equilibrium.
    - D. Always have a discontinuity equal to σ/ε_0 caused by the surface charge itself, regardless of other fields.

22. A positive charge q is placed above an infinite grounded conducting plane. Where are the induced negative charges located?

    A. Distributed throughout the bulk of the conductor, gradually decreasing with depth.

    B. Only on the surface of the conductor, since charges in a conductor reside on its surface.

    C. Forming a second "image" charge within the conductor's volume at the mirror location.

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    A. The potential will require an infinite series of polynomials in $\cos\theta$ to match the boundary conditions.

    B. The potential will only depend on $\cos^2\theta$ because other terms go to zero at infinity.

    C. The potential must be written as a finite sum of polynomials in $\cos\theta$, with more than just the $\cos^2\theta$ term.

    D. The potential must include some other functions besides polynomials in $\cos\theta$.

24. Inside a dielectric, how does the total electric field compare with the external field applied from outside?

    A. It still points in the same direction as the external field but is weaker in magnitude because the induced field partially cancels it.

    B. It points in the opposite direction because the polarization field dominates inside the dielectric.

    C. It becomes zero because the induced field completely cancels the applied one.

    D. It alternates direction throughout the dielectric due to internal dipole alignment.

25. When a dielectric material is inserted between the plates of a charged capacitor (with Q held constant), what happens to the energy stored in the field?

    A. It increases, because $U = \frac{1}{2}Q^2/C$ and the capacitance C decreases when the dielectric is added.

    B. It stays the same, because the total charge Q has not changed.

    C. It decreases, because $U = \frac{1}{2}Q^2/C$ and the capacitance C increases when the dielectric is added.

    D. It first decreases and then increases back to its original value as the dielectric fills the capacitor.

26. Which boundary condition on the magnetic field is always correct at an interface with no free surface current?

    A. $B_\perp$ is discontinuous.

    B. $B_\parallel$ is discontinuous.

    C. $H_\perp$ is continuous.

    D. $H_\parallel$ is continuous.

27. A charged particle moves through a region with only a magnetic field present. Which is true?

    A. The magnetic field does work on the charge.

    B. The speed of the particle changes but not its direction.

    C. The magnetic force is always parallel to the velocity.

    D. The magnetic force is always perpendicular to the velocity.

28. A charged particle enters a region where uniform **E** and **B** fields are both present. The particle's velocity is initially perpendicular to both fields. Which statement about the particle's kinetic energy is correct?

    A. Only the electric field can change the particle's kinetic energy.

    B. Only the magnetic field can change the particle's kinetic energy.

    C. Both the electric and magnetic fields can change the particle's kinetic energy.

    D. Neither field can change the particle's kinetic energy.

29. A particle with charge q moves with velocity $\mathbf{v}$ into a region with a uniform magnetic field $\mathbf{B}$, where $\mathbf{v}$ and $\mathbf{B}$ are almost (but not quite) parallel to each other. Which statement best describes the effect on the particle's momentum?

    A. The magnetic field changes the magnitude of the particle's momentum but not its direction.

    B. The magnetic field changes the direction of the particle's momentum but not its magnitude.

    C. The particle's momentum remains completely unchanged.

    D. The magnetic field changes both the magnitude and direction of the particle's momentum.

30. A uniform electric field **E** points upward and a uniform magnetic field **B** points into the page. A narrow beam of positive charges enters this region moving to the left, so that **v** is in the direction of $\mathbf{E} \times \mathbf{B}$. The particles in the beam all move in the same direction, but they have different speeds. As the particles move through the region, what happens?

    A. All particles travel in straight lines through the region, regardless of their speed.

    B. Slower particles are deflected in the direction of **E**, faster particles are deflected opposite to **E**, and for some velocity in between those two speeds, a particle can move without deflection.

    C. Slower particles are deflected opposite to **E**, faster particles are deflected in the direction of **E**, and for some velocity in between those two speeds, a particle can be deflected exactly perpendicularly to **E**.

    D. All particles follow the same circular path around **B**, independent of their speed.

31. When the magnetic field through a conducting loop changes with time, what induces an emf in the loop?

    A. The electric field created by the time-varying magnetic field.

    B. The magnetic field lines themselves exerting a continuous Lorentz force on charges.

    C. The loop's resistance converting magnetic energy directly into voltage.

    D. The static electric field produced by stationary charges along the wire.

32. If magnetic field lines pass through a stationary conducting loop and remain constant in time, what emf is produced in the loop?

    A. A steady emf proportional to the field strength.

    B. An alternating emf that reverses as the field penetrates the loop.

    C. None, because the magnetic flux is constant and does not change with time.

    D. A transient emf which arises from non-uniformity of the field in space.

33. In a region of space with no free charges and no conduction currents, the electric field is changing with time. According to Maxwell's equations, what does this imply?

    A. No magnetic field can exist because there is no current.

    B. A changing electric field produces a magnetic field in the same way a conduction current would.

    C. A magnetic field exists only if charges are accelerating nearby.

    D. The magnetic field must be zero unless $\nabla \cdot \mathbf{E}$ is nonzero.

Appendix C

Full Exam Difficulty and Discrimination

Item	Difficulty (p)	Discrimination (D)
1	0.70	0.286
2	0.85	0.143
3	0.65	0.429
4	0.75	0.571
5	0.45	0.714
6	0.95	0.000
7	0.90	0.286
8	0.75	0.429
9	0.95	0.143
10	0.90	0.143
11	0.95	0.000
12	0.80	0.429
13	0.55	0.571
14	0.35	0.714

Item	Difficulty (p)	Discrimination (D)
15	0.85	0.429
16	0.80	0.000
17	0.90	0.289
18	0.45	0.857
19	0.65	0.571
20	0.45	0.857
21	0.85	0.429
22	0.70	0.714
23	0.65	-0.286
24	0.65	0.571
25	0.50	0.289
26	0.55	0.429
27	0.90	0.249
28	0.65	0.429
29	0.95	0.143
30	0.60	0.571
31	0.75	0.571
32	0.85	0.429
33	0.70	0.571

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