AN INVESTIGATION OF RUBENS FLAME TUBE RESONANCES

by

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ABSTRACT

The Rubens flame tube is a teaching demonstration that is over 100 years old that allows observers to visualize acoustic standing wave behavior [H. Rubens and O. Krigar-Menzel, Ann. Phys. (Leipzig) **17**, 149 (1905)]. Flammable gas inside the tube flows through holes drilled along the top, and flames are then lit above. The tube is closed at one end and is driven with a loudspeaker at the other end. When the tube is driven at one of its resonance frequencies, the flames form a visual standing wave pattern as they vary in height according to the pressure amplitude in the tube. Although the basic performance of the tube has been explained [G. Ficken and C. Stephenson, Phys. Teach., **17**, 306-310 (1979)], this paper discusses the previously unreported characteristic of the tube's resonance frequency shifts. This study observed that this phenomenon involves an upward shift of the natural frequencies of the lower modes from what would ordinarily be expected in a closed-closed tube. Results from a numerical model suggest that the shift is primarily due to a Helmholtz resonance effect created by the drilled holes. The numerical model is explained and the numerical results are compared to experimental findings and discussed.

I. INTRODUCTION

Imagine you are a teacher of a physics class and you decide to introduce acoustics through a demonstration of the Rubens flame tube. You set it up, light the flames, and have the students predict where the resonance frequencies will occur based on the tube dimensions and the gas in the tube. A few students notice that the numbers are not coming out right; the resonance frequencies are not even close to what they predicted. You do not have an explanation.

This scenario did happen to the author's associates and provided the motivation for this research. This paper will introduce the flame tube, detail the methods of the research, and present results, pertinent discussion, and conclusions.

The introduction will discuss the history of the flame tube, its use as a classroom demonstration, previous research, and will give more explanation of the motivation for this research.

A. History

A German physicist named Heinrich Rubens discovered a way to demonstrate acoustic standing waves visually, using what he referred to as a "flame tube." He used a brass tube, 4 meters long, and 8 cm wide, that had 100 holes, 2 mm wide, drilled across the top. The holes were spaced 3 cm apart. The tube was filled with coal gas for 2 minutes and then flames were lit from the gas exiting through the holes on top. The tube was closed at both ends and driven at one of the ends with a tuning fork in a box (now such a tube is driven with a loudspeaker). Standing waves in the tube developed. At resonances of the tube, the standing wave was seen in the flames above the tube with pressure in the tube corresponding to flame height. Rubens published his results in 1905.[1]

B. Classroom demonstration

Because this demonstration provides a rare visual representation of sound waves, it naturally serves well as a teaching demonstration in the classroom setting of introductory physics or acoustics. When teaching about sound waves, it is common to talk about resonances in pipes that are harmonically related. The Rubens flame tube is suitable for a discussion of resonances because the resonances are easily seen: the flame height variation increases dramatically as resonance is reached. The Rubens flame tube is a demonstration of a simple behavior.

Being given the speed of sound of the gas in the tube as well as the length of the tube, students can, in theory, calculate the fundamental frequency of the system, as long as they know that the fundamental wavelength is twice the length of the tube and that the speed of sound in a medium is equal to the wavelength times the frequency. They can know where the resonances occur because the resonances are clearly visible; the flames reach a maximum height. In theory, the students can also calculate the speed of sound if they know the frequency input into the tube from the speaker and if they know the wavelength by measuring flame peak distances and then multiplying the wavelength and the frequency:

$$c = \lambda f \ , \tag{1}$$

where c is sound speed, λ is wavelength and f is frequency.

C. Previous research

Although this demonstration is more than 100 years old, relatively few studies have been published on the Rubens flame tube. However, the papers that have been published on the flame tube reveal important information about the tube: Jihui and Wang published an article discussing the relationship between flame height and pressure in the tube;[2] Spagna researched the flame behavior in depth, studying the flicker of the flame and the phase of the flicker with respect to the speaker;[3] Daw published an article detailing a two-dimensional flame table;[4] Ficken and Stephenson published an article explaining several interesting behaviors of the tube.[5] For example, they show that the flame maxima occur at pressure nodes in the tube and the minima occur at pressure antinodes. The mean gas flow out of the holes is greatest at the nodes, and the Bernoulli equation predicts this. This effect reverses for low gas pressures or high acoustic amplitudes, such that the flame minima occur at the nodes and the flame maxima occur at the antinodes.

D. Motivation

As previously mentioned, during operation of the tube, the authors noticed that the resonance frequencies that occur in the tube do not match what was predicted. They are shifted from the expected values and are not harmonically related. Others have observed this same phenomenon. We observed a 58 percent upward shift of one of the resonance frequencies. The cause of this resonance frequency shift is not immediately obvious to students seeing the demonstration nor perhaps to the teacher.

The hypothesis is that the holes are creating a Helmholtz resonance which shifts the modal resonance frequencies. [In this paper, a modal resonance is any resonance that has pressure variation (not the constant pressure resonance) or is not the Helmholtz resonance.] In order to test this hypothesis, we developed a numerical model of the tube with holes and constructed two tubes to compare observed and predicted results. The results compare favorably and confirm that the holes create the resonance frequency shift.

A Helmholtz resonator consists of a volume of gas in the bulk of a container acting as an acoustic compliance and a mass of air in the neck of the container acting as an acoustic mass. In the flame tube, the volume of gas or acoustic compliance is the tube interior. The acoustic masses of air in each of the drilled holes create many Helmholtz resonators in parallel along the tube. These resonators notably affect the behavior of the tube as we will show more fully later in the paper.

II. METHODS

This section will explain the numerical model and the experiment.

A. Numerical model

In order to better understand the tube and to test the hypothesis, we need a method of modeling the tube in order to compare it to observation. A tutorial of the equivalent circuit theory used to model the tube follows.

For lumped-element systems (i.e. systems where all dimensions are small compared to wavelength), equivalent circuits can be used to calculate quantities such as volume velocity, pressure, and thus, resonance frequencies. In our case, the acoustic pressure corresponds to voltage in the circuit, and the volume velocity corresponds to current. Because the tube's length is much greater than either of the other two dimensions and because the wavelengths of interest are not greater than the length, a waveguide circuit is used to account for changing acoustic parameters along the longer dimension. Waveguide circuits translate impedances from one end to the other much like the impedance translation theorem. They can also account for changing boundary conditions along the length of the waveguide, or in our case, holes along the top. In a waveguide circuit, two terms correspond to an arbitrary source and an arbitrary termination. The three other impedances make up the “T-network” in Fig. 1.

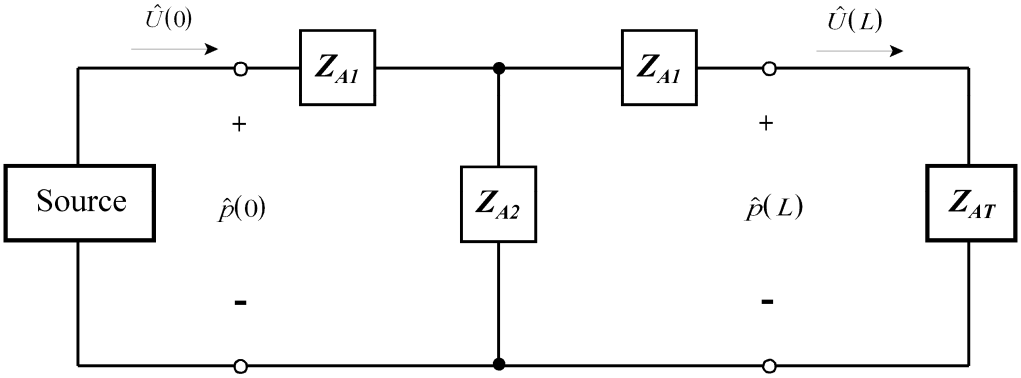

Figure 1 T-network with source and termination (Z_{AT}).

Equating the input impedance of this circuit to the impedance translation theorem, one finds the series impedance terms Z_{A1} and shunt impedance term Z_{A2}:

$$Z_{A1} = j\left(\frac{\rho_0 c}{S}\right)\tan\left(\frac{kL}{2}\right), \text{ and}$$

$$Z_{A2} = -j\left(\frac{\rho_0 c}{S}\right)\csc(kL), \tag{2}$$

where ρ_0 is the density of the gas, c is the speed of sound, S is the cross-sectional area of the tube, k is the acoustic wave number, and L is the length of the tube. This circuit can describe what occurs at the source and at the termination but not in between. If desired, one can also acquire acoustic quantities at points between the source and termination; however, a modification of the waveguide circuit is required. By coupling two waveguide circuits together, information can be obtained for interior points of the tube. The impedances Z_{A1} and Z_{A2} remain the same, but now there are two more impedance quantities to solve for in the second T-network:

$$Z_{A3} = j\left(\frac{\rho_0 c}{S}\right)\tan\left[\frac{k(L-x)}{2}\right], \text{ and}$$

(3)

$$Z_{A4} = -j\left(\frac{\rho_0 c}{S}\right)\csc\left[k(L-x)\right].$$

This arrangement models acoustics in standard tubes. Only one major change is needed to successfully model the holes across the top of the flame tube. These holes represent a change in impedance that can be accounted for by taking multiple T-networks and juxtaposing them together with shunt terms for each of the holes. The sound pressure and volume velocity can be modeled at the source, the termination, or any of the holes. One then couples two T-networks together between each hole impedance, source, or termination impedance, and the acoustic quantities can be obtained at any point along the tube. (See Fig. 2.) The volume velocity is represented by U, p is the acoustic pressure, and x is either the distance from the source or from the previous hole.

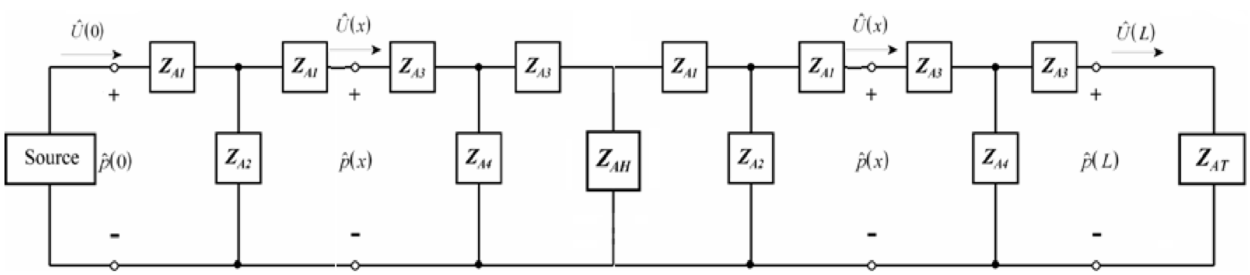

Figure 2 Waveguide circuit diagram for a tube with source, termination, and one hole (Z_{AH}).

An assumption is made that the frequency of interest is below the first cutoff frequency for cross modes in the tube. In this case, the cutoff frequency is 995 Hz.

The equivalent circuit theory is used to make a code that calculates the pressure along the length of the tube across a span of frequencies. The internal impedance of the source (the loudspeaker) is included and the volume velocity at the face is obtained using the Thiele-Small parameters given later on in the report. The frequencies at which the pressure amplitude is a local maximum are considered resonance frequencies. We use a computer to numerically calculate the pressure at every point along the tube at all the frequencies of interest by multiplying the impedance and the volume velocity together:

$$\hat{p}(x) = \hat{U}(x) \cdot Z(x)\,. \tag{4}$$

B. Experiment

In order to analyze the accuracy of the hypothesis, we compare the numerical model to experiment. The experiment required the construction of two new flame tubes to benchmark the model.

The two tubes differ only in the size of the holes for the flames. The tubes are 1.524 meters long, have a 2.6 centimeter cross-sectional radius, and are made of galvanized steel. There are 60 holes drilled along the top. One tube has 0.915 millimeter radius holes drilled across the top, and the other has 0.4575 millimeter radius holes, or half the size of the larger holes. Drilled in the center-side of each tube are 9.5 mm holes each with a short section of pipe used for the gas intake. An end cap was made for each tube that has a hole for a rubber stopper and a microphone (see Fig. 3).

A loudspeaker in a box was coupled directly to the tube at the other end. A 10.5 cm driver placed in a sealed box connects to the tube through a series of 2 cm wide circular cavities. A hinge and latch keep the box sealed during experimentation. (See Figs. 3 and 4.) The driver itself has Thiele-Small parameters: Fs=73.7 Hz, Qms=3.04, Mms=0.0097 kg and Cms=$4.8 \cdot 10^{4}$ m/N.

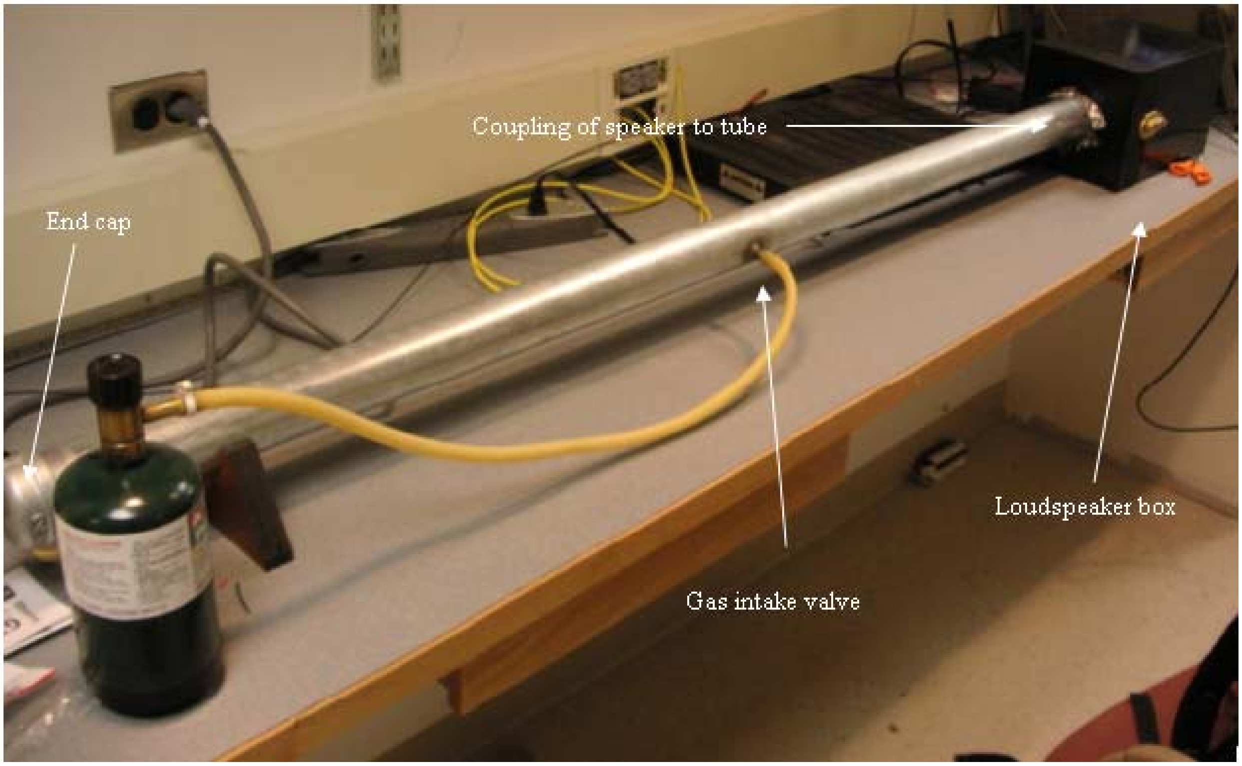

Figure 3 Flame tube setup.


Figure 4 Flame tube in operation.

Pressure maxima occur at the end of the tube away from the speaker at the resonance frequencies. We expect this because the cap on the end is rigid. Therefore, in order to compare the model to what is observed in the actual flame tube, a microphone is inserted into the end of the tube. Random noise drives the loudspeaker, and the microphone records the frequency response measurement of the pressure in the tube. In addition to measuring both tubes, this experiment includes frequency response measurements of the tubes using both propane and air inside the tube. The propane and air measurements give similar, supportive results. These measured frequency responses become the benchmark or observed data to test the numerical model.

Observations were also made pertaining to sound speed measurements. Knowing that flame peaks occur at every node in the pressure during normal operation, the distance between two peaks is one-half a wavelength. Observed data were taken by generating a flame pattern in the tube at a certain frequency and measuring the distance between flame peaks and multiplying the resultant wavelength by the frequency. This was done for several different modes.

III. RESULTS

This section will discuss the results of the numerical model and the comparison between the model and the observed data.

A. Numerical model

A graphic user interface helps one to easily visualize the results of the numerical model. The model is quite flexible. A user inputs into the model the relevant tube dimensions (tube radius, hole spacing, number and size), the speed of sound, and density of the gas. The model then outputs a graph of the magnitude of the pressure along the tube at whatever frequencies are chosen. This can be seen in Figs. 5 and 6. Note that the actual magnitude in decibels is irrelevant in all the remaining figures. The frequency is the more important quantity and the magnitude is only used relatively as a marker for where the resonance frequencies occur.

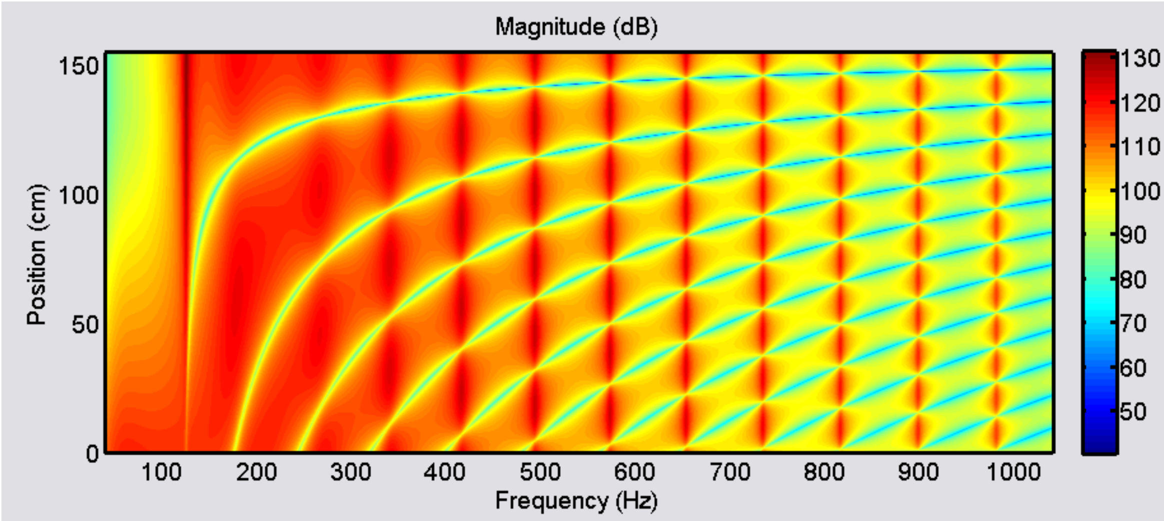

Figure 5 Graph of dB magnitude of pressure inside the tube from the speaker end (0 cm) to the cap end (150 cm) looking at frequencies from 40 Hz to 1040 Hz. The graph is modeled *with* the source.

The speaker significantly affects the overall response of the tube, especially decreasing the quality factor of the lower modal resonances. This is seen in the smearing of the modal resonance lines especially towards the cap end of the tube. The speaker also prevents the Helmholtz resonance from being a constant pressure resonance (i.e. there is constant pressure along the entire length of the tube) that a lumped parameter resonance should normally be. Figure 5 shows that the lowest resonance has a higher pressure amplitude at the cap end of the tube than at the speaker end (i.e. notice the increase in intensity that occurs in the left-most vertical line in the graph as one goes from 0 cm to 150 cm).

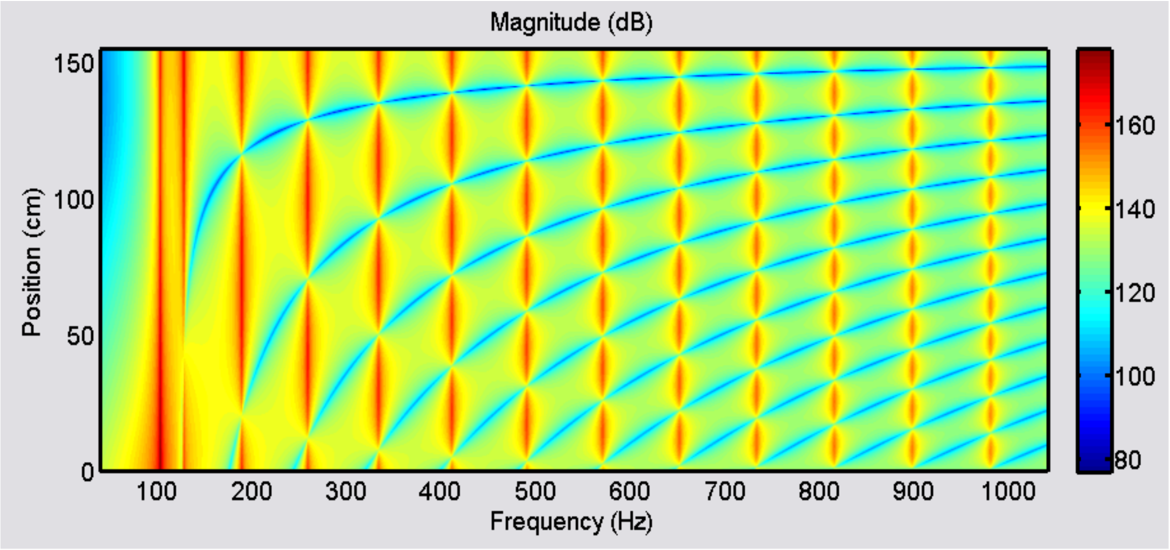

Figure 6 Graph of dB magnitude of pressure inside the tube from the speaker end (0 cm) to the cap end (150 cm) looking at frequencies from 40 Hz to 1040 Hz. The graph is modeled *without* the source but *with* the source cavity.

Figure 6 shows that by removing the speaker model, we increase the quality factor of the resonances and flatten the pressure response across the Helmholtz resonance. The pressure response is now more flat, but it is not perfectly flat because the model still includes the speaker cavity or box that couples to the tube, but not the speaker itself. The boundary conditions are still not uniform like in a closed-closed tube.

We also examine the magnitude of the resonance frequency shift that occurred by adding holes to a closed-closed tube. Figure 7 shows the model-generated graph of the frequency response of three tubes identical in every aspect except for the holes. One had no holes, a second had sixty smaller holes, and a third had sixty big holes, to match the experiment. This model ignores losses and excludes the loudspeaker and loudspeaker box, producing the response of the tube itself. Again, the magnitude is inaccurate: only the frequency is important.

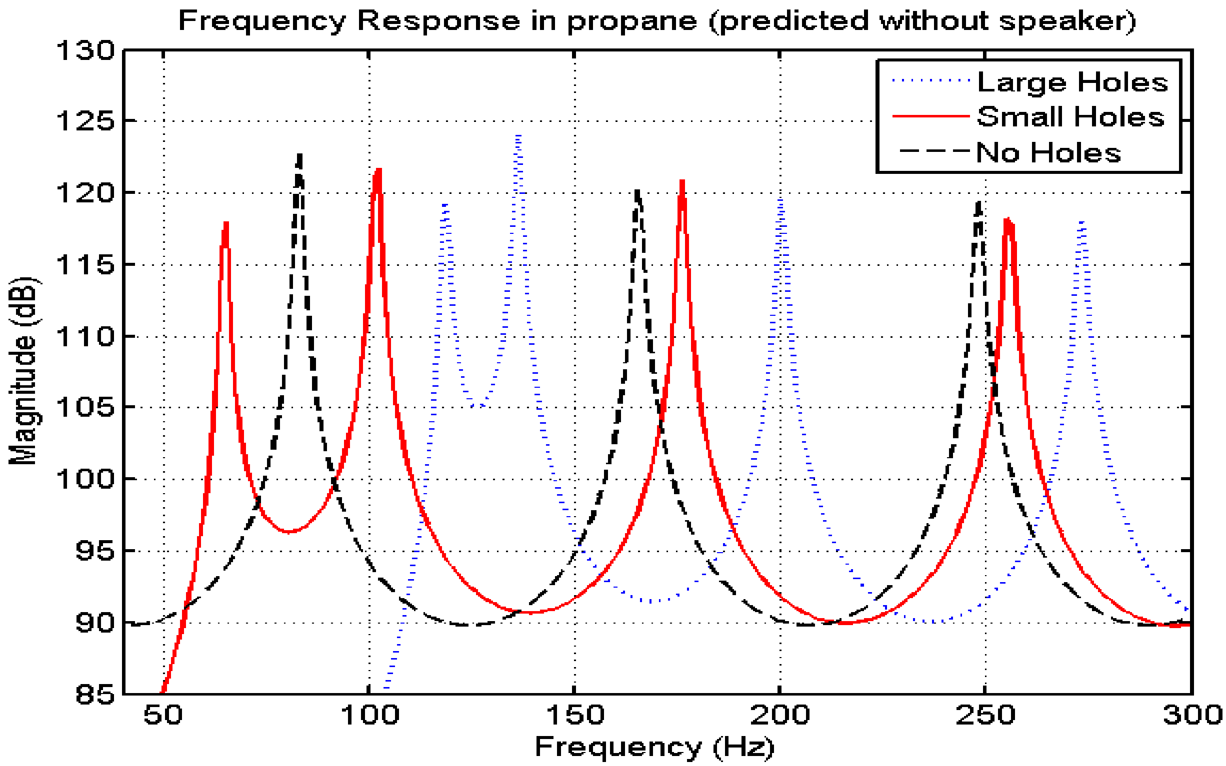

Figure 7 Model predicted frequency response at cap end for three cases of the tube: no holes, small holes, and large holes.

For both the large-holed and small-holed cases, the lowest peaks correspond to the Helmholtz resonance. The second lowest peak therefore corresponds to the first mode for the tubes with holes. The nonharmonic nature of the lower modes is evident from the lack of a peak at the frequency equal to the frequency separating the upper modes. For example, in the large-holed tube, the resonance frequencies starting from the third mode and going down are 273, 200, 136, and 118 Hz. The spacing of these modal frequencies is around 70 Hz. One expects a fundamental at 70 Hz and does not find it: there is a Helmholtz resonance at 118 Hz instead. For the first mode, a 64 percent shift occurs from a no-holed tube to a big-holed tube. Notice that the no-holed tube has a first resonance frequency of 83 Hz but the big-holed tube has its lowest resonance, the Helmholtz resonance, at 118 Hz. Because this resonance is the constant pressure, lumped parameter resonance, there can be no other resonances below it; therefore modal resonances must shift to allow for this Helmholtz resonance.

B. Comparison of predicted and observed

To compare the observed to the predicted, a frequency response was generated (see Fig. 8) from the numerical model by taking the pressure at the last point in the tube across all the frequencies and graphing the pressure versus frequency for the observed and the predicted. The focus of the graph was to see the alignment of the frequencies along the x-axis, so the magnitude scale is not accurate. This graph shows the observed and predicted with propane in the tube with the larger holes.

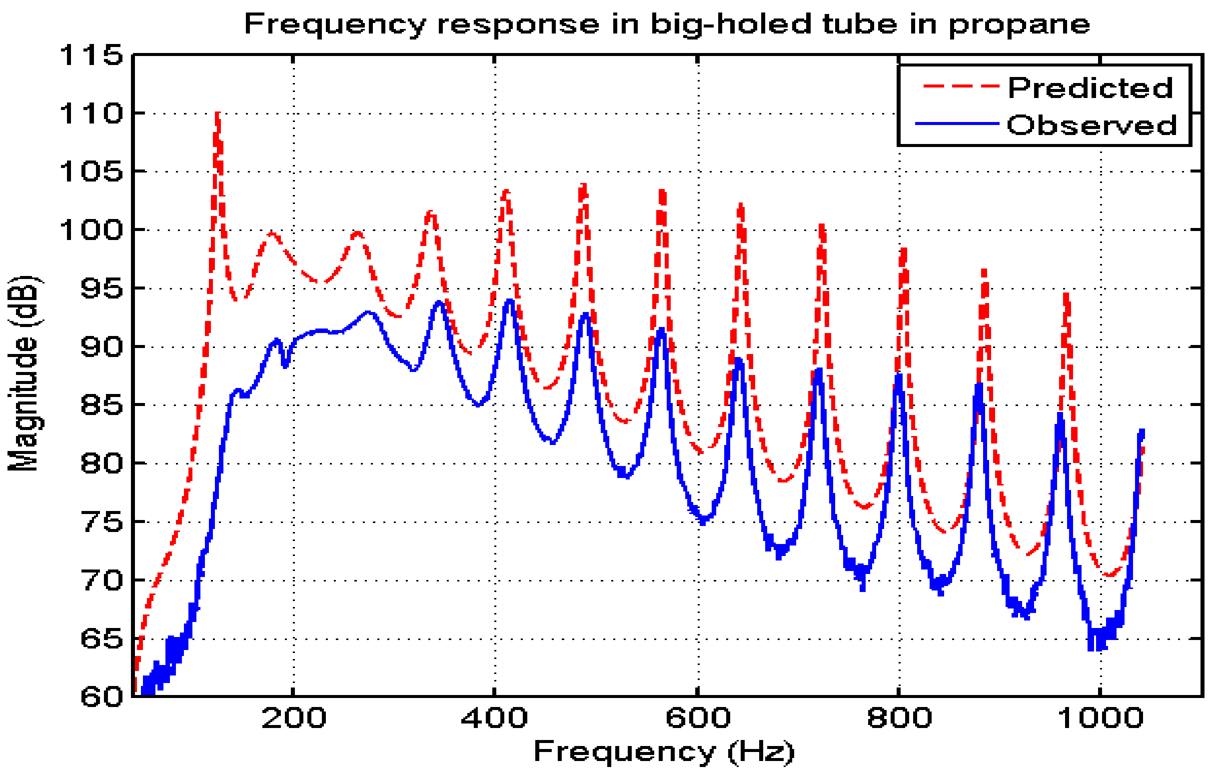

Figure 8 Frequency response at the cap end of tube comparing the observed response (tube in operation) to the model predicted response in big-holed tube.

The observed graph is not as smooth, but one can see that the peaks and valleys line up very well, showing agreement in resonance frequency prediction. Note that the lowest resonance peak for the observed response is quite small and does not match the quality factor of the corresponding peak in the numerical model. This is not understood; however, the peak does occur at the right frequency.

Similarly, good results are seen with the small-holed tube. (See Fig. 9.) This time, the Helmholtz resonance is much more visible in the observed data. There are some dips and bumps that occur in the observed data, absent from the predicted data (notably at 190 Hz and 310 Hz). Some behavior in the tube is causing these dips and bumps, and the numerical model does not account for the behavior. Nevertheless, the resonance frequencies are modeled well.

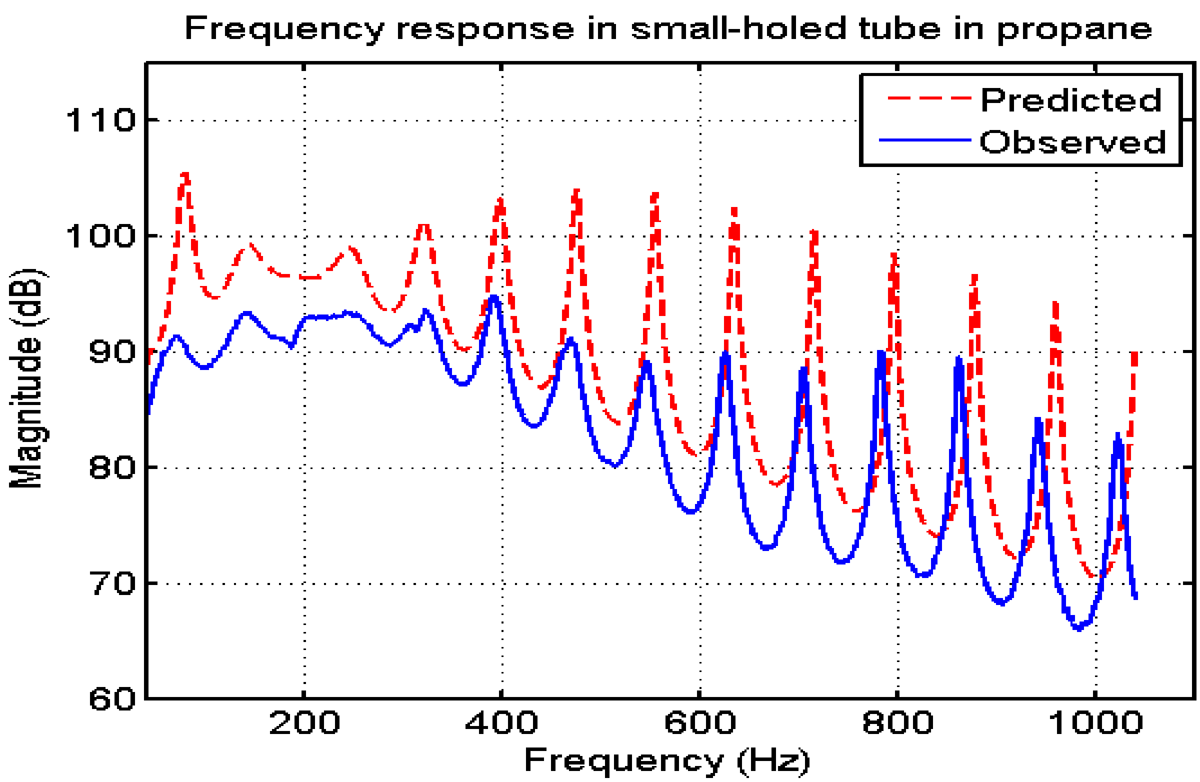

Figure 9 Frequency response at the cap end of tube comparing the observed response (tube in operation) to the model predicted response in small-holed tube.

Table 1 quantifies Figs. 8 and 9, showing both predicted and observed resonance frequencies for the small and big-holed tubes. It also gives error. The largest magnitude of error for the small-holed tube is 10 percent while for the big-holed tube it is 17 percent. This is for the Helmholtz resonance frequency.

Table 1 Predicted and observed resonance frequencies for both tubes and the error between the observed and the predicted

	Small Hole Res. Frequencies (Hz)		**Big Holes Res. Frequencies (Hz)**		**Small Holes**	**Big Holes**
Mode #	**Observed**	**Predicted**	**Observed**	**Predicted**	**% error**	**% error**
Helmholtz	**71**	**79**	**145**	**124**	**10**	**17**
1	**142**	**143**	**183**	**178**	**.70**	**2.8**
2	**242**	**246**	**274**	**263**	**1.6**	**4.2**
3	**323**	**321**	**342**	**335**	**.62**	**2.1**
4	**393**	**397**	**413**	**410**	**1.0**	**.73**
5	**470**	**475**	**488**	**486**	**1.1**	**.41**
6	**546**	**555**	**563**	**564**	**1.6**	**.17**
7	**626**	**635**	**640**	**643**	**1.4**	**.47**
8	**705**	**716**	**720**	**723**	**1.5**	**.41**

The percent error decreases with increasing frequency and is quite accurate for the higher modes, with typical errors of around 1 percent.

Hence, by including terms that account for the impedance of the holes and then calculating the resonance frequencies, the model accurately depicts the behavior of the tube, showing that the holes are creating the shift in resonance frequency.

V. DISCUSSION

The results clearly show that the Helmholtz resonance causes the shift in resonance frequencies. Where the Helmholtz resonance occurs and how much the resonance frequencies are shifted depend on the parameters of the tube. Both the model and the experiment show that by lowering the Helmholtz resonance, one lowers the percent shift in the modal resonance frequencies. In our case, the tube with smaller holes has a Helmholtz resonance that is much lower than the Helmholtz resonance in the tube with big holes (71 versus 145 Hz);

correspondingly, the resonance frequencies of the smaller-holed tube are not shifted as much. This is useful information for those interested in building their own tube. The tube designer controls how much the resonance frequencies are shifted.

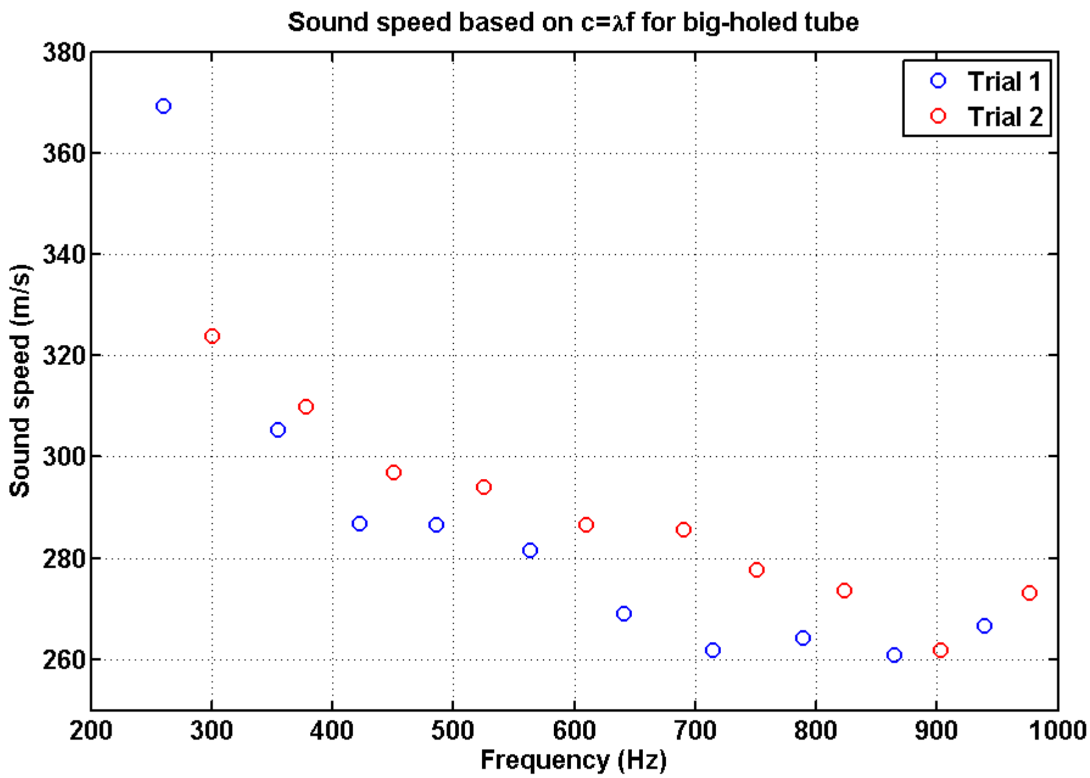

Figure 10 Calculated sound speed versus frequency using $c = \lambda f$. The distance between flame peaks is used to calculate λ and the frequency input into the tube from the signal generator is used for f.

The observed calculated sound speeds are incorrect. Equation (1) cannot be used if one uses the flame peak distances to calculate the wavelength; otherwise the result is erroneous. The sound speed predicted by Eq. (1), especially for the lower modes, is too high to be accurate, taking into account the gas inside the tube and its temperature. In Fig. 10, notice the high data points below 400 Hz: they are all in excess of 300 m/s, which is too fast for the temperature and density of the propane that was in the tube (this was verified by measuring the temperature of the propane inside while making these measurements). The sound speed profile should be constant across frequency because the speed of sound in a gas is dependent upon the medium and is independent of frequency. The sound speeds for the higher frequencies are closer to the actual sound speed. Therefore, the flame peak distance is not a reliable source for the wavelength in Eq. (1).

In addition, the numerical model was used to generate a calculated sound speed profile for the different resonance frequencies. As the frequency increases from the lowest resonance to the highest, the sound speeds generated from the numerical model exhibit an exponential decay-like trend that matches what is seen in the observed sound speeds (again, see Fig. 10): very high sound speeds in the low frequencies converging to the actual sound speed at high frequencies. The numerical model gives further evidence that the holes create this behavior. Consistent with the resonance frequencies, the sound speeds calculated for the higher modes converge to the actual sound speed in the tube based on the properties of the gas (propane, in this study). The holes do not significantly affect the higher modes.

VI. CONCLUSIONS

The Rubens flame tube serves well as a classroom demonstration, but calculating resonance frequencies or sound speeds is not a straightforward exercise of basic acoustics. Depending on how the tube is built, the phenomena may or may not be strongly present. For example, smaller and fewer holes will decrease the resonance frequency shift and will make the sound speed measurements more accurate. The tube can be used as a demonstration of standing waves in a closed-closed pipe or of parallel impedances and Helmholtz resonators, depending upon the circumstance. Also, a teacher could demonstrate mainly the higher modes, which are not affected as much, if he or she wanted to avoid the complicated behavior. Flame and gas flow properties could be another demonstration. The flame tube has been around for over 100 years but has been mostly unresearched. Professors and teachers should be aware of the complicated nature of the flame tube when demonstrating it and at least be able to refer to explanatory articles in order to better explain its behavior.

ACKNOWLEDGEMENTS

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[1]H. Rubens and O. Krigar-Menzel, "Flammenröhre für akustishe Beobachtungen (Flame tube for acoustical observations)," Ann. Phys., Lpz. **17**, 149 (1905).

[2]D. Jihui and C.T.P. Wang, "Demonstration of longitudinal standing waves in a pipe revisited," Am. J. Phys. **53**, 11 (1985).

[3]G. Spagna, Jr., "Rubens flame tube demonstration: A closer look at the flames," Am. J. Phys. **51**, 9 (1983).

[4]H. Daw, "A two-dimensional flame table," Am. J. Phys. **55**, 8 (1987).

[5]G. Ficken and C. Stephenson "Rubens flame-tube demonstration" Phys. Teach. **17**, 306 (1979).