Can The Large Synoptic Survey Telescope Detect And Identify Interstellar Comets?

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ABSTRACT

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From our understanding of planet formation we know that many comets are created and ejected, but we have yet to observe "interstellar" comets from other stars. A detailed estimation of the population of these comets has been recently determined. Those results concluded that based on their size and distribution that the Large Synoptic Survey Telescope (LSST) would be unlikely to see any interstellar comets beyond 5 AU. Our work takes into account the gravitational focusing of the Sun and the brightening of comets as they come closer to the Sun. We will more accurately describe the probability of realistically observing these close interstellar comets. Using numerical simulations we track the comets in their hyperbolic orbits about the Sun. From this simulation we determine the rate of visible interstellar comets and the unique characteristics of their paths across the sky. We conclude that the LSST will be able to detect and identify interstellar comets.

Keywords: interstellar comets, LSST, astrometry, comet

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Chapter 1

Introduction

1.1 Motivation for finding interstellar comets

Some of the comets we observe may not be from our solar system, but are passersby from distant solar systems. Comets that originate in other solar systems are called interstellar comets (ICs). Their existence is purely theoretical as we have never observed any ICs (Francis 2005; Kresak 1992). The possibility of discovering an alien comet is exciting. It would be the first discovery of any alien object and would bring many opportunities for scientific study.

ICs are predicted from our models of planet formation. During the early formation of a planetary system there is an accretion disk. This disk is made of dust and ice grains orbiting the central star. The grains collide with one another forming or accreting into larger and larger objects. Eventually this accretion process can create several objects some as large as planets. On occasion an object as large as a comet will pass by a planet and will exchange energy. This will accelerate the comet. If the comet reaches a velocity greater than the escape velocity of the solar system it will leave the solar system and begin traveling through interstellar space (Bally & Reipurth 2006). This model predicts that most of the objects that were not accreted into planets were ejected from their solar system. These interstellar comets may intersect our solar system and pass close enough to the earth to be detected.

The motivation for finding ICs is two fold. Discovering an IC would provide new observational opportunities and an IC would be an *in situ* sample of another solar system. If discovered we would be able to explore the differences between our solar system and the stellar system of the comet. Depending on the particulars of the discovered IC we could determine several of its properties: size, chemical composition, or structure. Determining the chemical composition, i.e. spectroscopy, of an object from another system would be powerful evidence to confirm and expand our understanding of the universe. No matter the particulars, discovering ICs would help us to place our solar system in galactic context.

The frequency at which we could observe ICs places limits on our models of planet formation. Because our current models predict that there are many ICs out in space the actual observational frequencies can help us to refine our models. By knowing the frequency at which we expect to observe ICs and then comparing the expected value to the actual observational frequency, we can adjust the planet formation models accordingly. By understanding ICs we can improve our understanding of planet formation.

1.2 Background on previous research

The assessments of how many ICs we expect to observe has changed significantly in the past five years. Earlier studies predicted very high numbers of observable ICs. McGlynn and Chapman 1989 determined that number density of ICs was, 10^{13} pc^{-3}. These earlier studies took what we know from the formation of our solar system and estimated how many comets our solar system ejected into interstellar space and then extrapolated for the number density of stars. Recently it has been shown that the actual mass density is more than five orders of magnitude less than we expected (Moro-Martín et al. 2009). Moro-Martín 2009 determined that the number density of ICs was at most 10^{10} pc^{-3} and possibly as low as 10^5 pc^{-3}. The major difference was that they took into account the fact that our Sun is larger than many stars and as a result our solar system ejected more mass than many others. Another study has recently shown that the space density of ICs is less than predicted by the earlier optimistic assessments (Jura 2011). Jura placed an upper bound on the space density of ICs by studying two white dwarfs and their chemical composition. They indicate that during the dwarf's cooling lifetimes less than expected numbers of ICs have collided with them. Their upper bound agrees with Moro-Martín 2009. These recent assessments help to explain why we have not observed any ICs to date.

A numerical simulation is needed to be able to consider all of the factors that play a role in detecting ICs. All previous assessments have been analytic in nature and as such it has been too difficult to consider all of the factors. Realistic detectability factors that are easier to incorporate into a numerical model include:

- Gravitational focusing: the effect of the Sun altering the trajectory of ICs
- Phase function: the effect of observing ICs at different angles
- Comet brightening: the effect when comets increase in brightness as they approach the Sun
- Sun's position: the position of the Sun in the sky at the time of observation
- Air mass: the amount of atmosphere the observing telescope must look through.

These realistic factors will be discussed in full detail in Chapter 2. A numerical simulation is needed because of the difficulty in analytic methods in incorporating these factors.

1.3 Available tools for detecting interstellar comets

Sky survey telescopes are relatively new and are capable of finding ICs unlike previous observatories. They are capable of finding ICs because they can scan the same region of sky periodically. Several sky survey telescopes have been constructed recently, the Catalina Sky Survey in 1998 and the Pan-STARRS in 2009 to name a few (Institute for Astronomy 2012; R.Hill 2012). These telescopes specialize in observing large portions of the sky in short periods of time. This type of telescope is well suited to finding ICs as opposed to other telescopes that focus in on smaller patches of the sky. With more and more sky survey telescopes being built the question of finding ICs has become more relevant.

The Large Synoptic Survey Telescope (LSST) is the best tool for finding ICs. The LSST is a sky survey telescope and will be able to observe objects much fainter than any previously built survey telescope. The size of the primary mirror is 8.4 m in diameter as compared to Pan-STARRS which has four 1.8 m diameter mirrors. The LSST will be able to detect objects down to 24.5 magnitudes in a single visit. It will be able to observe the entire night sky every three days in five different frequency bands (LSST Science Collaborations et al. 2009). The data from the LSST will also be immediately available to the public. The LSST is not built yet, but is projected to be in operation by 2020 and operate for ten years. Because of these qualification the LSST will be the best tool available for finding ICs.

1.4 Background concepts

The magnitude of a solar system body is a measure of how bright the object is in the sky. The magnitude scale is logarithmic and smaller values correspond to brighter objects. There are two ways to express magnitudes, absolute magnitude and apparent magnitude. Absolute magnitude is how bright the object is if it were observed at 1 AU from both the Sun and the Earth. Apparent magnitude is dependent on the distance the object is away. Therefore apparent magnitude is how bright the object appears in the sky. Using these two different ways of expressing magnitudes we can compare different objects as if they were at the same distance away, using absolute magnitude, or we can compare them as they appear in the sky currently, using apparent magnitude. Also note that we are talking about magnitudes for solar system objects only, there is different but related magnitude scale used for stars.

There are several coordinates systems used to describe where an object is in the solar system. Each of these coordinates systems has advantages for certain situations; we will use two specific systems in this thesis. The first are orbital elements. The orbital elements are a six part coordinate system that fully define the orbit of an object in a two body system. It is possible to convert from Cartesian coordinates using a position vector and a velocity vector to orbital elements and back. The advantage of the orbital elements is that they can be scaled linearly in time. One of the elements called mean anomaly can be scaled linearly in time to find the new position of the object, all other elements remain constant in time. This feature of orbital elements allows us to know the position and velocity of an orbiting object at any point in time. Two of the siz components of the orbital elements will be of particular instrest, eccentricity and inclination. Eccentricity is a measure of the shape of the orbit; values less than 1 indicate an ecliptic orbit while values greater than 1 correspond to hyperbolic orbits. Inclination is the angle of the orbital plane with respect to the ecliptic i.e the plane of the Earth's orbit. Inclination varies from 0° to 180°. Values close to 0° and 180° indicate the orbit is closely aligned with the ecliptic, but for the values close to 180° the object is orbiting in the opposite direction. Values close to 90° indicate the orbit is nearly perpendicular to the ecliptic. The other coordinate system we use deals with the position of object as seen in the sky. This system is just two dimensional, with components right ascension (r.a.) and declination (dec.) . The angles r.a. and dec. are a measure of how far east to west and north to south an object is in the sky, respectively. Right ascension goes from 0° to 360° and declination

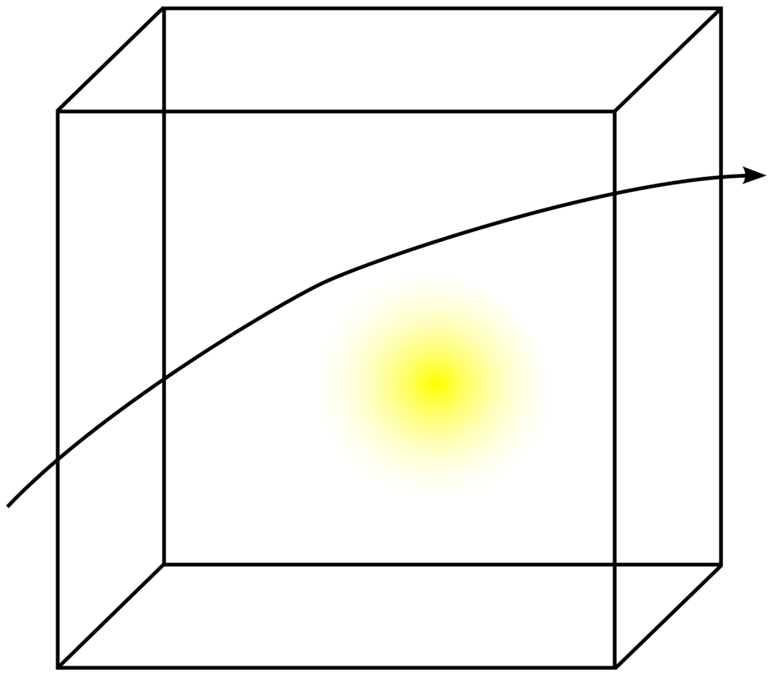

Figure 1.1 Drawing of a typical hyperbolic orbit about the Sun. ICs exhibit hyperbolic orbits, unlike other solar system bodies that exhibit elliptic orbits.

goes from 90° to -90°.

Nearly all objects in our solar system are in closed orbits, ICs are not. ICs have hyperbolic orbits. They are coming from other solar systems far away, will pass ours, continue on their way through space and never return. When we refer to the orbit of an IC we always imply that the orbit is hyperbolic and not elliptic, even if it is not explicitly stated. Fig. 1.1 illustrates a typical hyperbolic orbit about the Sun.

1.5 Our assessment

The assessment we have performed is numerical and contains two parts. The simulation models ICs in their hyperbolic orbits, determines which are visible, and calculates the number of ICs we expect the LSST to be able to observe per year. Then the simulation calculates the astrometry, i.e.

the position of the ICs in the sky as seen by the LSST, and determines features that distinguish ICs from other solar system bodies.

We determine the number of ICs we expect the LSST to be able to observe per year. To do so, the simulation takes two steps. The first step requires that we have a relation for the magnitude of an IC at any time; in section 2.2.2 we will discuss how we developed such a relation. For the first step the simulation numerically minimizes this magnitude relation for each IC and finds the time when each IC is brightest. Second, the simulation takes only the ICs that meet a threshold magnitude and steps them forward and backward in time, centered around the time of maximum brightness, to determine if there are any instants for which the IC is visible to the LSST. A rate is then calculated indicating how many ICs we expect the LSST to observe per year.

We perform astrometry on typical detectable ICs in an effort to determine how to identify ICs. To perform the astrometry of an IC we calculate its magnitude, r.a. , dec. , and the rate of change of the r.a. and dec. every three hours for a year before and after the time of maximum brightness. This data provides a picture of what the ICs would look like in the sky as it travels past the Earth. Using this astrometry data we determine which features identify ICs from other solar system bodies. In particular we need to determine if we can distinguish ICs from Oort cloud comets (OCCs) and near Earth objects (NEOs).

1.6 Overview of thesis

Can the LSST detect and identify interstellar comets? Yes. The LSST will be able to detect $0.01 - 10$ ICs in its lifetime and will be able to identify ICs by their hyperbolic orbits and mean velocities of approximately 120 arcsec hr^{-1} across the sky. This rapid motion across the sky makes the ICs unique so they can be distinguished, but most current algorithms for finding solar system bodies will not be able to detect them.

The rest of this thesis will follow a simple logical flow. Chapter 2 we will introduce the numerical simulation, its two components and give an overview of how it works. In section 2.2 we will explain in detail each component of the simulation. Section 2.3 will address the validity of the simulation by showing we have reproduced theoretical results and by demonstrating there are no erroneous anomalies in the simulation. Finally, in Chapter 3 we will present and discuss the results. Particularly we will detail how many ICs we expect the LSST to observe, the findings of our simulated astrometry and areas for future work.

Chapter 2

The numerical simulation

2.1 Overview of the numerical simulation

2.1.1 Determining the detectability of ICs

Determining the detectability of ICs requires two steps. First we numerically minimize a magnitude relation for each IC and find the time each IC is brightest. Second we step the ICs forward and backward in time checking several tests that determine if the IC is visible. The second step is only done for a subset of ICs that meet a threshold magnitude of 28 mags. Using this threshold gives the simulation a generous buffer just in case the minimization step finds a local minima instead of the global minimum. Once these two steps have been completed we are left with a set of ICs that are detectable by the LSST. From these IC we then calculate a rate of visible ICs per year that the LSST can observe.

To complete the first step of the simulation we need a magnitude relation that only depends on time. The magnitude of an IC is dependent on its position and several properties of the IC itself. We can calculate the position of the IC at any time in its orbits using orbital elements. The orbital elements are a six part coordinate system that fully define the orbit of an object in a two body system. The orbital elements of an IC can be determined from the initial position and velocity of the IC. Using the orbital elements we can advance the orbit linearly in time and then transform back to Cartesian coordinates to get the new position and velocity of an IC. With this coordinate transformation we can calculate the position of the ICs in their orbits about the Sun at any time. We can calculate the position of the IC at any time in its orbits using orbital elements. The orbital elements are a six part coordinate system that fully define the orbit of an object in a two body system. The orbital elements of an IC can be determined from the initial position and velocity of the IC. Using the orbital elements we can advance the orbit linearly in time and then transform back to Cartesian coordinates to get the new position and velocity of an IC. With this coordinate transformation we can calculate the position of the ICs in their orbits about the Sun at any time. The properties of the IC we need to know include the albedo, the phase function, and the comet brightening behavior of the IC. These properties all contribute a term to a complete magnitude equation. We will discuss how each of these properties affects the magnitude of an IC in section 2.2.2. Each of these properties depend only on the position of the object or are constant. This allows us to be able to build the magnitude relation that is only dependent on time.

To complete the second step of the simulation we perform the astrometry of an IC. To perform the astrometry we calculate the IC's magnitude, r.a. and dec. and the rate of change of the r.a. and dec. every three hours for a year before and after the time of maximum brightness . We also calculate the air mass and postions of the Sun and the Earth at each step, so we can determine if an IC is visible. The air mass of the ICs must be less than 2 for it to be visible. We will discuss how air mass in calculated in section 2.2.3. The Sun must also be more than 18° below the horizon for the sky to be dark enough to observe the IC. In section 2.2.4 we explain how we calculate the angle of the Sun with respect to the horizon. Using these tests we determine the subset of ICs that are visible to the LSST.

2.1.2 Determining the distinguishing characteristics of ICs

To determine the distinguishing characteristics of ICs we use the astrometry data that we calculated in the second step of the simulation. Using this astrometry data we determine unique characteristics of detectable ICs as they travel across the sky. We calculate several statistics about the ICs motion across the sky and determine what an IC would look like on average. We then compare the aggregate behavior of ICs to other solar system bodies like Oort cloud comets (OCCs) and near Earth objects (NEOs).

The Oort cloud is a spherical shell of comets at roughly 50 000 AU from the Sun. These comets are orbiting the Sun in closed orbits, but disturbances in the gravitational field can cause a few of these comets to fall towards the Sun. Many of these falling comets then pass the Sun and leave our solar system on hyperbolic orbits. All other comets in our solar system are in elliptical or closed orbits except for the some of the Oort cloud comets. Since the ICs have hyperbolic orbits they can be distinguished from other comets except for the Oort cloud comets. Therefore to identify ICs we just need to be able to distinguish them from Oort Cloud comets.

Near Earth objects are just what their name suggests; objects in our solar system that come very close to the Earth. These objects are typicallay come within about 1 AU of the Earth. Because NEOs do come so close to the Earth their motion on the sky is significantly affected by motion of the Earth. As a result it is easy to determine their orbits from their observed astrometry. ICs will also come close to the Earth so their motions across the sky will be similar.

2.2 Components of the numerical simulation

This section goes into the details of the different components of the numerical simulation. First we look at the initial conditions of the simulation. Next we discuss each terms in the magnitude relation of IC. Finally we explain how we calculate the air mass and the angle of the Sun with respect to the horizon.

2.2.1 Initializing the simulation

This section details how each of the initial conditions of the simulation are chosen. As discussed in chapter 2 the simulation is initialized by placing ICs in a cube and giving them appropriate radii and initial velocities. The Sun is also placed in the center of the cube and given its velocity as well.

The number and size of the ICs that are initially placed in the simulation cube is determined using a broken power law size distribution and a mass density. The size distribution defines the number of ICs we expect to exist of a given radius. The mass density is the amount of mass we expect exists in a given volume. The size distribution and mass density are combined by assuming that all ICs have the same density. As a result we can calculate the number of ICs that would exist in a given volume along with their respective radii. The simulation uses the variations of these distributions that Moro-Martín 2009 used. The equation for a broken power law is

$$N(r) \propto \begin{cases} r^{-q1} & \text{when } r < r_b \\ r^{-q2} & \text{when } r > r_b \end{cases}, \tag{2.1}$$

where r_b is the break point and $q1$ and $q2$ are the two slopes. This equation determines the number of objects that exists for a given size. Fig. 2.1 is a plot of the broken power law we use for the radii of the ICs for specific values of $q1 = 2$ and $q2 = 5$.

The initial velocities of the ICs has a small but not insignificant effect on the distribution of magnitudes observed. For the simulation we chose an initial random 3-dimensional velocity for the comets, we will call it v_0. We can determine the importance of v_0 by comparing the frequency of comets that come within 10 AU of the sun-earth distance. The sun-earth distance is defined as the minimum value of $(\Delta_{sun}^2 \Delta_{earth}^2)^{1/4}$ where Δ_{sun} is the distance to the Sun and Δ_{earth} is the distance to the Earth from the comet. About twice as many comets that have a v_0 of 5 km s^{-1} come

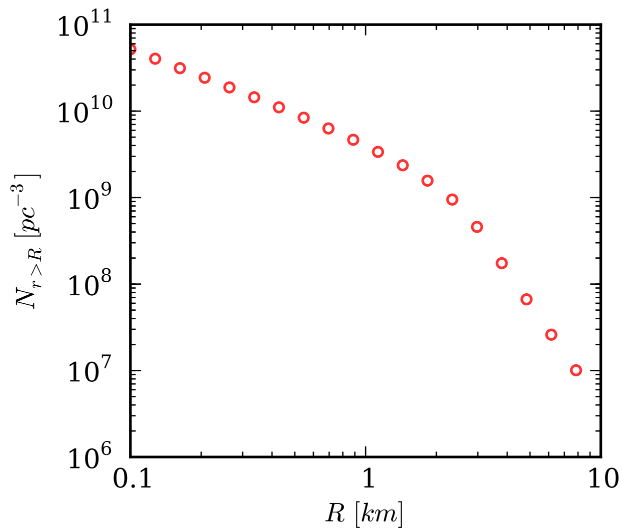

Figure 2.1 The number of ICs with radius greater than *R*. For this power law $q1 = 2$ and $q2 = 5$.

closer than comets that have a 30 km s^{-1} v_0. For velocities above 30 km s^{-1} there is no significant change in the sun-earth distance. Since this factor is significant we use a range of velocities.

The Sun has a velocity relative to the galatic Local Standard of Rest (LSR). The LSR is defined as the average motion of the nearby stars in the galaxy. There are several proposed values for the Sun's LSR velocity. Performing the same sun-earth distance test, as with the IC's velocity, using the different LSR velocities we found that there is no significant difference in the number of ICs that come within 10 AU of the Sun and the Earth. Therefore we concluded to use the most recent proposed LSR velocity for the Sun. The LSR velocity of the Sun is (11.1, 12.24, 7.25) km s^{-1} in the (U,V,W) galatic coordinate system (Schönrich et al. 2010).

2.2.2 Calculating the magnitude of an IC

To calculate the magnitude of an IC we take into account gravitational focusing, comet brightening and the photometric phase angle. Each of these factors contributes a term to the magnitude equation and will be explained in the following paragraphs.

As the ICs travel past our solar system the gravity of the Sun will tend to pull or focus them closer. The closer an object is to the Sun and the Earth the brighter it will be. To account for gravitational focusing we simply model the ICs in their appropriate hyperbolic orbits accounting for the mass of the Sun. We calculate the apparent magnitude V of an IC based on the distance from the Sun and the Earth using

$$V = H + 2.5\left[\log_{10}(\Delta_{sun}^2) + \log_{10}(\Delta_{earth}^2)\right] \tag{2.2}$$

where H is the absolute magnitude, Δ_{earth} is the distance from the IC to the Earth in AU and Δ_{sun} is the distance from the IC to the Sun in AU. The absolute magnitude H of an IC is its intrinsic brightness and will be discussed in detail later. This simple relation is a standard way to calculate the magnitude of a solar system body as a function of its position and absolute magnitude.

As comets approach the Sun they can become active and have a significant increase in brightness. There are two parts to consider in the model for comet brightening, first we account for the increase in the intrinsic brightness of the IC and second we account for the increase in brightness as it approaches the Sun.

To understand how we model the change in intrinsic brightness (i.e. absolute magnitude) we will discuss the difference between asteroids and comets, and how their absolute magnitudes are treated. In the context of this thesis, the only difference between comets and asteroids is that comets are active. This means that two objects of the same size and at the same distance away will have the same brightness if they are both inactive. When we refer to an IC as an asteroid, we mean the case when the IC is not active. The absolute magnitude of an object in the solar system is defined as the observed magnitude when the object is 1 AU away from both the Sun and the Earth. Because asteroids and comets of the same size have different observed magnitudes at 1 AU we use two different relations to define the absolute magnitudes of asteroids and comets. For asteroids, the absolute magnitude is represented as H, and for comets, the absolute magnitude is represented

Source	b_1	b_2
Kresak 1978	-0.20	2.10
Bailey & Stagg 1988	-0.17	1.90
Weissman 1996	-0.13	1.86
Sosa & Fernandez 2011	-0.13	1.20

Table 2.1 List of the parameters b_1 and b_2 from various sources.

as H_{10}. The absolute magnitude of an asteroid is defined as

$$H = \frac{\log_{10}(D) + 0.5\log_{10}(p) - 3.1236}{-0.2} \tag{2.3}$$

where D is the diameter of the IC in km and p is the albedo of the IC. The albedo of an asteroid is a measure of how reflective the asteroid is; it ranges from 1 to 0 where a value of 1 is interpreted as the asteroid is 100% reflective. We use an albedo of 0.06 in the simulation. The relation to determine H_{10} for comets is

$$H_{10} = \frac{\log_{10}(D) - b_2}{b_1}, \tag{2.4}$$

where D is again the diameter of the IC in km and b_1 and b_2 are parameters determined from observational data. Notice the albedo term is missing from Eq. (2.4). This is because we let the parameters b_1 and b_2 contain that information since they are determined directly from observational data. Several different values of b_1 and b_2 have been determined as noted in Tbl. 2.1. Eq. (2.3) is essentially a special case of Eq. (2.4), where $b_1 = -0.2$ and $b_2 = 3.1236$ with the added term for the albedo of an asteroid.

Once we have determined the absolute magnitude of the comets based on Eq. (2.4) we need to model how the comets get brighter as they approach the Sun. The model we use simply scales the apparent magnitude V as $1/\Delta_{sun}^{n}$ where n is an adjustable parameter called the photometric index. This model is widely used in the comet community (Sosa & Fernández 2011). The magnitude

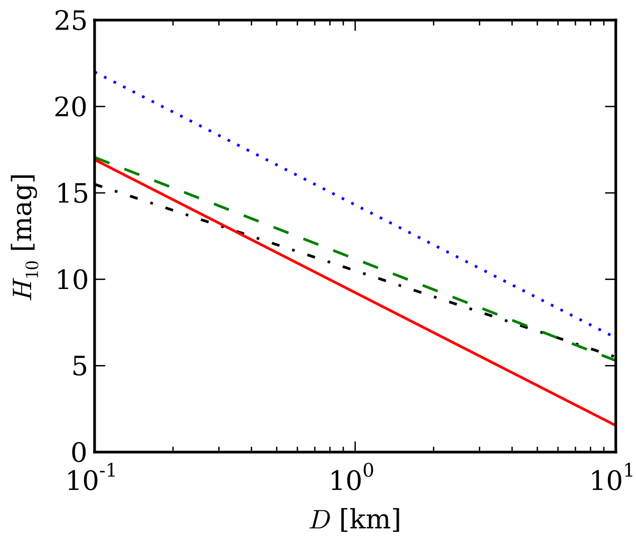

Figure 2.2 Plot of H_{10} for the different sources of $b1$ and $b2$. The lines correspond to the different sources as follows: Kresak 1978 (black, dot-dashed), Bailey & Stagg 1988 (green, dashed), Weissman 1996 (blue, dotted), Sosa & Fernandez 2011 (solid, red).

equation for comet brightening becomes

$$V = H_{10} + 2.5\left[\frac{n}{2}\,log_{10}(\Delta_{sun}^2) + \log_{10}(\Delta_{earth}^2)\right], \tag{2.5}$$

where H_{10} is the absolute magnitude determined by Eq. (2.4), Δ_{sun} is the distance from the IC to the Sun and Δ_{earth} is the distance from the IC to the Earth. When $n = 2$ Eq. (2.5) reduces to Eq. (2.2), i.e. the case without comet brightening. Comets typically have two different values of n, one for pre-perihelion approach and another of post-perihelion orbit. We use a pre-perihelion n of 5.0 and a post-perihelion n of 3.5 (Francis 2005). Large values of n correspond to brighter comets. Note that when $\Delta_{sun} = \Delta_{earth} = 1$AU, Eq. (2.5) reduces to H_{10} which is exactly what we want.

The photometric phase angle affects how bright the ICs are. The photometric phase angle is the angle between the Sun, Earth and the IC, with the Earth at the vertex. We use a phase function γ to adjust the brightness of the ICs based on its phase angle (Muinonen et al. 2010). The phase function also takes into account the scattering properties on the surface of the IC. The phase function we use is

$$\gamma = (1-G)\Phi_1(\theta) + G\Phi_2(\theta), \tag{2.6}$$

where G is a slope parameter, Φ_1 and Φ_2 are basis function to the phase curve, and θ is the phase angle. The value of G controls how steep the phase curve is; values close to 0 indicate a steep curve and values close to 1 indicate a shallow curve. We use a steep curve where $G = 0.05$, see Fig. 2.3. The basis functions are accurately approximated as follows:

$$\Phi_1(\theta) = \exp\left[-3.33\left(\tan\frac{\theta}{2}\right)^{0.63}\right], \tag{2.7}$$

$$\Phi_2(\theta) = \exp\left[-1.87\left(\tan\frac{\theta}{2}\right)^{1.22}\right], \tag{2.8}$$

where θ is the phase angle. To adjust the magnitude of an IC for its phase we use

$$V = H - 2.5\log_{10}(\gamma), \tag{2.9}$$

where H is the absolute magnitude and γ is the phase function.

Now that we have all of these different factors we can combine them into two equations, one with the comet brightening terms and one without. We will refer to these two equations as the comet magnitude equation and the asteroid magnitude equation. The comet magnitude equation is

$$V_{cb} = H_{10} - 2.5\log_{10}(\gamma) + 2.5\left[\frac{n}{2}\log_{10}(\Delta^2_{sun}) + \log_{10}(\Delta^2_{earth})\right], \tag{2.10}$$

where H_{10} is the absolute magnitude determined by Eq. (2.4), G is slope parameter to the phase curve, Φ_1 and Φ_2 are the basis function as defined above, θ is the phase angle, n is the photometric index, Δ_{sun} is the distance from the IC to the Sun and Δ_{earth} is the distance from the IC to the Earth. The asteroid magnitude equation is

$$V_{as} = H - 2.5\log_{10}(\gamma) + 2.5\left[\log_{10}(\Delta^2_{sun}) + \log_{10}(\Delta^2_{earth})\right], \tag{2.11}$$

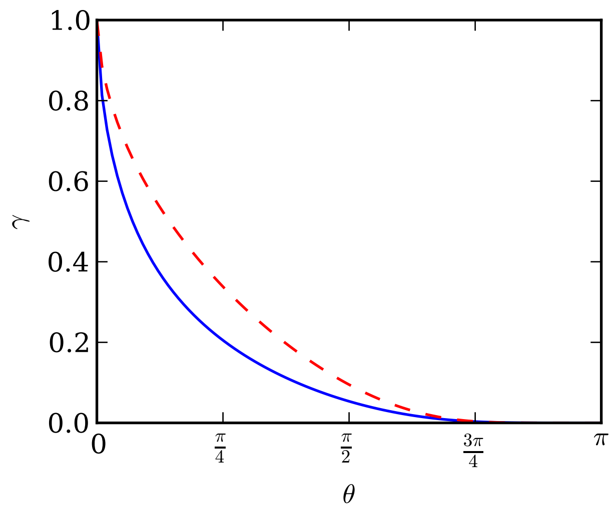

Figure 2.3 The phase function for the ICs. The solid blue line is when $G = 0.15$ and the dotted red line is when $G = 0.5$. We can see that the large value of G gives a more shallow curve.

where each of the parameters are the same as in Eq. (2.10) with the exception that H is determined by Eq. (2.3). Since Eq. (2.10) can result in magnitudes that are too large when $\Delta_{sun} > 1\text{AU}$ we define the magnitude of an IC as

$$V = \begin{cases} V_{cb} & \text{when } V_{cb} < V_{as} \\ V_{as} & \text{otherwise} \end{cases}, \tag{2.12}$$

remembering that smaller magnitudes correspond to brighter comets. Using this piecewise equation we ensure that a comet is never fainter than an asteroid of equivalent properties.

2.2.3 Calculating air mass

We calculate the air mass of ICs so we can accurately determine whether the IC is visible to the LSST or not. Air mass is a measure of how much atmosphere the LSST must look through to

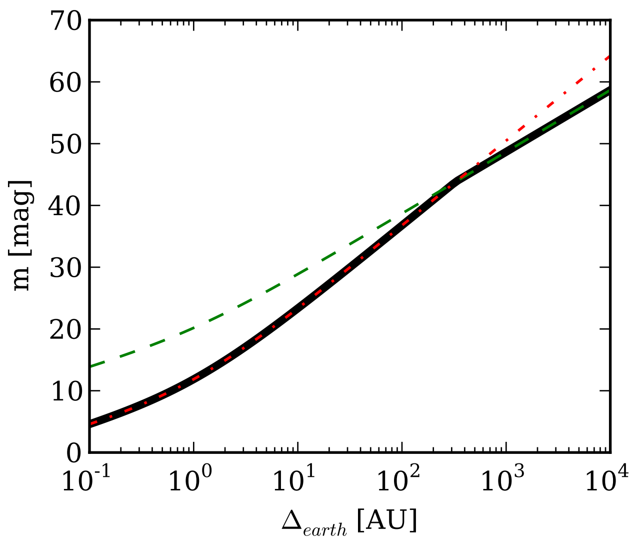

Figure 2.4 Asteroid magnitude vs. comet magnitude at opposition. The red dashed-dotted line represents the magnitude equation for a comet. The green dashed line is the magnitude of an asteroid. The modeled magnitude is the solid black line.

observe an IC. Air mass, A, is calculated as

$$A = \sec(\alpha) \tag{2.13}$$

where α is the zenith angle. The zenith angle is the angle between the zenith of the LSST and vector defined by the position of the IC and the center of the Earth. The zenith of the LSST is the vector that passes through the center of the Earth and the LSST. When the air mass is greater than 2 the IC is not visible either because there is too much atmosphere or the IC is below the horizon. Essentially limiting the air mass to be less than 2 defines a cone above the LSST where the ICs are actually visible.

2.2.4 Calculating the angle of the Sun with respect to the horizon

We calculate the angle of the Sun is with respect to the horizon of the LSST to determine if the sky is dark enough at the time of observation. To calculate this angle we use simple geometry. We work in a frame where the center of the Earth is at the origin. We negate the vector that defines the position of the LSST thus creating a vector that is normal to the horizon plane of the LSST pointing below the horizon. Using the dot product we calculate the angle between the vector to the Sun and the horizon vector. Then we subtract that angle from 90°, thus positive values indicate the Sun is below the horizon and negative values indicate it is above the horizon. If the Sun is more that 18° below the horizon, i.e. astronomical twilight, then it is dark enough for the IC to be seen.

2.3 Validity of the numerical simulation

2.3.1 Reproduction of theoretical results

Before we compare our results to the theoretical result of Moro-Martín we need to address a small error in the number density equation as derived by Moro-Martín. The number density equation is only valid for radii less then the break radius because of the limits of integration used during its derivation. A piecewise equation is needed to correctly define the number density for radii less than and greater than the break radius. We have made this correction and reproduced a plot of the number density of the ICs, see Fig. 2.5. We confirmed that this is correction is indeed accurate (Moro-Martín, personal communication). The result of this error is that there are essentially zero ICs beyond the break radius. Because we are only considering comet radii up to 10 km only the case where the break radius is 3 km is effected by this error. This indicates that comparisons to data for the 3 km break radius case will be expectantly different beyond 3 km.

The numerical simulation reproduces the results of Moro-Martín's work. To comfirm the validity of the model we ran the simulation with parameters that emulate the result of Moro-Martín. We set the mass of the Sun to zero, we did not use the phase function, we did not use comet brightening and we did not move the Earth in its orbit. Using these parameters we were able to reproduce the theoretical rates of visible ICs to the LSST. As can been seen Fig. 2.7 matches Fig. 2.6.

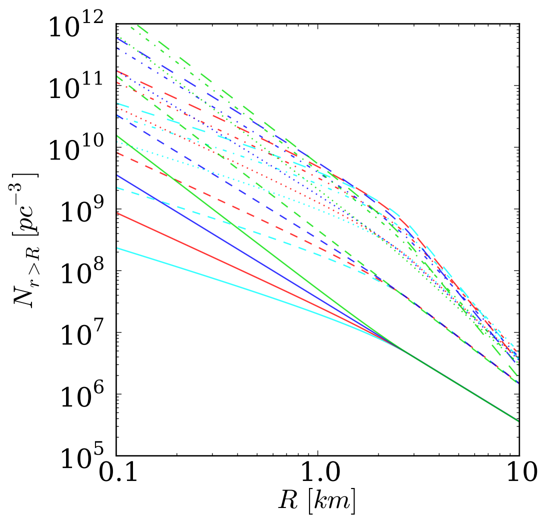

Figure 2.5 Correction to the number density plot from Moro-Martín. The colors and line types correspond to $q1 = 2.0$ (light blue), 2.5 (red), 3.0 (blue), and 3.5 (green); $q2 = 3$ (solid), 3.5 (dashed), 4 (dotted), 4.5 (dash-dotted), and 5 (long dashed).

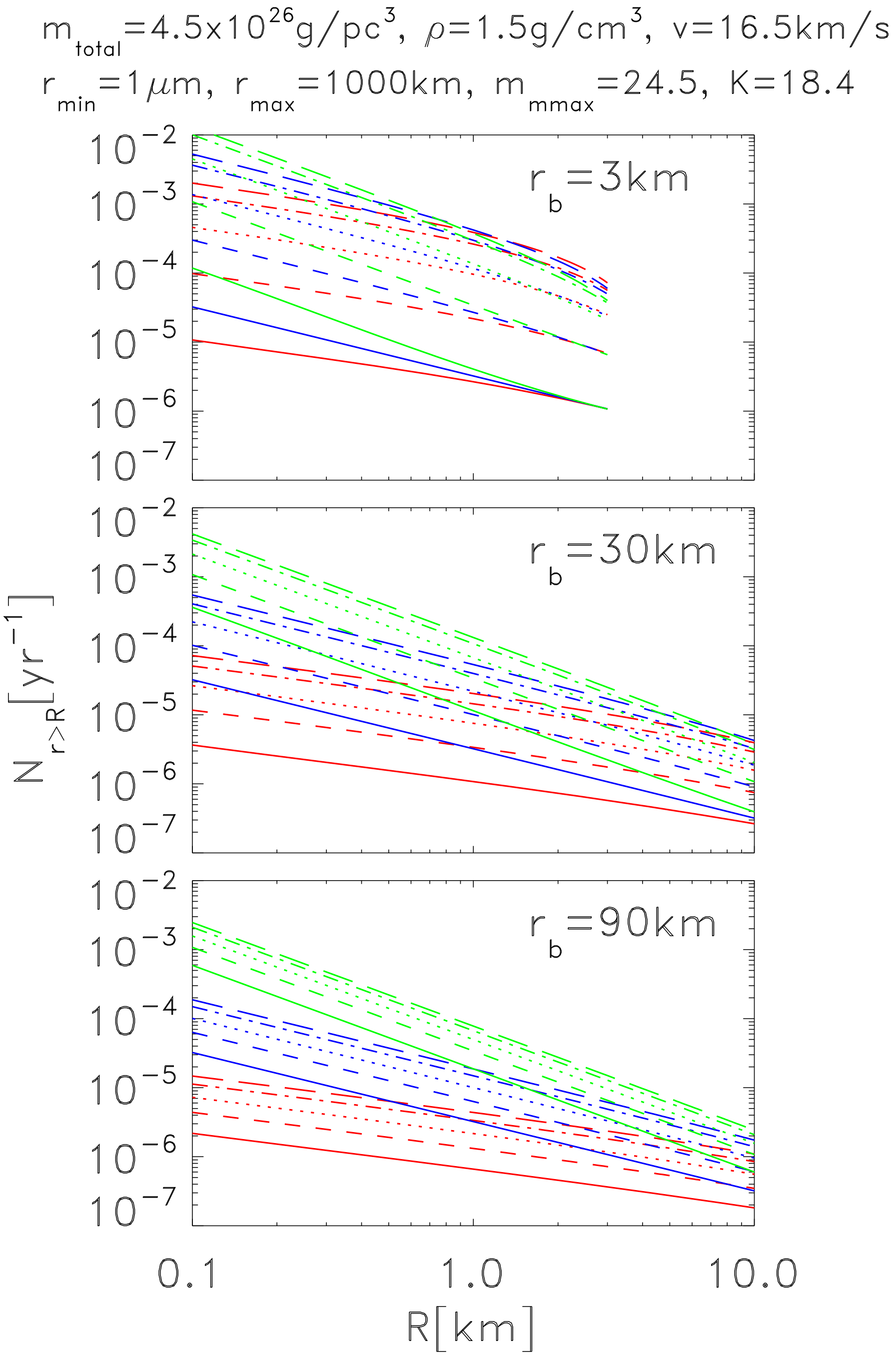

Figure 2.6 Plot of the number of visible ICs per year, taken from Moro-Martín 2009. The abrupt end of ICs in the $rb = 3$ km case is because of the small error in Moro-Martín work previously addressed. The correct behavior is seen in our results (Fig 2.7).

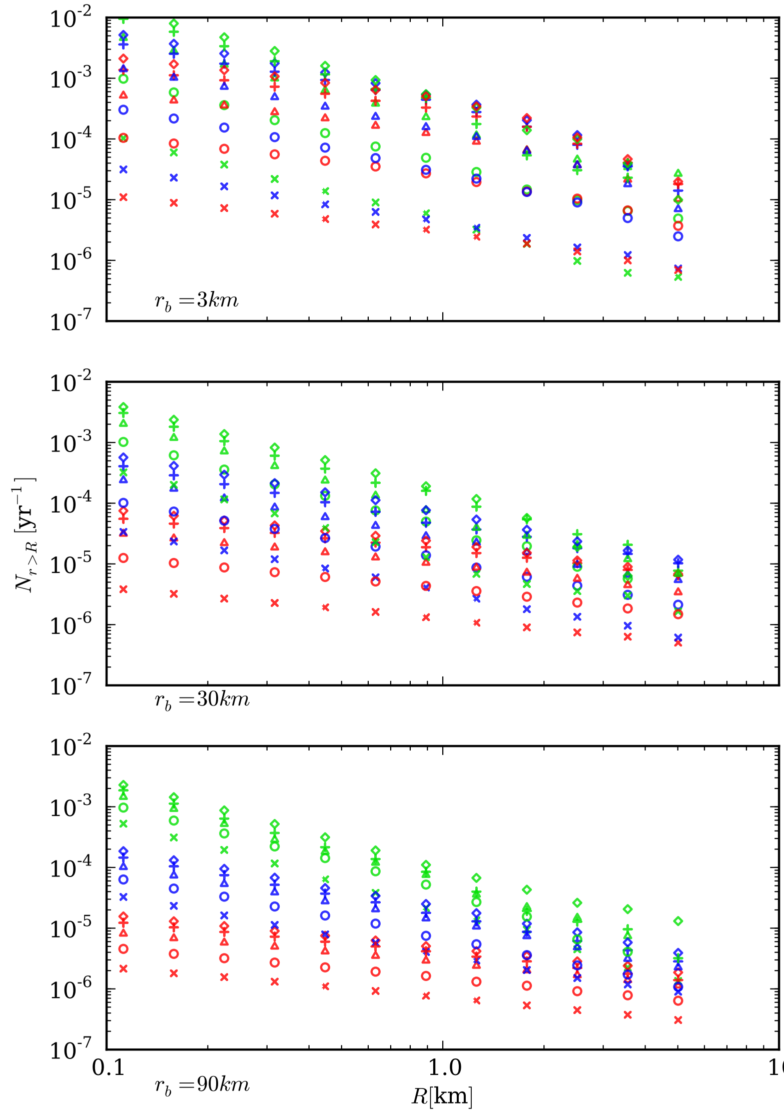

Figure 2.7 Plot of the number of visible ICs per year as calculated from the simulation. This plot matches Fig. 2.6. The colors and markers correspond to the values of $q1$ and $q2$ as follows: $q1 = 2.0$ (light blue), 2.5 (red), 3.0 (blue), and 3.5 (green); $q2 = 3$ ($\times$), 3.5 ($\circ$), 4 ($\triangle$), 4.5 (+), and 5 ($\diamond$).

Chapter 3

Results

3.1 Can the LSST detect ICs?

Yes, the LSST will be able to detect ICs. We ran the simulation for several different cases; Tbl. 3.2 summarizes the results. As can be seen in the table the LSST could be able to observe 0.1-10 ICs in its lifetime in the most optimistic case. Moro-Martín predicted that the LSST would only be able to observe $10^{-4} - 10^{-2}$ ICs in its lifetime. While the simulation is precise the input parameters to the simulation have large amount of uncertainty. As a result there are at least two orders of magnitude of uncertainty in the results. We can see that accounting for the gravity of the Sun and comet brightening this prediction has increased by a few orders of magnitude.

3.1.1 Effects of the input parameters

In this section we will discuss how much each of the parameters affects the results. A list of the parameters can be found in Tbl. 3.1. The nominal values of each parameter represent what we believe to be the most accurate values except for the values of q_1 and q_2. The values chosen for q_1 and q_2 represent the upper bound on the number density of ICs. The nominal case includes comet

Parameter	Description	Nominal Value
m_{total}	mass density of ICs	4.5×10^{26} g pc^{-3}
q_1	pre-break slope of the IC number density	3.5
q_2	post-break slope of the IC number density	5.0
r_b	the break radius of the IC number density	3 km
r_{min}	minimum radius of detectable ICs	0.1 km
ρ	density of ICs	0.5 g cm^{-3}
n_{pre}	pre-perihelion photometric index	5.0
n_{post}	post-perihelion photometric index	3.5
b_1	first absolute magnitude parameter	-0.13
b_2	second absolute magnitude parameter	1.2
v_0	initial velocity of ICs	30 km s^{-1}
G	phase function steepness	0.15
p	albedo of asteroid	NA

Table 3.1 List of input parameters for the simulation and their nominal values.

#	Difference from nominal values	N_{LSST}
1	none	0.57
2	$r_{min} = 0.01$km, $\rho = 1.5$ g cm^{-3}, $n_{pre} = 2$, $n_{post} = 2$, $b_1 = -0.20$, $b_2 = 3.1236$, $p = 0.06$, no gravity, no phase function	0.0023
3	$m_{total} = 4.5 \times 10^{30}$ g pc^{-3}	5900
4	$r_{min} = 0.01$km, $n_{pre} = 2$, $n_{post} = 2$, $b_1 = -0.20$, $b_2 = 3.1236$, $p = 0.06$	0.94
5	$v_0 = 5$ km s^{-1}	0.64
6	$b_1 = -0.20$, $b_2 = 2.10$	1.0
7	$b_1 = -0.17$, $b_2 = 1.90$	0.58
8	$b_1 = -0.13$, $b_2 = 1.86$	0.20
9	$q_1 = 2.0$, $q_2 = 3.0$	0.00029
10	$q_1 = 2.5$, $q_2 = 3.5$	0.0051
11	$G = 1$	0.82

Table 3.2 The number of ICs the LSST could observe in its lifetime for various different cases. N_{LSST} is the number of ICs the LSST could observe in 10 years. Each row changes a few different parameters for different situations showing how N_{LSST} changes. Each of the parameters and their nominal values are defined in Tbl. 3.1

brightening using the Sosa & Fernandez 2011 values for b_1 and b_2. Row 1 of Tbl. 3.2 lists how many ICs we expect the LSST to be able to observe in its lifetime based on the nominal values. Each subsequent row of Tbl. 3.2 changes a few of the parameters and lists how many ICs we expect the LSST to be able to observe based on the new set of parameters.

Rows 2 and 3 demonstrate how many ICs we expect to observe based on previous assessments. Row 2 is the Moro-Martín case. As can be seen when we run the simulation without the factors that Moro-Martín could not consider we get a very small number of visible ICs, 0.0023, that falls in the range they predicted. The third row uses a mass density close to what McGlynn and Chapman 1989 used in their assessment. This mass density predicts about 6000 visible ICs. As expected this early assessment predicts enough ICs that we should have seen them already.

Row 4 demonstrates how the results change assuming ICs are asteroids. To simulate the ICs as asteroids we change the comet brightening parameters, $n_{pre} = n_{post} = 2$ and change b_1 and b_2 to correspond asteroid values. We also set the albedo to 6% since the albedo is not defined for a comet. For the comet brightening cases we expect any comets less than 0.1 km to disintigrate, but for the asteroid case this is not so. For this reason we count ICs that get as small as 0.01 km. After making all of these changes we can see that the asteroid case is about the same as the nominal case with only a small increase. If we were to not count the smaller asteroids as visible then N_{LSST} for the asteroid case would be signifcantly less than the comet brightening cases. This effect can be seen in Fig. 3.1 where we have plotted the number of visible ICs per year for different comet radii.

Row 5 demonstrates the effect of the initial velocity of the ICs. As discussed in section 2.2.1 we expect the initial velocity of the ICs to have some effect on the number of visible ICs, where smaller velocities correspond to more visible ICs. After changing $v_0 = 5$ km s^{-1} we can see that the number of visible ICs increases but only slightly, from 0.57 to 0.64. Fig. 3.2 plots both the nominal case and the asteroid case for initial velocities of 5 and 30 km s^{-1}. The figure again

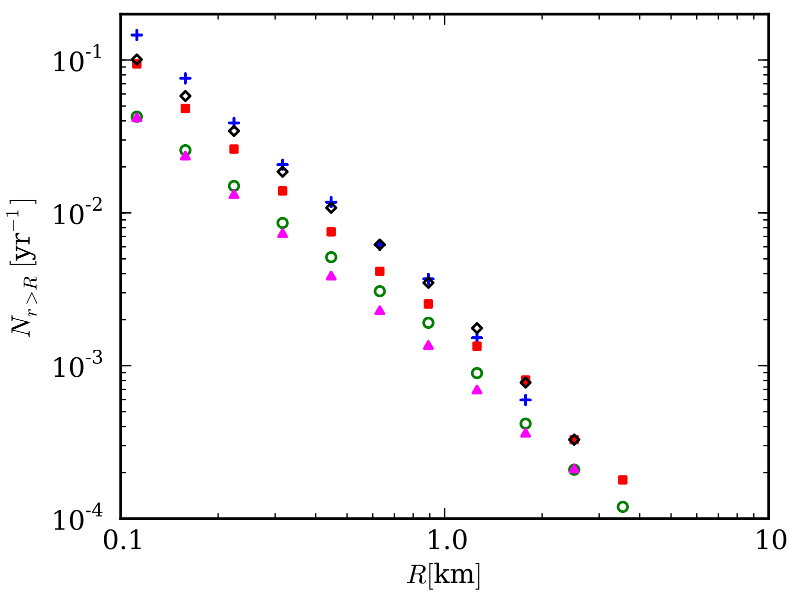

Figure 3.1 Comet brightening has a significant impact of the number of visible ICs. Here we can see the effect of the different parameters b_1 and b_2 for comet brightening cases compared to the asteroid case. The markers correspond to the different comet brightening cases and the asteroid case as follows: asteroid (green, $\circ$), Kresak 1978 (blue, $+$), Bailey & Stagg 1988 (red, $\square$), Weissman 1996 (magenta, $\triangle$), and Sosa & Fernandez 2011 (black, $\diamond$)

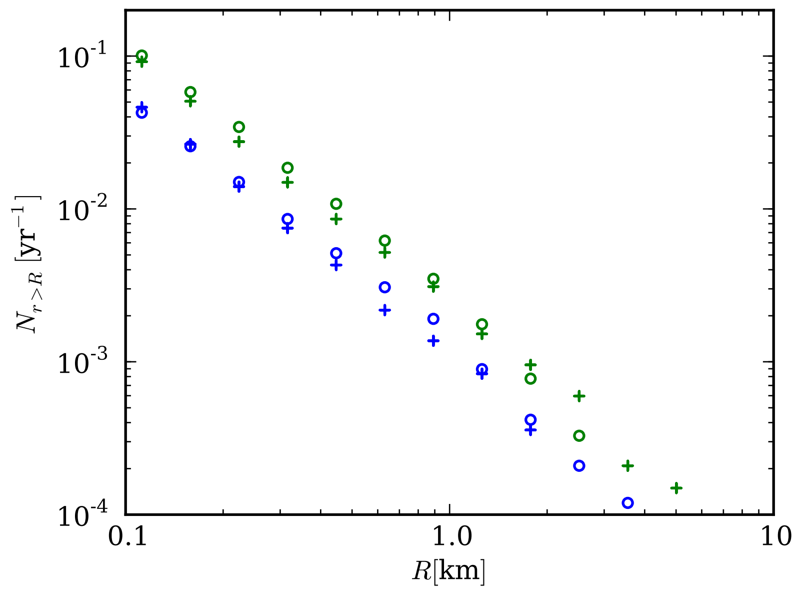

Figure 3.2 The velocity of ICs has little effect on the number of visible ICs per year. The '+' indicates a initial velocity of 5 km s^{-1} while the '○' indicates 30 km s^{-1}. The top pair of data points correspond to the nominal comet brightening case and the bottom pair correspond to the asteroid case.

demonstrates that the initial velocity has a little affect on N_{LSST}.

Rows 6-8 demonstrate the effects of the different comet brightening models. Rows 6-8 change the values of b_1 and b_2 corresponding with Kresak 1978, Bailey & Stagg 1988 and Weissman 1996 respectively. Since the b_1 and b_2 parameters contribute directly to the value of H_{10} for comets we can gain insight into the different cases by examining the value of H_{10} for each case. Fig. 2.2 plots the value of H_{10} for comets of different diameters. As can be seen in the figure H_{10} for the Kresak 1978 case drops below the Sosa & Fernandez 2011 case at small comet diameters. Because there are more smaller comets than larger ones this small difference in the values of H_{10} for different diameters results in an increase in N_{LSST} for the Kresak 1978 case. Since the other comet brightening cases are always greater than the Sosa & Fernandez 2011 case for values H_{10}, N_{LSST} is smaller for each case according to the differences in the values of H_{10}. Fig. 3.1 also shows the differences in the comet brightening cases by plotting the number of visible ICs per year for different comet radii.

Rows 9 and 10 demonstrate the effect of the number density of ICs. The values of q_1 and q_2 control the slope of the number density power law for the ICs. Row 9 uses values of q_1 and q_2 that are a lower bound on the number density and row 10 uses intermediate values; the nominal case is an upper bound on the number density. As can be seen the difference in the number density of the ICs has a very large effect on the number of visible ICs. Because of the large uncertainty in the values of q_1 and q_2 there is a large uncertainty in the final result as well.

Row 11 demonstrates the effect of the steepness of the phase function. Here we use $G = 1$ which defines the shallowest (i.e. brightest) possible phase function. As can be seen N_{LSST} increases by a small amount from the nominal case. A value of $G = 1$ is not realistic but demonstrates that the steepness of the phase function can only play a small role at best in the total result.

3.2 Can the LSST identify ICs?

Yes, the LSST will be able to identify ICs. After analyzing the astrometry data for the detectable ICs we found that their behaviors in the sky will be distinguishable. As previously discussed ICs have features of both OCCs and NEOs. ICs have hyperbolic orbits like OCCs and they come close to the Earth like NEOs. We believe that ICs will look like NEOs, but because ICs have hyperbolic orbits they will be distinguishable. ICs like NEOs have a high rate of motion across the sky because of how close they come to the Earth. The high rate of motion will make them different from OCCs. NEOs do not have hyperbolic orbits; this means if we find a comet that looks like an NEO but has a hyperbolic orbit then we may be able to conclude it is an IC.

Several orbital properties can be used to identify ICs; Tbl. 3.3 lists the mean and standard deviation of these orbital properties. We can see that ICs have high eccentricities corresponding

Property	Mean	Standard deviation
Eccentricity	3.47	3.07
Inclination	87.3°	41.6°
Declination	23.3°	31.4°
Right ascension	164.9°	103.2°
Declination rate	79.1 arcsec/hr	149.7 arcsec/hr
Right ascension rate	87.3 arcsec/hr	138.6 arcsec/hr
Geocentric distance	2.30 AU	1.58 AU
Air mass	1.67	0.31
Sun angle	32.5°	17.3°
Photometric phase angle	26.3°	15.9°
Apparent magnitude	23.0 mag	1.73 mag

Table 3.3 Statistics of various orbital properties of nominal ICs. These values were calculated only for detectable ICs and at the time of detection.

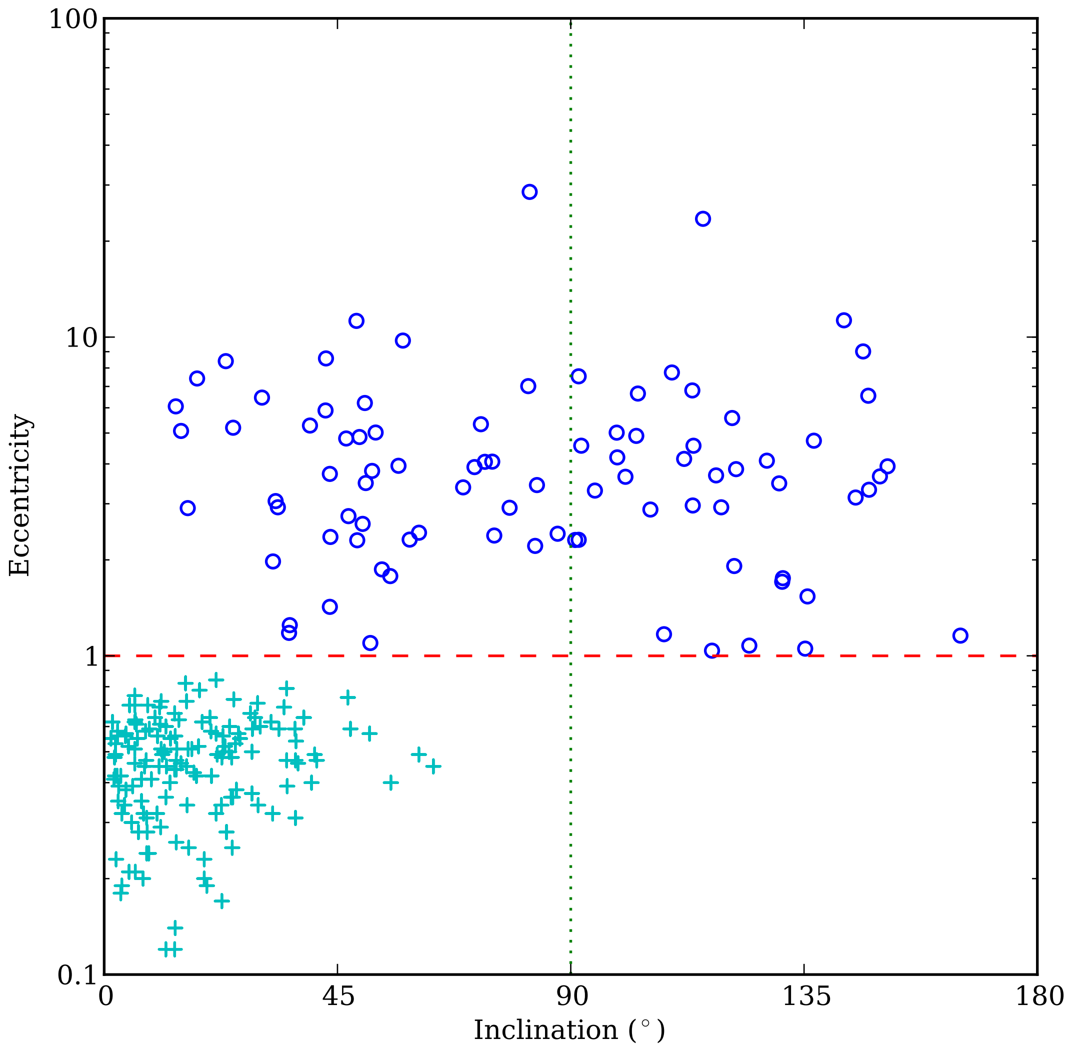

Figure 3.3 The distributions of NEOs and ICs are clearly separated in the eccentricity-inclination space. ICs are represented by the ○ symbol and NEOs by the + symbol. The red dashed line separates ecliptic orbits from hyperbolic orbits. The green dotted line indicates orbits that are perpendicular to the ecliptic plane of the solar system. As can be seen all NEOs have inclinations less than 90°; this means that they are all orbiting in the same direction as the Earth and approximately in the same plane. This is not the case for ICs that clearly cover the entire range of inclinations.

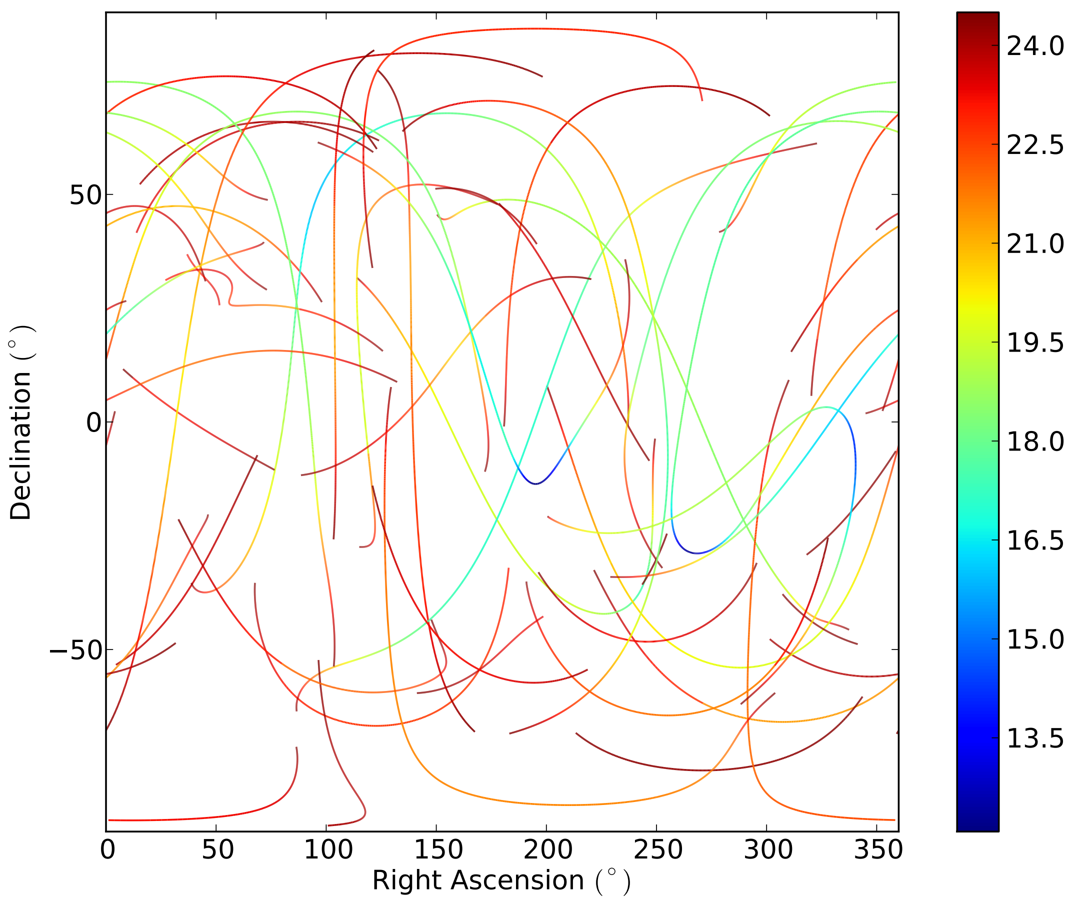

Figure 3.4 This plot shows the motions of the detectable ICs in the sky. We can see here that ICs could be discovered anywhere in the sky, there is no preferred direction. The colors of the paths correspond to the magnitudes of the ICs. The color bar on the right gives the scale of the colors as they correspond to magnitudes.

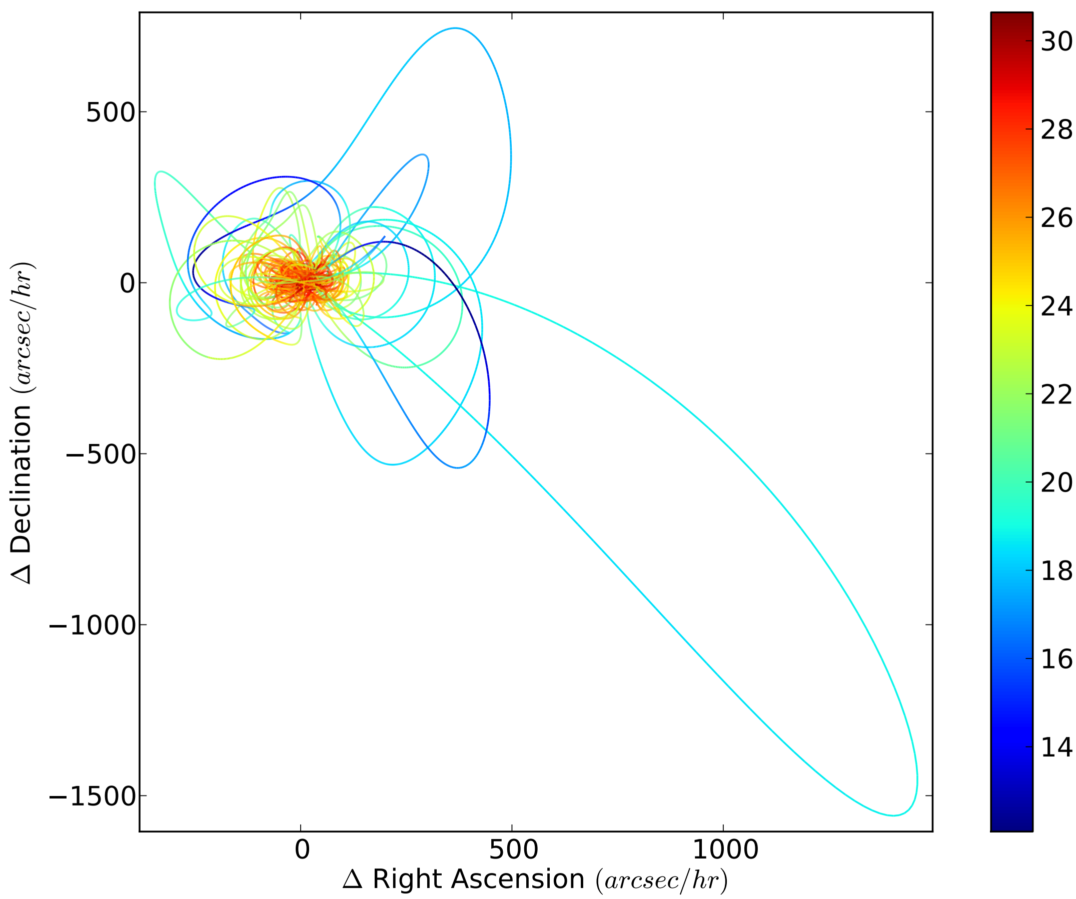

Figure 3.5 This plot shows the rates motion of the detectable ICs in the sky. As can be see ICs can have very large rates of motion across the sky. The colors of the paths correspond to the magnitudes of the ICs. The color bar on the right gives the scale of the colors as they correspond to magnitudes.

to hyperbolic orbits (i.e. greater that one) like we expect. Almost all objects in our solar system orbit the Sun in the same plane and in the same direction, but since ICs do not originate in our solar system they will not follow this same pattern. ICs have an average inclination of 87.3° and a large standard deviation of 41.6°; this indicates that ICs will be coming from almost any direction as expected. These high values of inclination and eccentricity make ICs unique from other solar system bodies. Fig. 3.3 demonstrates how NEOs and ICs differ for these two orbital properties.

As we have mentioned before the ICs have a high rate of motion across the sky. This can be seen by the rate of change of dec. and rate of change of r.a in Tbl. 3.3. These values represent only the magnitude of the rate of change only and not the direction. Then mean rate of change of dec is 79.1 arcsec hr^{-1} and 87.3 arcsec hr^{-1} for the rate of change of r.a. Together this corresponds to a mean rate of 117 arcsec hr^{-1} across the sky. Also the standard deviation is very large indicating that many ICs will be moving much faster than the mean velocity of 117 arcsec hr^{-1} across the sky, while others will be moving slower. Fig. 3.5 plots a few ICs and their rates of motion across the sky.

Algorithms exist that determine the six orbital elements of an object based on a range of observations. In other words the algorithms perform the inverse of the astrometry problem, i.e. they convert a range of r.a. and dec. points into orbital elements. As the algorithms are fed more data points they begin to constrain the valid values of the orbital elements, thus arriving at a more accurate solution. Once the solution has been constrained enough it can be compared to the orbital properties of ICs and determine if it is an IC or not. If the orbital element solution has and eccentricity greater than one and an inclination greater than 90° it is likely an IC. Using the listed orbital properties of the typical detectable ICs we can distinguish ICs from other solar system bodies.

3.3 Future Work

Current algorithms are not designed for finding ICs and therefore have a chance of missing them. The way most current algorithms find moving objects on the sky is by taking pictures of the sky and looking for dots that move across the sky relative to the background stars. The algorithm checks each picture or frame for these detections and, based on many physical constraints, is able to determine which detections are the same across frames. The problem is that to make the algorithms fast and efficient they only search in a given range for the next detection. If the object has moved out of that range from one frame to the next it will be discarded. Since ICs, move rapidly across the sky they run the risk of being discarded. Also many algorithms reject solutions that indicate the orbit is unbound, i.e hyperbolic. This allows the algorithms to be more efficient at finding bound comets but will make finding ICs nearly impossible. For future work we need to modify existing algorithms to be able detect ICs. If we do not modify existing algorithms we will likely not find any ICs.

The first step to being able to modify the algorithms would be to run existing algorithms on the our astrometry data. The astrometry data that we have produced for typical ICs is the kind of data these algorithms need to be able to search for ICs. One could feed the astrometry data for the brightest of the ICs to the algorithms and see how each performs. Based on the reasons explained above we believe that most algorithms will perform poorly. Once we have quantified how well the current algorithms perform, we can begin to make improvements. Because we have very detailed data on how a typical IC looks in the sky it will be possible to tailor these algorithms appropriately.

3.4 Conclusion

We conclude that the LSST will be able to detect and identify ICs. The LSST will be able to detect 0.01 – 10 ICs in its lifetime and will be able to identify ICs by their hyperbolic orbits and mean velocities of approximately 120 arcsec hr^{-1} across the sky. More work still needs to be done in order to actually discover an IC, but we now believe that doing so will be worth the effort. If we do not look for ICs we will not find them. Discovering an IC will provide new opportunities for research. Such as determining the spectroscopy of an IC, this would allow us to learn about the chemical composition of other planetary systems and put our solar system in galatic context. The frequency at which we observe ICs places limits on our planet formation models. Spending resources to find ICs, in order to determine the actual number that LSST observes, will be crucial in placing new limits on our planet formation models because we predict that the LSST will actually observe ICs in its lifetime. We should put forth an effort to develop algorithms that will enable the LSST to discover ICs.

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