Comparison of sound propagation in water-saturated sediment using different sediments in an Effective Density and Fluid model

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The fluid model of reflectivity (FM) is a simple model in which both the water and the sediment are presumed to be fluids defined by three properties: compressional sound speed (c), attenuation (a) and density. The Biot-Stoll model is more complex and takes into account that the sediment is actually a mixture of grains and water. Recent work has been done to simplify the Biot-Stoll model by Kevin Williams. In the EFD model, Williams derives an effective density and bulk modulus from the Biot model by introducing a simplification to the frame parameters of the Biot-Stoll model. Effective Density and Fluid models were each coded into MatLab and plots of reflectivity versus angle were created for 5 different sediment types at frequencies ranging from 100 hertz to 1,000,000 hertz. Both models sufficiently show the general trends of the sound as it bounces off of the water-sediment interface. Sand has a critical angle where all the sound is completely reflected for all frequencies. Clay is just the opposite in that it has an angle of complete intromission (all sound is absorbed into the sediment. Silt and sitly-sand have both 100% reflection, for all high frequencies, and 100% intromission for low frequencies.

Introduction

In the field of underwater acoustics much of the research in the past has been dedicated to shallow water applications and frequencies around the 1 kHz frequency band. It was believed that the Biot slow wave did not give much to the sound field being propagated at these levels. Since, research has turned to the higher frequency bands consisting of frequencies between 10 and 100 kHz. From this we have learned that the Biot Slow wave appears as a propagating acoustic wave in the water saturated sediment.[4]

These realizations lead to a greater comprehension of the behavior of sound propagation in water-saturated sand. Different phenomena have been observed previously and several efforts have been made to model these phenomena hoping to better explain the contributing factors. Acoustics researchers have been successful in creating models that chart sound propagation. This paper considers the model accepted by most scientists in the field as accurate, the Biot-Stoll model, and compares another model, the EDF model which simplifies some of the parameters used in the Biot-Stoll model. The purpose of this comparison is to determine the validity of the EDF model. In order to thoroughly draw an appropriate comparison, 5 sediment types have been identified with their parameters, and the reflectivity of sound at 5 different frequencies spanning the 10 to 100 kHz frequency range has been computed. Graphs showing this comparison have been developed.

Reflectivity from and the sound speed and the attenuation in a water-saturated sediment, depend on frequency. When the wavelength of the sound is somewhere on the same order of the grain size, then the sediment can no longer be treated as a fluid. There have been several models produced to help understand the interactions of high frequency acoustic signals to water-saturated sand. One model that is widely used for accuracy when representing these conditions is known as the Biot-Stoll model which is parameterized by13 physical properties of the sediments that effect how a sound wave interacts with a water-sediment interface and propagates in the sediment. The physical properties taken into account in this model are Porosity (β), Mass density of sand grains (ρ_s), Mass density of water in the pores (ρ_f), Viscosity (η), Permeability (κ), Tortuosity (τ), Pore size (a_{pore}), Bulk modulus of water (K_f), Bulk modulus of sand grains (K_r), Bulk modulus of frame (K_b), and Shear modulus of frame (μ). K_b and μ are complex and the real and imaginary parts are considered independently.[3] A brief explanation of each of the parameters will be given later.

Recent work has been done to simplify the Biot-Stoll model. Kevin Williams created his EDF by ignoring the frame properties in Biot-Stoll. Therefore K_b and μ are set to zero, and using the Kozeny-Carmen equation to relate the pore size to the value of κ, β, and τ, thus reducing the number of parameters from 13 to 8.The remaining 8 parameters are combined to represent an effective density (ρ_{eff}) of the water-saturated sediment. The effective density is used in a fluid model [4] along with a compressional sound speed and attenuation, also calculated from the physical properties, to describe acoustic interactions with the sediment. In reference 1, the sound speed dispersion, the attenuation, and the reflectivity calculated with the EDF model approximate the results obtained with the full Biot-Stoll for a sandy sediment.

Methods

-The Fluid Model of reflectivity

The fluid model of reflectivity (FM) is a simple model in which both the water and the sediment are presumed to be fluids defined by three properties: compressional sound speed (c), attenuation (a) and density. The reflection coefficient in this model at normal incidence is defined in equation (1) below.

$$R = \frac{Z_{sed} - Z_{water}}{Z_{sed} + Z_{water}} \tag{1}$$

Where Z_{water} is the impedance of the water and Z_{sed} is the impedance of the sediment. In simpler terms, the reflection coefficient is simply the ratio of the incident acoustic pressure p+ and the reflected pressure p-.

$$R = \frac{p_{refl}}{p_{inc}} \tag{2}$$

At angles other then that of normal incidence, the reflection coefficient is found as a function of the impedance and the incident angle using Snell's Law,

$$R = \frac{Z_{sed}\cos(\theta_{inc}) - Z_{water}\cos(\theta_{refl})}{Z_{sed}\cos(\theta_{inc}) + Z_{water}\cos(\theta_{refl})} \quad (3)$$

-The Biot-Stoll Model

The Biot-Stoll model is more complex and takes into account that the sediment is actually a mixture of grains and water. "It was successful in explaining dispersion and has been shown to predict different levels of scattering from the rough sand interface than found in previous fluid and visco-elastic models."[1] In the EDF model, Williams develops an effective density and bulk modulus which were derived from the Biot model. An explanation of the process he used will be described herein.

In reference 5, Stoll uses potentials defined in terms of the skeletal frame (**u**) and the water (**U**). Biot developed his equations for scalar potentials from these equations.

$$-k^2 H \Phi_s + k^2 C \Phi_f = -\omega^2 \rho \Phi_s + \rho_f \omega^2 \Phi_f \quad (4)$$

$$-k^2 C \Phi_s + k^2 M \Phi_f = -\omega^2 \rho_f \Phi_s + \frac{\omega^2 \alpha \rho_f \Phi_f}{B} + \frac{i \omega F \eta \Phi_f}{\kappa} \quad (5)$$

Where k is the acoustic wave number, α is the tortuosity, η is the pore fluid viscosity, κ is the permeability, ρ_f is the pore fluid mass density, ρ_s is the sediment particle mass density, and therefore

$$k = \frac{\omega}{c}, \quad \omega = 2\pi f$$

$$\rho = B\rho_f + (1 - B)\rho_s$$

$$H = \left[\frac{(K_r - K_b)^2}{D - K_b}\right] + K_b + \frac{4\mu}{3} \quad (6)$$

$$C = \frac{K_r (K_r - K_b)}{D - K_b}$$

$$M = \frac{K_r^2}{D - K_b}$$

$$D = K_r\left(1 + B\left(\frac{K_r}{K_f} - 1\right)\right)$$

The parameter F is the deviation from the Poiseuille flow as frequency increases. [1]

-Effective Density Model (EFD)

In the EFD model , Williams derives an effective density and bulk modulus from the Biot model by introducing the simplfication that the frame parameters Kb and mu are equal to zero. It is possible to make such an assumption because for sand sediments, the frame and shear moduli are much lower than the other moduli, rendering them insignificant.[1] Now equation (3) can reduce to

$$C := M = \frac{1 - B}{K_r} + \frac{B}{K_f}$$
$$H = C \qquad (7)$$

Williams observed that equations (4) and (5) could be simplified so that both equations are in the same terms (H). From these new equations a second order equation was derived.

$$k^2 = \frac{\omega^2 \rho_{eff}(\omega)}{H} \qquad (8)$$

This becomes

$$c = \sqrt{\frac{H}{\rho_{eff}(\omega)}} \qquad (9)$$

and $\rho_{eff}(\omega)$ is the effective density given by the following equation. [1]

$$\rho_{eff}(\omega) = \rho_f\left(\frac{\alpha(1 - B)\rho_s + B(\alpha - 1)\rho_f + \frac{i B \rho F \eta}{\rho_f \omega \kappa}}{B(1 - B)\rho_s + (\alpha - 2B + B^2)\rho_f + \frac{i B F \eta}{\omega \kappa}}\right) \qquad (10)$$

-Description of Parameters

To better understand the contributions of the different parameters, a short explanation of each one will be given here. *Porosity (β)* describes the amount of area occupied by the volume of the pores. It is expressed as the volume of the pores to the total volume of the element.[9] The *Mass density of* *sand grains (ρ_s)* is the measure of the density of individual particles within the element. The value does not vary much over a wide range of values, but the differences are notable at high frequencies. *Viscosity (η)* is a fluid property. It describes the degree to which a fluid resists flow under an applied force, measured by the tangential friction force per unit area divided by the velocity gradient under conditions of streamline flow.[8] Since the only fluid we will be dealing with is water, the value for viscosity will remain constant for all sediments. *Permeability (κ)* is the measure of the ability of a medium to conduct fluid flow.[9] Because permeability depends on frequency, it is possible that the values for permeability can vary quite a bit in the higher frequency range. *Tortuosity (τ)* compares the minimum path length of a contiguous path through the pore network to the straight line path.[9] *Pore size (a_{pore})* is the size of the pore between the grains of sand. This plays a significant role in the reflectivity and dispersion of sound at a water-sediment interface as the pore size and wavelength get closer to the same order of magnitude. We will find that pore size in sand, which is large in comparison to that of clay, plays a part in the phenomenon witnessed at higher frequencies. *Bulk modulus of water* is $K_f = c2*\rho_f$, where ρ_f is the mass density of the fluid in the pores. The *Bulk modulus of sand* (K_r) can sometimes be described as 'compressibility', and is believed to be the same for sand as quartz.[9]

Results

The models described above were coded into MatLab to ensure simple processing of the parameters presented in each of the models. Parameters for 5 different sediment types were obtained from both Williams and Hamilton. The sediment types chosen for this examination span a large variety of pore sizes. The properties for the five representative sediments are listed in Table1.

Parameters of Sediment Types												
	Mass Density of water (kg/m^3)	Bulk Modulus of Water (Pa)	Viscosity (kg/m/s)	Mass density of sand grains (kg/m^3)	Bulk Modulus of Sand grains (Pa)	Porosity	Permeability (m^2)	Tortuosity	Sound speed in water (m/s)	Density of water (g/cm^3)	Attenuation of water (dB/m/lambda)	Angle (degrees)
Williams' Sand	1000	2.25E+09	0.001	2650	3.60E+10	0.4	1.00E-10	1.25	1500	1	0	0-90
Hamilton's Sand	1000	2.25E+09	0.001	2710	3.60E+10	0.386	1.00E-10	1.25	1500	1	0	0-91
Hamilton's Silty Sand	1000	2.25E+09	0.001	2710	3.60E+10	0.528	1.00E-10	1.25	1500	1	0	0-92
Hamilton's Silt	1000	2.25E+09	0.001	2470	3.60E+10	0.606	1.00E-10	1.25	1500	1	0	0-93
Hamilton's Clay	1000	2.25E+09	0.001	2760	3.60E+10	0.775	1.00E-10	1.25	1500	1	0	0-94

Table 1: *This is a table representing the numbers used in the effective density coefficient program. The largest difference between each of the sediment types is the pore size.*

Sound impeding upon each new sediment interface exhibits unique behavior. Some consideration will be given to the behavior exhibited by each type of sediment, but for the purpose of this project, most focus will be given to the actual comparison of the two models.

The Fluid Reflection Model, in contrast, does not take as inputs the large number of parameters as does the EDF model. It is more concerned with two main parameters that help describe nature of the sound dispersion: The sound speed within the medium, and the attenuation. In this model, the impedance Z_{sed} is imaginary due to the nonzero attenuation in the sediment. This makes the wave number complex thus causing the sound speed to be complex which is a variable upon which Z_{sed} depends. This model also includes the angle of incidence (θ) and computes the transmission angle. (See Figures 2-6).

Discussion

Figures 1-5 show plots of each of the sediment types for both models. In the sand plots (Figs. 1 and 2), there is a critical angle above which almost 100% of the sound is reflected. One may notice a disparity in the representations. For example the EDF model does not show the critical angle as definitively as does the FM. This is because the EDF model has a stronger dependence on frequency.

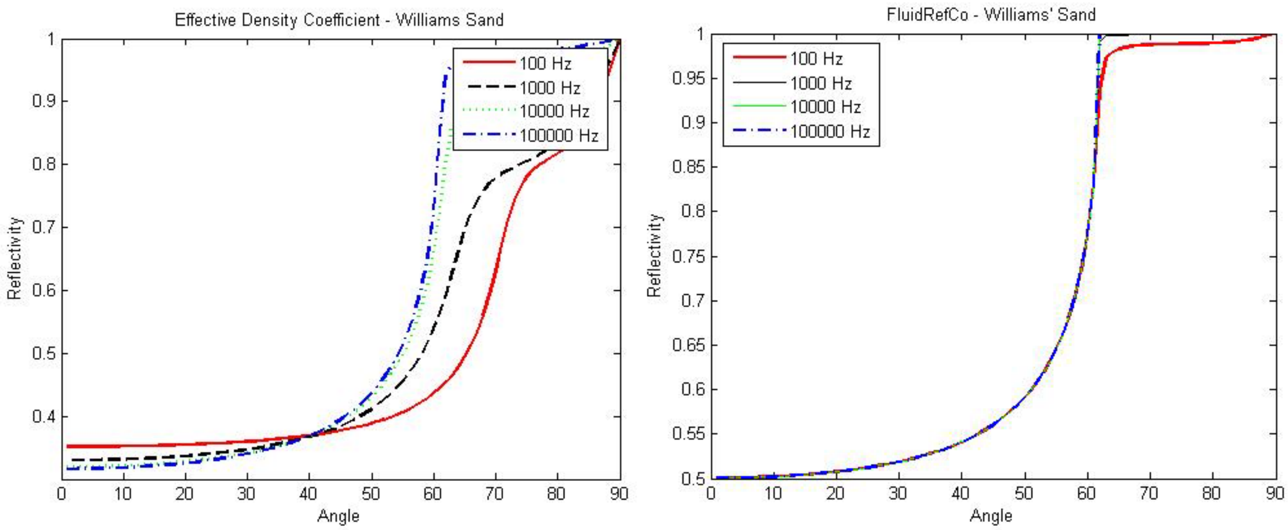

Figure 1: Modeled reflectivity values for the EFD (left) and fluid (right) models at four frequencies using the properties listed in Table 1 as Williams' Sand

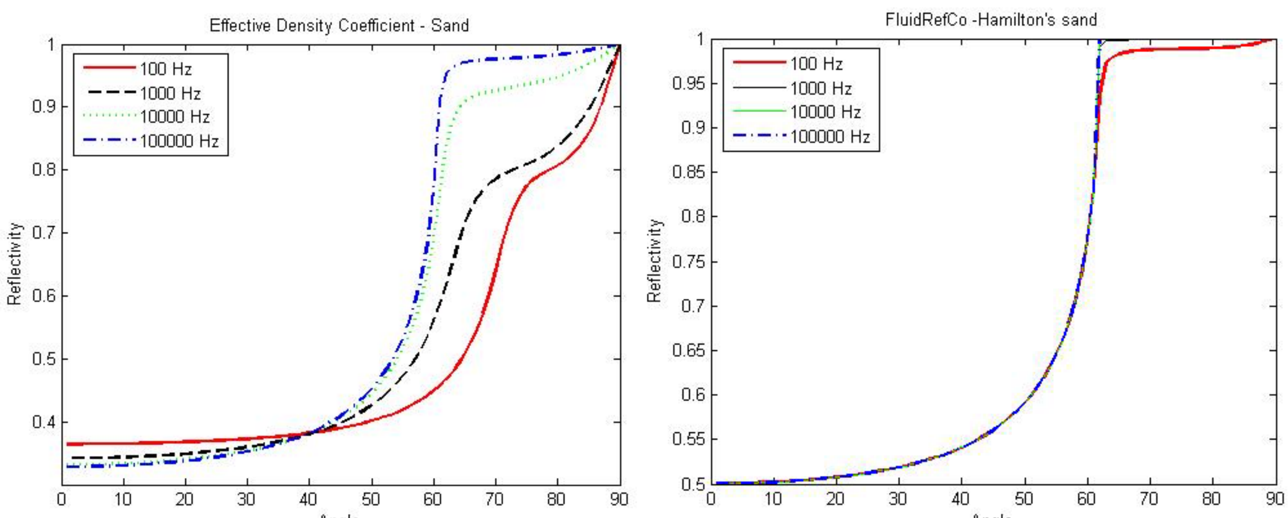

Figure 2: Modeled reflectivity values for the EFD (left) and fluid (right) models at four frequencies using the properties listed in Table 1 as Hamilton's sand.

At the interface for clay (Figure 3) there is an angle of intromission, meaning 100% of the sound is transmitted through the sediment at all frequencies. Again, the plots do not perfectly resemble each other. The EDF shows intromission at smaller angles for corresponding frequencies than does the FM.

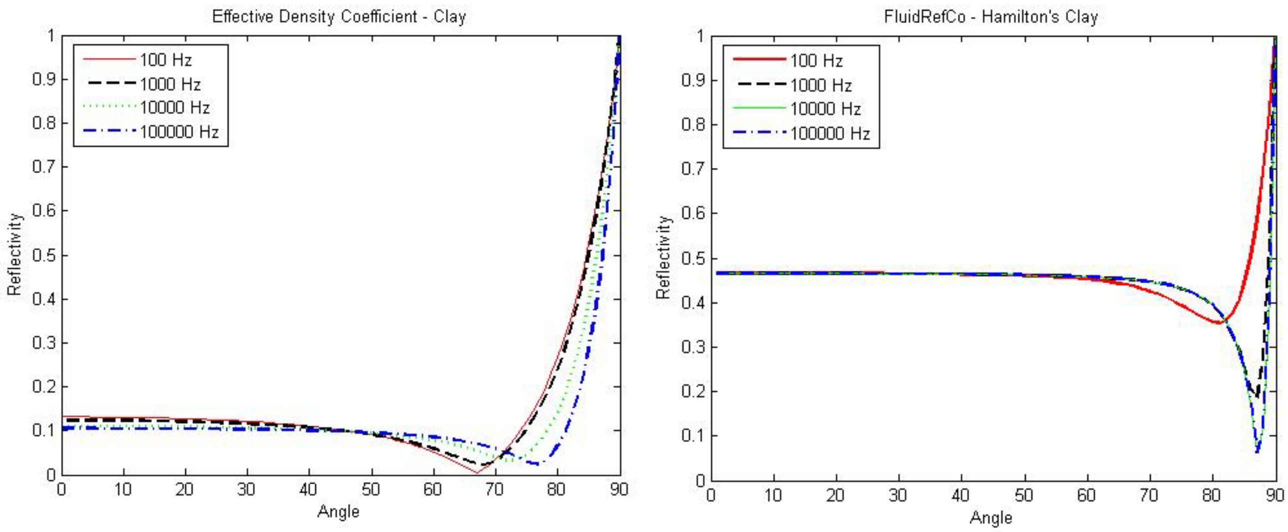

Figure 3: Modeled reflectivity values for the EFD (left) and fluid (right) models at four frequencies using the properties listed in Table 1 as Hamilton's Clay.

For silt and silty-sand, there is a critical angle for the high frequencies, and an angle of intromission at 100 Hz. This is likely due to the fact that for silty-sand and silt the wavelength of the sound is on the same order of the pore size. From this we learn that reflectivity is highly dependant on frequency.

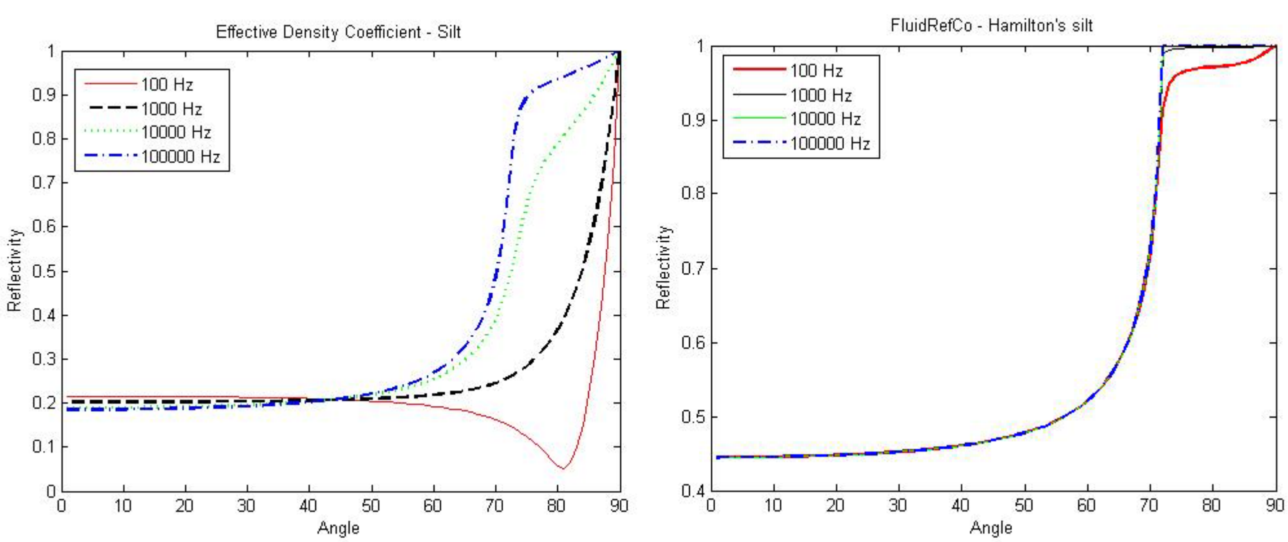

Figure 4: Modeled reflectivity values for the EFD (left) and fluid (right) models at four frequencies using the properties listed in Table 1 as Hamilton's Silt

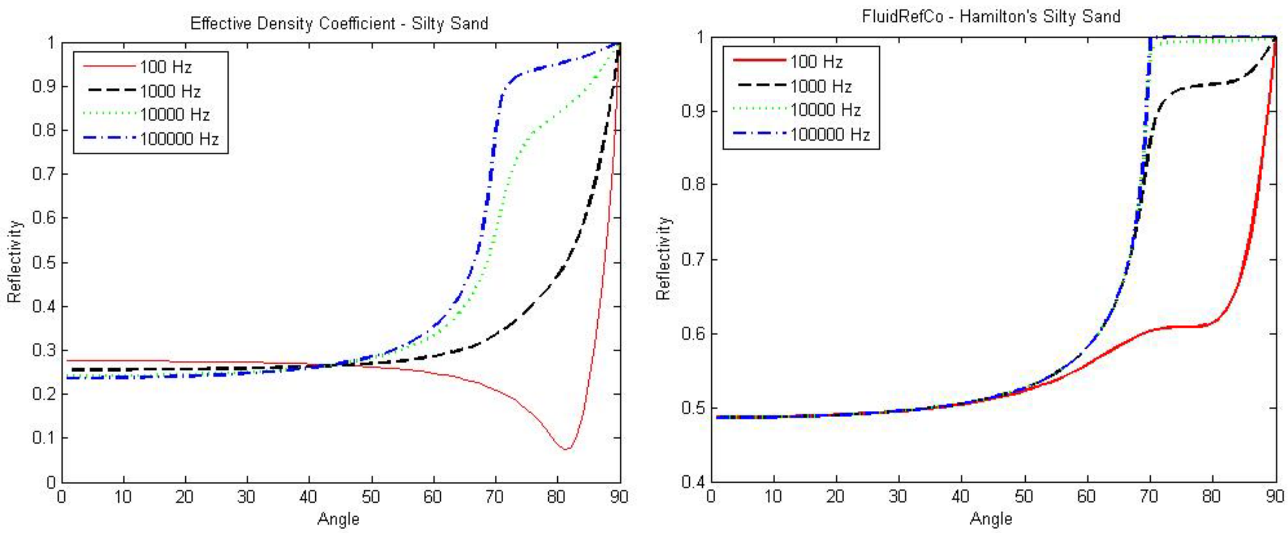

Figure 5: Modeled reflectivity values for the EFD (left) and fluid (right) models at four frequencies using the properties listed in Table 1 as Hamilton's Silty Sand.

Conclusion

Sound impeding on a water-sediment interface can exhibit very unique behavior. For high frequencies, there often is a critical angle of complete transmission. This is largely because the pore size of the sediment is somewhere on the same order as the frequency. For lower frequencies, there is, in many cases, an angle of intromission (all sound is transmitted into the sediment). Further understanding of this phenomenon would require a deeper analysis into the Biot slow wave.

The EDF and FM models both do a sufficient job in understanding the behavior of sound bouncing off of a water-sediment interface. Certain simplifications however, in the FM model reduce its level of validity. In each of the examples above it was significantly different then the corresponding Effective Density model. The main contribution to this difference is the fact that the Fluid Model only takes into account 3 main parameters and both the water and the sediment are presumed to be fluids defined by three properties: compressional sound speed, attenuation and density. Some critical information is lost by making such a hasty assumption.

References

[1] M. J. Buckingham, "A three-parameter dispersion relationship for Biot's fast compressional wave in a marine sediment," J. Acoust. Soc. Am. **116**, 769-776 (2004)

[2] K. L. Williams, "An effective density fluid model for acoustic propagation in sediments derived from Biot theory," J. Acoust. Soc. Am. **110**, 2276-2281 (2001)

[3] K. L. Williams and D. R. Jackson, "Bistatic bottom scattering: model, experiments, and model/data comparison," J. Acoust. Soc. Am. **103,** 169-181 (1998)

[4] N. P. Chotiros, "Biot model of sound propagation in water-saturated sand." J. Acoust. Soc. Am. **97**, 199-214 (1995)

[5] R. D. Stoll and T. K. Kan, "Reflection of acoustic waves at a water-sediment interface," J. Acoust. Soc. Am. **70**, 149-156 (1981)

[6] E. L. Hamilton, "Prediction of In-Situ Acoustic and Elastic Properties of Marine Sediments," Geophysics. **36**, 266-284 (1971)

[7] L. D. Hampton, "Acoustic Properties of Sediments." J. Acoust. Soc. Am. **42,** 882-890 (1967)

[8] http://dictionary.reference.com/search?q=viscosity

[9] M. Isakson and T. B. Nielsen, "The Relative Sensitivity of Reflection Loss on the Biot Parameters," Submitted to J. Acoust. Soc. Am. in 2005.

Appendix

Dispersion of Sediment Types			
Sand from Williams	**cp1 (km/s)**	**alpha1**	**Avg. attenuation**
100 hz	1.61E+05	-5.67E-05	
1000 hz	1.69E+05	-9.60E-04	
10000 hz	1.73E+05	-0.0036	
100000 hz	1.75E+05	-0.0118	
			-0.004104125
Sand from Hamilton	**cp1 (km/s)**	**alpha1**	**Avg. attenuation**
100 hz	1.62E+05	-6.15E-05	
1000 hz	1.70E+05	-9.75E-04	
10000 hz	1.74E+05	-0.0036	
100000 hz	1.75E+05	-0.0118	
			-4.11E-03
Silt from Hamilton	**cp1 (km/s)**	**alpha1**	**Avg. attenuation**
100 hz	1.50E+05	-2.73E-05	
1000 hz	1.55E+05	-9.63E-04	
10000 hz	1.59E+05	-0.0043	
100000 hz	1.60E+05	-0.0152	
			-5.12E-03
Silty Sand from Hamilton	**cp1 (km/s)**	**alpha1**	**Avg. attenuation**
100 hz	1.50E+05	-4.61E-05	
1000 hz	1.57E+05	-0.0012	
10000 hz	1.61E+05	-0.0048	
100000 hz	1.63E+05	-0.0165	
			-5.64E-03
Clay from Hamilton	**cp1 (km/s)**	**alpha1**	**Avg. attenuation**
100 hz	1.43E+05	-1.54E-05	
1000 hz	1.45E+05	-8.20E-04	
10000 hz	1.49E+05	-0.0049	
100000 hz	1.51E+05	-0.0188	
			-6.13E-03

-MatLab Code for FluidRefCo

```
%ELASREFCO Plane Wave Reflection Coef for Fluid/Elastic Interface

%-----------------------------------------------------------------
%Computes the plane wave reflection coeffecient for the Fluid/Elastic
%interface, based on work done by Brekhovskikh on pages 43 & 44.  It allows
%for complex Theta's.  Theta may be a vector, but currently, f may not.

%Usage:  R = ElasRefCo(Theta, f, rho1, cp, ap, cs, as, rho, c, a)
%   Theta - Angle of Incidence (from normal) in Radians
%   f - frequency in hertz
%   rho1 - density of elastic medium
%   cp - Compressional sound speed in elastic medium
%   cs - Shear sound speed in elastic medium
%   ap - Compressional attenuation in elastic medium (Np)
%   as - Shear attenuation in elastic medium (Np)
```

```
%   rho - density of fluid
%   c - Compressional sound speed in fluid
%   a - Compressional attenuation in fluid
%Note: He uses the exp(j[kr - wt]) notation... which is important in
%picking the standard for adding attenuation to the sound speed.
%----------------------------------------------------------------------

function R = FluidRefCo(Theta, f, rho1, cp, ap, rho, c, a)

if nargin == 7
     rho = 1.000;
     c = 1530;
     a = 0.0;
end

Theta=Theta*pi/180;  %Converts Theta to radians
%Convert sound speeds to their complex equivalents

w = 2.*pi.*f;

%Create complex sound speeds from attenuation data

%NOTE: Uses the exp(kr - wt) Notation!!!

k = w./c - i.*a;
kp = w./cp - i.*ap;
%ks = w./cs - i.*as;

c = w./k
cp = w./kp
%cs = w./ks;

%Calculate angles of transmission, via Snell's Law
Thetap = asin((cp./c)' * sin(Theta));
%Gammap = asin((cs./c)' * sin(Theta));

%Calculate Impedances of each wave

Z = (rho.*c)' * (1./cos(Theta));
Zp = kron((rho1.*cp)', ones(1,length(Theta))) ./cos(Thetap);
%Zs = kron((rho1.*cs)', ones(1,length(Theta))) ./cos(Gammap);

%Calculate Reflection Coef.

Rn = Zp-Z;
Rd = Zp+Z;
R = Rn./Rd;
```

-MatLab Code for EffDensCo

```
%ELASREFCO Plane Wave Reflection Coef for Fluid/Elastic Interface

%-----------------------------------------------------------------------
%Computes the plane wave reflection coeffecient for the Fluid/Elastic
%interface, based on simplification of Brekhovskikh's work done by Williams.
%It allows for complex Theta's.  Theta may be a vector, but currently, f
%may not.

%Usage:
%   Theta - Angle of Incidence (from normal) in Radians
%   f - frequency in hertz
%   rho - density of elastic medium
%   rhoeff - effective density
%   cp - Compressional sound speed in elastic medium
%   cs - Shear sound speed in elastic medium
%   ap - Compressional attenuation in elastic medium (Np)
%   as - Shear attenuation in elastic medium (Np)
%   rho - density of fluid
%   c - Compressional sound speed in fluid
%   a - Compressional attenuation in fluid
%Note: He uses the exp(j[kr - wt]) notation... which is important in
%picking the standard for adding attenuation to the sound speed.
% R is the reflection coefficient
% cp1 = computed sediment sound speed
% alpha1 = computed attenuation
%-----------------------------------------------------------------------
 function [R,cp1,alpha1] = EffDensCo(freq,Theta,para,cw,rhow,aw)

  %Convert sound speeds to their complex equivalents

beta =  para(6)     %Porosity
kappa = para(7) %permeability (m^2)
rho1 =  para(4)/1000 %Density of the grain (g/cm^3) (Specific gravity)
rhof =  para(1)/1000    %Density of the fluid within the sediment(g/cm^3)
(Specific gravity)
eta =  para(3)  %Absolute viscosity of the fluid (kg/m-s)
rmod = para(5)  %Bulk modulus of the grains (Pa)
fmod =para(2)   %Bulk modulus of the fluid (Pa)
alpha =  para(12)   %tortuosity (virtual mass parameter)

a = sqrt(8.*alpha.*kappa/beta)

%unit conversions
eta = eta *10;
rmod=rmod*10;
fmod=fmod*10;
kappa = kappa * 1e4;
a=a*100;

w = 2.*pi.*freq

epsilon = a.*sqrt(w.*rhof./eta)
```

```
rhotot=beta*rhof+(1-beta)*rho1

T = (-sqrt(i).*BESSELJ(1,epsilon.*sqrt(i)))./(BESSELJ(0,epsilon.*sqrt(i)))

F = (epsilon./4.*T)./(1-2.*i.*T./epsilon)

rhoeff = (rhof.*((alpha.*(1-beta).*rho1+beta.*(alpha-
1).*rhof+(i.*beta.*rhotot.*F.*eta./rhof./w./kappa))./(beta.*(1-beta).*rho1+(alpha-
2.*beta+beta.^2).*rhof+(i.*beta.*F.*eta./w./kappa))))

% calculate 'c and a' values (not water) in terms of the other twelve
% parameters (Biot)
num_angles=length(Theta);
H=((1-beta)/rmod+beta/fmod)^(-1)
%[vc1,vcs]= biotsoundspeed(freq,num_angles,para,cw)
vc1=sqrt(H/rhoeff)
%returns compressional and sheer, speed and attenuation

%Theta
freq, rhoeff, vc1/100, rhow, cw, aw

R = FluidRefCo1(Theta, freq, rhoeff, vc1/100, rhow, cw, aw);

%figure
%plot(Theta,abs(R))
%hold on
%plot(Theta*pi/180,abs(D),'--')

k1=w/vc1

cp1=w/real(k1)

alpha1=-imag(k1)
```