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Physics 492R
Capstone Project: Demonstration of properties of human hearing using a cochlear analogue

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Demonstration of properties of human hearing using a cochlear analogue

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I have constructed a device that demonstrates the spectral decomposition of waves according to spatial location along a surface analogous to the function of the basilar membrane of the mammalian cochlea. The design is based on previous work done by Robert Keolian at Pennsylvania State University.[1] I have modified Keolian's design to incorporate variable mass as well as stiffness and an electronically driven shaker, which allows the device to be driven with complex waveforms. The model can be used to demonstrate beating, masking, and other psychoacoustical phenomena that occur on the basilar membrane.

I. INTRODUCTION

Although there is much that is still not completely understood about mammalian hearing, a great deal can be explained by the function of the basilar membrane of the cochlea. The cochlea is the snail shaped organ of the inner ear. The basilar membrane divides the cochlear canal roughly in half as shown in figure 1. Georg Von Békésy originally observed a traveling wave that deposits energy of different frequencies at specific locations along the basilar membrane.[2] The basilar membrane varies in stiffness, mass, and width from its base to its apex. The basal end is the stiffest, thinnest and least massive, and the apical end is least stiff, widest and most massive. This means that the resonant frequency of the membrane differs along its length, with the basal end resonating at the highest frequencies and the apical end resonating at the lowest frequencies.[3] This creates an impedance gradient in the membrane that produces waves that always travel from base to apex.[3] Numerous mathematical but fewer physical models have been used to demonstrate the mechanics of this wave.[1, 2,4,5,6]

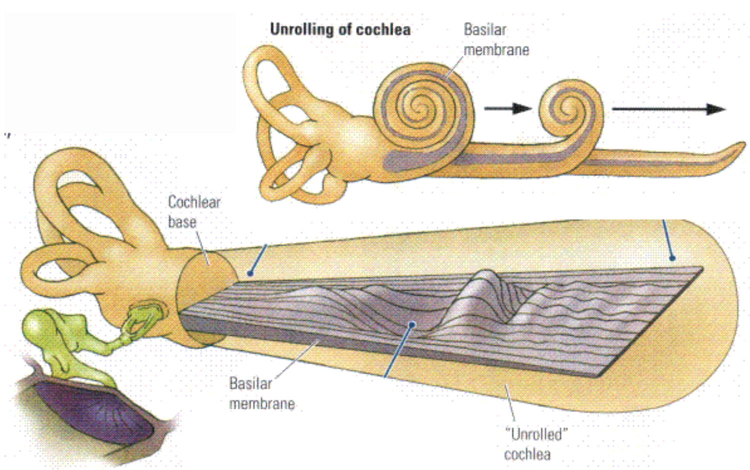

FIG. 1. The cochlea and basilar membrane of the inner ear.

I have built a physical model based on a design by Robert Keolian, a professor at Pennsylvania State University.[1] A polycarbonate twin walled sheet of the type typically used in architectural glazing is attached vertically to a clear PVC tube that is bent downward. The walls of the polycarbonate sheet are held apart by many thin parallel ribs. The sheet is cut at the top along an exponential curve. The PVC tube is bent downward on another exponential curve. This creates many vertical channels of varying height connected at the bottom in the PVC tube as shown in figure 2. The apparatus is filled with water up to a certain height in the vertical channels, h_0. The height of the air columns is then given by $h_a(x) = h_{a0}10^{x/D_a} + h_0$, and the height of the water columns is given by $h_w(x) = h_{w0}10^{x/D_w} - h_0$. For the physical model, $h_0 = 4.38$ cm, $h_{a0} = 0.63$ cm, $D_a = 61$ cm, $h_{w0} = 12$ cm, and $D_w = 1.74$ m. A driver is attached to the end of the PVC tube where the PVC pipe and the top of the polycarbonate sheet are closest. The other end of the PVC pipe is stopped with a rubber cork. A metal strip is connected to the top with small holes drilled above each of the channels. After the apparatus is filled with water a piece of electrical tape is placed over the metal strip sealing the air in each of the channels. The air trapped in the vertical channels acts as springs of varying stiffness, simulating the varying stiffness of the basilar membrane. The height of the water varies along the apparatus simulating the varying mass of the basilar membrane. When driven, traveling waves can be seen on the waters surface similar to traveling waves observed on the basilar membrane. A numerical model was used as a design guide for the physical model.

The physical model can be used in a classroom setting to help describe some of the basic physics of hearing, to demonstrate the function of the basilar membrane in the cochlea, and to explain how some psychoacoustical phenomena are created as a result of this function. It can also be used to show wave phenomena such as beating, resonance and evanescence.

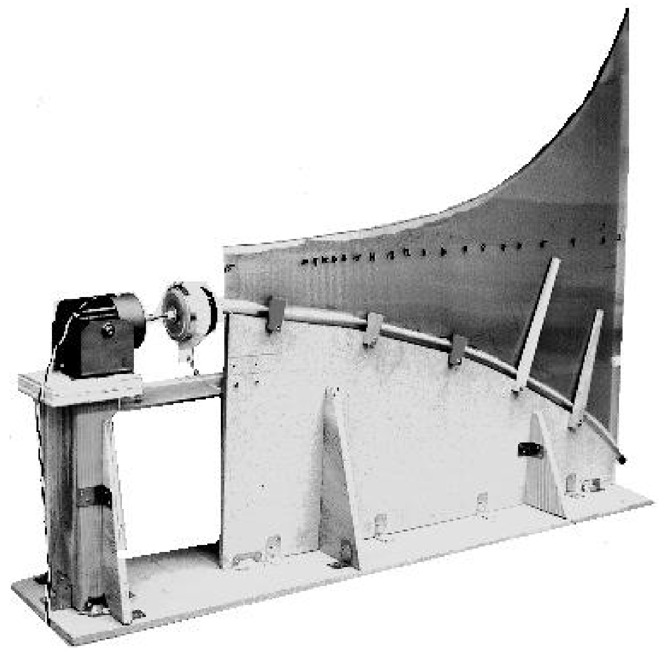

FIG. 2. The demonstration apparatus.

II. MATERIALS AND METHODS

A. MATHIMATICAL MODEL

I experimented with a numerical model that was a guide in the final design of the physical model. The wave equation of the apparatus with varying modulus and its derivation are described by Keolian.[1]

The derivation of the wave equation follows that of sound, starting from three differential equations: a force equation, an equation of continuity, and an equation of state. Assuming harmonic time variations we let $y(x,t) = Y(x)e^{jwt}$ be the horizontal displacement of the fluid in a small section of the bent tube at the base of the apparatus representing the scala, or fluid filled section of the cochlea. Let $z(x,t) = Z(x)e^{jwt}$ be the vertical displacement of fluid in the apparatus, and $p(x,t) = P(x)e^{jwt}$ be the excess oscillatory pressure at the base of the sheet as shown in figure 3. Newton's Force equation can be written

$$\frac{\partial p}{\partial x} + \rho \frac{\partial^2 y}{\partial t^2} = 0$$

where ρ is the density of water.

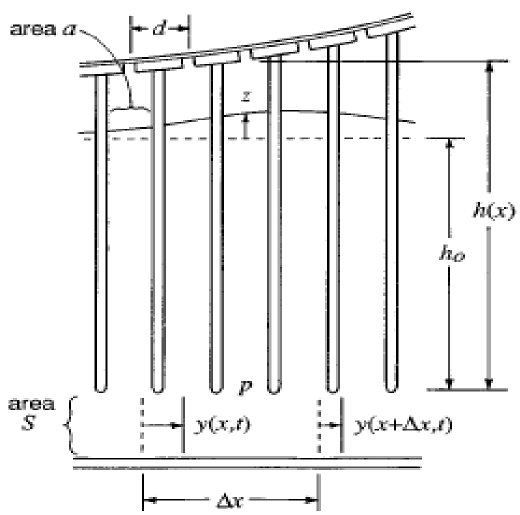

FIG. 3. Schematic detail of the vertical sheet and PVC tube. Equilibrium positions of the liquid are shown with a dotted line; displaced positions with a solid line. Taken from Keolian[1]

For sound the continuity equation relates the divergence of flow to the compression of the fluid. Here the fluid is virtually incompressible and is instead allowed to move up the vertical channels. If the distance between channels, d, is small compared to the section of the apparatus that we are considering, Δx, and to the local wavelength, then the fluid entering a short section is $Sy(x,t)$. Here S is the area of the PVC tube representing the scala. The volume that leaves at $x+\Delta x$ is $Sy(x,t)+S\Delta x\,\partial y(x,t)/\partial x$ and the volume going up the vertical channel is $z(x,t)\Delta xa/d$, where a is the area of the vertical channel. The continuity equation is a statement that the net volume of water in a small section of the apparatus does not change,

$$S\frac{\partial y}{\partial x}+\frac{za}{d}=0\,.$$

For sound, the equation of state relates the pressure fluctuations to the density fluctuations in the medium. In the apparatus the pressure at the base of the vertical channel is related to the force per channel area needed to move the water column and compress the gas. This relationship is given by

$$p=(K(x)+j\omega R-\omega^2 M(x))z/a$$

Where $K(x)$ is the spring constant of the gas trapped in the channel, R is the mechanical resistance of the channel, and $M(x)=\rho a h_w(x)$ is the mass of the water in the channel. For isothermal compressions the ideal gas law gives $K(x)=P_0 a/h_a(x)$, where P_0 is atmospheric pressure. The damping term, R, is related to the viscosity of water.

We now proceed by taking the derivative of the equation of state with respect to x and substituting into Newton's force equation, which give us

$$\frac{1}{a}\frac{\partial z}{\partial x}(K(x)+j\omega R-\omega^2 M(x))+\frac{z}{a}\frac{\partial}{\partial x}(K(x)+j\omega R-\omega^2 M(x))+\rho\frac{\partial^2 y}{\partial t^2}=0\,.$$

For gradual variations in K(x) and M(x) the second term can be ignored. Next we take the derivative of the continuity equation with respect to x and substitute into the above equation with the second term eliminated to obtain

$$-(K(x)+j\omega R-\omega^2 M(x))\frac{Sd}{a^2}\frac{\partial^2 y}{\partial x^2}+\rho\frac{\partial^2 y}{\partial t^2}=0 .$$

This is the wave equation which can be written in the form

$$\frac{\partial^2 y}{\partial x^2}-\frac{1}{c^2(x)}\frac{\partial^2 y}{\partial t^2}=0 ,$$

but it is more convenient if we let $y(x,t)=Y(x)e^{j\omega t}$. This gives us

$$\frac{\partial^2 Y}{\partial x^2}+\frac{\omega^2}{c^2(x)}Y=0$$

where

$$c^2(x)=\frac{B(x)}{\rho}=\frac{Sd}{a^2\rho}[K(x)+j\omega R-\omega^2 M(x)].$$

Here $B(x)$ is the effective bulk modulus. A wave traveling from base to apex sees K(x) get smaller and M(x) get larger; $c^2(x)$ decreases to zero where most of the waves energy is deposited after which point $c^2(x)$ become negative and the wave becomes evanescent.

Obtaining solutions numerically to this wave equation poses problems because the reflected wave always blows up at the apical end. We can greatly simplify the problem by splitting the equation into left and right going waves,

$$(\frac{\partial}{\partial x}+\frac{j\omega}{c(x)})(\frac{\partial}{\partial x}-\frac{j\omega}{c(x)})Y=0 .$$

Békésy observed only traveling waves in the cochlea, and as long as we are careful to avoid too drastic changes in impedance, we should see all the waves traveling from base to apex with no reflections.[1, 3] The basilar membrane is completely flaccid at the apical end so that any low frequency energy is absorbed and not reflected.[3] The apical end of the apparatus is not sealed with tape to replicate this condition. With this in mind, we are justified in throwing out the wave that travels from apex to base. We are left with

$$\frac{\partial Y}{\partial x} + \frac{j\omega}{c(x)} Y = 0$$

whose solution is given by

$$Y(x) = y_0 e^{\int_0^x -j\frac{\omega}{c(\xi)}\partial\xi}.$$

This gives us an analytical solution which is easily implemented to find responses of the physical model. Appendix A gives a Matlab M file which animates a solution for response when driven at two frequencies simultaneously.

B. PHYSICAL MODEL

Construction of the apparatus needs to be done with care. I will here outline my method of construction. Twin walled polycarbonate sheeting is obtainable from plastics manufacturers in various sizes. I bought a 4 ft X 6 ft sheet from Laird Plastics in Salt Lake City. The sheet comes with thin protective layers of plastic on each side. These should be left on for as long as possible during the construction to protect the surface from scratching and marring. The size of the channels formed by the ribs is 5.62 mm X 7.87 mm, though there is some variation in this. I mapped out the curves on the top and bottom of the apparatus by measuring and marking by hand. This is a tedious process. Special care needs to be taken not to crack the ribs between the channels when cutting the polycarbonate sheet along these curves, especially along the top curve. Cracks in the ribs will connect air columns making springs that are less stiff than intended. I found that a band saw works fairly well for this, while a jig saw tends to crack some of the ribs.

A slit that is just larger than the width of the polycarbonate sheet and the length of the bottom of the polycarbonate sheet needs to be cut in the PVC tube. A milling machine was used to do this. I used a shelf in the slit as shown in figure 4 to give the polycarbonate sheet something to rest on. The PVC tube is most easily bent by filling it with hot sand or heating the PVC with a heat gun while it is filled with sand until it is fairly pliable. Care should be taken that the tube is heated uniformly. Care must also be taken so that the tube is not pinched or twisted during the heating and bending process. The tube can then be bent to the desired shape. A form should be made ahead of time that the tube can be easily bent to. The slit in the tube should be cut ahead of the bending. Masking tape wrapped around the tube works well to keep the sand from spilling out the slit.

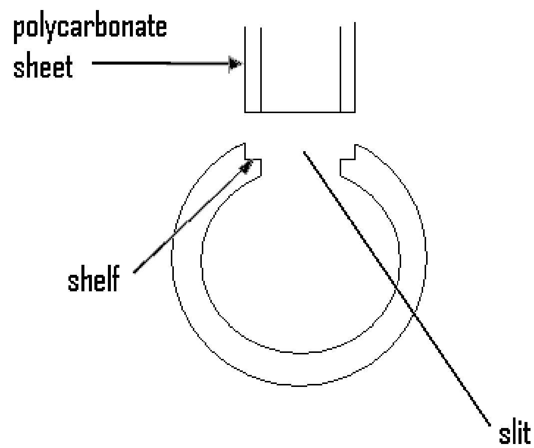

FIG. 4 Cross section of PVC tube with slit and shelf

The PVC tube is then glued to the polycarbonate sheet. The product I used is called, “Standard Weld On 16.” This will break apart under sufficient stress. If I were building another apparatus, I would search for better glue for this application. The tube can be held to the sheet by wrapping them together with masking tape and rubber bands.

If too many rubber bands are used, or the rubber bands are too tight, the ribs of the sheet on the opposite side of tube will crack because of the pinching of the rubber bands. This must be avoided. A good solution to this might be to place the metal strip on top so that the channels are not pinched. More rubber bands could then be used to secure the tube to the sheet. The PVC polycarbonate joint needs additional waterproofing after it is glued together. For this I used a product called "Amazing Goop," made by the same company that makes "Shoe Goo." A very generous application of this is needed along the PVC-polycarbonate joint. Excessive moving, jostling, and bumping will cause this joint to spring leaks that can be repaired by additional application of "Amazing Goop."

A long setting epoxy is used to adhere the metal strip to the top of the polycarbonate sheet. I used a 1/4 inch wide strip of aluminum and long setting "Loc-Tite" epoxy. The small holes that will rest over each channel need to be drilled in the aluminum before it is adhered to the polycarbonate sheet. Drilling the holes after the aluminum strip is attached will likely lead to puncturing the ribs between the channels. It is important that there is one hole resting above each channel. I used a clear piece of tape placed over the top of the vertical channels and marked the place where each hole should be. The tape was then moved to the aluminum strip and 1/16 inch diameter holes were drilled at each mark. After the holes are drilled a generous layer of epoxy is applied to one side of the aluminum strip. It is then necessary to clear the holes of epoxy with something like a toothpick. The metal strip can then be secured to the sheet using rubber bands. Great care must be taken so that the holes line up directly over the channels. When the epoxy sets additional epoxy is needed on the top edge of the polycarbonate sheet where it meets the aluminum strip in order to make the junction air tight. Some of the holes will probably have filled with epoxy while it was setting. Extreme care needs to be taken when clearing these holes so as not to puncture the ribs separating the channels.

I made the driver out of a few PVC pipe fittings. A 3 inch diameter section of pipe is connected to the 1/2 inch clear PVC tube by 3 inch to 1 ½ inch and a 1 ½ inch to 1/2 inch bushings. A small piece of plastic machined to reduce the driver diameter inside the bushings gradually is also inserted. Two clear plastic circles just less than 3 inches in diameter are bolted together on either side of a latex sheet. The latex sheet is held to the end of the 3 inch section of PVC with a hose clamp. The bolt is reverse threaded so that it can be attached to the bolt of the shaker by means of a small metal piece machined with reverse threads on the driver side and forward threads on the shaker side. This functions like a turnbuckle to connect the shaker to the driver. A small hole is drilled in the top and bottom of the driver so that air can escape from the top hole while it is being filled with water, and the water can be drained from the bottom hole. Flexible PVC tubing was glued into each of these holes so that they can be easily clamped and unclamped when needed.

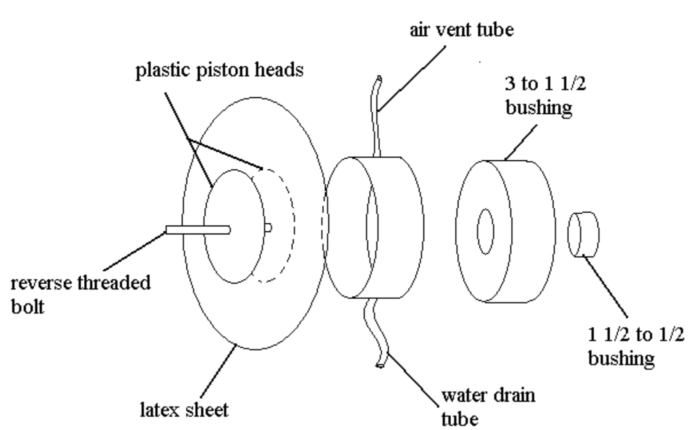

FIG 5. Driver assembly, the hose clamp and machined plastic inserted into the bushings are not shown

The apparatus is attached to a particle board base. A “shark fin” shaped piece that follows the curve of the bottom of the apparatus and stands vertically is attached to a flat base. The flat base sticks out about 1 ft 4 inches from the basal end of the apparatus to provide a space to mount the 4 inch X 6 inch post that will hold up the shaker. The base structure needs to be very sturdy so that the energy of the shaker is not dissipated in shaking the base. To strengthen the base I put two triangular shaped supports on either side of the shark fin, and one on each side of the post. The shaker is bolted onto a platform that sits on top of the post. The shark fin and post are mounted to the flat base by means of framing brackets. A stabilizing strut is also needed between the shark fin and the post. The apparatus is shown on top of the base in figure 1.

The cochlear apparatus is mounted to the base by 5 wooden brackets shown in Figure 6. The two brackets nearest the apical end are elongated to help stabilize the cochlear apparatus. The brackets are bolted together on either side of the base so that they squeeze the PVC tube and hold it in place.

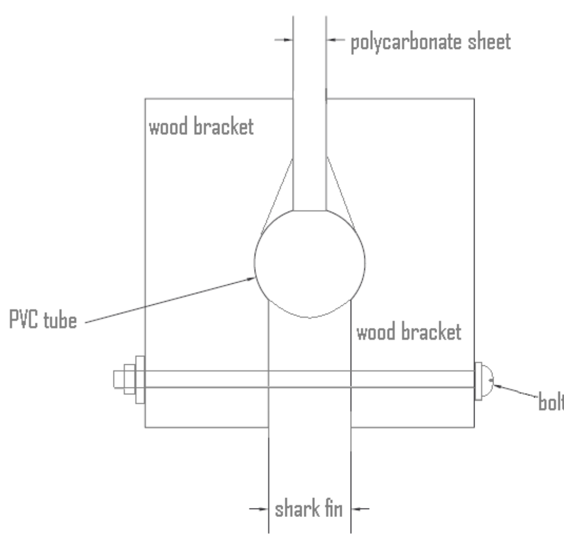

FIG. 6. Cross section of wooden bracket attaching cochlear apparatus to base.

The model is about 10 cm in height at the basal end and 1.2 m in height at the apical end, not including the particle board base. It is 4 ft long. The particle board base adds about 4 inches of height, and 1 ½ ft of length.

III. RESULTS

The completed model has an effective bandwidth from about 2 Hz to 35 Hz. The amplitude of maximum response at the apical end is about 2 ½ inches and at the basal end about 1/8 inch. Beating, evanescence, and the place principle (the maximum response of a sine wave corresponding to a specific place along the membrane) are all clearly evident throughout the entire bandwidth of the apparatus.

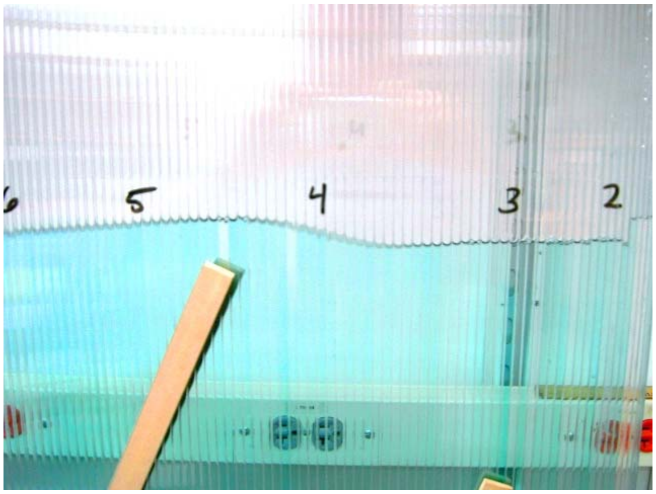

FIG. 7. Response of apparatus driven at 4 Hz.

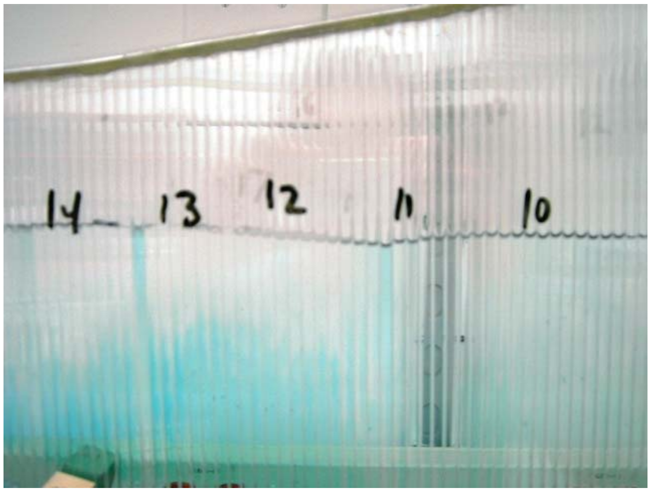

FIG 8. Response of apparatus driven at 12 Hz.

A critical band is a bandwidth of the hearing spectrum where multiple stimuli within the bandwidth have a significant effect on one another, and effect the way we perceive pitch, loudness, and tone color. This principle is responsible for much of the way we hear, and for the psychoacoustical effects of the basilar membrane. This is clearly demonstrated on the physical model. In connection with critical bands the model can show just noticeable differences in pitch, though just noticeable differences in loudness will be more difficult to observe. The model can also demonstrate masking, and how low frequencies are better at masking higher frequencies.

The ear does not perceive a tone unless a certain number of cycles (dependent on frequency) are present in the signal. We hear these signals as short clicks rather than short tones. The model also shows how the basilar response to short duration tones is broad with no apparent region of maximum response.

DISCUSSION OF RESULTS

The original design intent was for the apparatus to have a band width from 2 Hz to 60 Hz. However, it is difficult to observe visually on the model what is happening at frequencies above around 20 Hz. This makes the bandwidth above this region fairly useless as far as demonstration purposes are concerned. While we can achieve this bandwidth, response is better if we put less water in the apparatus than was originally intended. This means that the shaker used to drive the apparatus has less water to push, and less stiff springs to push against. This leads more visible, higher amplitude response patterns across the device. The sacrifice is bandwidth, but since the upper part of the bandwidth is fairly useless this is not a great loss.

The visibility of the waves and thus its usefulness as a classroom demonstration are greatly dependent on the amplitude of the waves traveling along the apparatus. Larger amplitude waves require greater stroke distance and a higher force rating of the shaker used. We have found that to get adequate amplitude in our waveforms that a shaker with 0.75 inches of stroke with a 13 lbf rating was required. An amplifier with sufficient power to drive the shaker at this force is also required. Anyone wishing to build a similar model should take the cost of the shaker into consideration. Other than the shaker the materials used are inexpensive.

One of Keolian’s design objectives was portability of the apparatus. In this design, a great deal of portability has been sacrificed in order to incorporate variable mass (to more accurately represent the properties of the basilar membrane), to increase the bandwidth, and to be able to show a greater variety of phenomena with electrical driving capabilities. Since a good deal of the bandwidth at higher frequencies is not useful, a more portable design could be used with electronically driven means, and a versatile demonstration device achieved.

The numerical model simulates very well what occurs in the physical model. This is further validation of the simplification I made by throwing out the wave that travels from apex to base. Use of the numerical model can be very helpful in the design of the physical model. Of course, the more fine tuned the numerical model, the more accurately it will represent the physical model. R depends on the mass of water in each column. In my numerical model R is an average and is constant across the apparatus. Those wishing to define R in terms of the mass of water in each column are referred to Keolian's derivation of the wave equation for the device[1]. Refinements to the numerical model could also be made at the driving end of the apparatus. This could be useful in determining stroke and force requirements for the shaker.

V. CONCLUSIONS

The physical model should be effective as a classroom demonstration. It can be used in physics courses to illustrate principles of wave mechanics, resonance, beating, and evanescence. It can be used in acoustics and speech, hearing, and language classes to help students understand cochlear function, and some psychoacoustics. Other results are possible as well and further investigation into these possibilities is encouraged.

One possible extension of these ideas would be to build a wave tunneling demonstration. This could be built in such a way as to show standing waves, evanescent waves, and traveling waves all at the same time. The apparatus would be similar to the one that demonstrates cochlear waves. The channels of the polycarbonate sheet would have a uniform short section, and uniform taller section, followed by another uniform short section. The device could be driven so that, in the short section, the wave speed is real, and in the taller section the wave speed is imaginary. This would create an evanescent wave in the taller section. The first shorter section could be constructed so that the reflections from the interface where the wave becomes evanescent create standing wave patterns in the first section. The third section would have the same wave speed as the second section. This would allow the evanescent wave to tunnel into the third section. An absorption boundary at the end of the third section similar to the cochlea would create traveling waves in the third section. A schematic of this design is shown in figure 9. This device could be very useful in helping students understand wave mechanics as well as having applications in quantum mechanics classes.

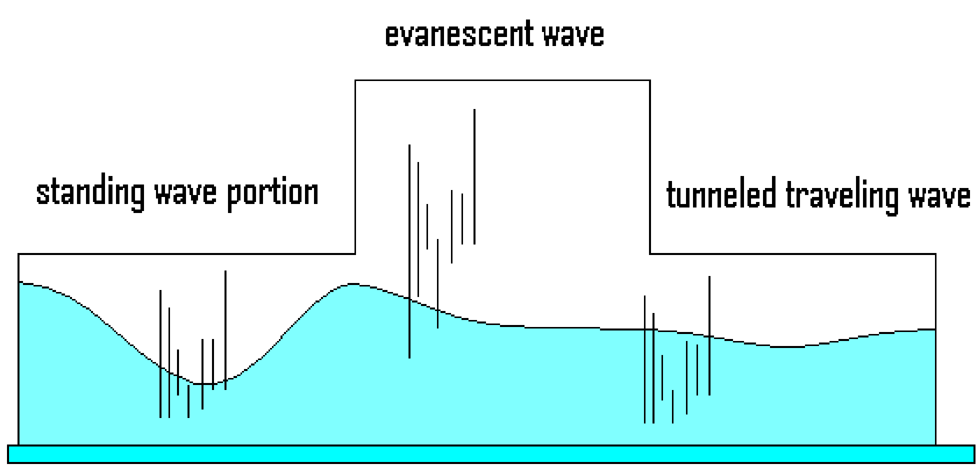

FIG 9. Schematic of a wave tunneling demonstration.

VI. ACKNOWLEDGEMENTS

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APPENDIX A

The following is Matlab code used to solve the one directional wave equation for two driving frequencies. The solution is animated.

```
clear
close all

set(0,'DefaultFigurePosition',[10   50  1000   500]);
x=0:0.001:1.2960;
inches=5/8;
S=(inches*0.0254)^2*pi/4;  %cross-sectional area
f1=10;
f2=20;
w1=f1*2*pi;
w2=f2*2*pi;
y1=.01;
y2=.002;
%parameters
a=3.16e-5;
d=6.45e-3;
h_0=0.203;
H=6.35e-3;
D=0.381;
R=0.0257*2;
rho=1000;
P_0=1.01e5;
hw=.12*10.^(x./1.74428);
M=rho*a.*hw;
ha=0.0063*10.^(x./0.6096);
K=P_0*a./ha;
c1=sqrt(S*d/a^2/rho.*(K+j*w1*R-w1^2.*M));
c2=sqrt(S*d/a^2/rho.*(K+j*w2*R-w2^2.*M));

for p=1:length(x)
    ic1(p)=sum(1./c1(1:p))*.001;
    ic2(p)=sum(1./c2(1:p))*.001;
end

Z1=j*w1*S*d*y1*exp(-j*w1.*ic1)/a./c1;
Z2=j*w2*S*d*y2*exp(-j*w2.*ic2)/a./c2;


t=0:.002:1;
for m=1:length(t)
    z1=Z1*exp(j*w1*t(m));
    z2=Z2*exp(j*w2*t(m));
    bar(x,real(z1)+real(z2));
    xlim([min(x),max(x)])
    ylim([-max(abs(Z1+Z2)),max(abs(Z1+Z2))])
    xlabel(['t = ',num2str(t)])
    pause(.01);
end
```