Dynamics of a Many-Spin System including Relaxation Effects : Phase Transitions, Spin Waves, and Magnetic Patterns

Soyoung Jung

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Manuel Berrondo, Advisor

Department of Physics and Astronomy

Brigham Young University

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ABSTRACT

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Soyoung Jung
Department of Physics and Astronomy
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We study the dynamics of multi-spin systems with energy dissipation with the Heisenberg model for anti-/ferromagnetism. Individual two-spin short-range interactions of magnetic dipoles give rise to coherent long-range behavior on a lattice structure. The spins are free to rotate and can arrange themselves in a parallel configuration in the ordered state. The local magnetic field acting on each spin arises as the result of the addition of nearest neighbor spins. Additional dissipative effects allow us to study the onset of ordered states as dynamical process. We include anisotropy to simulate the layered structure of the experimental samples and a long range interaction as an opposing force. As a result, we have been able to observe properties of magnetism in simulated 2D anti-/ferromagnetic lattice including the formation of domains, domain walls, spin waves, and magnetic pattern formation, which correlates well with experimental observations in thin magnetic films. We discuss how these results can sharpen the understanding of anti-/ferromagnetism and the dynamics of complex system by comparing the result with other complex system.

Keywords: many-spin system, spin dynamics, ferromagnetism, magnetic pattern, spin waves, magnetic domain walls

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Chapter 1

Introduction

1.1 Introduction to Spin Model for a Many-Spin System

Spin models refer to mathematical models used to explain anti-/ferromagnetism. Spin dynamics simulation using spin models has become a powerful tool for investigating dynamics of magnetic system. In 1953, Landau and Lifshitz introduced the basic dynamical equation for magnetization, based on phenomenological grounds that magnetic domains are formed by the interactions of spins. [1] In 1954, Gilbert [2] introduced a convincing form for the energy dissipation term to add on to the dynamical equation that Landau and Lifshitz had found. The combined form is now called the Landau-Lifshitz-Gilbert (LLG) equation, which is a fundamental dynamical system for applied magnetism. Since experiment cannot precisely observe dynamics of spin system, spin models provide a great way to study various magnetic properties.

We develop a spin model and simulate the dynamics of many-spin system using Heisenberg model with LLG equation, which is an essential stepping stone for studying ferromagnetism. The simulation not only reveals fascinating magnetic properties of anti-/ferromagnetism that are comparable to experimental results but also provides more insight to understand underlying principles of complex systems, such as the role of consensus/frustruation and pattern formation.

1.2 Single Spin Dynamics : LLG Equation

For a single spin, Landau-Lifshitz equation in dimensionless form is

$$\frac{d\mathbf{S}}{dt} = \mathbf{H} \times \mathbf{S} \tag{1.1}$$

where $\mathbf{H}$ is external field and $\mathbf{S}$ is a spin. This simply means that the spin precesses about the external magnetic field as its rotation axis. A simple analogy that can give more intuition is a pendulum rotating about the gravitational field described by

$$\Gamma = \mathbf{r} \times \mathbf{F} \tag{1.2}$$

where Γ represents the torque on the pendulum, $\mathbf{r}$ represents the position of the pendulum and $\mathbf{F}$ represents gravitational force exerted on the pendulum. The cross product in the equation represents the motion of the pendulum in the presence of gravitational field. Similarly a spin precesses around its external magnetic field. However, notice that this just gives the general idea of the motion of spin but cannot be completely analogous to the motion of a spin, since a pendulum can have two variables (position and velocity) but a spin only has one variable.

With equation 1.2, the spin precesses about its local magnetic field but never settles down to a fixed position. However, this is not the whole story of what happens in real life. Due to the transfer of energy from macroscopic motion to macroscopic thermal motion, there is always energy loss. Therefore, the friction or relaxation term that represents energy dissipation of spin so called Gilbert term is needed for the model [2]. After adding appropriate relaxation term, [2] the equation becomes

$$\frac{d\mathbf{S}}{dt} = \mathbf{H} \times \mathbf{S} + \lambda \mathbf{S} \times (\mathbf{S} \times \mathbf{H}) \tag{1.3}$$

The second term in LLG equation therefore represents energy dissipation or relaxation term. Now, the LLG equation becomes a nonlinear partial differential equation even in the case of a single spin variable. The method of solving the non-linear equation will be shown in a later chapter. (See Sec. 2.2)

LLG equation is valid both for single spin and many-spin system. Therefore, in the present work, we will use LLG equation combined with Heisenberg model so the spin simulation exhibits a complex behaviour.

1.3 Classical Heisenberg Model with LLG Equation

This Section will explain the concepts that constitute the classical Heisenberg model with LLG equation, including the concept of spin and structure of a many-spin system.

First thing to know is how spins are represented. In the classical Heisenberg model, spins are understood as tiny magnetic dipoles or tiny magnets of microscopic to subatomic dimensions that produce magnetic field. These magnetic dipoles (spins) are represented in form of vectors, which have direction and magnitude in 3D space. They are free to rotate in any direction. However, the length of spins is always preserved. These dipoles in form of vectors do not have the quantum property of spin and this is why it is called "classical" Heisenberg model.

Next ingredient in the Heisenberg model is the lattice structure. we use a discrete lattice to represent a many-spin system. The magnetic dipoles (spins) have fixed positions on the lattice structure. They are like arrows whose tails are pinned at each node of the lattice. The lattice can have any dimension, but we choose two dimensional square lattice to keep the model simple and yet have enough complexity to show rich dynamics of spins. (See Fig. 1.1) Therefore, the many spin system is represented by the array of spin vectors, which eventually forms 3 by nx by ny matrix where 3 comes from three components (x, y and z) of spins vectors, nx and ny represent the

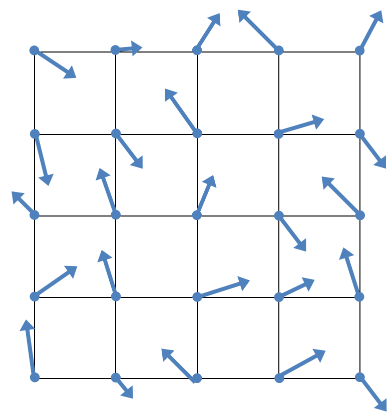


Figure 1.1 Spins are fixed on a two dimensional lattice structure but they are free to rotate. Different spin lengths appearing on the figure represent that the spins are three dimensional.

width and the length of the 2-d lattice.

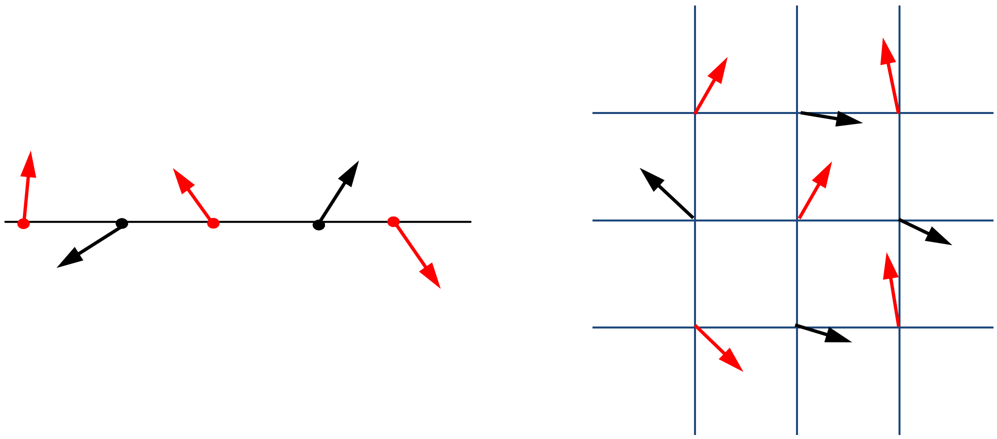
The Heisenberg model is governed by the Hamiltonian

$$\mathscr{H} = \sum_{ij} J_{ij} \mathbf{S}_i \cdot \mathbf{S}_j \tag{1.4}$$

where J_{ij} represents the characteristic parameter that determines anti- or ferromagnetism and $\mathbf{S}_j$ labels the four nearest neighbor (NN spins) for the target spin $\mathbf{S}_i$. The equation of motion that describes how a spin evolves in time is given by

$$\frac{d\mathbf{S}_i}{dt} = -J(\mathbf{H}_i \times \mathbf{S}_i) \tag{1.5}$$

where $\mathbf{S}_i$ represents a spin on the ith index of the lattice and $\mathbf{H}_i$ represents the local magnetic field for a spin. J is the characteristic parameter that determines whether the spin behaves as ferromagnetic or anti-ferromagnetic, assuming that it is the same for every pair of spins. It is important to notice that this equation preserves the spins length to be one as we mentioned before. The proof that the equation of motion preserves the spins length to be one is provided in the Appendix.



One dimensional lattice Two dimensional lattice

Figure 1.2 There are two NN spins for each spin on one dimensional lattice and four NN spins for each spin on two dimensional lattice. On the figure, black spins are NN spins for red spins, and red spins are NN spins for black spins.

Next we include the relaxation term based on the LLG equation, which gives

$$\frac{d\mathbf{S}_i}{dt} = -J[\mathbf{H}_i \times \mathbf{S}_i + \lambda \mathbf{S}_i \times (\mathbf{S}_i \times \mathbf{H})] \tag{1.6}$$

where λ in the added term is the energy dissipation term to determine how fast the energy is dissipated from the spin. Notice that adding relaxation term does not change the fact that the spins length is preserved.

As noticed in the equation of motion (LLG equation), the local field is what determines the motion of spins and it comes from the interaction among the spins in a many-spin system. Since each spin is a magnetic dipole, it produces its own magnetic field around it which affects nearby spins. At the same time, the spin is affected by the magnetic field produced by the nearby spins. Therefore, the local magnetic field is formed by the nearby spins. In the Heisenberg model, NN spins are chosen to form the local magnetic field to determine the motion of the spins, where NN spins refer to the spins that have the shortest distance from the given spin. For example, there are only two NN spins for each spin on one dimensional lattice. There are four NN spins for each spin for two dimensional lattice (North, South, East, West) and six NN spins for three dimensional lattice (North, South, East, West, Up and Down). (See Fig. 1.2) As spins are affected by the local magnetic field constituted with addition of NN spins, they tend to either align or anti align themselves with the local magnetic field depending on the property of the spins whether it is ferromagnetic or anti-ferromagnetic. This property is determined by the sign of J. Positive J gives the spins a ferromagnetic property and negative J gives spins an anti-ferromagnetic property.

The Heisenberg model with LLG correction is a highly non-linear and complex many-spin system. However, because of its high complexity and non-linearity, its analysis is challenging but rewarding due to rich dynamical structure it exhibits such as magnetic phase transition, domain formation, domain wall formation, spin waves, and magnetic domains patterns.

1.4 Overview

We simulate the dynamics of many-spin system with classical Heisenberg model with relaxation effect (LLG equation). We solve the nonlinear coupled equations of motion with stereographic projection which is the projection of a unit sphere on a stereographic plane. Stereographic projection of spins transforms the non-linear equation of motion into a linear equation and therefore gives the exact solution without any approximation error for the single-spin case. The stereographic projection and linearization of equation of motion will be explained and discussed in detail in Sec.2.2

The algorithm to calculate the dynamics of spins is simple to understand in that it only includes vector addition (finding local field) and rotation (precession of spins) operation. The algorithm for each time step will be further discussed in Sec.2.3.

Moreover, we include anisotropy to spins so that the model can simulate some thickness of magnetic material even though the model is only two dimensional. How we define and simulate anisotropy in detail can be found in Sec.2.4. We conclude that giving anisotropy effectively enables to simulate the surface of bulk material.

Also, we include long range interaction to observe magnetic pattern formation. The long range interaction represents the influence from all the spins on the lattice except for NN spins for our case. We use magnetic dipole-dipole interaction as the long range interaction and it reveals that long range interaction opposes the nearest neighborhood spin interaction. We consider these two opposing interactions to be consensus and frustration. Consensus implies that the spins completely agree in that they align or anti-align themselves (NN spin interaction only), whereas frustration implies that the two different interactions (NN spin interaction and long range interaction) oppose each other so that it causes the spins to exhibit a rich complexity and variety. In adding long range interactions, we choose only some spins to represent long range interaction to keep the simplicity of the spin model whereas in practice all the spins should be included. More details are discussed in Sec.2.5.

With all these tools, it is possible to simulated rich spin dynamics including phase change, domain formation, domain wall observation, spinwaves, and magnetic pattern formation. Even though each piece that constitutes the model is not new, the pieces are put together in a way that has never been put together before with beautiful simplicity. Despite the simplicity, this model describes properties of anti-/ferromagnetism and dynamics of spins very well. (See Fig. 1.3) Especially magnetic pattern formation favorably agrees with the experimental result of magnetic thin film pattern formation.

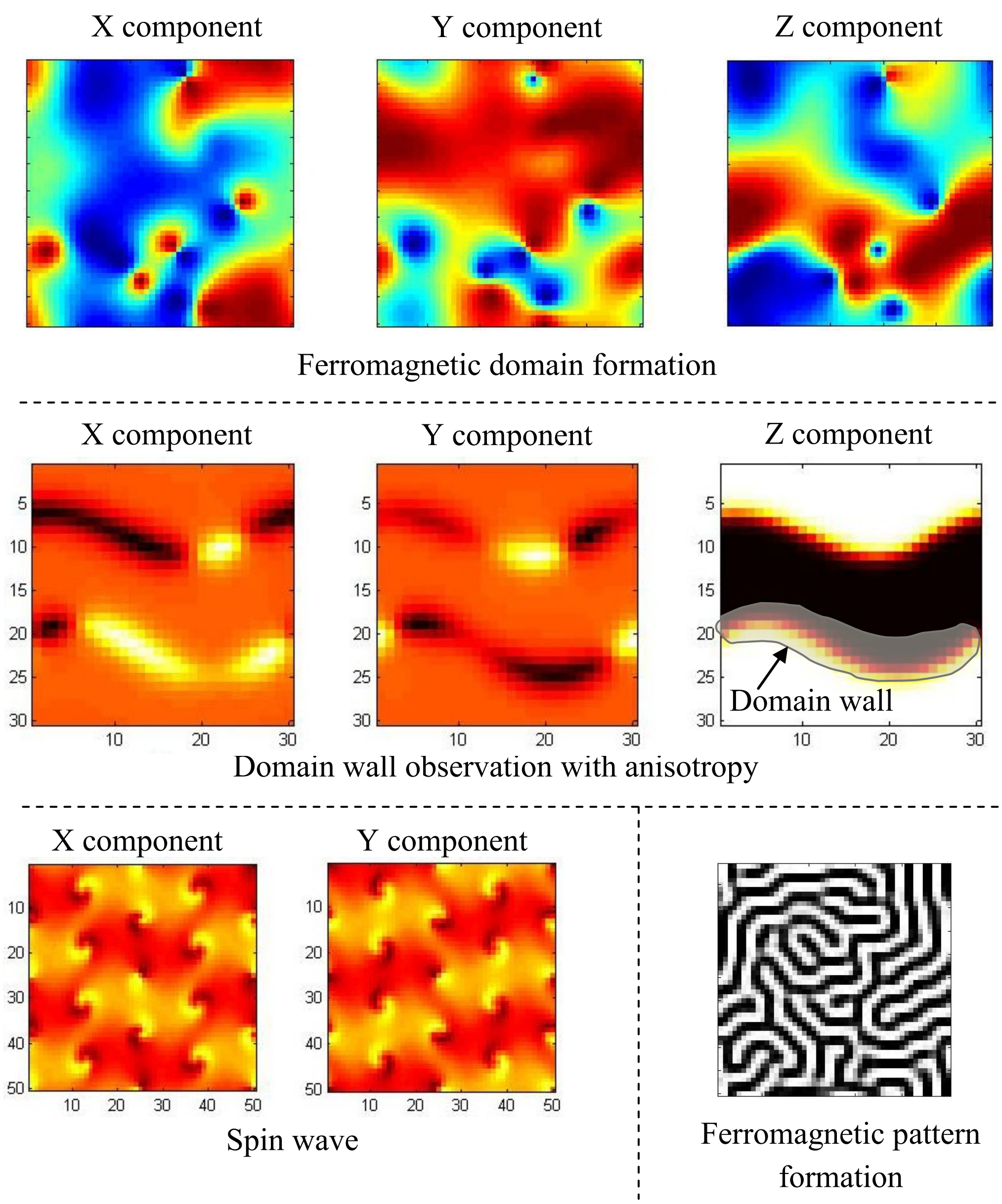


Figure 1.3 Results for spin simulation using the Heisenberg model with LLG equation. For anti-/ferromagnetic phase transition and domain formation, see Sec.3.1. For Domain wall observation, see Sec.3.2. For spin wave, see Sec.3.3. For ferromagnetic pattern formation, see Sec.3.4.

Chapter 2

Method

In this chapter we discuss how we solve the differential equation to present the dynamics of a many-spin system. We also include anisotropy and long range interaction to make the model more realistic and observe patterns that appear as a result.

2.1 Integration Method for the Equation of Motion without Energy Dissipation Term

The equation of motion with Gilbert energy dissipation term for a $\mathbf{S}_i$ on the lattice structure is

$$\frac{d\mathbf{S}_i}{dt} = -J[\mathbf{H}_i \times \mathbf{S}_i + \lambda \mathbf{S}_i \times (\mathbf{S}_i \times \mathbf{H}_i)] \tag{2.1}$$

where $\mathbf{S}_i$ is the spin on ith position of the lattice, $\mathbf{H}_i$ is local magnetic field for $\mathbf{S}_i$, and λ is a dissipation coefficient. Equation 2.1 is highly non-linear and is certainly complicated to solve. Therefore, before we explore how to solve this non-linear equation, it is desirable to solve the equation of motion without dissipation namely

$$\frac{d\mathbf{S}_i}{dt} = -J(\mathbf{H}_i \times \mathbf{S}_i) \tag{2.2}$$

This form of equation is what we eventually end up solving once we transform the non-linear equation of motion to a linear equation using stereographic projection. This will be explained in detail in later sections. In this section we discuss the solution for a linear differential equation for a many-spin system without energy dissipation term, since it provides the basic for solving the non-linear equation of motion.

Looking at the first order linear equation 2.2, it is easy to notice that we just need to integrate the left and right hand sides to get the solution for spin. However, before we start exploring the integration method, it is important to explain how we represent the spins on the lattice structure so that what we do is clear.

Spins are represented as vectors. They have direction and magnitude and are free to rotate in 3D space. That is, they have x , y and z components. Also spins must satisfy the basic assumption for classical Heisenberg model that the magnitude is always conserved and chosen to be one, or in other words $S_x^2 + S_y^2 + S_z^2 = 1$ where S_x, S_y S_z indicate each component of a spin. These spins are assumed to be fixed on a lattice structure. For our purpose, we choose a 2D square lattice so that the model can be relatively simple but still allows to describe the richness of the dynamics of the spins. This setting can be visualized as a two dimensional chess board with identical length arrows whose tails are pinned on each node and point in any 3D direction. (See Fig. 2.1) Those arrows are expressed as vectors with x,y, and z components with their orientation at each node of the lattice. Since each position of the node on the lattice is specified with indices, the whole lattice-arrows system becomes a big $3 \times nx \times ny$ matrix or array of vectors, where 3 comes from three components of spin vectors, nx represents the number of nodes along the width of the lattice, and ny represents the number of nodes along the length of the lattice. The many-spin system is now represented as a matrix. Representing spins on lattice structure with matrix form has a major advantage that it is possible to do the calculation all at once. That is, there is no need of going through the list of all the spins to solve the equation as each time step advances. Calculating the

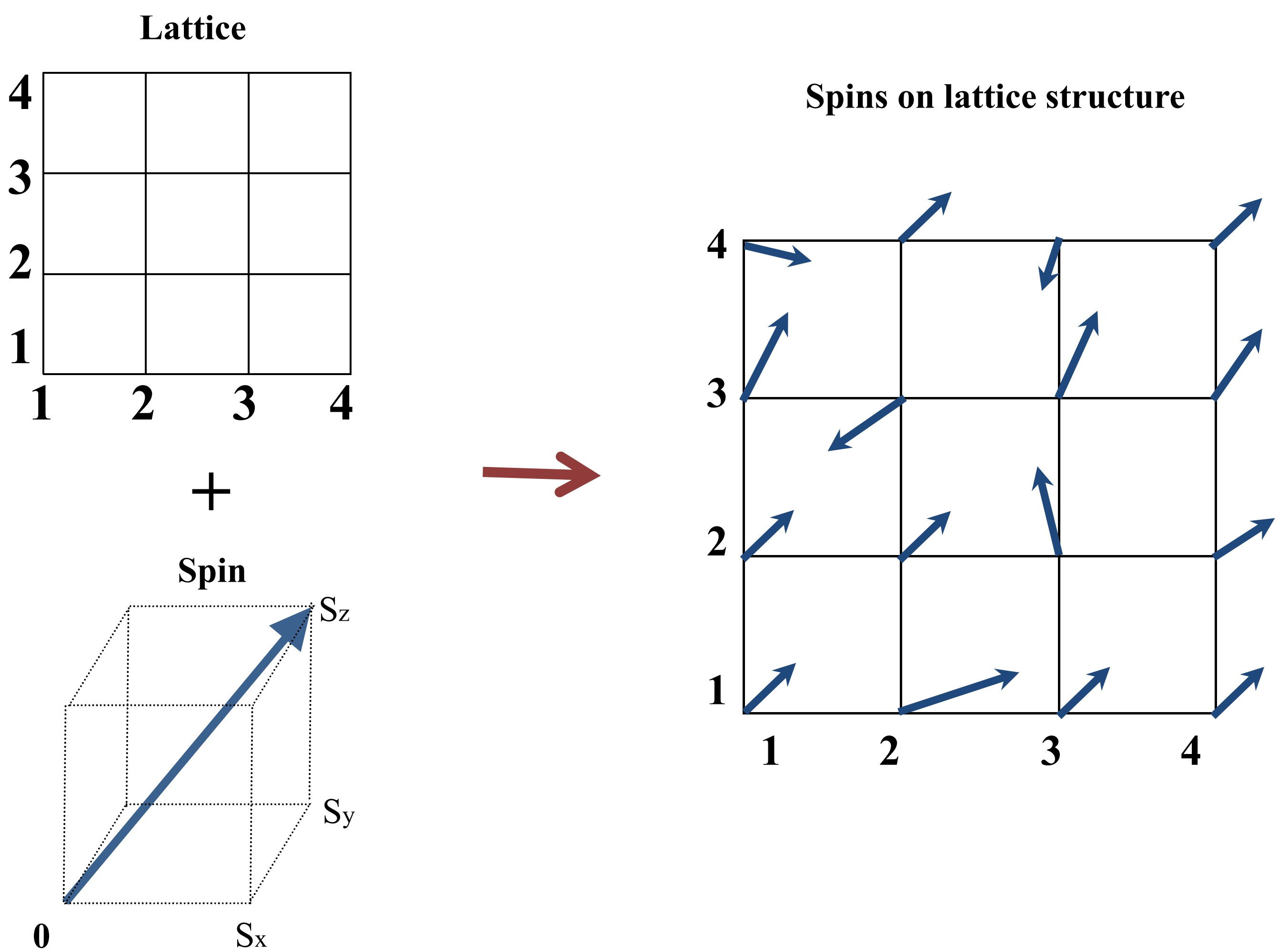


Figure 2.1 The lattice is 2D square lattice. Individual spin is a vector which has *x*,*y*, and *z* compoenet. Then the spins are pinned on each node of the lattice. Different length of spins indicates that the spins are not 2D object.

local magnetic field for each spin, integrating, and rotating for each spin can be done with one matrix operation without taking too much time.

Understanding how the spins are represented, it is now appropriate to look at the equation of motion. The dynamics of Heisenberg model without relaxation effect is governed by the equation 2.2. Defining $-J\mathbf{H}_i\times$ to be the operator ξ_i the equation becomes

$$\frac{d\mathbf{S}_i}{dt} = \xi_i \mathbf{S}_i \tag{2.3}$$

The solution for this differential equation is easy and well know to be

$$\mathbf{S}_i(t+\Delta t) = e^{\Delta t \xi_i}\mathbf{S}_i(t) \tag{2.4}$$

or

$$\mathbf{S}_i(t+\Delta t) = e^{-J\Delta t \mathbf{H}_i \times}\mathbf{S}_i(t) \tag{2.5}$$

Knowing that $d\mathbf{S}_i/dt = -J(\mathbf{H}_i \times \mathbf{S}_i)$ tells us that the spin precess about its local magnetic field (see Sec.1.2), it is apperent that the exponential factor in the solution for spin (Eq.2.5) represents the rotation of the spin about its local magnetic field through a time step. [3] The cross product maps a vector into another vector. Hence it can be expressed as 3×3 antisymmetric matrix

$$\hat{\mathbf{n}}\times = \begin{pmatrix} 0 & -n_z & n_y \\ n_z & 0 & -n_x \\ -n_y & n_x & 0 \end{pmatrix},$$

where n_x, n_y, and n_z represent the x, y, and z component of the vector $\hat{\mathbf{n}}$. The exponential of the corresponding matrix in Eq.2.5 gives us the rotation in matrix form and therefore the whole solution is now in matrix form.

The next step is to define and find the local magnetic field. This is made up of the magnetic field arising from nearest neighbor (NN) spins and the external field. The magnetic field arising

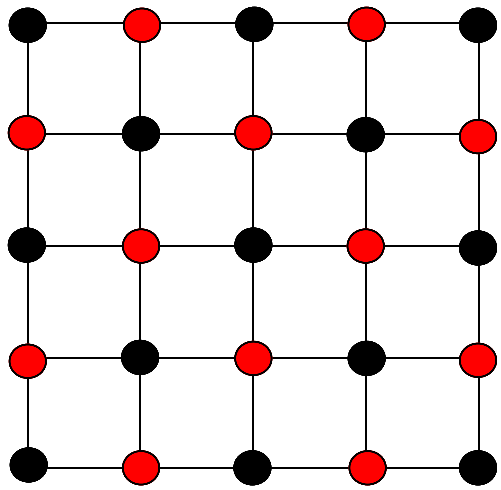


Figure 2.2 One of the sub-lattices includes all the NN spins on the other sub-lattice.

from NN spins is just the addition of NN spins. Therefore, the total magnetic field for each spin corresponds to the addition of NN spins plus the external field, namely

$$\mathbf{H}_i = \sum_{j}^{NN} \mathbf{S}_{ij} + \mathbf{H}_{ext} \tag{2.6}$$

where $\mathbf{S}_{ij}$ is the NN spins for $\mathbf{S}_i$ and $\mathbf{H}_{ext}$ is the local effective external field.

Once we have the solution for an individual spin $\mathbf{S}_i$, we need to apply the corresponding operator to every spin on the lattice in order to simulate the dynamics of the many-spin system. As the solution is applied to the lattice represented as a matrix, there is a trick to simplify the process. The trick consists of dividing the lattice into two sub-lattice structures. Dividing the lattice into two sub-lattices (black and red) provides a major advantages in that it provides an easier way to calculate the magnetic field from NN spins. Notice that each sub-lattice has all the NN spins in the opposite sub-lattice. (See Fig. 2.2) Therefore, finding the local magnetic field for the spins on a black spot in the figure can be achieved quickly by using the red sub-lattice points and *vice versa*.

However, dividing the lattice into two sublattices raises a different problem. Since the lattice is divided into two sublattices, $\mathbf{H}$ appearing in the solution

$$\mathbf{S}(t+\Delta t) = e^{-J\mathbf{H}\times}\mathbf{S}(t) \tag{2.7}$$

has to be separated into two parts. Notice that $\mathbf{S}$ and $\mathbf{H}$ corresponds to the spin matrix and the local magnetic field matrix with size $3 \times nx \times ny$. Since the local magnetic field is divided into two parts, there are two calculations to do in order to find the total $\mathbf{H}$ for each time step, as well as finding the final solution which involves putting the two parts together. Then, if two sub-lattices are labeled red and black, $\mathbf{H}_{total} = \mathbf{H}_{red} + \mathbf{H}_{black}$, and $\mathbf{S}(t+\Delta t) = e^{-J(\mathbf{H}_{red}+\mathbf{H}_{black})\times}\mathbf{S}(t)$. This, however, cannot be written as a product of exponential as $e^{-J\mathbf{H}_{red}\times}e^{-J\mathbf{H}_{black}\times}$, because the two exponents do not commute. [3] To solve this problem we approximate this as a product of three exponential operators valid to second order. That is,

$$e^{(\mathbf{A}+\mathbf{B})\tau} \simeq e^{\mathbf{B}\frac{\tau}{2}}e^{\mathbf{A}\tau}e^{\mathbf{B}\frac{\tau}{2}} \tag{2.8}$$

Now the solution becomes

$$\mathbf{S}(t+\Delta t) = e^{-J\frac{\Delta t}{2}\mathbf{H}_{red}\times}e^{-J\Delta t\mathbf{H}_{black}\times}e^{-J\frac{\Delta t}{2}\mathbf{H}_{red}\times}\mathbf{S}(t) \tag{2.9}$$

This exponential form preserves the magnitude of each spin vector. We are now ready to move on to solve the non-linear LLG equation.

2.2 Solving the Non-linear Landau-Lifshitz-Gilbert(LLG) Equation using Stereographic Projection

Now the energy dissipation (Gilbert relaxation term) is added and the equation of motion becomes,

$$\frac{d\mathbf{S}}{dt} = -J[\mathbf{H}\times\mathbf{S} + \lambda\mathbf{S}\times(\mathbf{S}\times\mathbf{H})] \tag{2.10}$$

Adding energy dissipation term makes it possible for the spins to stabilize at a certain orientation as time evolves because the energy dissipation term gives a damping effect. λ is the energy dissipation constant that determines how fast the energy is dissipated. Thus, larger λ means the spins will stabilize themselves faster. The equation of motion is not a linear differential equation

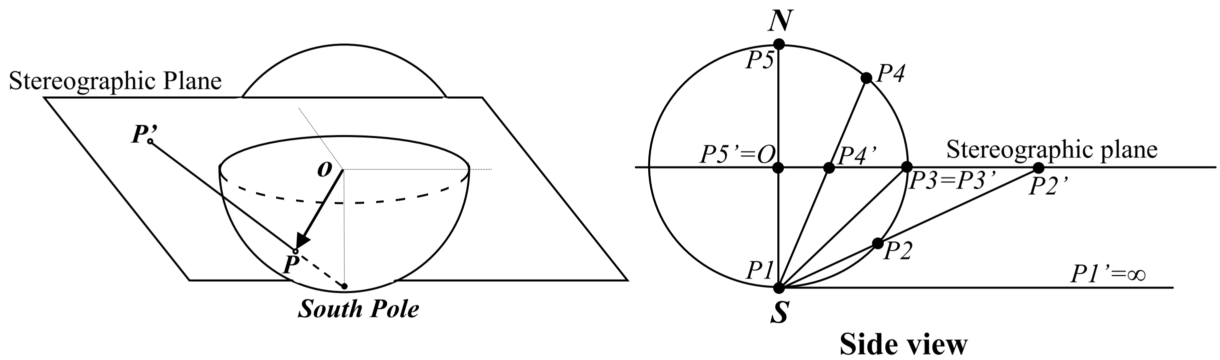


Figure 2.3 Stereographic projection is projection of a unit sphere onto the stereographic plane. The figure on the right is an example of stereographic projection of a spin, where $\mathbf{P}$ a spin and $\mathbf{P}'$ is the projected spin on the stereographic plane. The figure on the right is more examples of projection with side view of unit sphere and the stereographic plane. $\mathbf{P1}$, $\mathbf{P2}$, $\mathbf{P3}$, $\mathbf{P4}$ and $\mathbf{P5}$ are spins and $\mathbf{P1}'$, $\mathbf{P2}'$, $\mathbf{P3}'$, $\mathbf{P4}'$ and $\mathbf{P5}'$ are projected spins on the stereographic plane. Therefore, if a spin points to South pole ($\mathbf{P1}$), the projected spin onto the Stereographic plane is infinity ($\mathbf{P1}'$). On the other hand, if a spin points to North pole ($\mathbf{P5}$), the projected spins is zero ($\mathbf{P5}'$)

anymore. Therefore, to solve this non-linear equation we use a trick which is stereographic projection proposed by Lakshmanan and Nakamura [4],which reduces the non-linear damping term $\lambda\mathbf{S}\times(\mathbf{S}\times\mathbf{H})$ to a linear term. While Lakshmanan and Nakamura derive their solution by stereographic projection onto the complex plane, [4] we use a conformal transformation method because it gives a simpler form of the solution which also has the additional advantage of working with real quantities.

The stereographic projection maps a unit sphere onto the stereographic plane, which is the plane perpendicular to the local magnetic field that cuts through the origin of the unit sphere. When a spin is oriented from the origin of the unit sphere, the spin vector should exist somewhere on the surface of the unit sphere since the length of the spin is always one. Then the projected vector lies on the stereographic plane where the line that goes through both the South pole of the sphere and the spin vector pierces the stereographic plane. (see Fig. 2.3) Therefore, the following rules should hold.

1. If the spin vector points toward the North pole, the transformed 2d vector should be 0

2. If the spins vector points toward the South pole, the transformed 2d vector should be ∞.

3. If the spins vector points toward the equator of the sphere, the tip of the transform coincides with the tip of the spin vector.

A simple conformal transform that satisfies all those rules is

$$\mathbf{r}' = \frac{2}{\mathbf{r}+\hat{\mathbf{n}}} - \hat{\mathbf{n}} \tag{2.11}$$

where $\mathbf{r}'$ is the transformed 2d vector of the unit vector $\mathbf{r}$ given in terms of a fixed unit normal vector $\hat{\mathbf{n}}$ from the center of the sphere(orgin) to the North pole. Then we can use this conformal transformation to project the spins on the stereographic plane as is defined by $\hat{\mathbf{n}}$. Define

$$S^2 = \mathbf{S}\cdot\mathbf{S} = 1 \tag{2.12}$$

and

$$S_n = \mathbf{S}\cdot\hat{\mathbf{n}} \tag{2.13}$$

Also define the components of $\mathbf{S}$ parallel to and perpendicular to $\hat{\mathbf{n}}$

$$\mathbf{S}_{\parallel} = S_n\hat{\mathbf{n}} \tag{2.14}$$

and

$$\mathbf{S}_{\perp} = \mathbf{S} - \mathbf{S}_{\parallel} \tag{2.15}$$

Let ω be the transformed spin vector $\mathbf{S}$ and $\hat{\mathbf{n}}$ a unit vector for the local magnetic field, which also defines the stereographic plane. Then the conformal transform becomes

$$\omega = \frac{2}{\mathbf{S}+\hat{\mathbf{n}}} - \hat{\mathbf{n}} \tag{2.16}$$

which can be also written as

$$\omega = \frac{2(\mathbf{S}+\hat{\mathbf{n}})}{(\mathbf{S}+\hat{\mathbf{n}})^2} - \hat{\mathbf{n}} \tag{2.17}$$

Applying the definitions from above, we get

$$\omega = \frac{\mathbf{S}_\perp}{S_n + 1} \tag{2.18}$$

$$\mathbf{S} = \frac{2(\omega + \hat{\mathbf{n}})}{1 + \omega^2} - \hat{\mathbf{n}} \tag{2.19}$$

This will be used to recover the spins vectors after solving the linearized equation using the stereographic plane. The purpose of this transformation is to linearize the equation of motion in order to solve it. Therefore now we can use the stereographic projection to solve equation 2.10. We start by re-writing equation 2.10 with the BAC-CAB identity $\mathbf{A} \times (\mathbf{B} \times \mathbf{C}) = \mathbf{B}(\mathbf{A} \cdot \mathbf{C}) - \mathbf{C}(\mathbf{A} \cdot \mathbf{B})$. Then we have

$$\dot{\mathbf{S}} = -J(\mathbf{H} \times \mathbf{S} - \lambda(\mathbf{H} - S_n H \mathbf{S})) \tag{2.20}$$

where the dot notation is used for time derivative. Also from equation 2.18 we have

$$\dot{\mathbf{S}} = \dot{S_n}\omega + (1 + S_n)\dot{\omega} + \dot{S_n}\hat{\mathbf{n}} \tag{2.21}$$

Taking the dot product of $\hat{\mathbf{n}}$ with equation 2.20 gives, after some simplification

$$\dot{S_n} = \lambda H(1 - {S_n}^2) \tag{2.22}$$

Now we can substitute equation 2.21 with equation 2.22 and equation 2.20 to get

$$-\mathbf{H} \times \mathbf{S} + \lambda(\mathbf{H} - S_n H \mathbf{S}) = \lambda H(1 - {S_n}^2)\omega + (1 + S_n)\dot{\omega} + \lambda H(1 - {S_n}^2)\hat{\mathbf{n}} \tag{2.23}$$

After simplification and applying equation 2.14 and equation 2.15 we are left with

$$\dot{\omega} = -J\mathbf{H} \times \omega - \lambda H \omega \tag{2.24}$$

which is the stereographic differential equation for spins including the relaxation effect. Equation 2.24 is a linear differential equation and also is in the form that we are familiar with. Using the method introduced in the previous section, it is not a hard task to find the solution for ω. The solution is

$$\omega(t + \Delta t) = e^{-\Delta t J H \hat{\mathbf{n}} \times} e^{-\lambda H \Delta t} \omega(t) \tag{2.25}$$

2.3 Dynamics of Spins

This section will explain how the dynamics of spins is obtaining using the algorithms that we found. It will be a short section but an important one. All the elements are laid down and ready to describe the dynamics of the spins so it is next possible to find the dynamics of the spins. The initial condition for all the spins in the lattice, the time step, the external magnetic field, and the characteristic parameter J are assumed. We impose periodic boundary condition in general and one time step refers to the cycle of finding local field for the spins and rotating them according to the local field. Then the final algorithm is the following.

1. the initial spin values on the lattice are given.

2. the local magnetic field for each spin can be found by adding the NN spins for each spin. Here we take advantage of the separation of the lattice into two sub-lattices to move all the spins in one of the sum-lattices simultaneously.

3. find the corresponding $\hat{\mathbf{n}}$ given its each local magnetic field.

4. transform spins to ω with stereographic projection using equation 2.18

5. using the solution that we found equation 2.25, calculate new value of ω for given dt.

6. find the new spins using equation 2.19.

Repeating this procedure will reveal the dynamics of the spins. The result can be expressed in different ways. In our research, the result is presented as plots and animations to describe and study the dynamics of spins.

2.4 Anisotropy

The lattice structure that we choose to work with is two dimensional. This is because it keeps the model quite simple but still provides enough intricate to represent the complex motions and richness of a many-spin system. However, in reality there is no such thing as two dimensional many-spin system since there should be inevitably some thickness for any kind of magnetic material. Even for a very thin plate of magnetic material, the thickness is much larger than the diameter of an atom, which means magnetic materials –even a very thin plate– should be considered as a three dimensional object. Therefore, it is appropriate to include some thickness in the spin model to give more reality. Adding additional dimension to make three dimensional lattice could be done and the main features of the model such as the method of getting local magnetic field and the equation of motion are still the same and valid. However, it would also make the model more complicated and reduce the simplicity which is one of the main advantages of the model. Therefore, we instead add anisotropy to simulate some thickness of the material.

Consider ferromagnetic materials in a form of thin plate. That is, length and width is much larger than the thickness of the plate. Consider a system where there is no external field present. Due to the thin plate structure of the material, there are many more spins along x and y direction than in z direction. Therefore, much more energy would be consumed to align all the spins in x or y direction altogether than it takes for the spins in z direction to align altogether. Hence it is easier for the spin to align themselves oriented in direction normal to the surface (z direction) since it takes less energy to do so. Therefore, there exists an anisotropic effect for those thin materials in the direction normal to the surface. In other words, we are simulating the surface of a bulk material with anisotropy.

We first need to define what anisotropy exactly means in this spin model: spins have a tendency to align themselves into a specific direction rather than the others. In the case of thin material, the anisotropy is in z direction, normal to the surface.(see Fig.2.4) Now the question is how anisotropy can be represented in the lattice structure. Simply speaking, anisotropy means that the spins tend to be influenced more from one specific direction of the lattice. In this specific case the z component of local magnetic field is the more influential component of the local field . That is, spins feel z component of the magnetic field to be stronger than what it really is. Therefore, if the magnetic

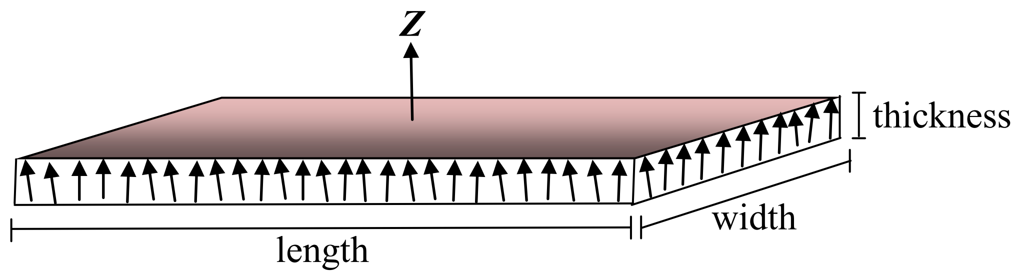


Figure 2.4 For a plate-like magnetic material, length and width of the material is much greater than the thickness of the material. Therefore, it is much easier for the spins to align themselves in the z direction (ferromagnetic case). Therefore, we are able to simulate the surface of the bulk material with thin thickness.

field for isotropic many-spins system is

$$\mathbf{H} = \begin{pmatrix} H_x \\ H_y \\ H_z \end{pmatrix},$$

where H_x, H_y, and H_z represents x, y, and z components of magnetic field, what anisotropic spins will "feel" is

$$\mathbf{H}_{anisotropy} = \begin{pmatrix} H_x \\ H_y \\ kH_z \end{pmatrix},$$

where k is a real number larger than 1. Therefore in our simulation, anisotropy can be achieved by adding weight toward z component of the local magnetic field. This will add anisotropy in z direction. Anisotropy in other direction is also possible by changing the weight of the other components.

2.5 Long Range Interaction

In addition to anisotropy, long range interaction is also needed in a more realistic model. The spins in the classical Heisenberg model are governed by the nearest neighbor interaction. This means that we consider only four nearest neighbor spins to be involved in the dynamics of each spin on the lattice and all other spins on the lattice are ignored. However, all those ignored spins also form their own magnetic field around them and it should affect other spins as well even though the magnitude of the influence should be much less than that of NN spins. Since the number of other spins is much larger than just four NN spins, even very weak interaction from non-NN spins can be add up to something that matters for the dynamics of spins. Therefore, including the interaction from non NN spins is also important to build a more realistic many-spin model along with anisotropy.

Given that the spins we are considering are really magnetic dipoles, we choose a magnetic dipole-dipole interaction to present the long range term. Consider the equation for magnetic dipole-dipole interaction expressed as

$$H = \frac{C}{{r_{ij}}^3}[3(\mathbf{S}_i \cdot \hat{\mathbf{r}}_{\mathbf{ij}})(\mathbf{S}_j \cdot \hat{\mathbf{r}}_{\mathbf{ij}}) - \mathbf{S}_i \cdot \mathbf{S}_j] \tag{2.26}$$

where C is a constance, $\mathbf{r_{ij}}$ is the vector between two dipoles at positions i and j, and $\mathbf{S_i}$ and $\mathbf{S_j}$ are dipoles at positions i and j on the lattice. With anisotropy in z direction the spins in the lattice tends to point either in the z or $-z$ direction. Therefore the dot product part becomes negligible because the vector $\mathbf{r}$ lies on the lattice surface and vector $\mathbf{S}$ points perpendicular to the surface of the lattice. Now there is only one term left in equation 2.26 and this becomes

$$H = -\frac{C}{r_{ij}^3}\mathbf{S}_i \cdot \mathbf{S}_j \tag{2.27}$$

Comparing this with the Hamiltonian for the regular classical Heisenberg Model, which is

$$\mathscr{H} = \sum_{ij} J_{ij}\mathbf{S}_i \cdot \mathbf{S}_j \tag{2.28}$$

reveals that they have the same form but with different coefficient. First of all, long range interaction has a factor of $\frac{1}{{r_{ij}}^3}$ which means that the strength of long range interaction is less than the NN spins interaction with factor of ${r_{ij}}^3$. We do expect the long range interaction to be a minor correction for the dynamics of the spins. Also notice that there is an extra minus sign. In the equation of motion, J is the characteristic parameter that multiplies the dot product to indicate if the model is ferromagnetic or anti-ferromagnetic. Then, the minus sign in the long range interaction term indicates that, for a ferromagnetic material, the long range interaction is anti-ferromagnetic. Therefore, adding long range interaction will have an antagonistic effect in the ferromagnetic model which we call "frustration". This frustrated system exhibits a richer structure and dynamics than the model without long range interaction.

Chapter 3

Result

In this section we will discuss our findings.

3.1 Phase Transition

An exciting moment during the development of this work was when the spin model with energy dissipation term showed the anti-/ferromagnetic phase transition for the first time. As explained in a previous chapter, the spins on the lattice structure stabilize themselves to equilibrium state where all spins align or anti-align themselves depending on the sign of J. This phase transition with domain formation is easy to observe by assigning colors to the numerical values of each of the spins on 2d lattice. Each component of spin has a numerical value between -1 and 1, since the magnitudes of spin vectors are chosen and conserved to be one. Therefore, taking advantage of the plotting options in Matlab that assigns different colors to different numerical values, each component of each spin is plotted separately with different colors according to its numerical value to show the formation of domains and magnetic phase transitions. Gray scale, hot scale, and jet scale are used for convenience in different occasions. The color bar in the figure shows the assigned values and the corresponding color. (See Fig. 3.1)

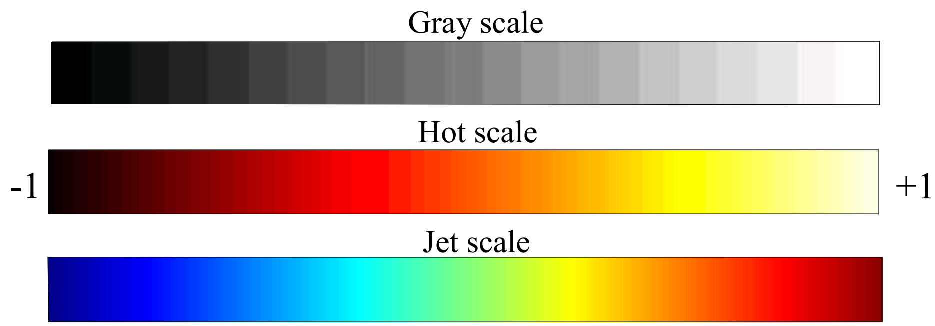


Figure 3.1 Each component of each spin on the lattice structure will be plotted with different color according to its different numerical value between $+1$ and -1. Gray scale, hot scale, and jet scale are used for convenience in different occasions.

In the spins model without any anisotropy or long range interaction involved, the model shows ferromagnetic property with $J > 0$. Starting from a random configuration, there is no distinct region where the spins form similar orientation (domains). As time steps advance, the spins start to form little patches of regions where they align themselves together, making small domains first. Those domains grow larger and combine themselves and eventually form larger domains for the lattice as a whole. With a very long running time, the lattice eventually becomes one big domain. This should be the expected result since it makes sense that the spins achieve the minimum energy state when they align themselves all together with ferromagnetic tendency. (See Fig.3.2)

Another case is when $J < 0$. (See Fig. 3.3) As explained before, this means that the spins have anti-ferromagnetic property. The model again starts from the random configuration. Soon it is apparent that the spins start to anti-align themselves to each other to form chessboard-looking configuration. In this case, with anti-ferromagnetic property, there are no domains formed. As time step advances, the lattice becomes a complete chessboard, which does makes sense again in terms of energy minimization that the lattice as a whole achieves lowest energy state as they all anti-align themselves to each other.

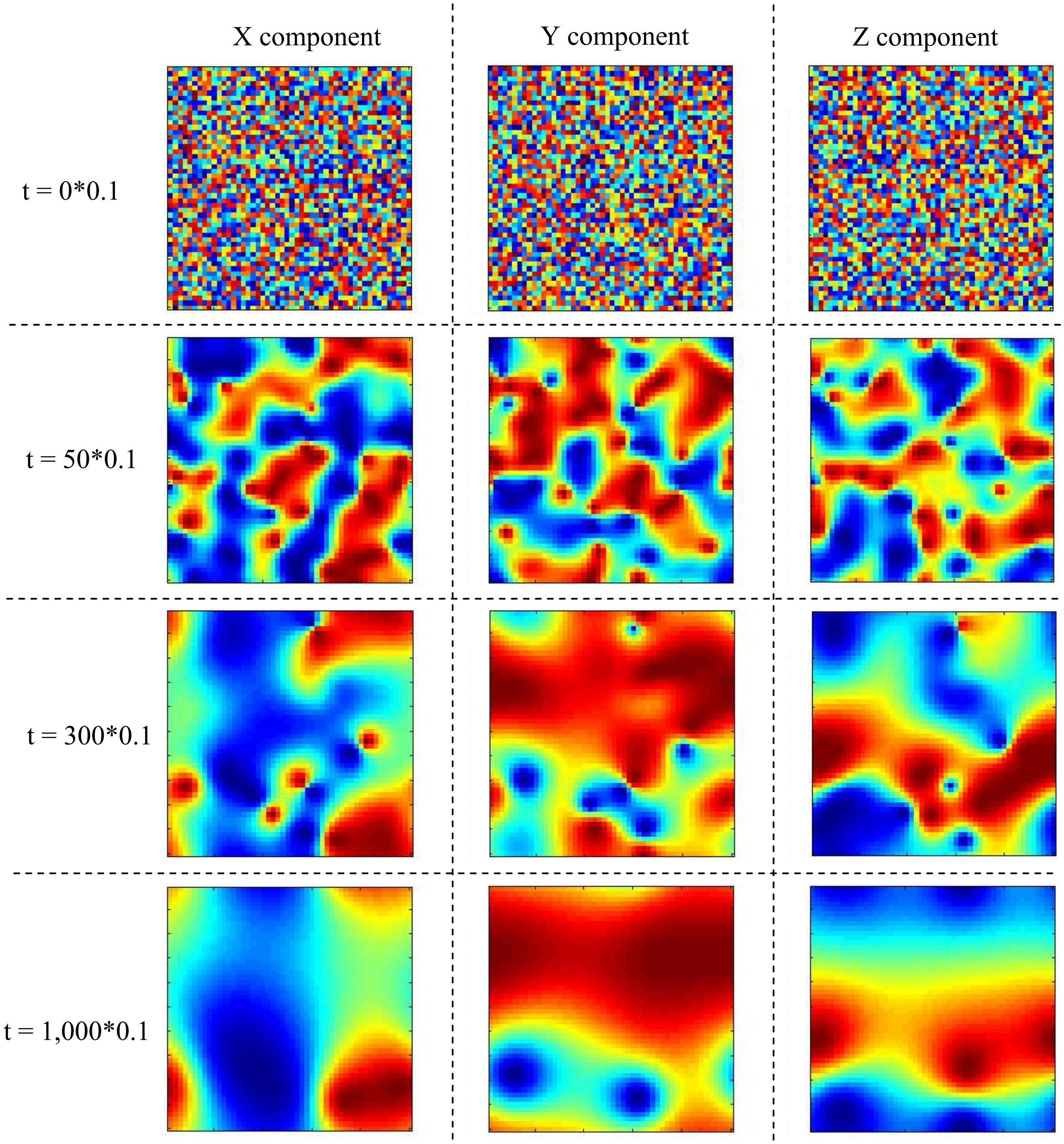


Figure 3.2 $J = 1$, $\lambda = 1$, no anisotropy, and no external field, 50×50 lattice. Each plot shows x, y, and z components of spins at times steps of 0, 50, 300, 1,000. Starting from completely random configuration without any domains formed, spins form small domains at the beginning. Later on, those domains combine themselves together to form larger domains, showing ferromagnetic property.

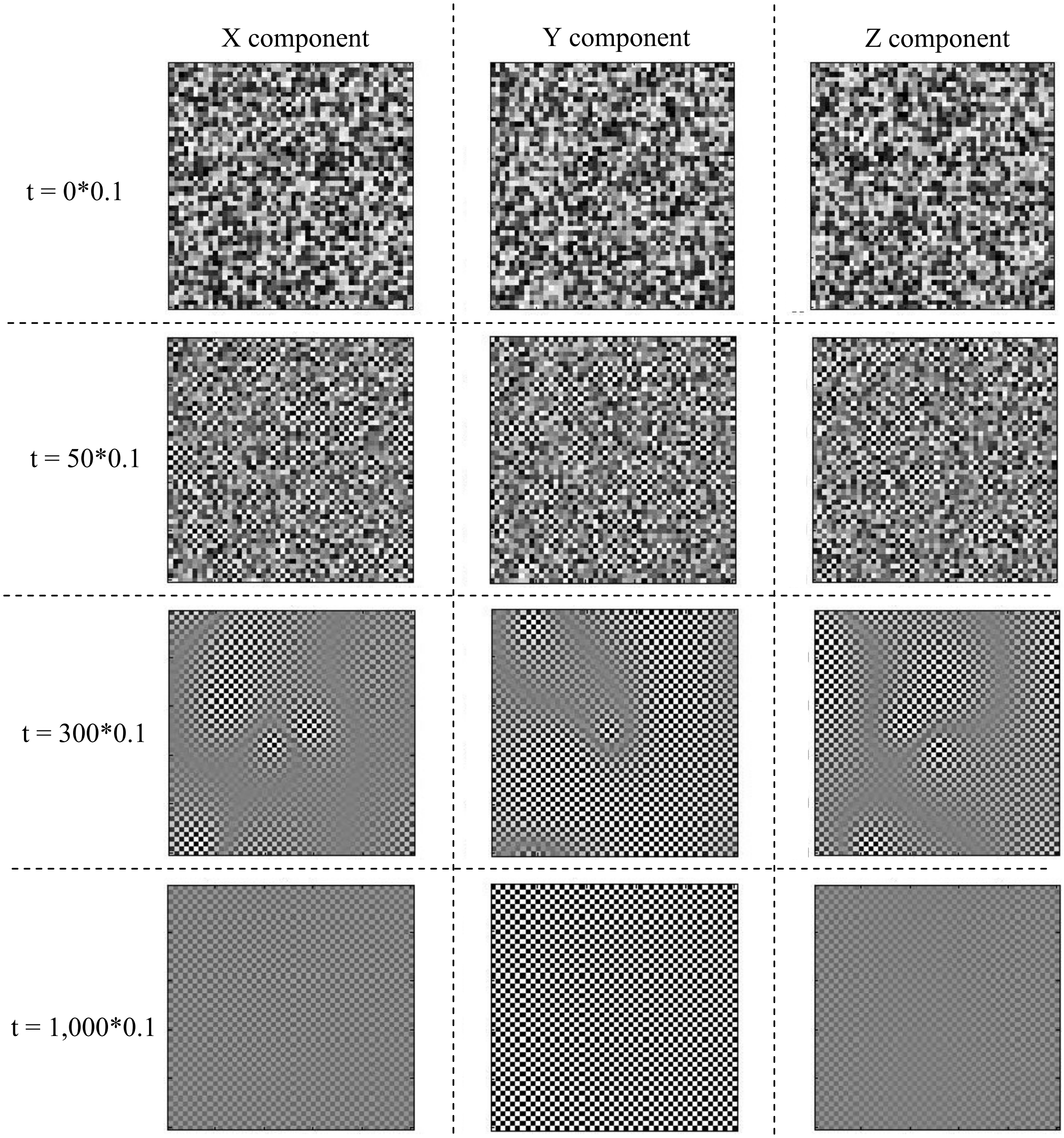


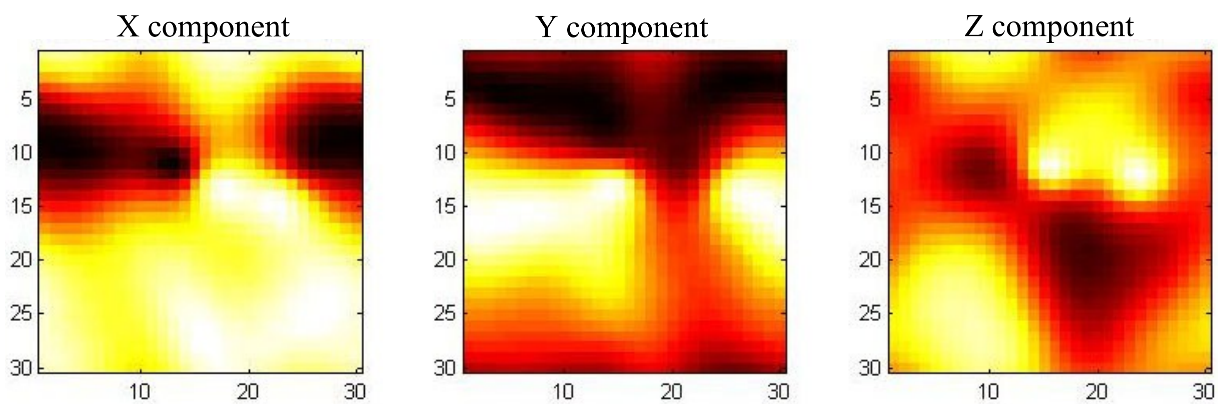
Figure 3.3 $J = -1$, $\lambda = 1$, no anisotropy, and no external field, 50×50 lattice. Each plot shows x, y, and z components of spins as time evolves. Starting from completely random configuration, spins anti-align themselves as time evolves and eventually completely anti-align with each other.

3.2 Domain Wall Formation

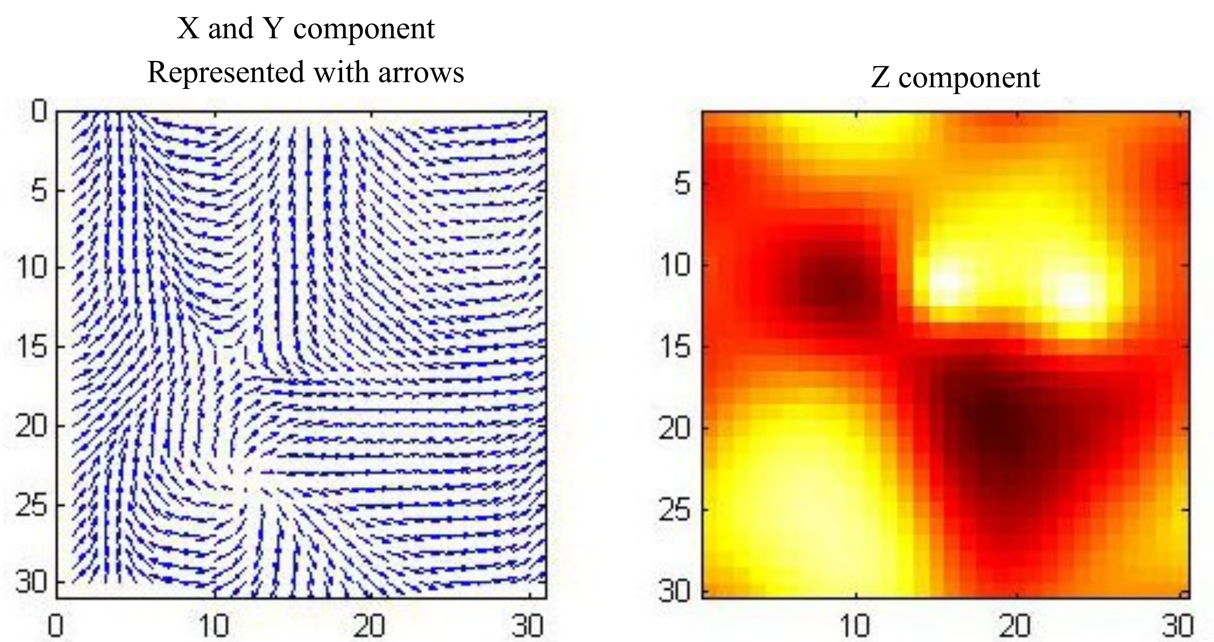
Domains are formed when $J > 0$. Between these domains, there exist relatively narrow regions that mark the boundaries of the domains. They are called 'domain walls'. It is also an interesting task to observe domains walls from the spin model. First of all, consider an isotropic case where there is no simulated thickness in the model. Observing each component of spins, it is apparent that each component form domains. However, it is very subtle and not easy to determine and recognize domains in the three dimensional sense where all three components are combined together. Domains walls are also hard to recognize since domains in each component of the spins usually occur in different places. It is possible to observe how the domains look like in the isotropic case with the quiver command in Matlab so that x and y components of the spins are plotted together with form of arrows that have magnitude and direction plotted next to the z component of spins. However, it is still too subtle to observe and pin out clearly where domains and domain walls are. (see Fig.3.4)

Even though domain walls in the isotropic case are too subtle to determine, anisotropic case is different. Since spins have a tendency to align themselves into a certain direction, it is much easier and apparent to observe domains walls in the anisotropic case. (see Fig.3.5)

One important role of domain walls is determining the dimension of the lattice it represents. There are two different kinds of domains walls: Neél wall and Bloch wall. Neél wall is where the spins rotate on the same plane as they change their orientation from one domain to another domain and it indicates that the lattice structure is two dimensional. On the other hand, Bloch wall is where the spins do not lie on the same plane as they rotate their orientation from one domain to the other domain and hence it indicates that the lattice is three dimensional.(See Fig.3.6) Therefore, if the spins model with anisotropy truly can simulate the surface of bulk material, that is, if the lattice can be considered to simulate thin three-dimensional material, the domains walls should be Block walls. Therefore, observing domain walls can tell us whether adding anisotropy indeed represent



(a) Isotropic case



(b) Isotropic case with x and y component of spins represented with arrows

Figure 3.4 $J = 1$, $\lambda = 1$, no anisotropy, no external field. (a) It is apparent that domains are formed, but it is hard to tell where the exact domain walls are. (b) x and y components are represented as arrows to give more idea how the domains look like for isotropic case. However, it is still hard to point out where exact domain walls are.

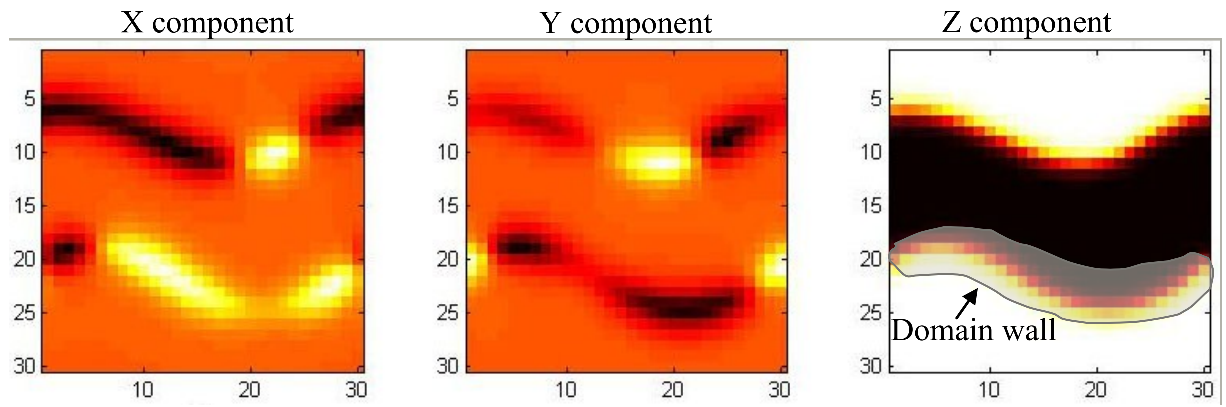


Figure 3.5 $J = 1$, $\lambda = 1$, $k = 1.2$, no external field. Since spins have a tendency to align into a certain direction, most of x and y components of spins are close to zero, except for where domain walls appear. Therefore, domain walls are apparent.

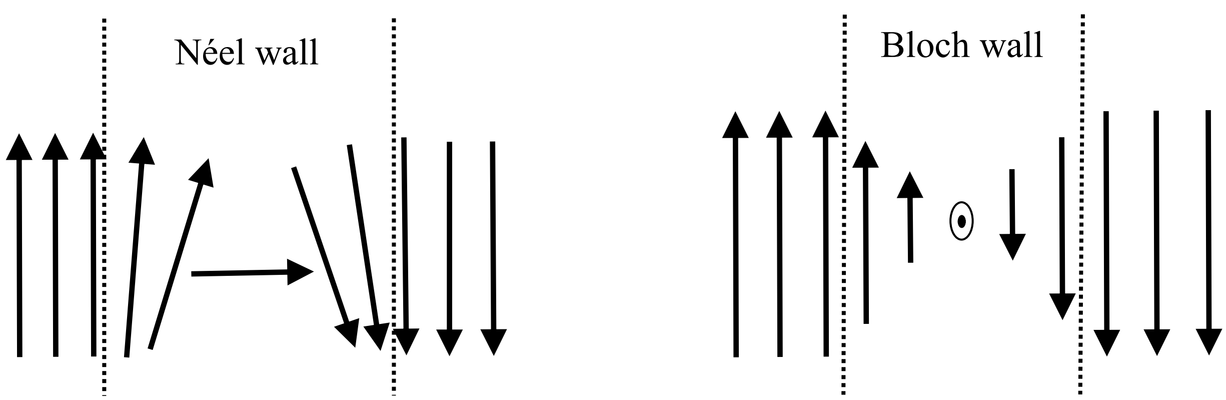


Figure 3.6 Neél wall is where the spins rotates on the same plane as they change their orientation from one domain to another domain and it indicates that the lattice structure is two dimensional. Block wall is where the spins do not lie on the same plane as they rotate their orientation from one domain to the other domain and hence it indicates that the lattice is three dimensional. Shorter spins in Bloch wall indicates that the spins are rotating in direction away from the page.

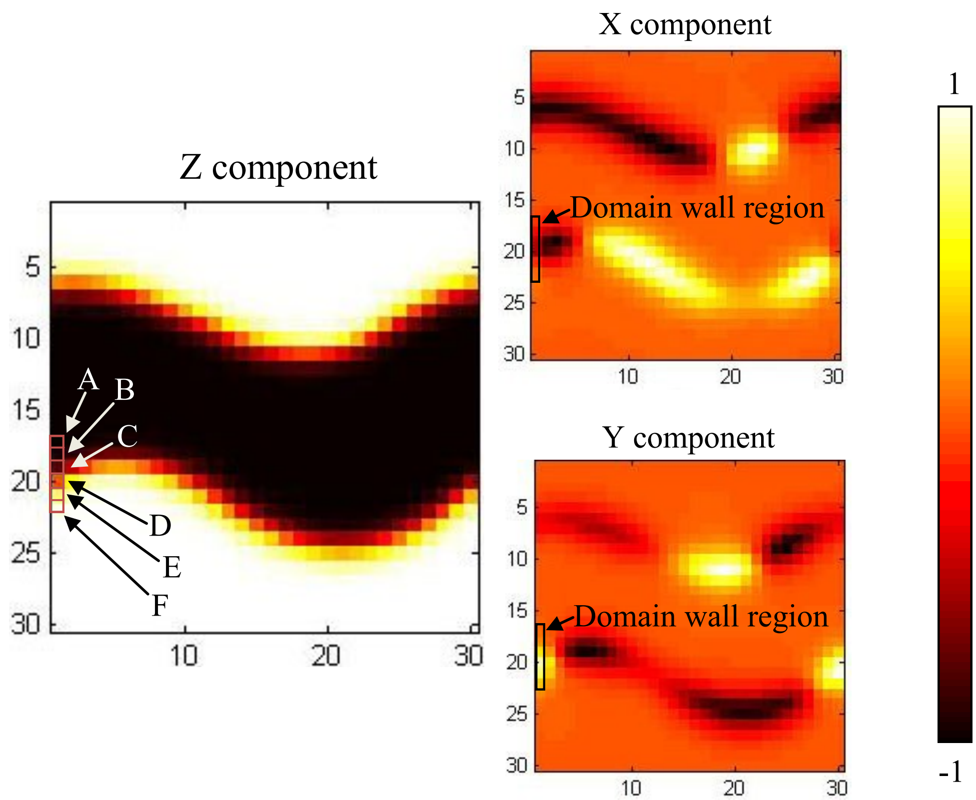


Figure 3.7 $J = 1$, $\lambda = 1$, $k = 1.2$, no external field. **A**, **B**, **C**, **D**, **E**, and **F** are vectors that are chosen from the specific positions indicated on the figure, across the domain wall to decide if the domain wall is Néel wall or Bloch wall.

some thickness of bulk material in the model.

Determining the type of walls is not too hard. We can use the property that if vectors **A**, **B**, and **C** are on the same plane then $\mathbf{A} \cdot (\mathbf{B} \times \mathbf{C}) = 0$. First, take spins across the domain wall including a couple of spins from one domain and the other domain. (see table 3.1) Then calculate dot and cross product of three vectors in a row in order. If the products come out to be non-zero, it provides evidence that the lattice indeed simulates 3-D lattice. (See Fig. 3.7) Table 3.2 is the result of products that are performed and it indicates that the spins model with anisotropy has Bloch domain walls, and therefore provides evidence that the spin model simulates the surface of bulk material.

Table 3.1 Vectors across the domain walls from figure.3.7

A	$(-0.1129, 0.0363, -0.9929)$
B	$(-0.2508, 0.1364, -0.9584)$
C	$(-0.4866, 0.4225, -0.7647)$
D	$(-0.5237, 0.8512, -0.0341)$
E	$(-0.1435, 0.6885, 0.7109)$
F	$(0.0466, 0.3336, 0.9416)$

Table 3.2 Dot and cross products to determine the type of domain wall

$\mathbf{A} \cdot (\mathbf{B} \times \mathbf{C})$	0.0153
$\mathbf{B} \cdot (\mathbf{C} \times \mathbf{D})$	0.0776
$\mathbf{C} \cdot (\mathbf{D} \times \mathbf{E})$	0.0358
$\mathbf{D} \cdot (\mathbf{E} \times \mathbf{F})$	-0.0694

3.3 Spin Waves

Spin wave is another interesting phenomenon that can be observed in the spin model. Spin waves are the propagating disturbances in the ordering of magnetic material. To observe spins waves, we first use a one dimensional lattice where the spins line up on a single line. However, spin waves cannot be observed with random initial configuration of spins. Rather, they needs to be in an symmetric pattern as initial condition. The initial condition is set to be in sinusoidal. Then as times step evolves, the wave-like behavior is observed.

This can be done in a two dimensional lattice as well. The initial condition is that the spins need to be in sinusoidal initial condition both in x and y direction of the lattice. In this case it shows richer complexity and patterns.(See Fig. 3.9)

3.4 Ferromagnetic Pattern Formation with Anisotropy and Long Range Interaction

The magnetic phase transition shows the characteristics of anti/ferromagnetic material. Thus it also points to the possibility of modeling more realistic magnetic material and indeed we are able to simulate the surface of thin magnetic material and its response to an external magnetic field. One example of magnetic material is Co-Pt magnetic thin film. Co-Pt thin film, under no external magnetic field influence, forms maze-like magnetic domain patterns shown in Fig.3.10. [5] Knowing that the spin model that we have developed can represent different magnetic features, it is possible to simulate the magnetic pattern which can be observed in Co-Pt magnetic thin films. This domain pattern does not appear in the isotropic classical Heisenberg model. Therefore, there should be more aspects that have to be included to the spin model so that it can simulate the pattern shown in magnetic thin film.

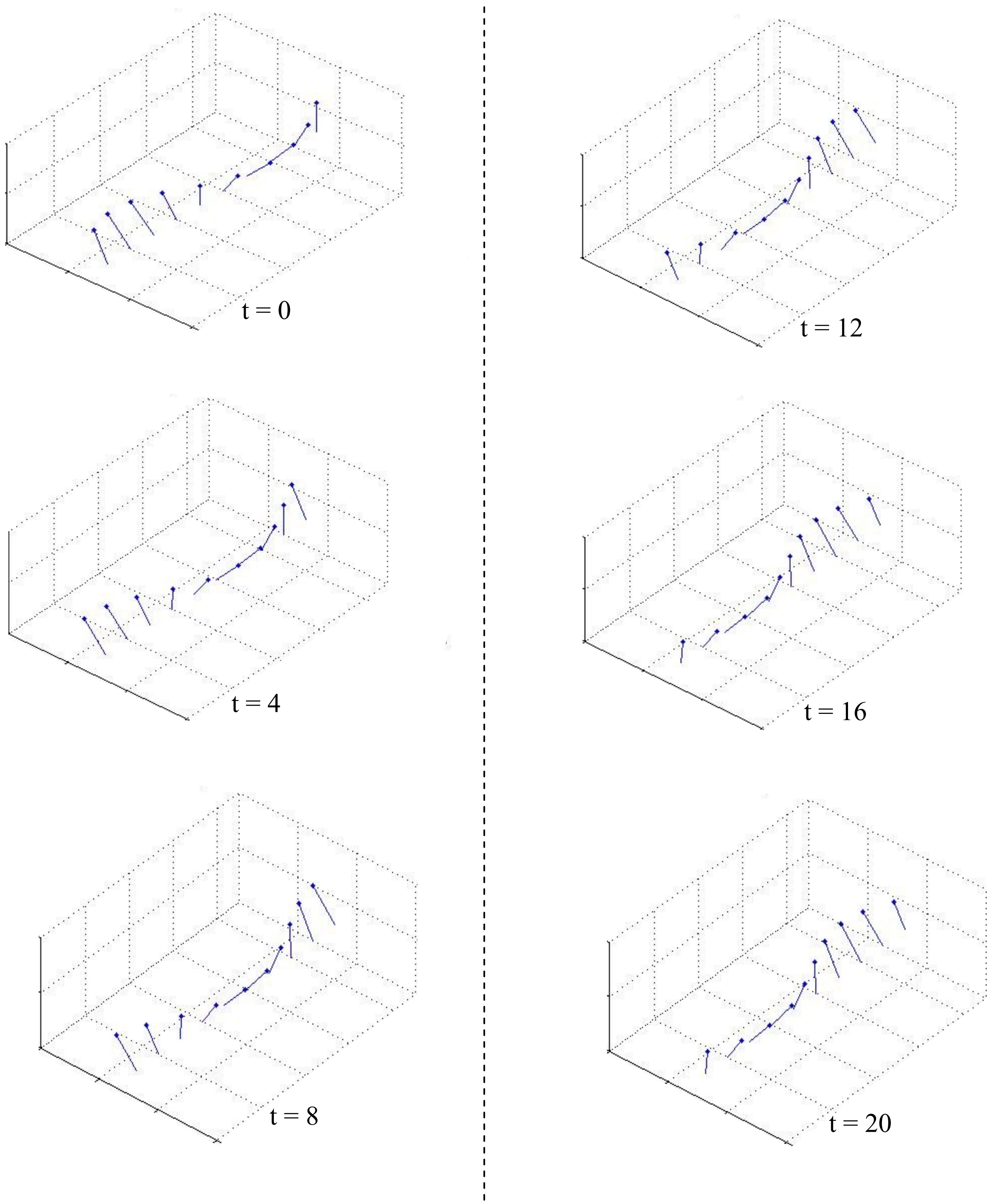


Figure 3.8 The spins model also can simulate 1D spinwave. To simulate spinwave, initial x and y components of spins are set to form a sinusoidal wave that have a period of the length of the lattice.

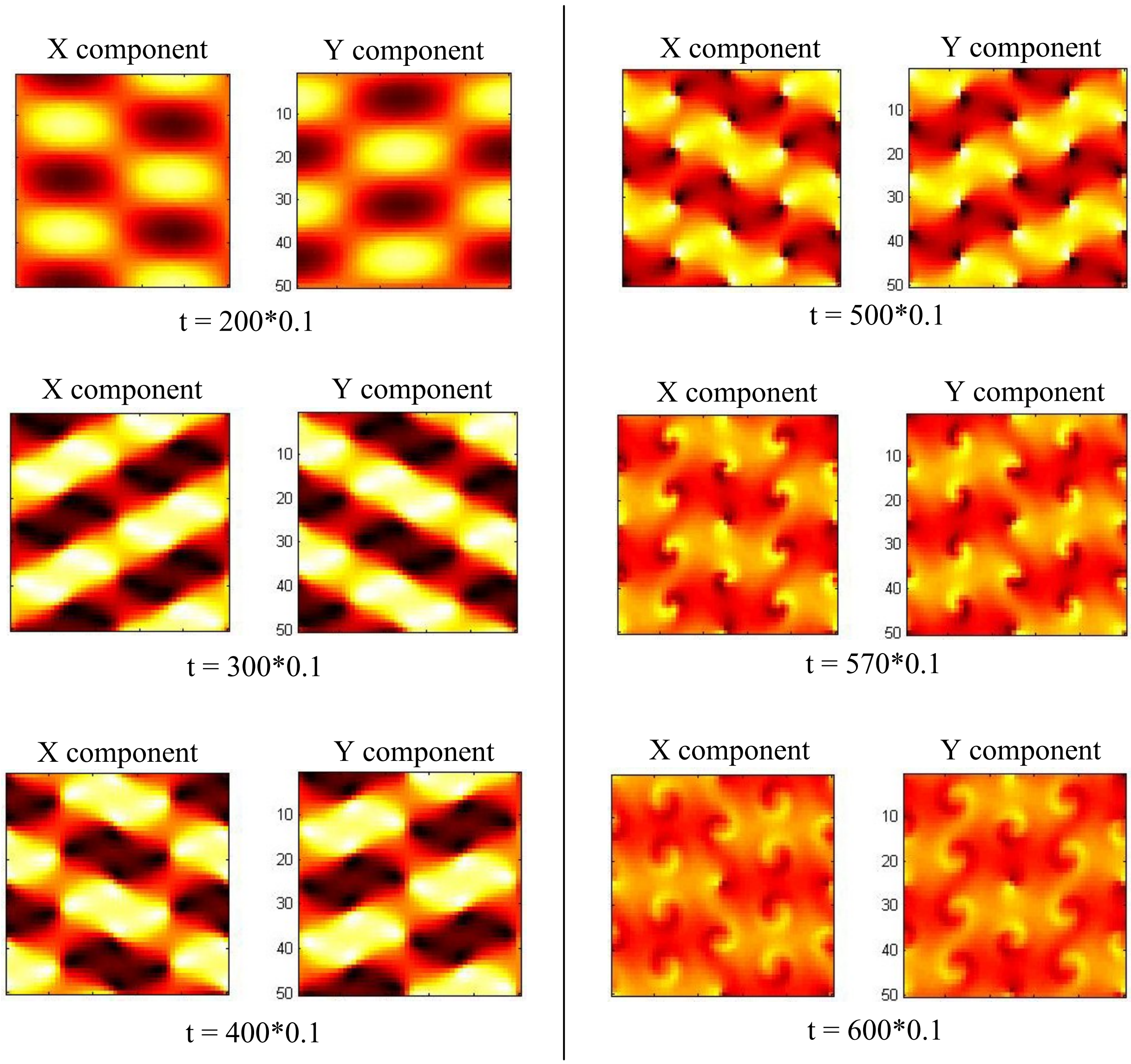


Figure 3.9 The spins model also can simulate 2D spinwave. To simulate spinwave, initial x and y components of spins are set to form a sinusoidal wave that have a period of the length of the lattice. z components of the spins are arranged so that they have value that is close to one. 50×50 lattice.

Figure 3.10 Co-Pt thin film, under no external magnetic field influence, forms maze-like magnetic domain patterns.[Westover, personal communication]

First thing to add is anisotropy. In our spin model the lattice is only two dimensional. However, even a very thin plate-like material should have some thickness much greater than the size of an atom. Therefore, anisotropy is needed to mimic some thickness of the material so that the spin model can simulate the surface of a bulk material. As explained in Chapter 2, the anisotropy happens in the z direction, defines as normal to the area of the plate. The width (x direction) and the length (y direction) is much greater than the thickness (z direction) of the material and therefore it gives rise to the anisotropy in z direction. (see Fig.2.4; see section 2.4 for more detailed explanation for anisotropy)

Adding anisotropy to the model is not enough. The long range interaction should be also included to give results comparable to the experimental patterns. The magnetic dipole-dipole interaction is chosen to represent the long range interaction and it turned out that the long range interaction acts as opposing effect against the NN spin interaction (frustration). This predicts that the spin model would have much richer and more complex dynamics when including both consensus and frustration. (see section 2.5 for more details and explanation for long range interaction.) It would be ideal to include all the spins in the lattice in calculating the long range interaction between spins. However, there are $nx \times ny$ number of spins in the lattice as well as the periodic

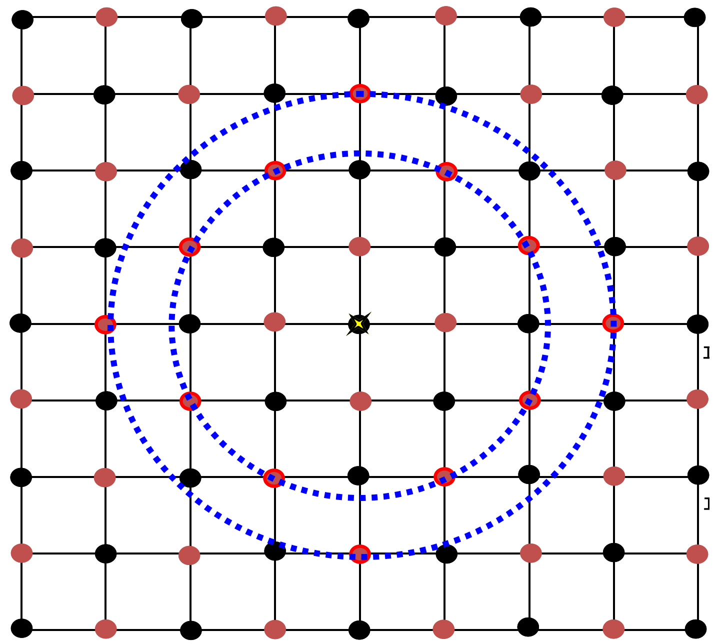


Figure 3.11 To represent long range interaction, some spins are chosen to represent long range interaction. For the black spins in the middle of the lattice, 12 red spins which cross the blue dashed circles are chosen. NN spins cannot be used for the long range interaction. Due to the integration method, only spins on red sublattice can be chosen to represent long range interaction. Those 12 spins are chosen such a way that it still keeps the simplicity of the model but is enough to represent the rich dynamics of the many-spin system.

boundary condition. Including all of them in calculating the long range interaction would be too complicated and time demanding. A possible alternative (which we did not explore) is Fourier Transform of the dipole-dipole interaction back and forth. To keep the simplicity of the model, we chose a selected number of spins on the lattice as representation of the long range interaction. This frustration induces enough antagonistic effect to produce the desired magnetic patterns. The sub-lattice structure is taken into account and twelve spins are chosen in such a way that it still keeps the simplicity of the model but is enough to represent the rich dynamics of the many-spin system. (see Fig.3.11)

Fig.3.12 shows the time evolution of spins with anisotropy in the z direction and long range interaction as frustration against consensus NN spin interaction. The spins are located on a 50×50

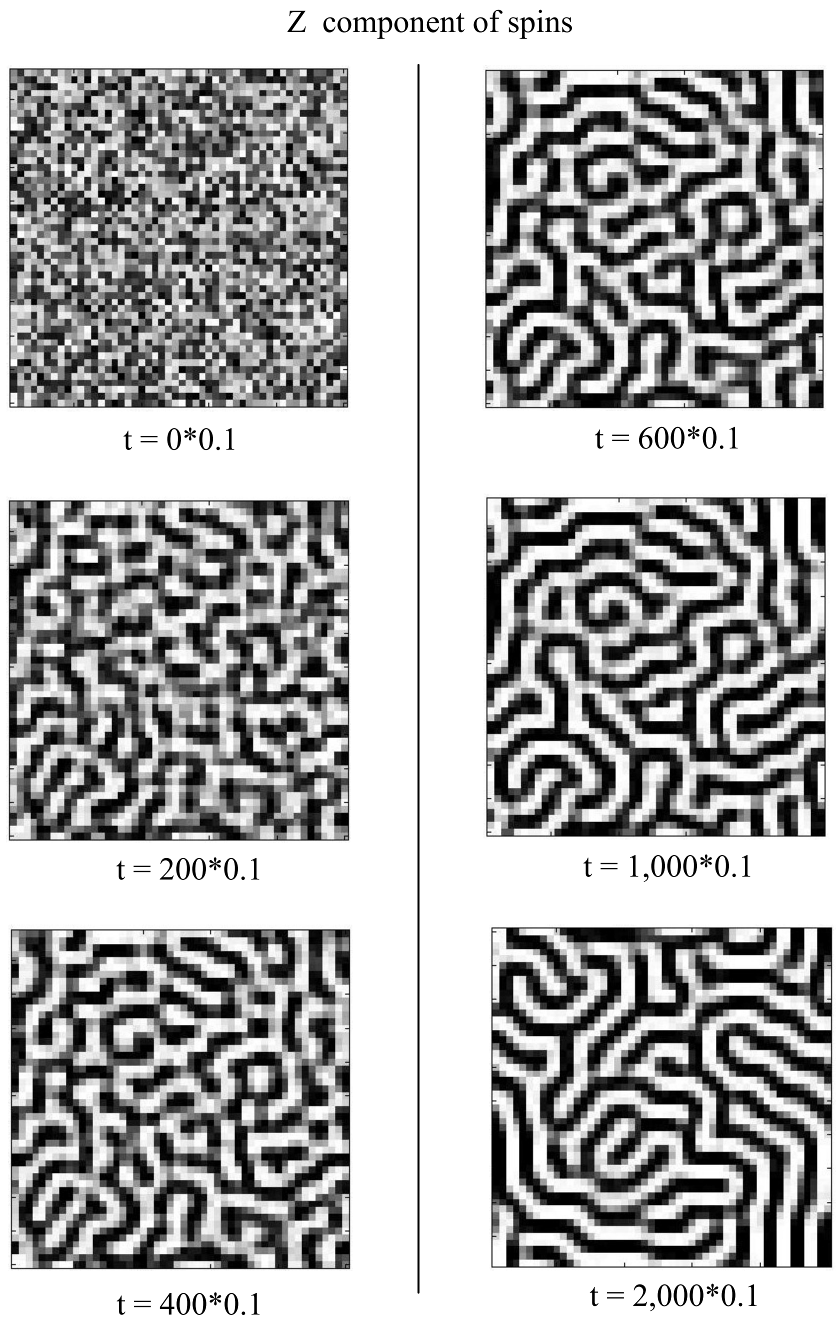
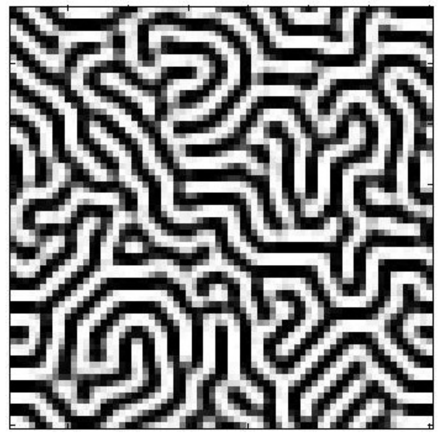


Figure 3.12 $J = 0.05$, $\lambda = 0.5$, $k = 1.5$, no external field, 50×50 lattice. The initial orientation for the spins are chosen to be random. There is no particular pattern formed at the beginning. As times goes, it is apparent that the spins form a maze-like pattern.

mfm image



Spin simulation
70 × 70 lattice

Figure 3.13 Comparison between the spin simulation and an image taken from Co-Pt thin film.[Westover, personal communication] The maze-pattern they both show is very similar to each other.

2D lattice structure. As always, the initial orientation for the spins is chosen to be random. There is no particular pattern formed at the beginning. As time evolves, it is apparent that the spins form a maze-like pattern. The contrast of NN spin ferromagnetic interaction and long range anti-ferromagnetic interaction is apparent in the pattern. Suppose one spot on the lattice is picked: along the direction of the maze the color does not change, which means it shows ferromagnetic feature along the maze path. That is, there is a path or direction along the maze pattern where the spins have the same orientation, representing a ferromagnetic phase in that particular direction. On the other hand, in the direction across the maze pattern, the spins change their orientation with opposite orientation which indicates anti-ferromagnetic behavior. In general terms we can say that the ferromagnetic properties along the maze path comes from the NN spin interaction while the anti-ferromagnetic property across the maze path arises from long range interactions. As both interactions co-exist and combine together, it results in a rich and complex dynamics and patterns. This is definitely comparable to experimental result. Fig.3.13 illustrates the comparison between the spin model and an image taken from Co-Pt thin film. [Westover, personal communication] The maze-pattern they both show is very similar to each other.

This is not the only result that is comparable to the experimental findings. In the experimental research group using Co-Pt thin film, they apply an external magnetic field to the thin film in the perpendicular direction to the surface of the film and observe the resulting magnetic domains on the film. The applied external field starts from zero and increase until all the magnetic moment of the film is aligned with the external magnetic field. The external magnetic field is further decreased back to zero until the maze-like pattern domain is recovered on the film. As the external magnetic field increases, the pattern on the magnetic film shows little circular bubbles before it get saturated satisfying that all the magnetic moments align with the external field. As the external magnetic field go around the loop (starting from no magnetic field, maximum magnetic field and back to zero magnetic field) the magnetization of the film shows hysteresis, which means the magnetization history of increasing external magnetic field and decreasing external magnetic field follows completely different paths. (see Fig.3.14) The same procedure is followed on our spin model simulation. First, the model starts with no external magnetic field and its dynamics is trailed until the maze-like pattern is formed and stabilizes. Then an increasing external magnetic field is applied until the spins get saturated and they all align with the external magnetic field. Then the magnetic field is decreased back to zero. The maximum external magnetic field is -0.3 and it took 1000 times steps for the external magnetic field to reach its maximum value. The maximum external magnetic field remained with the same value of -0.3 for 500 times steps and then decreased back to zero in 1000 time steps. (The external magnetic field is applied in negative direction for convenience) Fig.3.14 shows how the patten changes in the spin model as the external magnetic field changes for both experiment and spins simulation. The spin model indeed shows that the pattern on the lattices turns into a bubble pattern as the external field increases the same way as the Co-Pt thin film does. It gets saturated as the external magnetic field reaches its maximum, and then recovers the maze-like pattern as the field decreases back to zero as Co-Pt thin film does. It

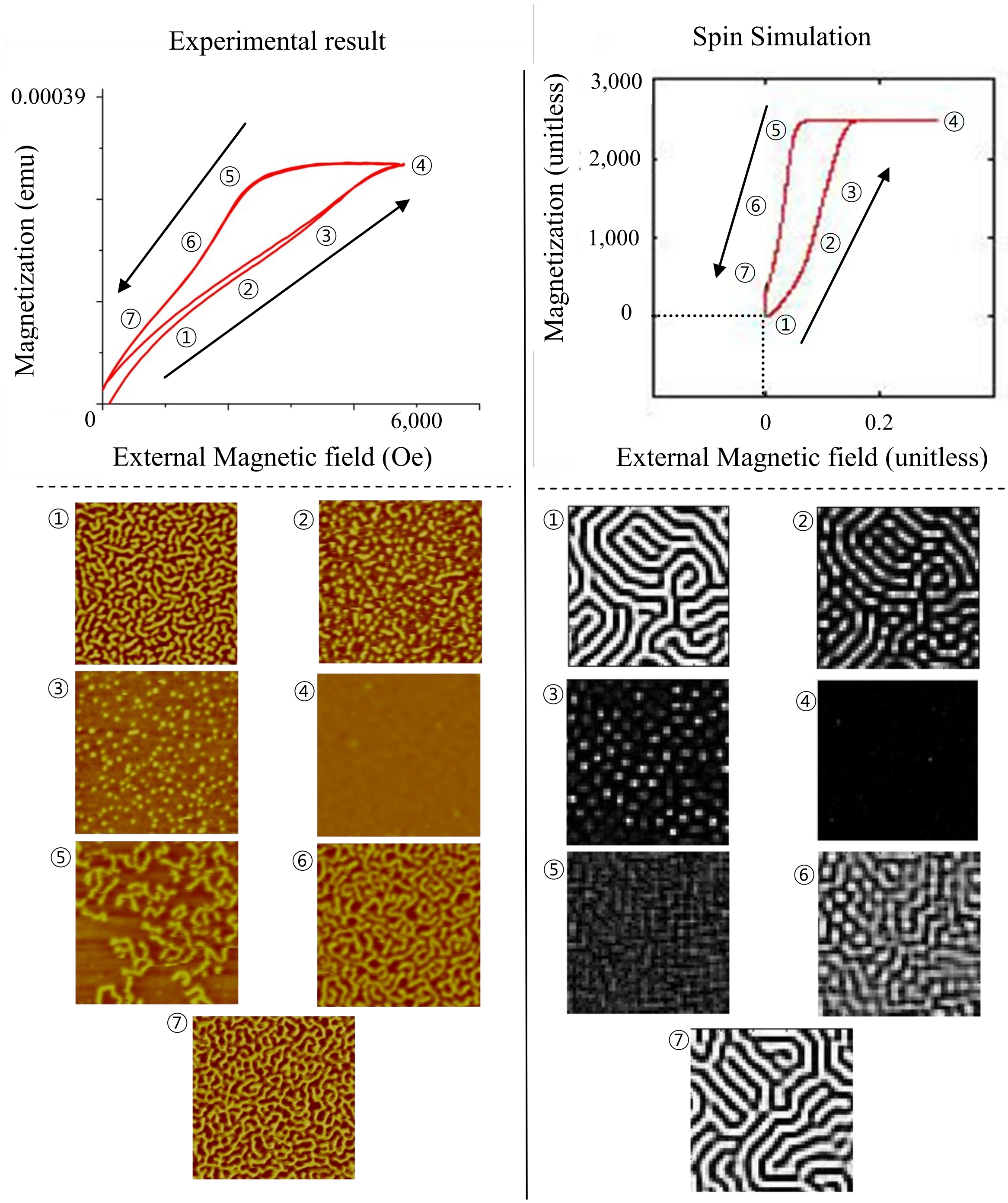
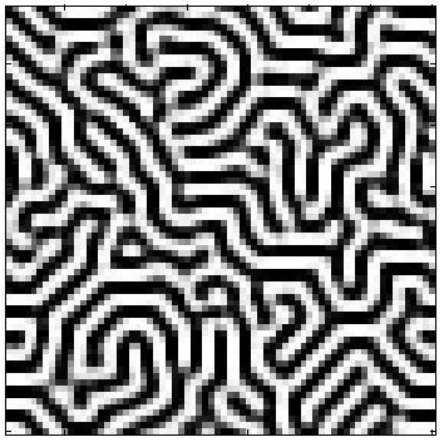


Figure 3.14 The figure on the left is experimental result and the figure on the right is the spin model simulation. External magnetic field is applied to the Co-Pt thin film experimentally and spins model until they get saturated, which means all the spins align with the external field. External magnetic field started with zero, is increased until saturation and then is decreased back to zero again. The magnetization is also measured for both Co-Pt film and the spin model. Spins simulation shows comparable result showing hysteresis, which means magnetization path as the external field increases is different from the magnetization path as the external field decreases. Different patterns for different stages in applying external field is also comparable to experimental result.[Westover, Personal communication]

corresponds to the experimental behavior in an amazingly similar way.

Along with observing how the pattern changes the magnetization of the spin model is also observed. Magnetization for the spin model is simply the vector addition of all the spin values on the lattice. Since we use size 50×50 lattice, there are 2500 spins on the lattice. Each spin value can vary between -1 to 1. Therefore, the maximum magnetization of the lattice is 2500 and the minimum is -2500. Fig.3.14 shows how the magnetization changes as the external field changes. As expected, at the beginning the magnetization is zero with maze-like pattern. As the external field increases, the spins start to align themselves along with the external field and the magnetization is not zero anymore but moves toward the preferred direction. An interesting point here is that the spin model also shows the hysteresis loop that is observed in the experiment: the magnetization history of increasing external magnetic field and decreasing magnetic field does not take the same path. Rather, once the spins are saturated, it stays at the saturation for a while before it falls off even though the external magnetic field has already started to fall off. Fig.3.14 also shows the comparison of magnetization for the experimental result and the spin-model. It is striking that the hysteresis that the model exhibits is as real as the experimental result. Despite its simplicity, the spin model corresponds well with what experiments for magnetic thin film shows.

Finding results that are comparable to the experiment is indeed an exciting moment! However, there is something else to be mentioned about the patterns encountered. The pattern that we observe resembles Turing pattern very much. (see Fig.3.15) This is a pattern that can be usually obtained in the dynamics of chemical reactions, among other phenomena. The differential equations used to produce Turing pattern are quite different from the LLG equation that we use in the spin model. Yet, very similar pattern is formed. Therefore we ask ourselves whether there are more fundamental pattern forming principles lying behind those different differential equations. This is a plausible proposal since the pattern that we see cannot be similar by accident. They seems like to share so much similarity that looking at them leads us to think that there should be more funda-



Spin Model
70×70 lattice

Turing pattern

Figure 3.15 The pattern from the spin model and Turing model have so much similarity even though they have very different form of pattern causing differential equations. [6] It leads us to ask a question if there is more fundamental principles lying behind those different differential equations.

mental principles underlying those two different equations. This tantalizing conjecture entices us to compare and study the dynamics of the many-spin systems with different (but related) pattern forming differential equations.

Chapter 4

Conclusion

We have demonstrated that the spins model using simple algorithm can exhibit rich and complex dynamics of many-spin system including magnetic phase transitions, the appearance of magnetic domain walls, spinwaves, and magnetic pattern formation. The coupled LLG equations effectively show how the dynamics of the spins is governed. Using the Heisenberg model and a matrix presentation of the many-spin system with 2D lattice enables us to simplify the process for simulating the many-spin system. Using stereographic projection makes it possible to solve the non-linear LLG equation without approximation error so that we can simulate the rich dynamics of a many-spin system. As we solve and simulate, we keep the algorithm simple in such a way that simplicity becomes a great advantage of the model that we developed. As a result, anti-/ferromagnetic phase transitions, domain walls, spinwaves are well simulated and observed to provide more information on the many-spin system. Including anisotropy and long range interaction to build a more realistic model also proves that it can simulates thin magnetic material by exhibiting magnetic pattern formation and hysteresis of magnetization as an external magnetic field is applied. Despite its simplicity, the spin model successfully simulates various anti-/ferromagnetic properties. This model can be useful to understand more about anti-/ferromagnetism and can answer questions that cannot be demonstrated in experimental results.

Furthermore, especially the pattern formed from the model raises more questions about pattern formation in connection with Turing patterns and pattern formation in various other scientific fields, and whether there could be more universal and underlying principles for pattern forming mechanism.

Appendix A

Conserved spin length

The motion of equation for Heisenberg model preserves the spin length.

Consider

$$\frac{dS^2}{dt} = \mathbf{S} \cdot \frac{\mathbf{S}}{dt} + \frac{d\mathbf{S}}{dt} \cdot \mathbf{S} \tag{A.1}$$

where

$$S^2 = \mathbf{S} \cdot \mathbf{S} \tag{A.2}$$

Then,

$$\frac{dS^2}{dt} = 2\mathbf{S} \cdot \frac{\mathbf{S}}{dt} \tag{A.3}$$

We already know that

$$\frac{d\mathbf{S}}{dt} = -J(\mathbf{H} \times \mathbf{S}) \tag{A.4}$$

Therefore,

$$\frac{dS^2}{dt} = 2\mathbf{S} \cdot \frac{\mathbf{S}}{dt} = -2J\mathbf{S} \cdot (\mathbf{H} \times \mathbf{S}) = 0 \tag{A.5}$$

Then, we can conclude that

$$\frac{dS^2}{dt} = 0 \tag{A.6}$$

and

$$S^2 = k \tag{A.7}$$

where k is constant and therefore, the spins length is always conserved. We choose k to be one.

The resulting 0 in A.6 is from $\mathbf{S} \cdot (\mathbf{S} \times \vec{l})$, where $\vec{l}$ is any vector. Whatever $\vec{r}$ is the resulting product is zero and therefore, $\frac{dS^2}{dt} = 0$. Since the relaxation term also has $\mathbf{S}\times$ term, LLG equation with relaxation term also preserves the spin length.

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