Particle Interferometry to Understand The Quantum Entity

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by
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I hereby submit the following senior thesis entitled *Particle Interferometry to Understand the Quantum Entity* for the approval and acceptance of the Department of Physics and Astronomy, Brigham Young University, Provo, Utah. With this submission, I also acknowledge and express appreciation to Dr. Jean-Francois Van Huele for many hours of encouragement and instruction. Also to Dr. J. Dean Barnett for allowing me use of his birefringent crystal and various other scientific instruments. I also thank the Department of Physics and Astronomy for funding my research during the summer of 1993.

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Introduction

"Complementary Variables" is a term used in quantum physics to refer to mutually exclusive measurements. These measurements stem from aspects of reality that can not exist simultaneously. Let me illustrate the problem with an analogy taken from every-day experience: Suppose I claim to be writing this paper at a workstation in a small computer lab, in the B.Y.U. ESC building. Suppose I also claim to be on a surfing vacation in California. Either one of these statements has the potential of being independently true; however, they are mutually exclusive so that truthfulness in one implies falsehood in the other.

There are three elements that form the previous contradiction. One element is the fact that I am indivisible. If I were divisible then half of me could be typing this paper while the other half is on vacation. This would allow both of the previous statements to be true. This condition is referred to as quantization. A quantum is a discrete indivisible unit. People are quantized, and so it makes sense to talk about one, two, or three people; however, it doesn't make sense to talk about 1.5, or 2.3 people. I have been told that the average American family has 2.3 children, but I've never seen the average family, or an average family, or even a normal family.

The second and third elements of this contradiction stem from localization. Localization relates to the size of an object. A small object is highly localized, while a large object is extended in space. One way to defeat the contradiction is to assume that I am so large, that I can be in Provo, Utah and California at the same time. Another possibility is that these locations are so close together that anyone could comfortably be simultaneously in both of them. The three part contradiction is:

1) I am indivisible,

2) I am localized,

3) I am in two remotely distant places at the same time.

This three part contradiction lies at the heart of the paradoxical experiments discussed in this paper.

Quantization: Suppose you are attacked by a pack of dogs at night. The first question that comes to your mind might be "what is the smallest possible pack of dogs?" How many of the dogs will you have to kill or chase off before you are safe from harm? Suppose the pack is divided into two groups, and one group runs off after a cat. Now suppose half of the remaining dogs are hit by a dump truck while chasing you across the highway. Through this process, we can reduce the number of dogs down to the fundamental dog unit. We discover that the smallest possible dog unit is one dog. It takes at least one dog to bark, bite, chase cats, and do all of the things that are associated with being a dog. Like dogs, many common objects are quantized.

The Particle concept is closely related to the principle of quantization. The relationship stems from the fact that even though many common objects are quantized, most are divisible. For instance, the dog is the smallest unit of dogs, but we can divide an individual dog into its parts: the heart, brain, legs, lungs. None of these parts by itself is a dog, but a dog is composed of all of these parts. Many of these parts can be considered quantized. For instance, the heart is quantized, or comes as a unit, but it can be divided into various components.

The ability to divide objects into smaller components leads us to question whether matter is itself quantized. Suppose we take a piece of stone and hit it with a sledge hammer breaking it into little pieces. Now take one of the small pieces and hit it with the hammer smashing it to a fine gravel. If we continue this process of smashing smaller and smaller fragments of matter, will we eventually find a piece so small that it can't be broken into smaller fragments? One milestone in answering this question was achieved by John Dalton in the early 1800's. Dalton hypothesized that all common matter is composed of atoms. The atom is the smallest particle that maintains all of the chemical properties of an element[1]. From his work we discover that the elements are quantized around individual atoms; however, atoms are themselves divisible into protons, neutrons, and electrons. So far, the search for the smallest unit of matter continues. Electrons seem to be fundamental, but protons and neutrons appear to be divisible.

As the scientific community discovered increasingly small particles, scientists invented the concept of a point particle. A point particle is an idealization of smallness. It is so small that it can be at a single point in space.

A Wave in its simplest sense, is a repetition. Some common examples are the waves of the ocean, sound waves, or the vibrations of an earthquake. In each of these examples, the wave travels through some medium: solid, liquid, or gas. Sound waves consist of repetitive areas of high and low pressure in air, the waves of the ocean contain a repetitive pattern of crests and valleys in water, and earthquakes are repetitive movements of solid matter. In each of these examples, the wave is limited by its medium. The waves of an earthquake travel from deep within the earth to the surface and along the surface. The waves of the ocean travel the length of the ocean, but they stop at the shore. Sound waves travel throughout the atmosphere, but are incapable of traveling to outer space. It was once believed that all waves were limited to the medium in which they traveled and were incapable of existing outside of that medium.

The discovery that light is an electromagnetic wave created an interesting problem. If all waves are limited to the medium in which they exists, and if light is observed to travel through space, then space must be filled with some fluid. This fluid became known as the luminiferous ether[1]. Light was believed to travel through the ether in a way comparable to sound traveling through air. The existence of ether provided another interesting possibility: it might be used to measure the velocity of the earth. It is analogous to sticking your hand out of the window of an automobile while driving down the street. You can tell that the car is moving by the feel of the breeze on your hand. In a similar way, a person on earth should be able to measure the breeze caused be the motion of the earth through the ether.

The Michelson-Morley Interferometer was designed for such an experiment(see section 1.3 and figure 1-10). Albert Michelson believed he could orient the two arms of the interferometer with one parallel to the movement of the earth through the ether and the other perpendicular to it. He could then compare the speed of two beams of light traveling down the arms of the interferometer. A beam traveling perpendicular to the motion of the earth should have a different velocity than one traveling parallel to it. This would allow Michelson to measure the speed and direction of the earths movement through the ether.

Although numerous attempts were made, this experiment was never successful in detecting any movement of the earth through the ether. The negative outcome of the Michelson-Morley experiment implies that ether doesn't exist. Empty space is empty. (The Michelson-Morley experiment also failed to find an ether frame of reference. This non-existence of an absolute frame of reference is the foundation of Einstein's special theory of relativity.)

Waves gained their independence in the late 1800's. Until then all waves were believed to travel through some medium. The discovery that light is a wave, the fact that light travels through empty space, and the experimental indication that empty space is empty demonstrates that some waves, like light, can exist independent of any medium. To understand the significance of this discovery, imagine watching some children playing with a jump rope in a gymnasium. The movement of a jump rope is a well understood wave phenomenon. Now suppose the wave were to become independent of the jump rope. Imagine watching the wave leave one end of the rope. The rope becomes limp and falls to the floor. The wave is free to travel throughout the gymnasium. Now suppose the wave enters another jump rope on the opposite side of the gymnasium. A lifeless rope suddenly springs into action.

As another example, imagine what would happen if the waves of the ocean were to become independent of water. Rather then stopping at the shoreline, the waves would continue on their journey. Suppose you were sitting near the ocean drinking a glass of milk. The waves from the ocean would travel from the shore to your cup and cause the milk in your cup to move about. They would also travel to the nearby lakes and ponds and cause waves to appear in every local body of water. These examples may seem bizarre, but they are similar to what is commonly believed to happen with light and other forms of electromagnetic waves. For instance, a radio station creates wave movements in the electrons of its transmitting antenna. These waves are believed to leave the station antenna and travel through the local countryside. As they pass by individual homes, they cause the electrons in radio antenna to move, allowing reception to the local radio stations. These same waves are capable of traveling into space, and provide communication between the earth and various space craft.

From these examples, it is clear that electromagnetic waves are fundamentally different from other, elastic waves. With the knowledge that waves can exist in empty space, it is possible to define the perfect wave. As was stated previously, a wave in its simplest sense is a repetition. The wavelength is defined as the distance from one repetition of the pattern to the next. We can define a perfect wave as one in which a pattern is repeated perfectly every wavelength with no beginning or end. For this reason, a perfect wave must be able to travel through empty space. The waves of the ocean, for instance, can't be perfect waves because their pattern ends at the shoreline. Any wave that depends on some medium for it's existence can't be perfect. It is possible, on the other hand, to picture a perfect electromagnetic wave. This wave would extend to every point of space and would have no beginning or end. The wave pattern would perfectly repeat itself every wavelength.

We have considered two diverging fields of study. The description of the particle has become increasingly small, eventually arriving at the point particle: a mathematical description of matter so small that it has no size at all. The description of the wave has become increasingly large, eventually filling the entire universe with its repeating pattern. These two concepts enjoyed peaceful coexistence for many years and successfully explained many phenomena known to man. The common material objects were believed to be composed of various combinations of protons, neutrons, and electrons, which are essentially point particles. In this way, most of the material world was described as various arrangements of point particles.

Unlike particles, waves don't have definite locations. A violin may have a definite location in an orchestra, but everyone in the room hears the sound produced by the violin. Therefore, even though the violin is in one place, the sound it produces fills the hall. This is also true of the radio waves produced by radio stations. The radio station is in one place, but the waves produced by it fill the countryside. This is one of the main differences between particles and waves. If an object is made of particles, then we can determine the location of each particle in the object. The size and shape of the object are determined by the distribution of its particles.

Waves don't have definite locations; however, through the mathematical process of Fourier analysis, any wave pattern can be decomposed into a linear combination of perfect harmonic waves. The harmonic waves have no beginning or end and fill all of space. They are examples of perfect waves because their patterns are perfectly repeated throughout space without variation. An interesting comparison exists between the wave and particle descriptions. In the particle description, material objects are arrangements of protons, neutrons and electrons. The size and shape of the object are determined by the arrangement of the infinitely small point particles. The wave description is similar. Any wave pattern can be decomposed into some combination of infinitely big harmonic waves. The size and shape of the wave pattern are determined by the arrangement of harmonic waves.

Suppose we have an object of some moderate size. We haven't determined whether it is composed of particles or whether it is a wave phenomenon and we want to consider both possibilities. Let the object fill an area with sides of length x, y, and z. For a particle composition we may ask how many particles are contained in the object. To answer this question, we can calculate the volume of the object from the length of the sides and then multiply the volume by the particle density to get the number of particles in the object.

A similar question can be asked about wave phenomena. If some wave phenomenon is contained in an area with sides of length x, y, and z, then we may ask how many unique harmonic waves are required to compose this phenomenon. A detailed investigation of this question requires the use of Fourier analysis. A rough approximation can be achieved by using the relation $\Delta x \Delta k \geq \frac{1}{2}$. In this relation Δx is the length of one of the sides of the area in which the phenomenon is contained, and Δk represents the spread in wave numbers ($k = 2\pi/\lambda$, where λ is the wavelength). The $\Delta x \Delta k_x \geq \frac{1}{2}$ relation tells us that the size of a wave phenomenon in space is inversely proportional to the spread of wave numbers composing the phenomenon. This means that a wave phenomenon that exists in a small region of space is composed of a greater variety of harmonic wavelengths than a wave phenomenon filling a larger region of space.

To understand why, remember that the point particle is infinitely small and the harmonic wave is infinitely large. Where the point particle is infinitely small, it takes either more of them, or they need to be more spread out, to fill a larger area. The earth, for instance, is made of more particles than a mosquito. Harmonic waves, on the other hand, are infinitely large. More of them, with a large spread in wavelengths are required to fill a smaller region of space, and fewer harmonic waves with a small spread in wavelengths will fill a larger region in space.

The clear separation of the particle and wave concepts came to an end in the early 1900's when Albert Einstein discovered that in some situations, light behaves as a particle. Light had previously been thought of as a wave phenomenon which could be explained using wave techniques; however, the wave theories had fallen short in predicting the behavior of light in some situations. In Einstein's formulation, he treated light as a particle rather than as a wave phenomenon. His method was successful. In 1924 Louis De Broglie postulated that if a wave phenomenon like light displays particle-like characteristics, then particles might also have wave-like characteristics. The particle nature of wave phenomena, and the wave nature of particles have been experimentally established, and we now understand that all forms of matter have both particle and wave characteristics.

The marriage of the particle and wave concepts leads to the three part contradiction described at the beginning of this section. The three parts are:

1) The system is an individual quantum,
2) The system is a particle phenomenon,
3) The system is a wave phenomenon.

Any two of these claims are consistent. For instance, if claim two is true, then we are referring to a particle phenomenon, or something composed of individual particles. Claims one and two together are interpreted to mean that the system is an individual particle. Claim three states that the system is a wave phenomenon, or something composed of harmonic waves. Claims one and three together are interpreted to mean that the system is an individual harmonic wave. Claims two and three are also consistent if we disregard claim one. If claim one is false, then the system might be a conglomeration of many particles and waves. The system may have enough particles that it isn't infinitely small, and enough harmonic waves that it isn't infinitely large.

The contradiction arises from all three statements being true at the same time. It is impossible to imagine how a single quantum of matter can simultaneously be an infinitely small particle and an infinitely large harmonic wave. The currently held solution which was most eloquently expressed by Niels Bohr, and is illustrated in many experiments, is the following: it is never possible to observe the particle and wave nature at the same time. This is the meaning of "complementary", all three aspects of wave-particle duality are observable depending on the environment, but are not realized simultaneously.

The following paper contains four sections. Section one pertains to single particle interferometry, and contains the results of the experiments I conducted in 1994. Subsections 1.1, 1.2, and 1.7 contain basic information which will be of interest to the general reader. Subsections 1.3 - 1.6 contain experimental arrangements and results which will be of more interest to the experimentalist. These experiments only require basic scientific equipment, and provide good demonstrations of the subject.

Section two is written for the general reader, and is an introduction to the topics discussed in section three. Section three begins with the EPR experiment which presents the fundamental question of the remaining sections. Subsections 3.2 and 3.3 are designed to provide the reader with some necessary background for understanding the experiments discussed in sections 3.4 - 4.1. Subsections 3.4 - 3.6 are not in depth, but rather provide a brief overview of the important results obtained form each experiment. All of section three is written for a general reader.

Section four begins with the GHZ experiment which provides the best solution to the EPR problem. Subsection 4-1 is not written for a general reader. The mathematical formulation of the GHZ experiment, which is included, requires a sound understanding of quantum mechanics. Subsection 4-2 is intended for the general reader and contains a creative explanation of the collapse of the wave function. Finally, in an epilogue, I address a theological implication of quantum theory.

1. Single Quantum Interferometry

1.1 The Young Double-Slit Experiment and Wave-Particle Duality

The Young Double-Slit experiment provides a good demonstration of wave-particle duality. In the double-slit experiment, as represented in figure 1-1, a beam of coherent light is directed toward a screen containing two parallel slits. A second screen is placed a short distance beyond the first. The slits must be close together and the beam wide enough to illuminate both slits equally. Some light passes through the equally illuminated slits and forms an interference pattern on the second screen. This pattern is the result of wave interference and is a manifestation of the wave nature of light.

With improved measuring techniques, it is now possible to observe the marks left by individual photons of light striking the second screen. These photons are a manifestation of the particle nature of light. In figure 1-3, we see two slides labeled (a) and (b), containing results of a double-slit experiment[2]. Only a few photons appear in slide (a) due to a short exposure time. Slide (b) displays the results of the same double-slit experiment, but with a longer exposure time. In addition to photons, the double-slit experiment can also be used to display the wave-particle duality of electrons. Figure 1-4 slides (a)-(e), display progressive results of a double-slit experiment involving electrons[3].

As stated in the introduction, only two of the three components of wave-particle duality can be observed simultaneously. The three components are:

1) quantization.

2) particle nature.

3) wave nature.

The quantized particle nature of matter is clearly visible in figure 1-4(a), where only a few electrons appear in the image. As the number of electrons increases, the quantization becomes less apparent. The image appears as a conglomeration of particles and waves from which both the wave and particle nature of matter are visible. Finally, as the number of electrons continues to increase, the particle nature becomes less apparent and a clear wave pattern appears on the screen. In this experiment, the number of electrons in the image determines which of the three components of wave-particle duality are visible. The particle nature is visible when only a few electrons appear, and the wave nature is visible when many electrons appear.

Much about the quantum entity can be learned from the patterns that are formed by the two-slit experiment; however, what we don't see in this experiment is far more important than what we do see. Suppose we conduct a two-slit experiment in which a single quantum is released from a source, travels through the two slits, and produces a single speck on the screen. This single speck of matter is clearly a quantized particle when it strikes the screen;

however, is it in this state when it passes through the two slits? This is a difficult question to answer because we can't observe it directly. We only see the image on the screen, and from it we have to determine what happened through the entire experiment.

One might think that yes, the particle is in the same state throughout the experiment. As depicted in figure 1-2, one can imagine a single particle leaving the source, passing through one of the slits, and arriving at the second screen to produce the single speck visible in the pattern. The incorrectness of this assumption, which we will discuss in the next section, is one of the most astounding aspects of quantum mechanics and may constitute the foundation of all of the other counter-intuitive quantum mechanical results; for according to Richard Feynman, "every paradoxical result of quantum mechanics can be demonstrated by the Young Double-Slit experiment"[4].

The first paradoxical result we will investigate is the question as to which of the two slits the particle went through. Although the following experiments are quite different in design, they are similar in that in each experiment the particle is given two possible paths to follow in order to arrive at its destination. In each experiment, we want to discover which path the particle takes.

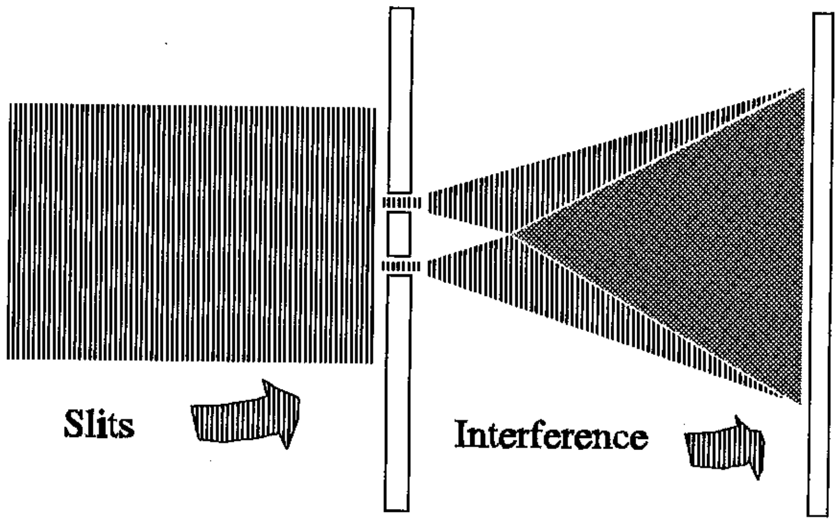


Figure 1-1. The Double-Slit Experiment.

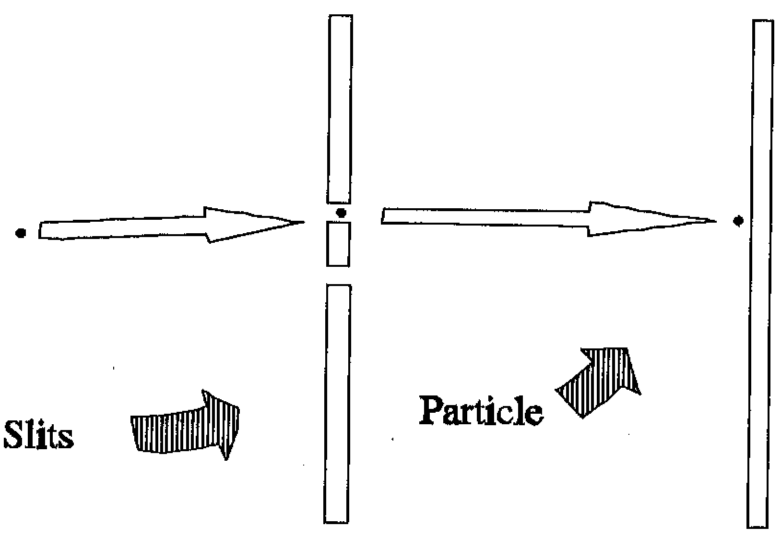


Figure 1-2. The Single Particle Interpretation.

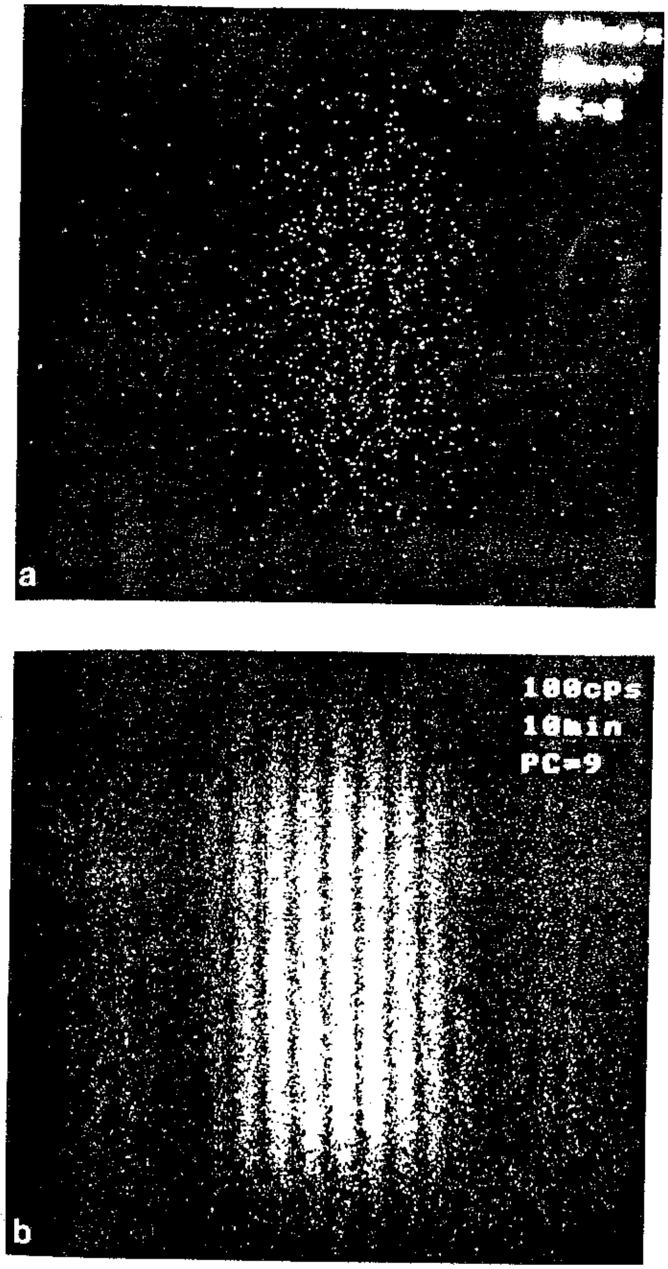


Figure 1-3. Results of a photon double-slit experiment.

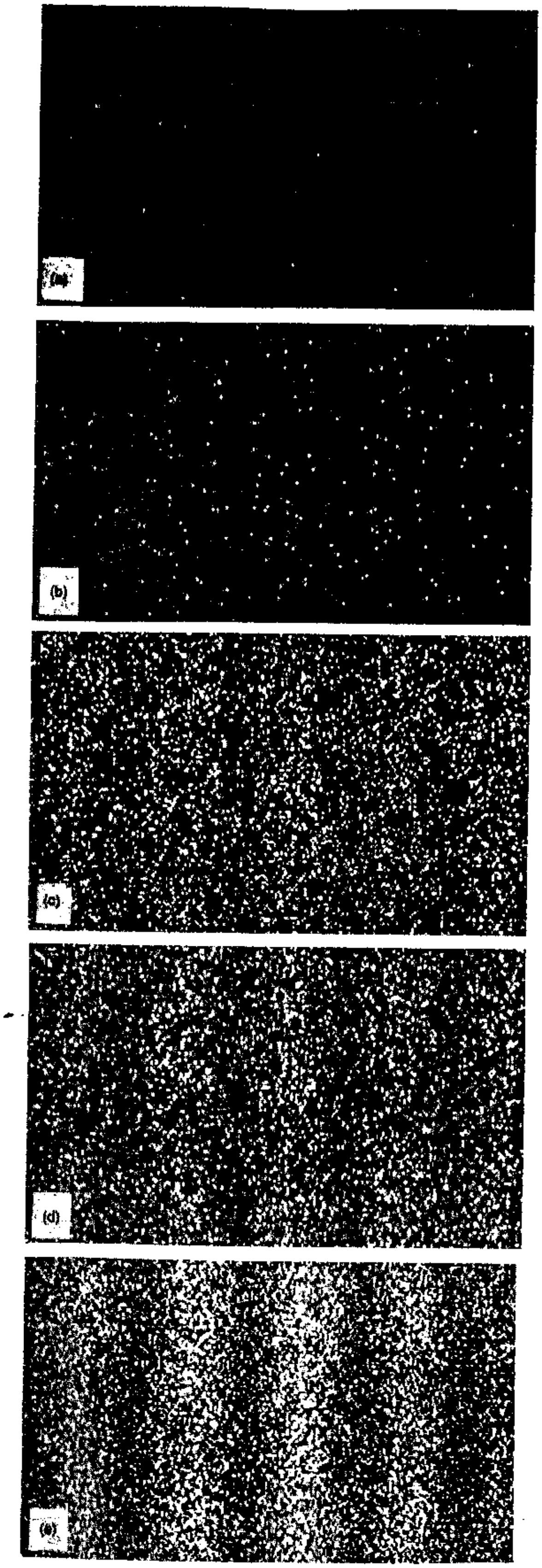


Figure 1-4. Results of an electron double-slit experiment.

1.2 Mach-Zehnder Interferometry

The following description illustrates the main points of an experiment conducted by P. Granger, G. Roger, and A. Aspect[5]. In this experiment, a beam of particles produced at source *S*, passes through a 50-50 beam-splitter to produce two beams, one transmitted and one reflected (see figure 1-5). I will label the transmitted beam B_1 and the reflected beam B_2. These beams are directed to two detectors labeled D_1 and D_2. The detectors are designed to count the number of individual particles that arrive at them. The experiment begins as a stream of particles is emitted from the source. At first the two detectors randomly register counts, but as time progresses, we notice that approximately half of the particles produced at the source are registered at D_1 and the other half are registered at D_2. From these results, we may conclude that each particle travels from the source to the beam-splitter. At the beam-splitter, the particle has a fifty-fifty chance of being either reflected or transmitted. It is something like the flip of a coin. If it is reflected, then it will register at D_2, but if it is transmitted, it will register at D_1.

Now add two mirrors which reflect the beams to a second 50-50 beam-splitter as depicted in figure 1-6. The two beams are reunited in the second beam-splitter and after passing through are directed to the particle detectors D_1 and D_2. Half of B_1 is transmitted through the second beam-splitter to D_2 and the other half of B_1 is reflected to D_1. B_2 is also split into two parts, one transmitted to D_1, and the other reflected to D_2. Unlike the first experimental arrangement, the particles are no longer equally divided between the two detectors. It is possible to arrange this experiment in such a way that B_1 and B_2 interfere destructively at one of the detectors, and constructively at the other detector. Aspect discovered, while conducting this experiment, that the distribution of particles between the two detectors, is a function of the difference of path length of B_1 and B_2. In figure 1-7 we see graphs of the number of particles arriving in the detectors, labeled MZ1 and MZ2, as a function of path difference. The path difference is displayed in terms of channels, where each channel corresponds to the photons wavelength divided by fifty. Below the graphs are two depictions of the Mach-Zehnder interferometer, one interferes destructively at D_2, and the other at D_1.

Suppose we construct the experiment with appropriate path lengths so that the beams interfere destructively at D_1 and constructively at D_2 as depicted in figure 1-6. With this arrangement, all of the particles are detected in D_2 and no particles are detected in D_1. We can now consider three variations of this experiment. One variation is to place an object in the path of B_2 so that this beam isn't able to complete the journey to the second beam-splitter (see figure 1-8). Another variation is to place the object in the path of B_1 instead of B_2. Both of these variations have the same effect. Each particle travels from the source to the first beam-splitter where it has a fifty percent chance of being reflected and a fifty percent chance of being transmitted. If it proceeds in the blocked path, then it will be restrained and never arrive at the second beam-splitter or the detectors. Only the unblocked beam arrives at the second beam-splitter, and without interference from the blocked beam is divided equally between the two detectors. Thus, when an object is blocking the path of one of the beams, each particle in the other beam has a fifty percent chance of being detected at D_1, and a fifty percent chance of detection at D_2. If the object blocking one of the beams is removed from the experiment, then each particle must be detected at D_2, and not at D_1 (compare figures 1-6 and 1-8). Another possible variation of the experiment is to block both beams with objects, thus stopping both beams before they arrive at the second beam splitter. With both beams obstructed, neither of the detectors will signal the arrival of particles, because all of the particles are blocked before reaching the detectors.

We can now use the detectors to determine which possible variations of the experiment might be in use. For instance, if a particle leaving the source isn't detected in either of the detectors then we know that one or both of the paths was blocked. Or, if a particle leaving the source arrives at D_2 then we know that either one or both of the paths was open. The interesting case is when a particle is detected at D_1. In this case we gain simultaneous information about both paths. We know that one of the two paths was closed and the other was open. This is true because if both paths were open, destructive interference wouldn't allow any particles to go to D_1, but if both paths were blocked, then the particles couldn't arrive at either of the detectors. This results in two non-classical consequences. First, a single particle detected at D_1, tells us that one of the paths was blocked, and the other was open. Which means that a single particle is able to give simultaneous information about two different paths. This would be impossible from a classical point of view, because the classical particle can only travel one of the paths and isn't aware of whether the other path is blocked or not.

The second point is that the classical particle would have to travel in the unblocked path in order to arrive at the detectors. To better understand this point, consider another variation of the experiment in which a third detector is placed in the path of B_2 as depicted in figure 1-9. By comparing figures 1-6 and 1-9 we see that there are two ways of determining if D_3 is in place. One way is that D_3 will begin to register counts as particles arrive in it. The second way we can know that D_3 is in place, is that D_1 will begin to register counts. From a classical point of view, this is extremely unusual, because the classical particle arriving in D_1 would have traveled in beam B_1. So the particle, which was detected at D_1, would classically travel in the unblocked path. Elitzur and Vaidman[6] call this an *interaction-free measurement*, because a particle traveling in path B_1, is able to signal the existence of an object in path B_2, without interacting with it.

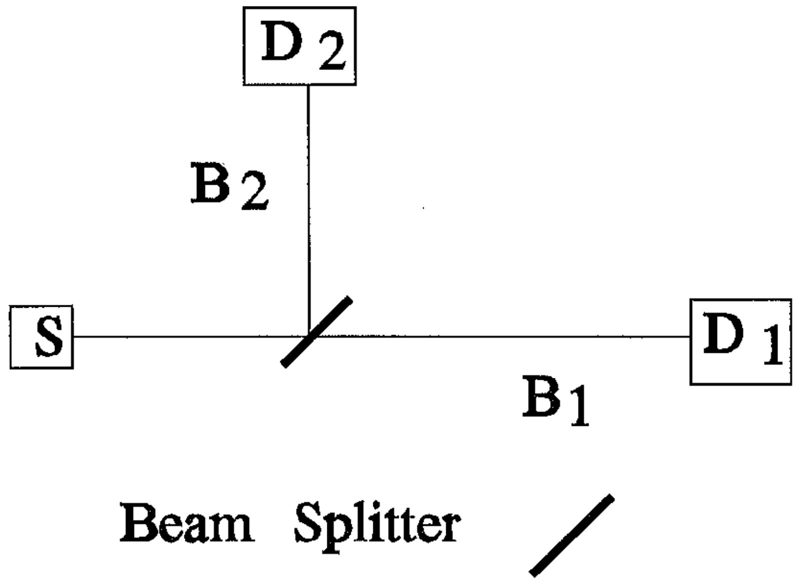


Figure 1-5. Beam division by a half silvered mirror.

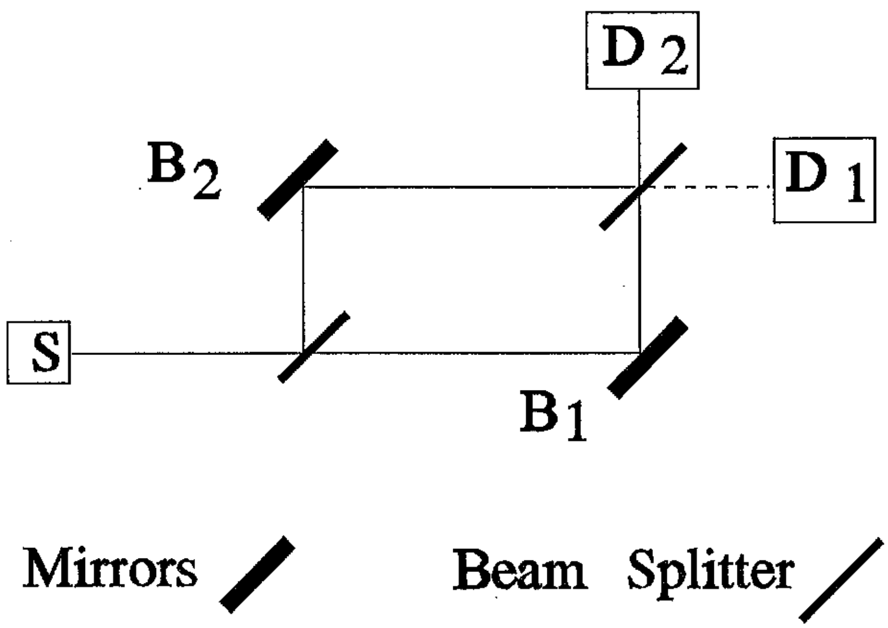
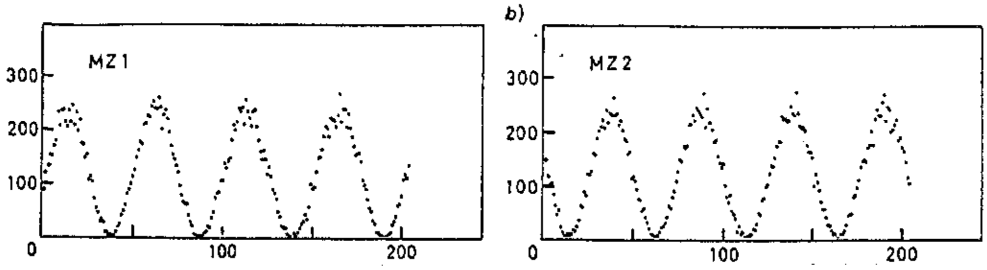


Figure 1-6. The Mach-Zehnder Interferometer.



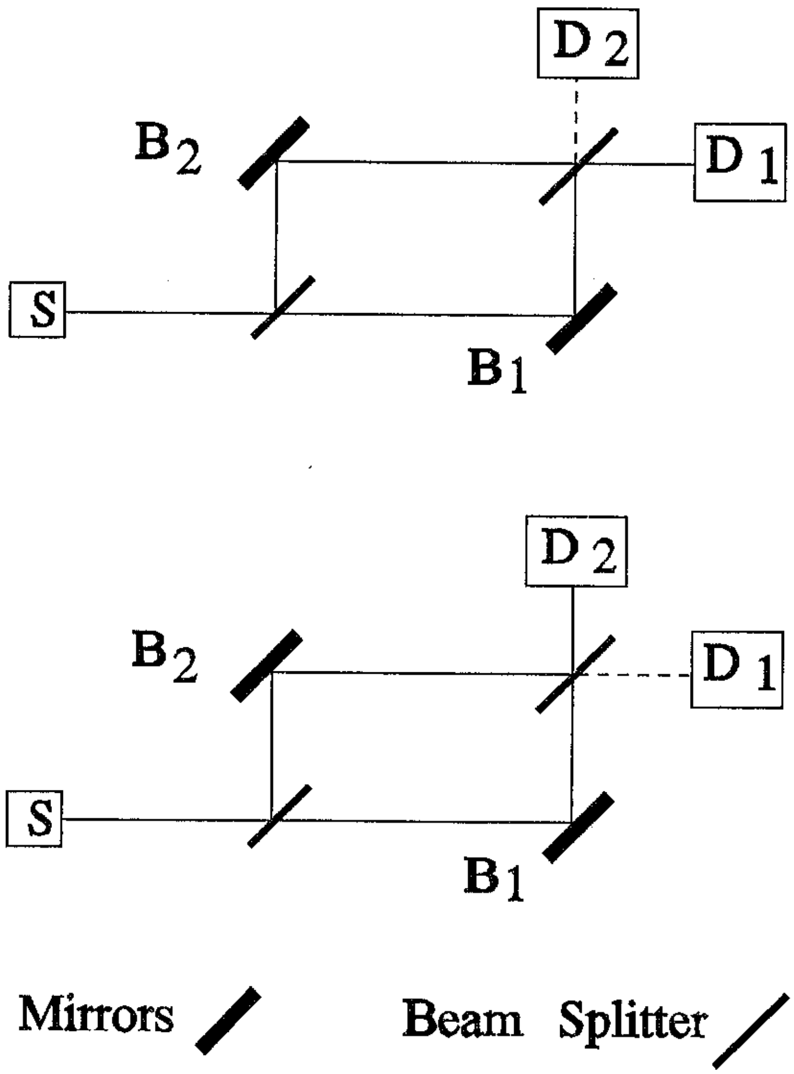


Figure 1-7. Results of varying path difference.

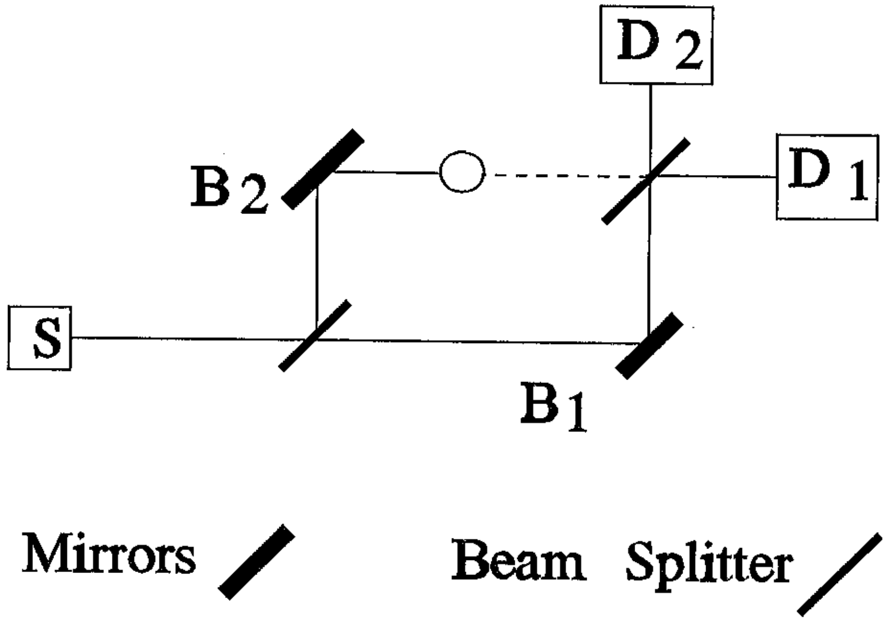


Figure 1-8. Interferometer with blocked arm.

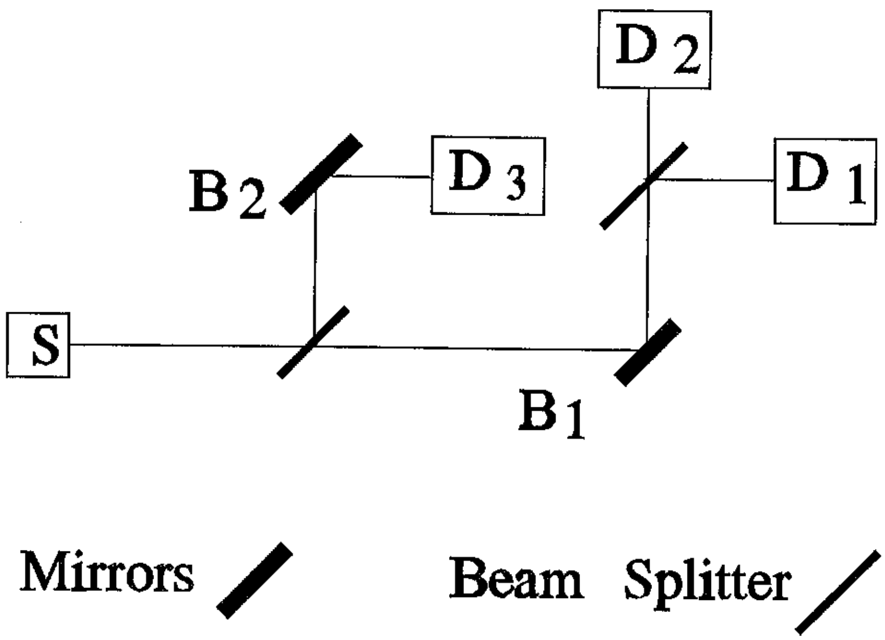


Figure 1-9. Interferometer with three detectors.

1.3 Michelson-Morley Interferometry

All of the basic aspects of the previous experiment can also be demonstrated with a modified apparatus in which the Mach-Zehnder interferometer is replaced with a Michelson-Morley interferometer (See figure 1-10). The revised experiment only requires one beam-splitter, instead of the two beam-splitters of the Mach-Zehnder type experiment. The beam passes from the source to the beam-splitter where it is divided into a reflected and a transmitted beam. I will label the two paths *A* and *B*. The two beams after traveling separate paths are reflected back to the original beam-splitter. Half of *A* is transmitted through the beam-splitter and back to the source. The other half of *A* is reflected toward a screen. *B* is also split into two parts, one transmitted to the screen, the other reflected back toward the source. On the screen, the reflected part of *A* is combined with the transmitted part of *B* to create an interference pattern in the shape of a target.

As individual photons arrive at the screen, interference directs them away from some regions of the screen and towards others. As demonstrated in figures 1-3 and 1-4, the regions of the screen that receive many photons become bright areas of the pattern, and the regions of the screen that receive few photons become dark areas. For this reason, the overall pattern represents interference effects experienced separately by each photon. The pattern is not a result of photons interacting with each other, but is caused by each photon acting individually.

I was able to conduct this experiment using an *Optics Technology inc*, model 176, Michelson-Morley interferometer. The source beam was produced by a *Spectra Physics*, model 155 SL-1, helium-neon laser. I also placed a 3 cm focal length convex lens over the aperture of the laser to cause divergence. The beam passed from the laser through the lens, and then traveled 9.5 cm to the beam-splitter. The two arms of the interferometer were approximately 9.5 cm and 10.2 cm in length. After recombination at the beam-splitter, the recombined beam produced an interference pattern on a white wall 170 cm away. The interference pattern looked like a target, with circling rings of light and dark areas.

I came to the conclusion that the dark rings were nodal areas, and that destructive interference was guiding the photons away from the dark rings and toward the light rings. Therefore, the pattern on the wall is a result of the combination of *A* and *B* in the beam-splitter. To test this hypothesis, I placed a piece of paper in the path of *A*. With paper blocking this beam, I observed that the pattern disappeared and in its place was a formless smear of illumination. I removed the paper from *A* and placed it in *B*. This also caused a similar looking illuminated smear. At this point, I placed a second piece of paper in the other beam so that both beams were now blocked. With both beams blocked, I noticed the wall was completely dark. Neither the target pattern, nor an illumination smear appeared on the wall. I now removed both pieces of paper and the target pattern reappeared on the wall (see figure 1-10).

I noticed that at the center ring of the target pattern was a nodal spot approximately 5 mm in diameter. This dark spot became illuminated whenever one of the two beams was blocked with paper. However, when both beams were blocked the 5 mm spot became dark as was the entire wall. I took note of this 5 mm nodal area, and concluded that a single photon appearing in this area is sufficient evidence that one of the arms of the interferometer is blocked and the other is open. If both arms are open, destructive interference restrains the photons from this region. If both arms are blocked, the entire wall is dark; therefore, an individual photon in this region is able to provide simultaneous information about two different paths.

I had also noticed previously, that each time I placed a piece of paper in one of the beams, that the blocked beam caused a spot of illumination on the paper. With my classical intuition, I assumed that this spot represented scattered photons from the blocked beam. In other words, when the paper was placed in *A*, I assumed that the photons in *A* were either absorbed by the paper, or scattered off of the paper in all directions causing the visible spot on the paper. With all of the photons in *A* being either absorbed or scattered, I decided that the photons in *B* were solely responsible for the smear of illumination on the wall. This leads to the paradox discussed: A single photon striking the wall informs me that one of the paths is blocked. But classically the photon giving the signal must have traveled in the unblocked path and couldn't have interacted with the paper in the blocked path.

To conclude this experiment, I placed a piece of paper so that it only blocked the bottom half of *A*. With this piece of paper in place, I noticed that the bottom half of the target pattern had been replace by an illumination smear, while the top half still displayed the target pattern. I then moved the paper to the right half of *A*, and noticed that the right half of the pattern was an illumination smear, while the left half was the target pattern. I found that I could cause any part of the target pattern to be replaced by an illumination smear, by placing the piece of paper in an equivalent position in one of the beams (see figure 1-11). I put the paper back in its original position so that it only blocked the bottom half of *A*. I then placed a second piece of paper so that it blocked the left side of *B*. With these two pieces of paper in place, the image on the wall was divided into four quadrants with the target pattern appearing in the upper right. the lower right and upper left quadrants displayed the illumination smear, and the lower left quadrant was completely dark(see figure 1-11). I traced this pattern with an ink pen. Figure 1-12 is the trace of the size and approximate shape of the pattern.

The upper right pattern represents an area where both beams are open and interfering, and the dark quadrant in the lower left represents an area where both beams are blocked. There are no dark nodal areas in the upper left and lower right quadrants. From this we see that a single photon in any of the nodal areas of the target pattern is evidence that that part of one of the beams is blocked, and that that part of the other beam is open.

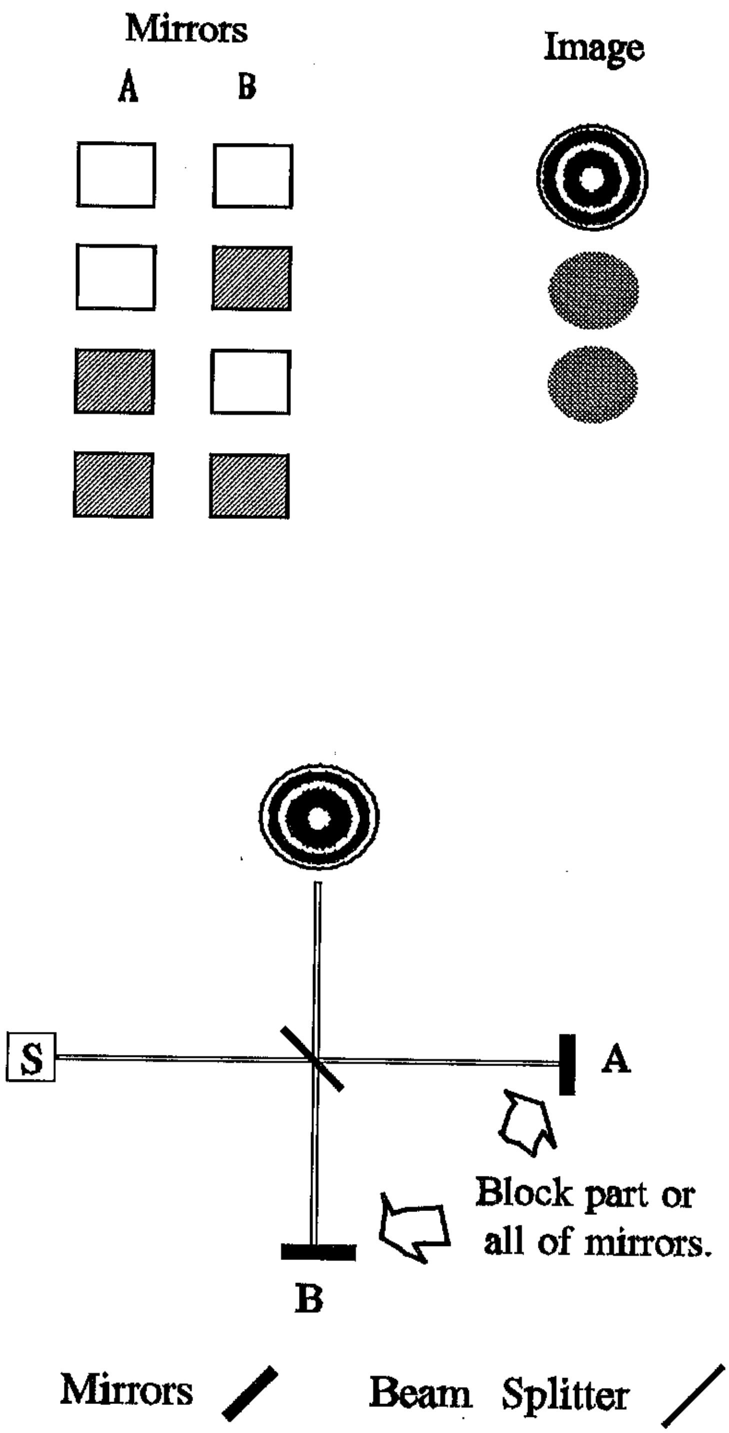
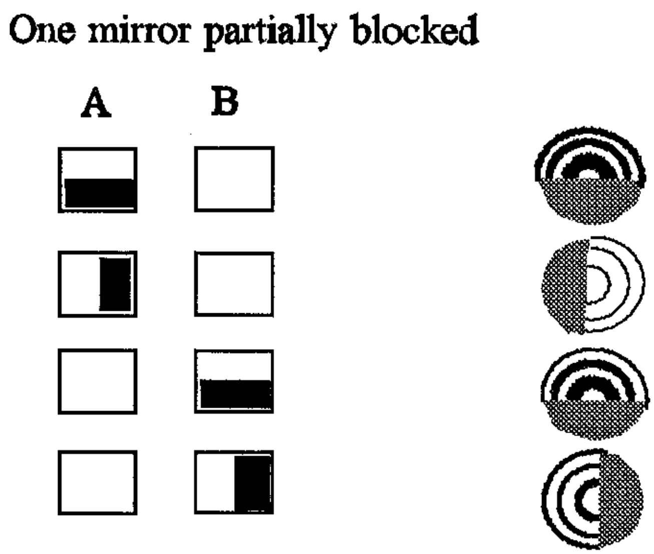


Figure 1-10. The Michelson-Morley interferometer.



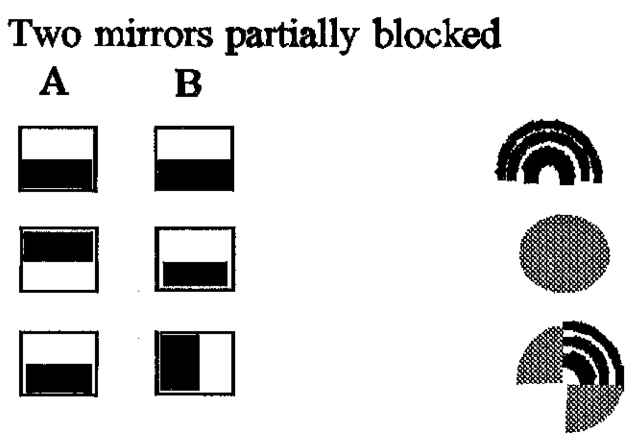


Figure 1-11. Illustrations of experimental results.

Figure 1-12. Ink pen sketch of four quadrant pattern.

1.4 Michelson-Morley Interferometry With Birefringent Crystal

I then created a variation of this experiment by placing a birefringent crystal in the emergent beam to separate it into its component polarizations (see figure 1-13). Two target patterns appeared on the wall. Figure 1-15 contains a photograph of the twin patterns on an 8.5-11 inch piece of paper. With the use of a polarized lens, I verified that the target pattern on the left was composed of vertically polarized light, and the pattern on the right was composed of horizontally polarized light. I discovered that placing a horizontally polarized lens in either *A* or *B* caused the target pattern on the left to become an illumination smear. I also discovered that a vertically polarized lens in either *A* or *B* caused a similar effect on the pattern on the right. At this point, I took a second polarized lens and discovered that polarizing both *A* and *B* vertically caused the pattern on the right to become completely dark while polarizing both *A* and *B* horizontally caused the pattern on the left to become completely dark. When one of the arms of the interferometer was polarized horizontally and the other vertically, both target patterns were replaced by illumination smears (see figure 1-14). After this, I blocked one of the beams with a piece of paper and achieved the other results displayed in figure 1-14.

From these observations, I conclude that a single photon appearing in the nodal areas of the target pattern on the left allows us to determine that the vertical polarization is free to travel in one of the arms of the interferometer and is blocked in the other. Thus, a single vertically polarized photon is able to provide simultaneous information about two different paths at the same time. It is also true that a single photon appearing in the nodal areas of the target pattern on the right allows us to determine that horizontally polarized light is blocked in one arm of the interferometer and is free to travel in the other. These facts are established by a similar argument as in the previous experiment. That is, one arm must be blocked and one open because if both were blocked then no light would arrive in the pattern, and if both were open, then destructive interference would guide photons away from the nodal areas.

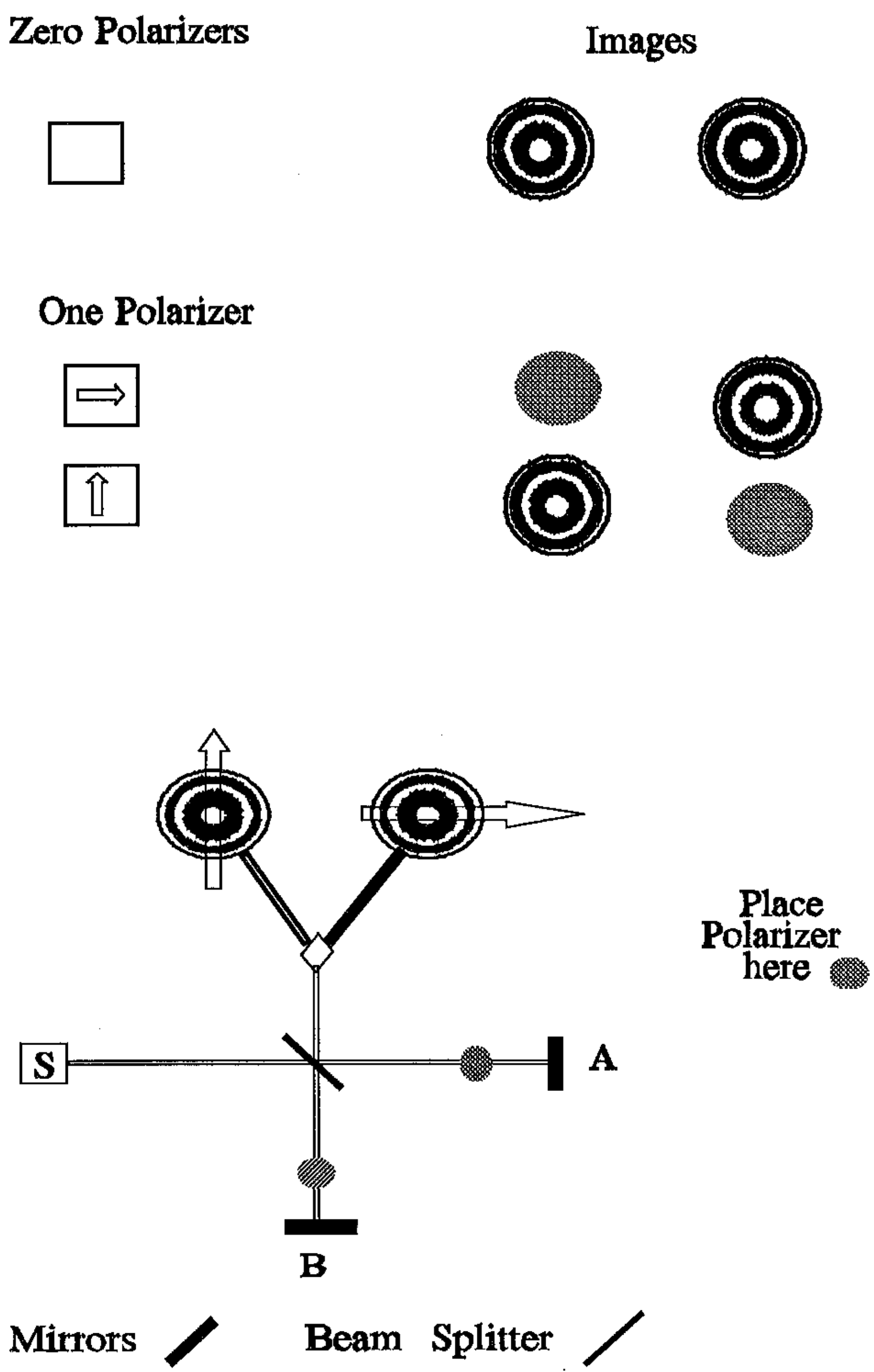


Figure 1-13. Interferometer with birefringent crystal.

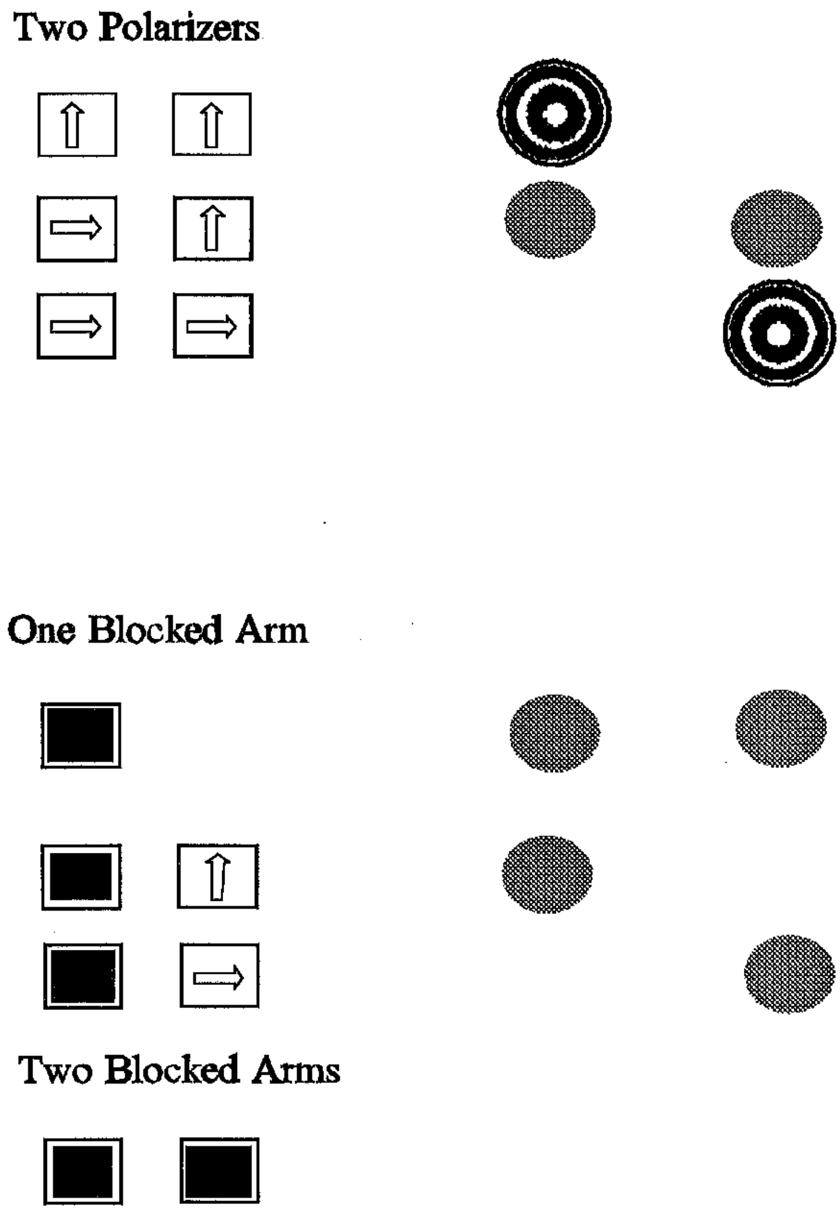


Figure 1-14. Illustrations of experimental results.

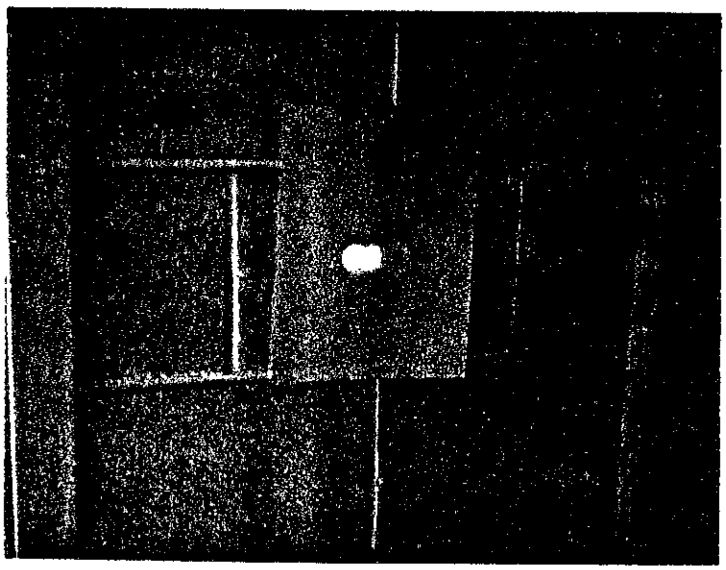


Figure 1-15. Photograph of Birefringent pattern.

1.5 Michelson-Morley Interferometry With Birefringent Crystal and Quarter-Wave Plate

As a final variation, I placed a quarter wave plate (oriented at a 45 degree angle to the horizontal) in the path of *B*, so that the beam would have to pass through it twice, thus rotating the horizontal and vertical polarizations(see figure 1-16). I then placed various polarizers in front of the source and in front of *A* and achieved the results displayed in figures 1-16 and 1-17. I observed that target patterns only appeared when the source light was circularly polarized. This is because a circularly polarized photon is in a superposition state with a 50% chance of being horizontally polarized, and a 50% percent chance of being vertically polarized. After passing through the beam splitter, it is in a four part superposition state, with a 25% chance of being horizontally polarized in *B*, a 25% chance of being vertically polarized in *B*, a 25% chance of being horizontally polarized in *A*, and a 25% chance of being vertically polarized in *A*. I will label these states A_h, A_v, B_h, B_v. As the two states in *B* pass through the quarter-wave plate, their polarizations are switched so that B_h becomes vertically polarized and B_v becomes horizontally polarized. With these new polarizations, they are able to interfere with the *A* states, so that A_v and B_h produce the target pattern on the left, while B_v and A_h produce the target pattern on the right.

This variation of the experiment has the unique feature that not only is each target pattern the result of two different paths, but it is also the result of two different polarizations. In the first version of the experiment, the pattern was the result of A-B, one photon traveling two different paths. In the second experiment the interference pattern on the left is the result of A_v-B_v, and the pattern on the right is the result of A_h-B_h. In each of these cases we are dealing with one photon, two paths, same polarization. In the current version the patterns are the result of A_v-B_h and B_v-A_h, one photon, two paths, different polarizations. And thus we see that in this variation of the experiment, a single photon in one of the target patterns is able to provide simultaneous information about two different polarizations in two different arms of the interferometer.

For example, suppose the source is right circularly polarized. In this case, a single photon appearing in a nodal area of the target pattern on the left alerts us to the fact that either the vertically polarized light in A or the horizontally polarized light traveling toward mirror B is blocked, and the other is open. If both were blocked then the pattern would vanish, and if both were open then destructive interference would guide the photon away from the nodal area. Thus from a single photon we gain information about two different paths and two different polarizations.

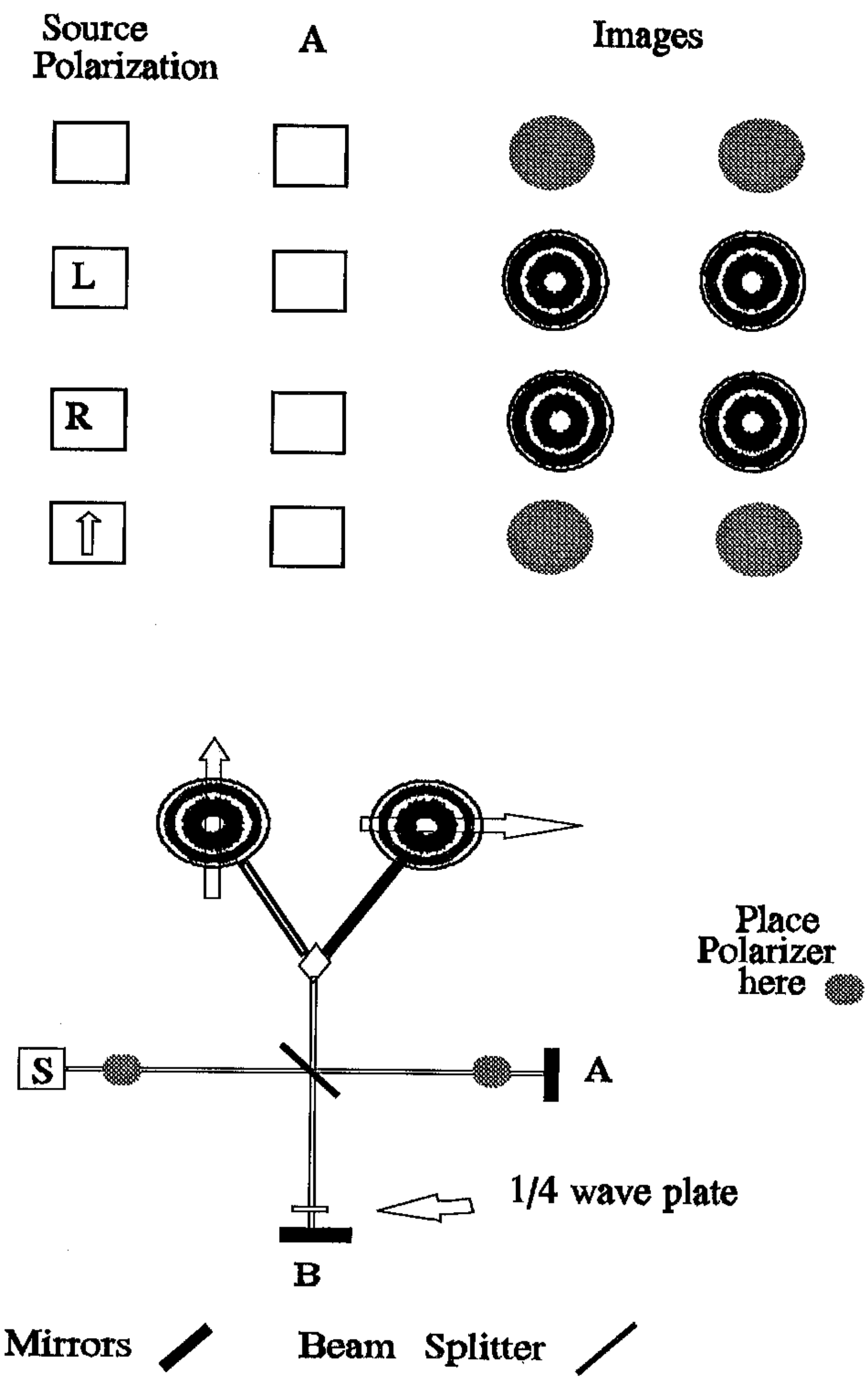


Figure 1-16. Michelson-Morley interferometer with birefringent crystal and quarter wave plate.

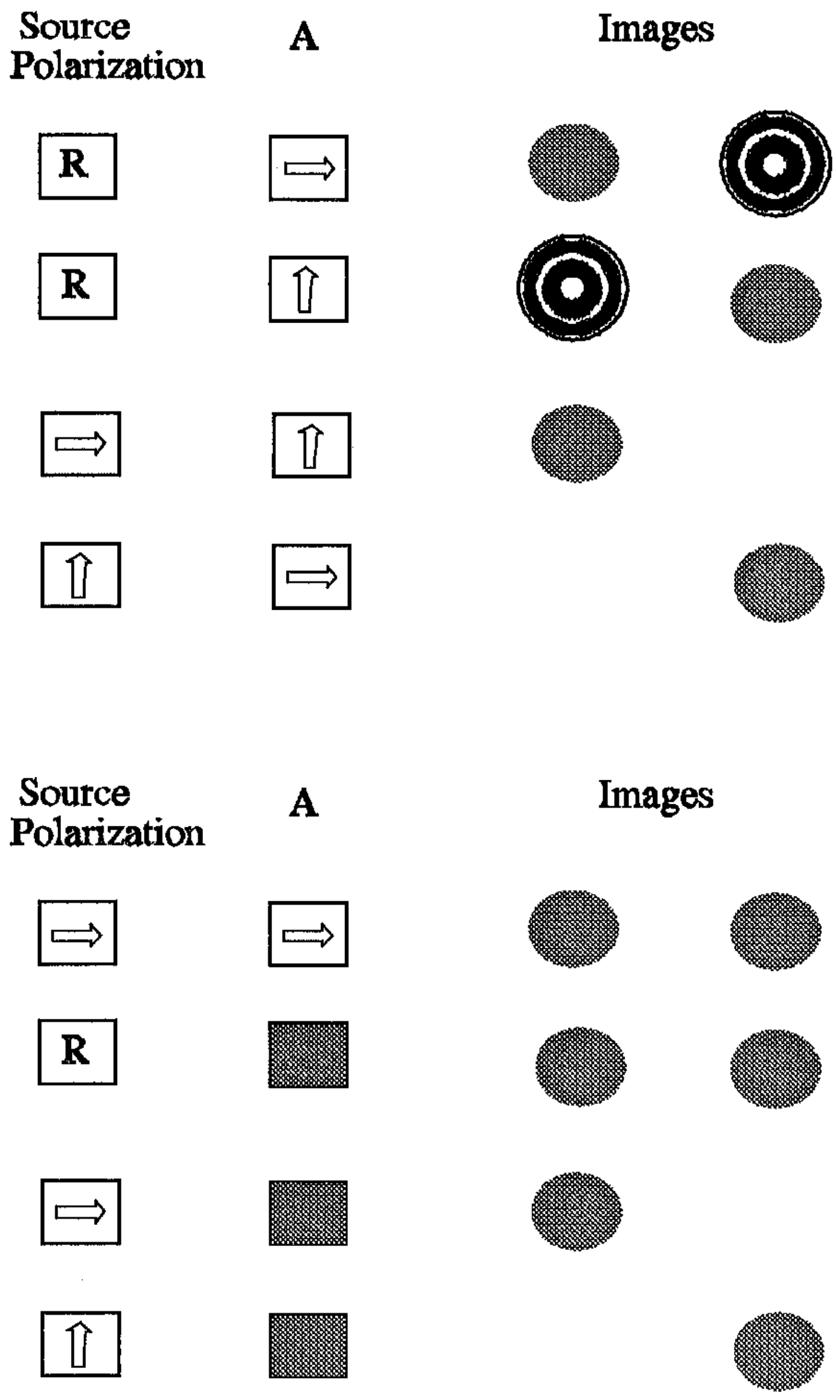


Figure 1-17. Illustrations of results.

1.6 The Young Double-Slit Experiment

I was able to construct a double-slit experiment using the same *Spectra Physics*, model 155 SL-1, helium-neon laser as in the previous experiment. I placed a 4.30 mm diameter glass rod 8 mm beyond the aperture of the laser. The glass rod was positioned horizontally, with its axis perpendicular to the beam of the laser to produce vertical divergence. The double slit was placed 8 mm beyond the glass rod with the length of the slits running vertically (see figure 1-18). Each slit was 0.15 mm wide, and the slits were separated by 0.25 mm (see figure 1-19). I turned the laser on and shifted the slits back and forth in front of the laser beam until a clear interference pattern appeared on a wall 75.5 cm away (see figure 1-20). By observing the illumination of the two slits, I affirmed to my satisfaction that the clearest pattern did in fact correspond with equal illumination of the two slits. I positioned the double slit so as to provide the clearest interference pattern.

I now desired to block one of the slits so as to observe the effect it would have on the interference pattern. Because of the closeness of the slits, it was difficult to block only one of the slits without blocking the other. To solve this problem, I placed a 0.4 mm wide single slit on top of the double slit. The single slit was tilted 18 degrees in the clockwise direction from the vertical position of the double slit, and the single and double slits intersected at the laser beam(see figure 1-21). This allowed the single slit to block the upper part of the left double slit and the lower part of the right double slit. The single slit, when in position, altered the interference pattern so that the upper and lower portions of the pattern were replaced by illumination smears, while the center part still maintained the original pattern. Figure 1-22 displays the actual size and approximate shape of this pattern as traced onto an 8.5-11 inch piece of paper. This pattern was the result caused by the single slit blocking the upper part of the left double slit and the lower part of the right double-slit, so that the upper illumination smear was the result of only the right double slit and the lower illumination smear was the result of only the left double slit. The pattern was still visible in the center where both slits were still open to produce interference.

This experiment has the same two non-classical effects as the previous two experiments: first, a single photon striking the wall in a nodal area of the interference pattern informs us that one of the slits is blocked, and the other is open. This implies that a single photon can give information about both slits. Secondly, the classical photon must travel through the unblocked slit and should be unaffected by the status of the other slit. I removed the single slit, and observed that the original interference pattern appeared on the wall.

Instead of the single slit, suppose we now take a polarized lens and cut it in half. Now place one half so that the polarization axis is vertical, and the other half so that the polarization axis is horizontal. We can place these two halves in front of the two slits so that the beam passing through one slit is polarized vertically, and the beam passing through the other slit is polarized horizontally. As with the previous experiment, the closeness of the slits will make it difficult to cover only one slit with the vertical polarized lens and the other slit with the horizontal polarized lens. Once again, to solve this problem we could place the polarized lenses at an angle with the double slit so that most of the left slit, except for the bottom is polarized vertically, and most of the right slit, except for the top, is polarized horizontally. With this arrangement, the interference pattern will appear at the top and bottom where both slits are polarized in the same direction; however, in the center where one slit is vertically polarized and the other is horizontally polarized, the interference pattern will be replaced by an illumination smear.

Although it might not be clear yet, this result is similar to the effect of placing the single slit over the double slit. The single slit destroyed the interference pattern by blocking one of the double slits. The remaining double slit wasn't able to produce the interference pattern alone. The polarized lenses have a similar effect. The polarized lenses partially block one of the slits, that is, the vertically polarized lens blocks horizontally polarized light, and the horizontally polarized lens blocks vertically polarized light. Therefore, a vertically polarized photon can only pass through the slit with the vertically polarized lens, and a horizontally polarized photon can only pass through the slit with the horizontally polarized lens. Each photon must be either horizontally or vertically polarized and for this reason, only capable of passing through one of the slits.

On the other hand, when both slits are polarized vertically, then a vertically polarized beam can pass through both slits while a horizontally polarized beam isn't able to pass through either slit. When both slits are polarized in the same direction, an interference pattern will appear on the wall. When one slit is polarized in one direction and the other slit is polarized in a direction perpendicular to the first, no interference pattern will appear.

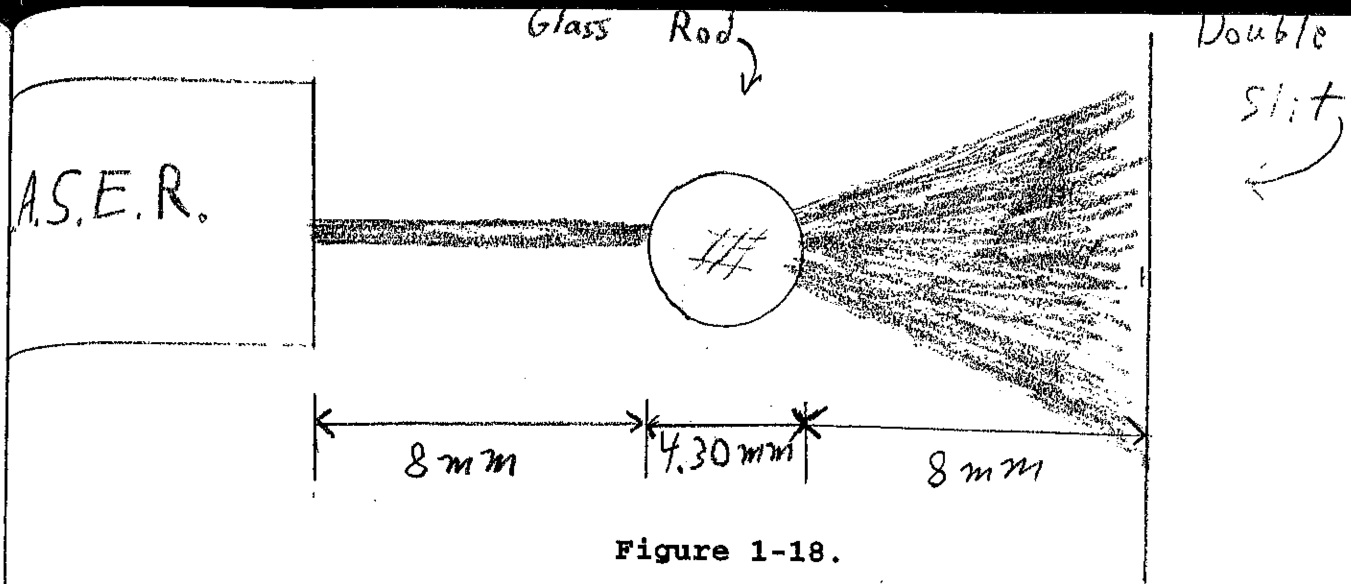


Figure 1-18.

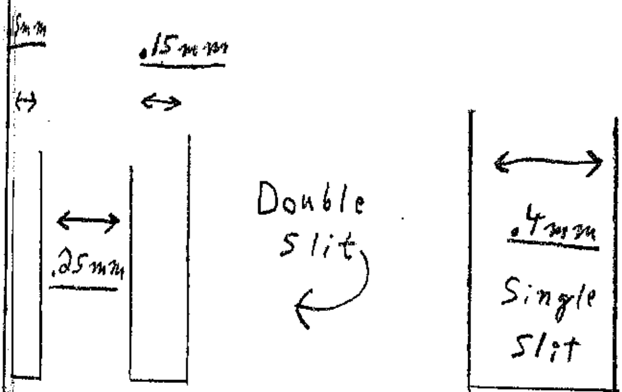


Figure 1-19.

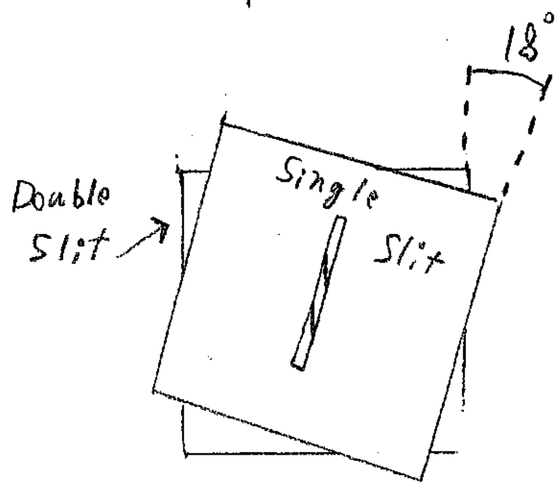


Figure 1-21.

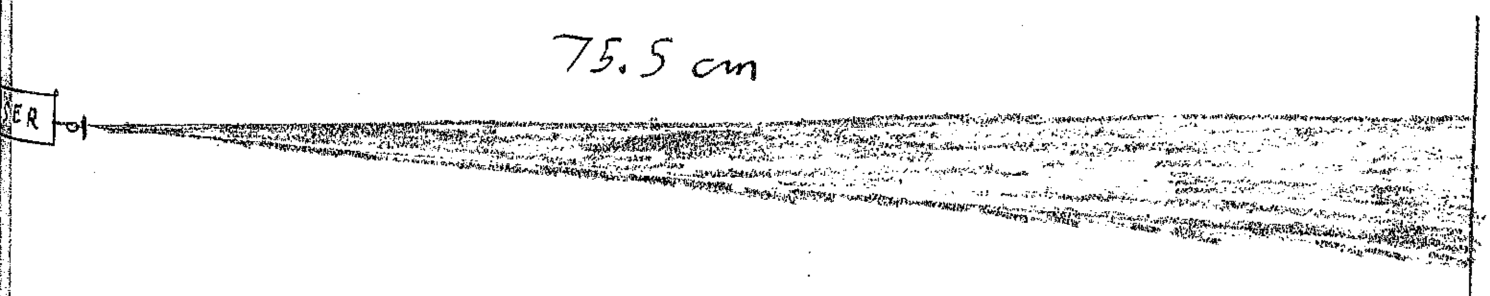


Figure 1-20.

Figure 1-22. Pen Sketch of double-slit pattern.

1.7 Conclusion

From the double-slit experiment, and the other related experiments, we see that the same photon which produces a single point on a screen is capable of providing simultaneous information about two different paths. This is in contrast with the intuitive assumption made in figure 1-2, where we assume that the particle only passes through one of the double-slits. We must account for the single particle's ability to gain simultaneous information about both paths. Consider the following possible explanations.

One explanation is that the particle and wave nature are separable. The particle-wave relationship can be compared to a surfer on the ocean. The wave spreads out and passes through both slits while the particle only passes through one slit. After passing through the slits, the wave forms the interference pattern, and the particle is guided by the wave. In figure 1-23 individual particles are represented as being carried along with the wave.

A another explanation can be derived from the previously mentioned three components of wave-particle duality:

1) "it" is an individual quantum.
2) "it" is a particle phenomena.
3) "it" is a wave phenomena.

Only two of these aspects of matter can be observed at one time, and it may be that the quantum can only experience two of these aspects of matter at one time. In this explanation, we must introduce the idea of possible realities. A single quantized particle is released from the source. As it travels to the slits, it loses the characteristic of quantization and becomes a conglomeration of possible particles and waves. At this point it is experiencing the second and third aspects of wave-particle duality, but not the first. As the group arrive at the two slits, some of the possible particles pass through the slits. After passing through the slits, the possible particles interfere with each other either destructively or constructively. When the possible particles arrive at the second screen, one is chosen through some mysterious process to become the real particle, and all of the other possible particles vanish .

This sudden selection of one of the possible particles to become the real one is known as collapse of the wave function. Through the process of constructive and destructive interference, we can determine which possible particles are more likely than others to become the real one, however we can never be sure which will be selected. For this reason, quantum mechanics is known as a probabilistic theory: we are never absolutely sure which possible particle will become real. Figure 1-24 is a representation of a multitude of possible particles before the collapse of the wave function, and figure 1-25 depicts a single particle after the collapse of the wave function.

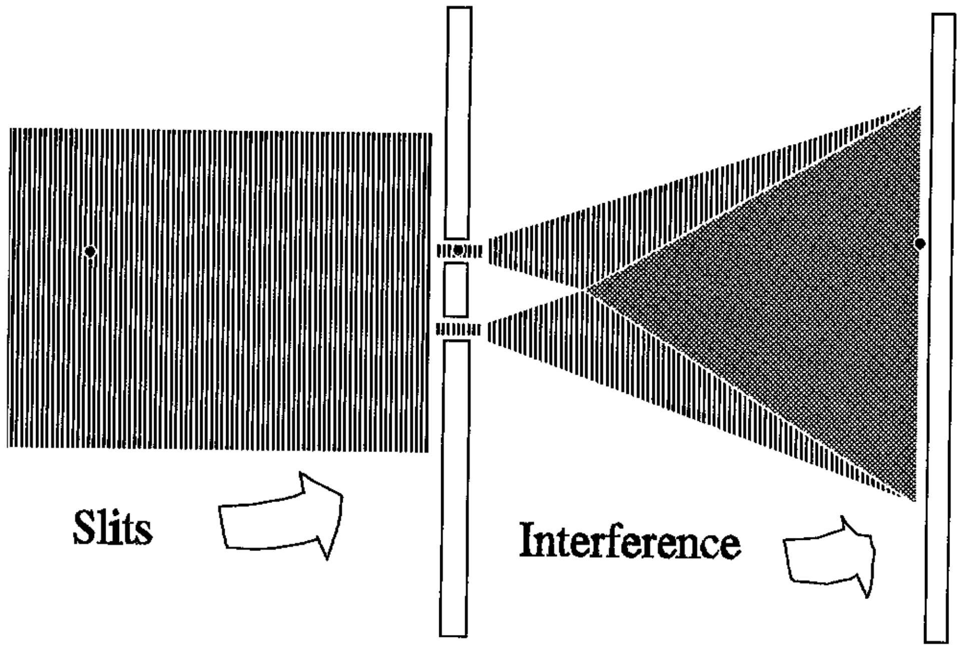


Figure 1-23. Wave-Particle Separability.

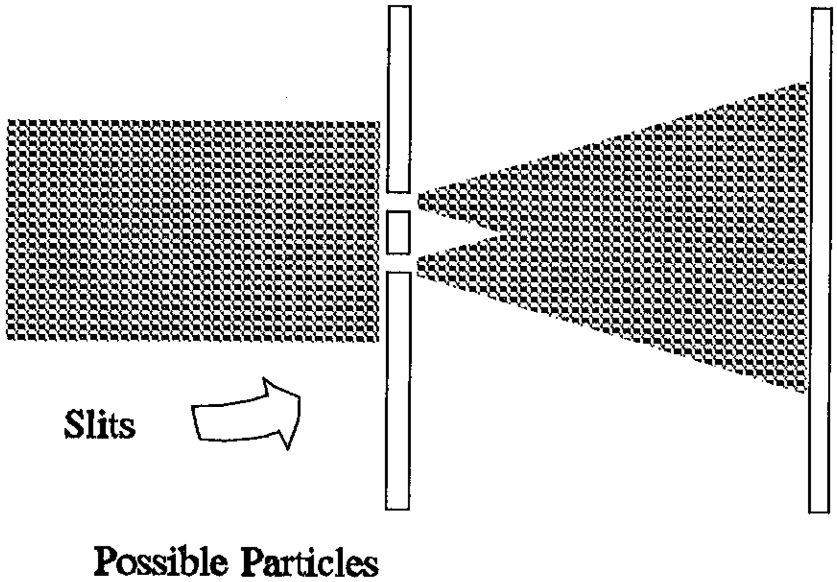


Figure 1-24. Before Collapse of Wave Function.

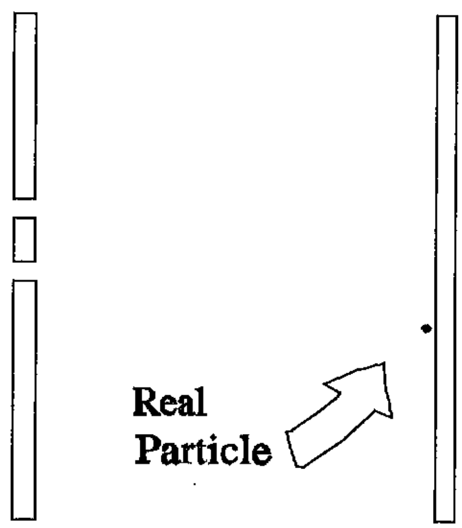


Figure 1-25. After Collapse of Wave Function.

2. An Introduction to Coupled Systems

2.1 An Escaped Politician

Before I continue, I would like to illustrate a few points with a very common and believable analogy. Suppose a wellknown politician is cast into prison for ethics violations. After a few days, he pays one of the guards to look the other way while he climbs under the fence. The local community is immediately alerted to the dangers of having an unethical politician on the loose, and everyone is on the lookout. Fortunately for the law enforcement team, our politician's behavior is quite predictable. He is observed to run in a straight line, neither turning to the right or the left, or changing speed, until he encounters one of the local citizens. When he encounters a citizen, he slips out of view, and then chooses a new direction to run until his next encounter. Observing this behavior, the chief of police calls in a team of physicists to help capture the convict. The physicists immediately recognize the behavior.

Our politicians behavior can be summed up with a simple mathematical inequality. The inequality is: $\Delta x \Delta v_x \geq$ constant where Δx represents our uncertainty about where the politician is, and Δv_x represents our uncertainty about where he is going. The product, $\Delta x \Delta v_x$, is always greater than some constant. The reason for this is the following: when our politician is in a densely populated area, he is seen by many citizens and this allows us to have a good idea of where he is; however, with each encounter he changes direction and this makes it difficult to know where he is going. On the other hand, when our politician is in a sparsely populated area he has few encounters and spends most of his time running in a straight line. With few changes in direction his course is more consistent; however, as the time between sightings increases, we are less certain about where the politician is. Therefore, it is impossible to know exactly where the politician is and where he is going at the same time. When we see him his course is uncertain, and when we don't see him, his location is uncertain.

2.2 The Photon Burst Experiment

Imagine a single particle of dust placed in an extremely large empty container. The inside of the container is under perfect vacuum and is free of anything accept the particle of dust. We may now ask the question: Where in the container is the particle? To answer this question, we line the interior of the box with light sources that give off bursts of photons at the press of a button. Also, the boxes interior is lined with the necessary viewing cameras and equipment. Whenever we want to know where the particle is, we press the button and flood the container with photons. The photons provide us with the ability to see the particle of dust.

The behavior of the particle will turn out to be similar to the behavior of the politician in the previous analogy. The similarity stems from the interaction between the particle and the photons. Each time we want to know where the particle is, we flood the container with photons. The photons allow us to see the particle, but they also collide with the dust particle and change its direction of travel. As long as the container is dark, the particle travels in a straight line according to the principle of inertia. When the container is flooded with photons, the photons continually collide with the dust particle changing its direction of motion. Much like the politician, the motion of the particle becomes more uniform with fewer photon encounters, but with few encounters we are uncertain about its position. With many encounters, its location is seen, but its direction of travel becomes uncertain.

This situation is illustrated in figures 2-1 through 2-6. Figure 2-1 represents the particle immediately after a photon encounter. The photon encounter allows us to know where the particle is, but the effect of the photon on the particle causes its motion to become uncertain as represented by the arrows. Each arrow represents one possible motion of the particle, and we can refer to them as possible motions. As time passes, the particle moves in some uncertain direction causing us to become uncertain about where it has moved to. In figure 2-2 we are still uncertain about the particle's motion, but we are now also uncertain about its position. We can refer the cluster as possible particles, where each dot represents a possible location where the real particle might be. As time passes, the particle continues to move in some uncertain direction causing us to be increasingly less certain about its position as represented in figures 2-3 and 2-4. Finally, in figure 2-5 we are completely uncertain about the particles position and so we press the button releasing another burst of photons. These photons penetrate the cluster of possible particles, allowing one of them to be the real particle and causing all of the other possible particles to disappear(see figure 2-6).

We must remember, as explained in section 1.7, that these possible particles represent more than our uncertainty about where the real particle is. In section 1.6 we discover that in the double-slit experiment, a single real particle is able to gain simultaneous information about two slits in two different locations. In one explanation, this occurs by virtue of the possible particles passing through both slits and then providing information to the real particle which appears after the collapse of the wave function. If this explanation is true, then the possible particles represent more than our uncertainty about where the particle is, they represent some fundamental uncertainty in nature. In a sense, nature doesn't decided where the real particle is until the collapse of the wave function.

In addition to the possible particle, the possible motions also represent a fundamental aspect of nature. The importance of possible motions arises from the discovery that $p = k\ (h/2\pi)$. In this equation, p represents momentum, k represents the wave number which is inversely proportional to the wave length, and $(h/2\pi)$ is a conversion constant. According to this relation, each possible motion of the particle is directly associated with a possible harmonic wave. So each dot in figures 2-1 through 2-6 represents a possible particle at one of the locations where nature may choose the real particle to be, and each arrow represents a possible harmonic wave with one of the motions that nature may choose for the real particle to have. The possible harmonic waves are to be understood in the same way that the possible particles are. They represent nature's fundamental indecision about where the particle is going. The particle's motion continues to be uncertain until one momentum is selected through collapse of the wave function.

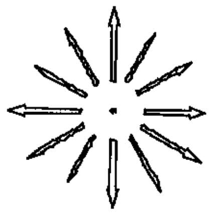


Figure 2-1.

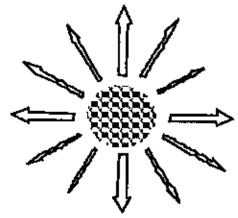


Figure 2-2.

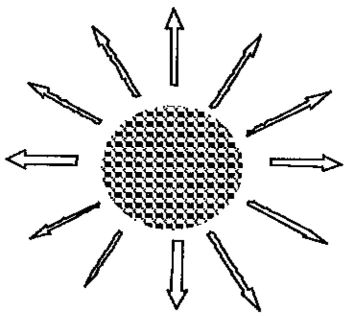


Figure 2-3.

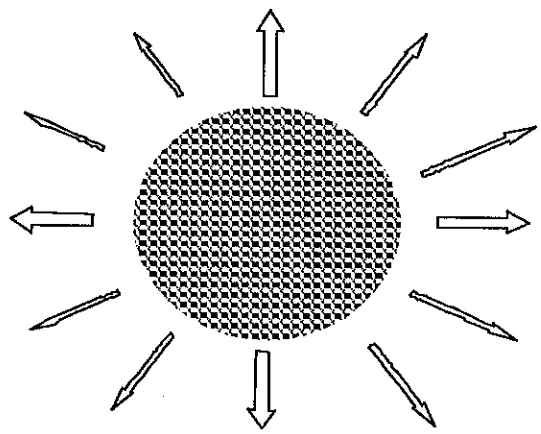


Figure 2-4.

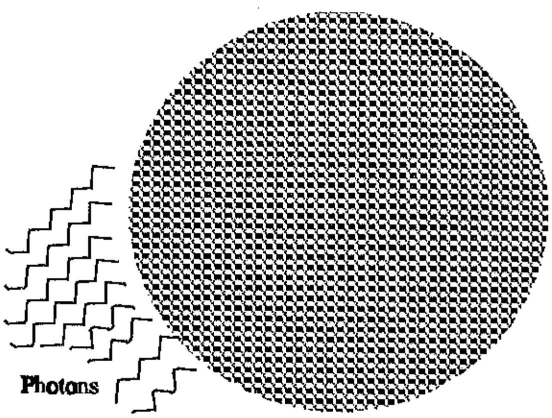


Figure 2-5.

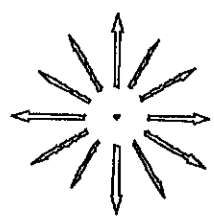


Figure 2-6.

2.3 The Single Slit Experiment

Consider another experiment in which a coherent beam of particles is directed toward a screen. The screen has a single slit cut in its center. A second screen is placed a short distance behind the first. This experiment is represented in figures 2-7 and 2-8. Figure 2-7 represents three stages the particle progresses through in this experiment. In the first stage, the particle hasn't arrived at the single slit yet. We know the direction of motion of the beam of particles, but we don't know where the particle is in the beam. The many dots in stage one are possible particles representing different possible locations of the real particle. The single arrow represents the single harmonic wave motion of the beam. During this stage the location of the particle is uncertain, but the motion is certain.

In stage two, the beam has arrived at the first screen. Upon arrival, the wave function of possible particles collapses. If the real particle turns up in the slit then it is able to pass through and continue its journey to the second screen. The first screen only has one slit cut in it and so every particle that arrives at the second screen must have passed through the slit. In this way, we know the location of the particle as it passes through the first screen. Unfortunately, this gain in information about the location of the particle is made by giving up knowledge about the motion of the particle. As the particle passes through the slit, it interacts with the slit causing its motion to become uncertain. The three arrows represent possible harmonic wave motions of the beam. As the spread in possible particles decreases, the spread in possible harmonic waves increases.

After passing through the slit, the single particle travels in the uncertain directions causing its location to become uncertain. By the time the particle arrives at stage three, its location has become uncertain again. The real particle may turn up almost anywhere on the screen. The many dots in stage three represent the various locations where the particle may appear.

Stage four represents the experiment after the final collapse of the wave function on the second screen. By reasoning backward, we can now trace out the path of the particle through the entire experiment. We can deduce its location in stage one from the fact that it passed through the slit in stage two. And from its locations in stages two and three, we can deduce the path that it traveled between these two stages. Thus we can retrospectively remove all of the uncertainties.

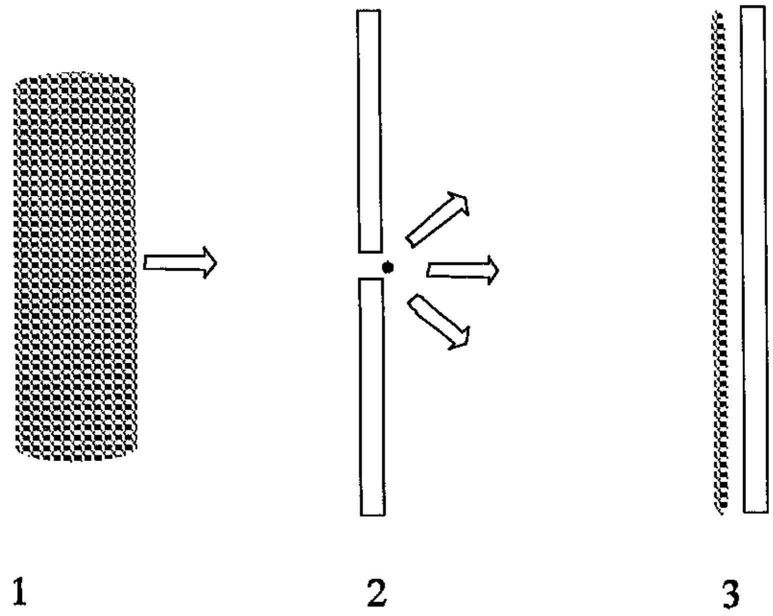


Figure 2-7. Three stages of the single slit experiment.

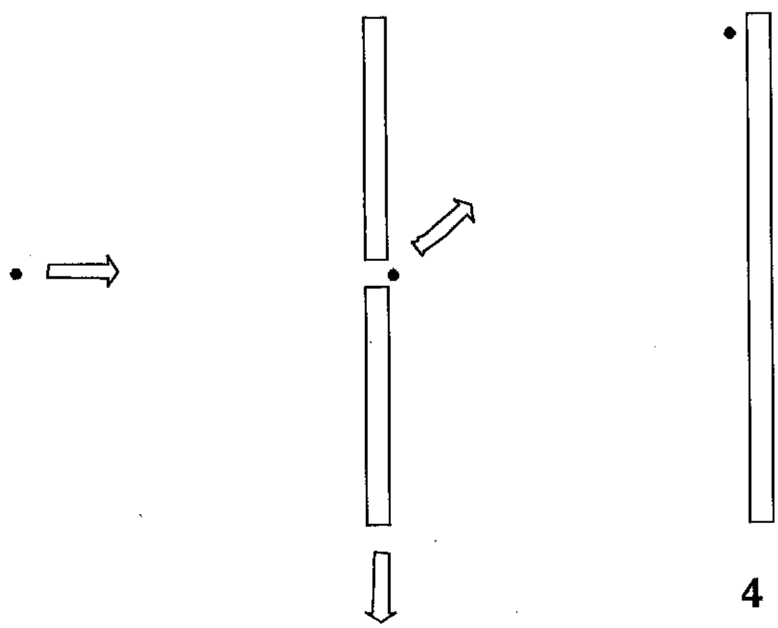


Figure 2-8. After collapse of the wave-function.

2.4 Heisenberg Uncertainty

In both the single slit and photon burst experiment, the interaction with other matter had two effects on our particle:

1. The position became more certain,
2. The momentum became less certain.

In the photon burst experiment, the photons interacted with the particle allowing us to see it and determine its position. However, they also caused the particle to change its direction making its momentum uncertain. The single slit in the other experiment had a similar effect. In stage one we were uncertain about the location of the particle, but in stage two we were able to determine the location of the particle by the fact that it passed through the single slit. This caused the position to become certain, but the interaction with the slit caused its momentum to become uncertain. It is a general rule of nature, that anything that causes the position of a particle to become more certain will also cause its momentum to become less certain, and anything which causes the momentum to become more certain will cause the position to become less certain. This relation is mathematically formulated as : $\Delta x \Delta p_x \geq h/4\pi$, and is known as one of Heisenberg's uncertainty principles. Location and momentum are examples of complementary variables. As explained in the introduction, complementary variables refer to aspects of nature that can't be simultaneously realized. They are such because any attempt to measure one, eliminates the possibility of measuring the other.

2.5 Possible Particle Pairs

There is a dilemma associated with the claim that anything that causes the position of a particle to become more certain will cause its momentum to become less certain. This dilemma arises from the fact that momentum is a conserved quantity. A conserved quantity must remain constant in an isolated system. For according to Newton, an object will continue to move with constant momentum in a straight line until acted upon by an external force. Also, for every force there is an equal and opposite force; therefore, the change in momentum of one interacting body must exactly equal the negative of the change in momentum of the other interacting body. If the change in momentum of one interacting body is uncertain, then the change in the momentum of the other interacting body is also uncertain but must be equal.

What does all of this mean? Consider an experiment in which a single stationary particle divides through natural processes into two separate particles as represented in figure 2-9. According to conservation of momentum, the sum of the momentums of the two daughter particles must equal that of the parent particle. Since the parent particle was stationary, the two daughter particles must have equal momenta in opposite directions. The pairs of arrows lettered A, B, and C in figure 2-9, represent some of the possible momenta. Whichever direction one particle is moving, the other particle is expected be moving in the opposite direction.

After a short period of time, the particles will have moved in some uncertain directions. We don't know which way the particles have gone, but we expect them to go in opposite directions. This leads to possible particle pairs as represented in figure 2-10. We can be sure that upon measuring the locations of the particles we will find them on opposite sides. This is an astounding claim when we consider that before the wave functions collapse, each possible particle has an equal probability of becoming one of the real ones; however, as soon as one wave function collapses, its real particle is located. This leaves only one possible location where the other real particle can be measured, and that is on the opposite side. This condition is referred to as coupling.

When two real particles are coupled together, their possible particles imitate the Boy Scout buddy system. Each possible particle has a buddy, and selection of one of the possible particles to be real when one of the wave functions collapses, guarantees that its buddy will also be selected when the other wave function collapses. It turns out that in addition to particles, other objects can be coupled. Consider the previously mentioned single slit experiment illustrated in figures 2-7 and 2-8. In the second stage of figure 2-7 we see that the particle's interaction with the slit has caused its momentum to become uncertain. But momentum is a conserved quantity, and so in order for the particle to change directions while traveling through the slit, the screen containing the slit must also begin moving to absorb the change in momentum of the particle. This is represented in figure 2-8 by the downward arrow below the first screen. The direction of motion of the particle coming out of the single slit isn't certain until it strikes the second screen. Therefore, the momentum of the particle and the momentum of the first screen are coupled until the particle strikes the second screen. When the particles wave function collapsed, as represented in figure 2-8, the particle was chosen to be in a certain spot on the second screen, and this only leaves one possible motion of the first screen, which in this case turned out to be downward.

Much of our current understanding about the quantum entity is a consequence of a lively debate between Albert Einstein and Niels Bohr[7]. Einstein played the part of the devil's advocate by trying to disprove some of the fundamental concepts of quantum theory such as Heisenberg's uncertainty principles. Niels Bohr was the defender of truth who ceaselessly repelled Einstein's attacks. Einstein developed a number of thought experiments to demonstrate the flaws in quantum theory, but Bohr was usually able to discover an error in each of Einstein's thought experiments. One notable exception was the EPR experiment. This experiment was published in the 1930's and stood for over 20 years before anyone was able to discover its flaws. The EPR experiment and subsequent experiments developed to disprove EPR are the topic of section 3.

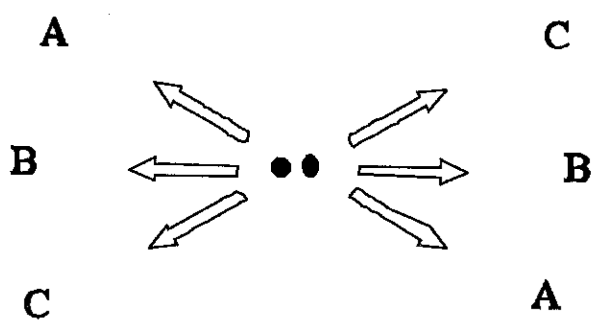


Figure 2-9. Real particles, possible momentum pairs.

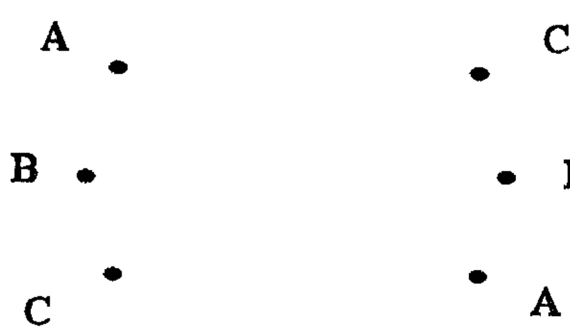


Figure 2-10. Possible Particle Pairs.

3. Two Quantum Interferometry

3.1 The E.P.R. Experiment

Albert Einstein didn't like many of the concepts of quantum theory, and one concept he especially didn't care for was the idea of complementary variables. He said that the description of the world given by quantum mechanics is incomplete, and he invented many thought experiments hoping to expose its failures. One 1935 paper by A. Einstein, B. Podolsky and N. Rosen[8]; points out an apparent contradictions between the complementary variables concept and particle coupling.

According to Heisenberg's uncertainty principle, the exact position and momentum of a particle can't be simultaneously known. This is because a particle's momentum is related to its wave nature, and its position is related to its particle nature. When we measure the momentum, the particle becomes wave-like and loses its particle-like characteristics. Its position becomes uncertain. On the other hand, when we measure its position, the particle maintains its particle-like characteristics, but doesn't display wave-like behavior. Its momentum becomes uncertain. Also, according to particle coupling, two or more particles can be coupled together so that the outcome of a measurement on one particle, guarantees an outcome of a measurement on the other particle. This was demonstrated in the previous sections possible particle pairs experiment where two particles were coupled so that their momenta had to be equal and in opposite directions.

The following experiment, proposed by Einstein et al, is known as the EPR experiment. Suppose that the momentum and position of two particles are coupled together so that a measurement of the momentum of one particle allows us to predict the momentum of the other particle, and a measurement of the position of one particle allows us to predict the location of the other particle. Suppose we allow the particles to travel a great distance apart so that they no longer interact, and then we perform various measurements on the particles. I will label the particles P_1 and P_2.

If we measure the position of P_1, we can use the result of the measurement on P_1 to predict the position of P_2. According to the EPR authors, "If, without in any way disturbing a system, we can predict with certainty...the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity." This means that the ability to predict the location of P_2 implies that P_2 is in fact at that location. The authors felt that the ability to predict a measurements outcome was as good as making the measurement. This implies that the wave functions of both particles collapse upon measurement of one particle. One particles wave function collapses by virtue of being measured, and the other particles wave function collapses by virtue of being predictable.

Also, instead of choosing to measure the position of P_1, we could have measured the momentum of P_1. Doing this would allow us to predict the momentum of P_2. The authors hypothesized that at the time of measurement, the particle P_2 is too far away from P_1 to be influenced by the measurements being made on P_1; therefore, the authors believed that both the position and momentum of P_2 must be certain at the time of departure from P_1. The position of P_2 must be certain because it can be determined by measuring the position of P_1, and the momentum of P_2 must be certain because it can be determined by measuring the momentum of P_1. Since P_2 is too far away to be influenced by measurements made on P_1, both the momentum and position of P_2 must be predetermined. This is in direct conflict with the Heisenberg uncertainty principle which claims that position and momentum can't be simultaneously certain.

As a third point, suppose we measure the position of P_1, and the momentum of P_2. We can now predict the position of P_2 from the measurement made on P_1, and we can predict the momentum of P_1 from the measurement made on P_2. This seems to imply the existence of hidden variables. Hidden variables are hidden aspects of nature that determine the outcome of measurements. We might imagine that upon departure from each other, the two particles agree upon some hidden variables. These hidden variables determine the final location and momentum of the particles. According to this theory, the position and momentum of a particle are determined before collapse of the wave function, but are only revealed to us after the collapse. This would imply that complementary variables don't represent fundamental indecision on the part of nature, but rather represent our inability to discover natures decisions.

3.2 Types of Particle coupling

Since all EPR experiments require coupled particles, it is appropriate to discuss various ways in which particles may be coupled together. One form of coupling involves uncertainty in momentum. This form is discussed in section 2.5 and represented in figures 2-9 and 2-10. As explained, this form of coupling results from conservation of momentum. Uncertainty in momentum and position satisfy the Heisenberg relation $\Delta x \Delta p_x \geq h/4\pi$.

Time and energy are also complementary variables and are related by a second Heisenberg uncertainty equation: $\Delta t \Delta E \geq h/4\pi$. This equation is interpreted to mean that the more certain we are about when an event occurred, the less certain we are about how much energy was involved. Much like momentum and position, time and energy uncertainty stem from the particle and wave natures of matter. The energy is associated with the wave nature, and the exact time of events is associated with the particle nature. To understand why, consider standing on the shore of an ocean and asking yourself when the ocean waves happened. This question doesn't make sense because the waves are an ongoing process with no beginning or end. Perfect waves, have no beginning or end in time as well as no beginning or end in space. They fill all of space and time with their perfect repeating pattern.

With this in mind, consider an event like the splitting of a particle into two daughter particles as represented in figure 3-1. If the reaction is displaying particle like behavior, then it will occur in an instant. The time of the reaction will be certain but the energy will be uncertain. But if the reaction displays wave like behavior, then the energy of the reaction will be certain but the time will be uncertain. This is represented in figure 3-1, by the three possible times A, B, and C. After the daughter particles are formed, they begin to move away from each other. Because of conservation of momentum, they move away with equal speed in opposite directions. As time passes, the distance the particles have traveled will depend on when they were formed. If they were formed earlier, they will have had more time to move further away. Also, the speed of the particles will affect how far they have moved. Faster particles will move further apart than slower ones.

We may now introduce possible particles in time. The three possible particle pairs labeled A, B, and C in figure 3-2, represent three possible distances the particles may have moved apart. Due to uncertainty in time, we may not be sure when the daughter particles were formed, but we do know that they were formed at the same time and are traveling with the same speed. Therefore, however far one particle has traveled in one direction, the other particle must have traveled the same distance in the opposite direction. Before the collapse of the wave function, each particle has some probability of being real. When the wave function collapses and one particle becomes real, then we can be certain that its buddy will also be real if measured. The real pair represent the actual time of the reaction.

We may combine these two forms of uncertainty to gain an overall picture. Because of momentum uncertainty we are unsure about the speed and direction of the two daughter particles, and because of time uncertainty we are unsure about when they were created. But we do know that they were created at the same time and are moving at the same speed in opposite directions. The overall effect is represented in Figure 3-3. Each possible particle represents one possible location where the particle may have moved to. Because of coupling, each possible particle is paired with one on the opposite side. After the wave functions collapse, two real particles have moved the same distance in opposite directions as represented in figure 3-4.

In addition to momentum and time coupling, spin can also be used in EPR type experiments. Spin is a recently discovered aspect of nature and can only be explained by quantum theory. Spin can be compared to the rotational motion of a particle, however spin is vitally different in that it stems from the wave nature rather than the particle nature of matter. The polarization effects of light are a result of the spin of individual photons. Spin can be measured around any axis, but only comes in discrete amounts. Half spin particles only allow two possible outcomes of measurement:

+1/2 and -1/2. This means that if we choose some axis in space, and measure the spin of a group of half spin particles, each particle will have a spin of either +1/2, or -1/2 around that axis.

Spin is conserved on any axis; however, the spin in one direction is complementary with respect to any perpendicular direction. This means that measuring the spin along some given axis causes the spin along any perpendicular axis to become uncertain. For example, in cartesian coordinates, measuring the spin around the z axis will cause the spin around x axis and y axis to become uncertain. It is this complementary relationship that allows spin measurements to be used in EPR type experiments.

Consider the previous event in which a single particle divides into two daughter particles. Suppose the total spin of the parent particle is zero. Because of conservation of spin, the spins of the daughter particles must sum to zero along any axis. This is known as the singlet state. Suppose the daughters are half-spin particles. This situation is represented in figure 3-5 in which there are two cases labeled A and B. Because they are half spin particles, the only allowed values are +1/2 and -1/2. If one of the particles has a spin of +1/2 then the other must have a spin of -1/2 so that the spins will sum to zero. Thus the only possible spin orientations for two half-spin particles in the singlet state are those given as A and B in figure 3-5.

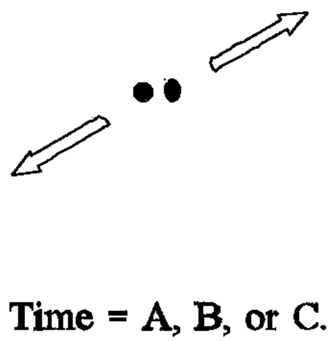


Figure 3-1. Time uncertainty.

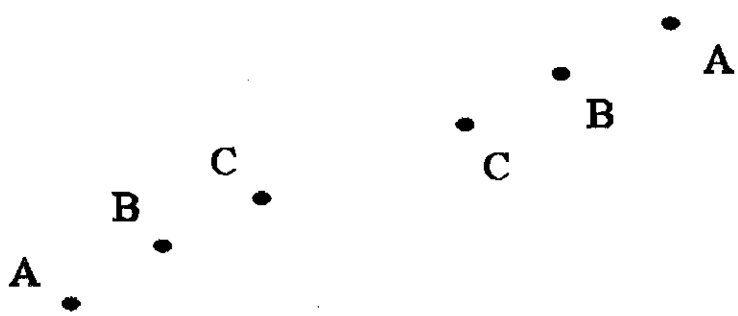


Figure 3-2. Possible Particle Pairs.

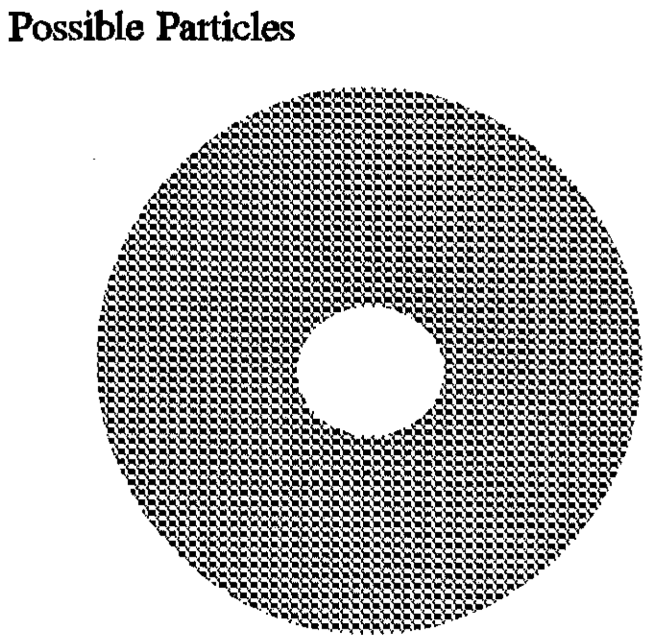


Figure 3-3. Before collapse of wave function.

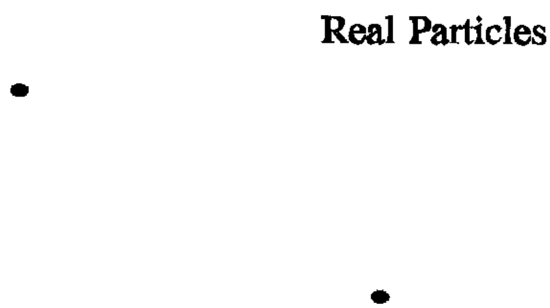


Figure 3-4. After collapse of wave function.

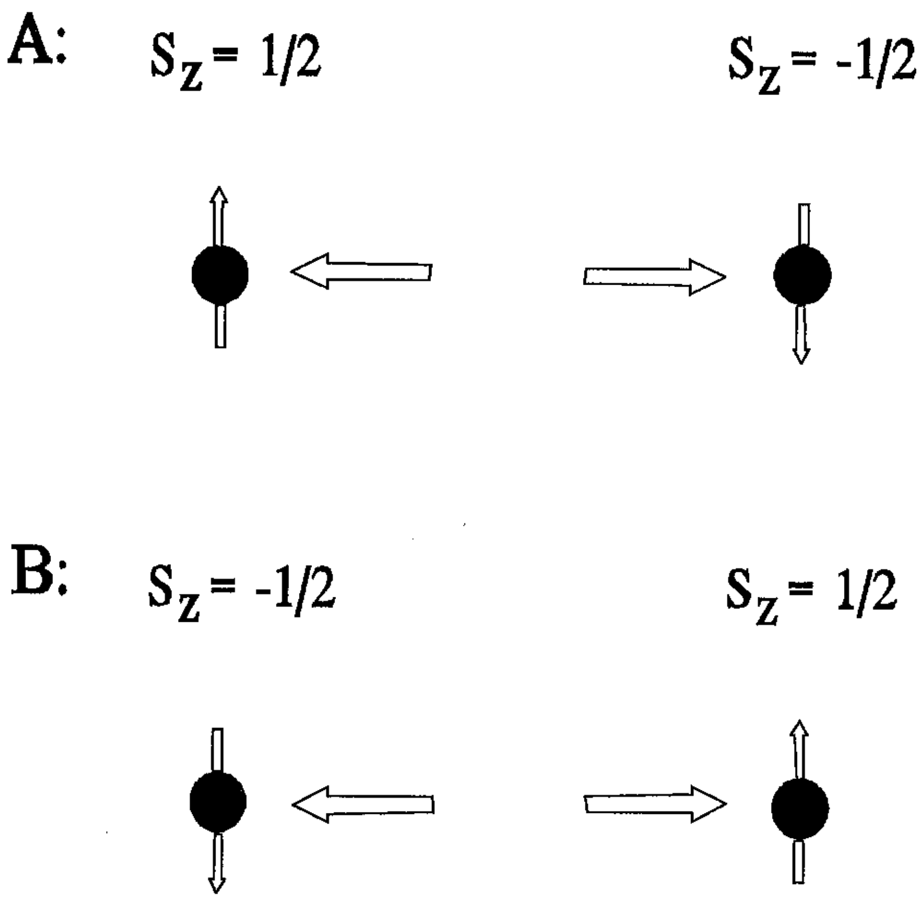


Figure 3-5. Singlet coupling of half spin particles.

3.3 Sources of Coupled Pairs

A few methods for producing coupled particles are commonly referred to. One is to use systems of particles produced by the decay of a single particle. This method is discussed extensively in section 2.5. A single particle divides into two particles. Due to conservation laws of physics, the energies, momentums, spins, and other conserved quantities of the daughter particles must sum to the energy, momentum, spin, etc. of the parent particle.

A second source of coupled particles is photon emission during the cascade of an electron within an atom. This method is illustrated in figure 3-6, in which a single electron cascades from a moderately stable energy state E_3, to an extremely unstable state E_2 and then to the ground state E_1, thus emitting two photons A and B. Photons emitted within a few nanoseconds of each other through this process, have been observed to display polarization coupling[9].

A third method of producing coupled particles arise from the process of parametric fluorescence or down conversion represented in figure 3-7. In this process, each photon in a laser beam is split into two daughter photons while passing through a nonlinear crystal. These daughter photons are observed to display coupling with respect to momentum[10].

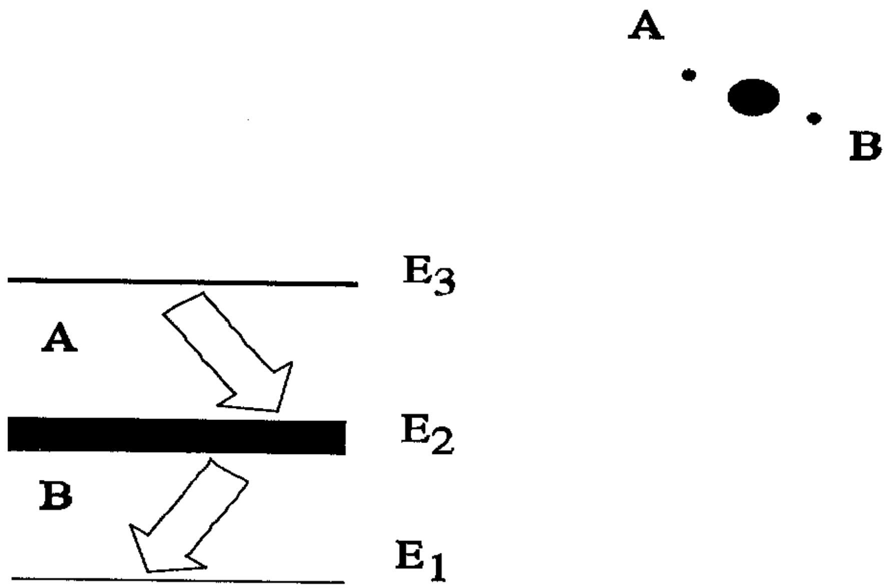


Figure 3-6. Photon production by electron cascade.

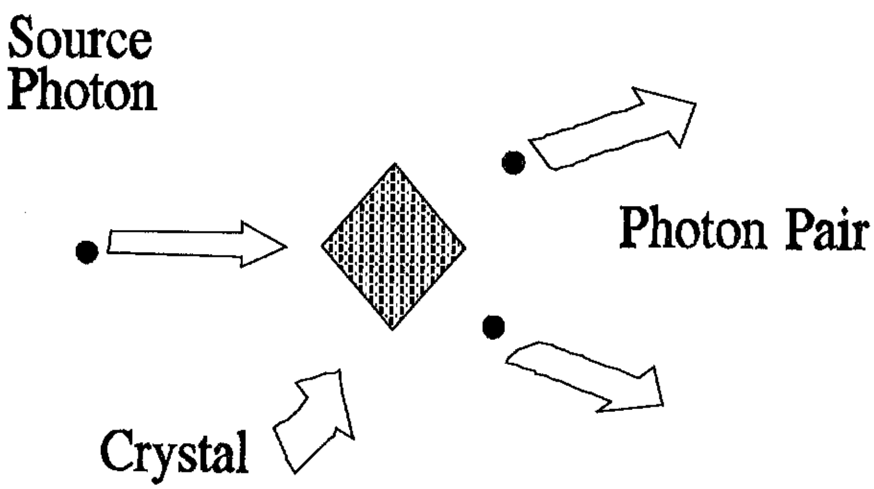


Figure 3-7. Parametric down-conversion.

3.4 Bell's Theorem

In 1964, J.S. Bell proposed a thought experiment in rebuttal to the EPR experiment[11]. Unlike the EPR experiment, Bell considered particles that were coupled together with respect to spin. He didn't explicitly state how these particles were to be produced, but he did state that they must be half spin particles coupled in the singlet state.

The essential idea behind Bell's experiment is represented in figure 3-5. Two spin half particles are produced in the singlet state so that their spins sum to zero around any axis. The two possible combinations are labeled A and B in the figure. If we measure the spin of one particle along some axis, then we can predict the spin of the other particle along the same axis with absolute certainty.

If we interpret this experiment using an EPR type hidden variables theory, then we may reason that during the measurement process, the two particles are too far apart for one to be affected by measurements made on the other. Since we can predict the spin of the particle on the left along any axis by measuring the spin of the particle on the right along that axis, we may falsely assume that the spin of the particle on the left is set by hidden variables which were determined before the particles separated.

By considering this experiment, Bell discovered a mathematical discrepancy between the predictions of quantum mechanics, and the predictions of hidden variable theories. Both theories agree that the spins of the two particles must sum to zero around any common axis. However, quantum mechanics and hidden variable theories disagree on the outcome, if we measure the spin of one particle around an axis, and then measure the spin of the other particle around an axis pointing in a slightly different direction. Bell discovered that hidden variables contain certain restrictions which have become known as Bell's inequalities. If Bell's inequalities are violated, then the hidden variable theories are false.

According to Bell, "It is the requirement of locality, or more precisely that the result of a measurement on one system be unaffected by operations on a distant system with which it has interacted in the past, that creates the essential difficulty." Bell discovered that according to quantum mechanics, measurements made on particles in coupled systems must instantaneously affect the other particles in the system regardless of distances. This effect is called *action at a distance*, and contradicts the EPR idea that particles are only affected by their immediate surroundings.

A version of Bell's experiment was conducted by Stuart J. Freedman and John F. Clauser[12]. In this experiment, coupled photons were produced by the atomic cascade of calcium. Photon pairs were coupled in the singlet state so that their spins would sum to zero around a common axis. Photons are spin-1 particles, and spin-1 particles allow three possible spin measurement outcomes which are 1, 0, and -1. However, relativistic concerns forbid some of the spin solutions causing them to be usable in the place of spin-1/2 particles as represented in figure 3-8. Freedman and Clauser found that their data violated Bell's inequalities, thus favoring quantum mechanics rather than hidden variable theories. They claim, "Our data, in agreement with quantum mechanics, violate these restrictions (Bell's inequalities) to high statistical accuracy, thus providing strong evidence against local hidden-variable theories."

Approximately 10 years later, a second Bell type experiment was conducted[13]. This experiment was designed to determine if coupled particles are immediately affected by the measurements made on distant particles as predicted by quantum mechanical action at a distance, or if the particles require time to become informed of the measurements according to Einstein's causality. In spite of its deficiencies, the authors felt that their experiment supported the quantum mechanical predictions. As stated by the authors, "A more ideal experiment with random and complete switching would be necessary for a fully conclusive argument against the whole class of supplementary-parameter (hidden variable) theories obeying Einstein's causality. However, our observed violation of Bell's inequality indicates that the experimental accuracy was good enough for pointing out a hypothetical discrepancy with the predictions of quantum mechanics. No such effect was observed."

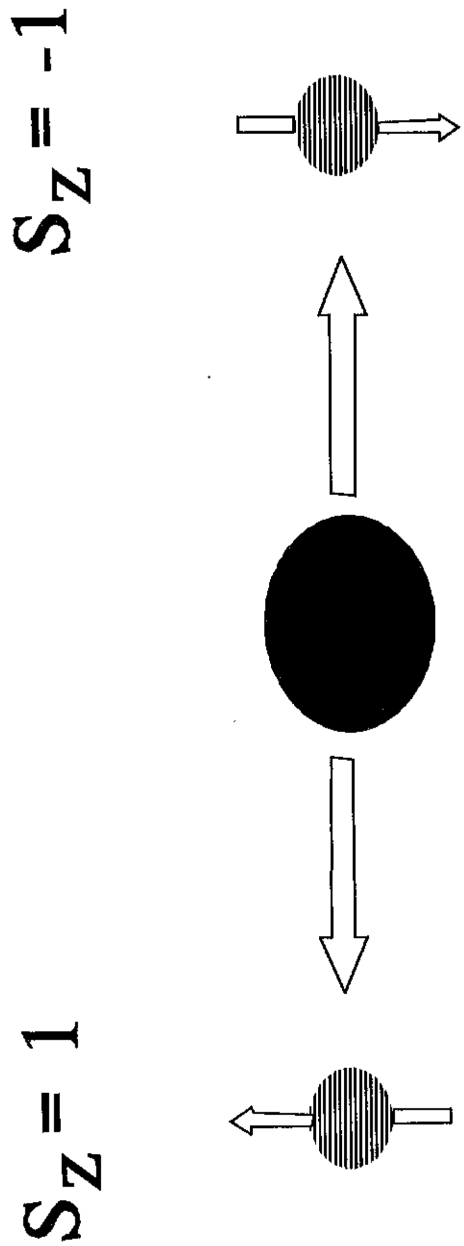


Figure 3-8. Bell type photon experiment.

3.5 Two particle Interferometry

In addition to Bell type spin-coupling experiments, other methods have been developed to test Bell's inequalities. A method referred to as two particle interferometry is similar to EPR in that it uses particles which are coupled with respect to momentum[14]. As illustrated in figure 3-9, a beam of photons is produced by the source S. These photons pass through a parametric down converter crystal in which each photon is divided into two daughter photons. The daughter photons are coupled with respect to momentum such that if one photon is in beam B_1, the other must be in B_3. Or if one is in beam B_2, the other must be in B_4.

A real photon passes through the parametric down-converter and is divided into two real daughter photons. We may picture one of the daughter photons being divided into possible photons in beams B_1 and B_2, and the other daughter photon being divided into possible photons in beams B_3 and B_4. The possible photons are paired off through coupling. Each possible photon in B_1 is paired with a possible photon in B_3, and each possible photon in B_2 is paired with a possible photon in B_4. After traveling separate paths, the beams B_1 and B_2 are reunited in a beam-splitter, and the beams B_3 and B_4 are reunited in a different beam-splitter. After the beam-splitters, the beams travel to detectors where the wave functions collapse. One of the real particles turns up in either D_1 or D_2. The other real particle turns up in either D_3 or D_4.

Surprisingly, there are no single particle interference effects in which a particle can be made to favor one detector similar to the effects illustrated in figure 1-7. With no single particle interference, detectors D_1 and D_2 each have an equal fifty-fifty probability of receiving the real particle, and D_3 and D_4 each have an equal probability of receiving the other real particle. Thus, there is no way of determining which detector will receive a real particle.

Even though there aren't single particle interference effects, there are two particle interference effects which demand certain correlations between the detections. By adjusting the path lengths of B_1, B_2, B_3, and B_4 it is possible to prevent certain combinations of detections. For instance, we may adjust the path lengths so that it is impossible for the pair of real daughter photons to be detected in D_1 and D_3, or to be detected in D_2 and D_4. This arrangement leads to the interesting result that if we register a particle in D_4, we can be certain that the other particle will register in D_1. Or if we register the particle in D_3, then we can be certain that the other particle will register in D_2. We have no way of knowing which detector the first particle will show up in, but once it is detected, the other particle is guaranteed to turn up in the correlated detector. This ability to predict where the other particle will turn up at is similar to the ability in the Bell experiment to predict the spin one particle by knowing the spin of the other. Both quantum mechanics and the hidden variable theories agree one the outcome for this arrangement of path lengths. On the other hand, there are certain path difference relationships for which the quantum mechanical and hidden variable theories predict slightly different results.

Such two-particle interferometry experiment was conducted by J.G. Rarity and P.R. Tapster[15]. In this experiment, the source photons were produced by a 413.4-nm wavelength krypton-ion laser. The photons were down converted by a deuterated potassium dihydrogen phosphate crystal. Data from this experiment violated Bell's inequality supporting the quantum mechanical interpretation over the hidden variable interpretation. The authors state, "In conclusion, we have demonstrated for the first time a violation of Bell's inequality based on phase and momentum, rather than spin."

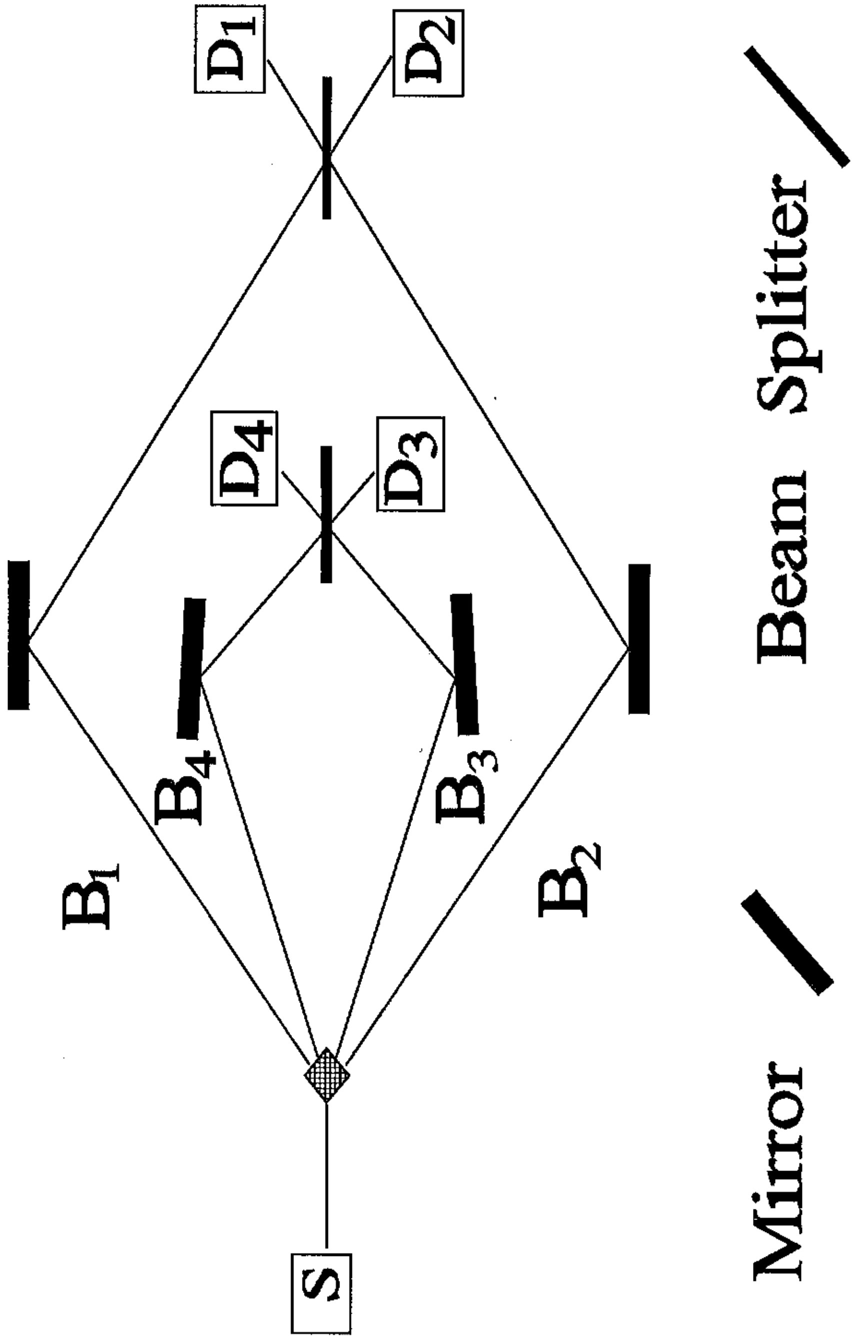


Figure 3-9. Two-particle interferometry.

3.6 Fransen Interferometry

Fransen interferometry is another method developed to test Bell's inequalities[16]. The Fransen interferometer is unique in that it is designed to take advantage of coupling in time rather than of spin or momentum. In the Fransen interferometer, as represented in figure 3-10, two particles are created through some process at source S. Due to time uncertainty, we don't know when the particles are formed, but we know that the particles are created at the same time. After creation, the two daughter particles are separated and directed down separate beams. One daughter particle travels to the beam on the right side of the interferometer, and the other travels to the beam on the left. Both arms of the interferometer contain half silvered mirrors placed a short distance from the source. These half silvered mirrors divided the two beams into two possible paths, one short and one long. The particle on the right has a fifty percent chance of proceeding in the long path B_3, and a fifty percent chance of proceeding in the short path B_4. The other daughter particle has a fifty percent chance of proceeding in B_2 and a fifty percent chance of proceeding in B_1. The paths B_1 and B_2, and also B_3 and B_4, are reunited in second beam splitters and directed toward detectors.

Like the previous experiment, it is possible to choose path lengths of B_1, B_2, B_3, and B_4, such that two particle interference occurs. More explicitly, it is possible to select path lengths such that for detections in D_1 and D_3, the short paths interfere destructively and the long paths interfere destructively. This forbids the particles from both taking short paths or from both taking long paths. It is impossible to register simultaneous detections in D_1 and D_3, because destructive interference forces one particle to take the short path and the other to take the long path causing them to arrive at the detectors at different times.

There are also choices of path lengths for which quantum mechanics and hidden variable theories predict different outcomes leading to Bell type inequalities. In August of 1989, two groups published results of Fransen type experiments[17]. By using photon pairs produced through the process of parametric down-conversion both groups were able to obtain the expected interference; however, neither group was able to obtain sufficient accuracy in their data to prove a violation of Bell's inequalities. The results were inconclusive.

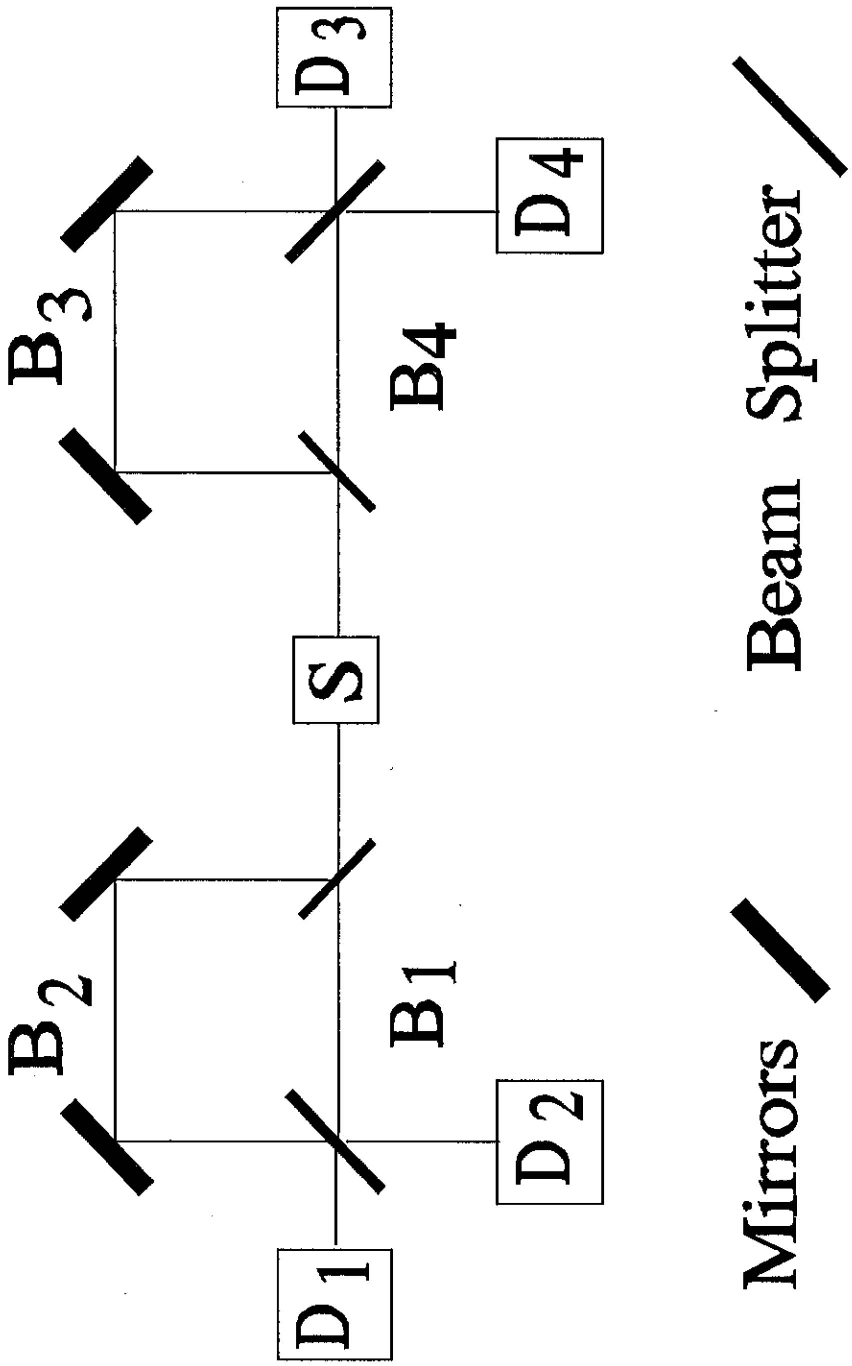


Figure 3-10. Fransen interferometry.

4. Many Quantum Interferometry

4.1 The GHZ Experiment

A recently developed four particle Bell type experiment[18] has a much simpler mathematical formulation in which the discrepancy between quantum mechanics and the hidden variable theories stems from an equality rather then an inequality. This improvement is known as the GHZ experiment. It is similar to Bell's experiment in that it deals with particles coupled with respect to spin, but there are some differences. One difference is that GHZ begins with a spin-1 particle in the state $|1\ 0\rangle$ rather than a spin-0 singlet state $|0\ 0\rangle$. This is an important distinction, because the singlet state $|0\ 0\rangle$, has zero spin with respect to any axis. Which means that the choice of directions in the previous Bell experiments was arbitrary. In contrast, the state $|1\ 0\rangle$, only has zero spin along one predefined axis. The spin in other directions is uncertain. In this experiment, we must choose a predefined z-axis along which the original particle is defined as having 0 directional spin. This particle divides into two spin-1 daughter particles. Conservation of spin allows the spin-1 daughter particles to be in one of three uncoupled states:

$$c_1|1\ 1\rangle|1\ -1\rangle,\ c_2|1\ 0\rangle|1\ 0\rangle,\ \text{or}\ c_3|1\ -1\rangle|1\ 1\rangle.$$

One method for finding the coefficients involves considering a different situation in which a spin-1 particle in the state $|1\ 1\rangle$ divides into two spin-1 particles:

$$|1\ 1\rangle = a_1|1\ 0\rangle|1\ 1\rangle + a_2|1\ 1\rangle|1\ 0\rangle, \tag{1}$$

with normalization $$|a_1|^2 + |a_2|^2 = 1. \tag{2}$$

Applying the ladder operator $S_+ = S_{1+} + S_{2+}$ to (1), produces:

$$0 = a_1(\sqrt{2})|1\ 1\rangle|1\ 1\rangle + a_2(\sqrt{0})|1\ 2\rangle|1\ 0\rangle$$
$$+ a_1(\sqrt{0})|1\ 0\rangle|1\ 2\rangle + a_2(\sqrt{2})|1\ 1\rangle|1\ 1\rangle,$$
$$0 = a_1 + a_2, \tag{3}$$

substituting $c = a_1$, we obtain:

$$|1\ 1\rangle = c(|1\ 0\rangle|1\ 1\rangle - |1\ 1\rangle|1\ 0\rangle), \tag{4}$$
$$|c|^2 = 1/2. \tag{5}$$

If we now apply the ladder operator $S_- = S_{1-} + S_{2-}$, to (4):

$$\sqrt{2}|1\ 0\rangle = c(\sqrt{2}|1\ -1\rangle|1\ 1\rangle - \sqrt{2}|1\ 0\rangle|1\ 0\rangle$$
$$+ \sqrt{2}|1\ 0\rangle|1\ 0\rangle - \sqrt{2}|1\ 1\rangle|1\ -1\rangle),$$

we obtain the correct form of the GHZ experiment:

$$|1\ 0\rangle = c(|1\ -1\rangle|1\ 1\rangle - |1\ 1\rangle|1\ -1\rangle), \tag{6}$$
$$|c|^2 = 1/2.$$

From this we see that the $|1\ 0\rangle|1\ 0\rangle$ solution isn't allowed and only two solutions $|1\ -1\rangle|1\ 1\rangle$ and $|1\ 1\rangle|1\ -1\rangle$ participate. At this point in the experiment, each of the two daughter particles divide into two half-spin particles. As represented in figures 4-1 and 4-2, only two uncoupled spin combinations are allowed. Either both half-spin particles on the right are positive as represented in figure 4-1, or they are both negative as represented in figure 4-2. Thus the complete solution is:

$$|\Psi\rangle = c(\downarrow_1\downarrow_2\uparrow_3\uparrow_4 - \uparrow_1\uparrow_2\downarrow_3\downarrow_4), \tag{7}$$

where each arrow represents a half-spin particle, numbers 1 and 2 being on the left in the figures, and 3 and 4 on the right:

$$\downarrow\downarrow\uparrow\uparrow = |1\ -1\rangle|1\ 1\rangle = \text{figure 4-1},$$

$$\uparrow\uparrow\downarrow\downarrow = |1\ 1\rangle|1\ -1\rangle = \text{figure 4-2}.$$

We may now solve for various expectation values. For some arbitrary direction $n_1(\theta,\phi)$, the matrix form $n_1 \bullet \sigma_1$, where σ is the pauli matrix, is defined with elements $a_{11} = \cos\theta$, $a_{12} = (\sin\theta)e^{-i\phi}$, $a_{21} = (\sin\theta)e^{i\phi}$, and $a_{22} = -\cos\theta$. The arrows represent two dimensional vectors: $\uparrow = (1\ 0)$, and $\downarrow = (0\ 1)$. The expectation value of the spin of the first half-spin particle in direction n_1 is:

$$\langle\Psi|(n_1 \bullet \sigma_1)|\Psi\rangle$$

$$\langle c(\downarrow\downarrow\uparrow\uparrow - \uparrow\uparrow\downarrow\downarrow)|(n_1 \bullet \sigma_1)|c(\downarrow\downarrow\uparrow\uparrow - \uparrow\uparrow\downarrow\downarrow)\rangle$$

$$= \langle c(\downarrow\downarrow\uparrow\uparrow)|(n_1 \bullet \sigma_1)|c(\downarrow\downarrow\uparrow\uparrow)\rangle - \langle c(\uparrow\uparrow\downarrow\downarrow)|(n_1 \bullet \sigma_1)|c(\downarrow\downarrow\uparrow\uparrow)\rangle$$

$$- \langle c(\downarrow\downarrow\uparrow\uparrow)|(n_1 \bullet \sigma_1)|c(\uparrow\uparrow\downarrow\downarrow)\rangle + \langle c(\uparrow\uparrow\downarrow\downarrow)|(n_1 \bullet \sigma_1)|c(\uparrow\uparrow\downarrow\downarrow)\rangle$$

$$= \langle c|(-\cos\theta_1)(1)(1)(1)|c\rangle - \langle c|(\sin\theta_1 e^{-i\phi_1})(0)(0)(0)|c\rangle$$

$$- \langle c|(\sin\theta_1 e^{i\phi_1})(0)(0)(0)|c\rangle + \langle c|(\cos\theta_1)(1)(1)(1)|c\rangle$$

$$= -\langle c|c\rangle (\cos\theta_1) + \langle c|c\rangle (\cos\theta_1) = 0 \qquad (8)$$

By a similar process, it can be shown that the expectation value is zero for any single particle, meaning that there are no single particle interference effects. Each particle has an equal probability of being $\uparrow$ or $\downarrow$ along any axis. We may now solve for the expectation value of the product of the spins of the first two particles in arbitrary directions n_1 and n_2:

$$\langle \Psi | (n_1 \bullet \sigma_1)(n_2 \bullet \sigma_2) | \Psi \rangle$$

$$\langle c(\downarrow\downarrow\uparrow\uparrow - \uparrow\uparrow\downarrow\downarrow) | (n_1 \bullet \sigma_1)(n_2 \bullet \sigma_2) | c(\downarrow\downarrow\uparrow\uparrow - \uparrow\uparrow\downarrow\downarrow) \rangle$$

$$= \langle c(\downarrow\downarrow\uparrow\uparrow) | (n_1 \bullet \sigma_1)(n_2 \bullet \sigma_2) | c(\downarrow\downarrow\uparrow\uparrow) \rangle$$

$$- \langle c(\uparrow\uparrow\downarrow\downarrow) | (n_1 \bullet \sigma_1)(n_2 \bullet \sigma_2) | c(\downarrow\downarrow\uparrow\uparrow) \rangle$$

$$- \langle c(\downarrow\downarrow\uparrow\uparrow) | (n_1 \bullet \sigma_1)(n_2 \bullet \sigma_2) | c(\uparrow\uparrow\downarrow\downarrow) \rangle$$

$$+ \langle c(\uparrow\uparrow\downarrow\downarrow) | (n_1 \bullet \sigma_1)(n_2 \bullet \sigma_2) | c(\uparrow\uparrow\downarrow\downarrow) \rangle$$

$$= \langle c | (-\cos\theta_1)(-\cos\theta_2)(1)(1) | c \rangle$$

$$- \langle c | (\sin\theta_1 e^{-i\phi_1})(\sin\theta_2 e^{-i\phi_2})(0)(0) | c \rangle$$

$$- \langle c | (\sin\theta_1 e^{i\phi_1})(\sin\theta_2 e^{i\phi_2})(0)(0) | c \rangle$$

$$+ \langle c | (\cos\theta_1)(\cos\theta_2)(1)(1) | c \rangle$$

$$= \langle c | c \rangle \ (\cos\theta_1)(\cos\theta_2) + \langle c | c \rangle \ (\cos\theta_1)(\cos\theta_2)$$

$$= (\cos\theta_1)(\cos\theta_2) \tag{9}$$

From (9) we see that there are two particle interference effects, but only in the predefined z direction. This should be clear from the figures. Each particle has a definite z-direction spin in figure 4-1, and a different one in figure 4-2. By knowing the z component spin of one of the half-spin particles, we can ascertain which of the two cases we are in, 4-1 or 4-2, and from this knowledge we can predict the z-component spin of every other particle in the system. Notice, that this only applies to the predefined z-axis. Knowing the x or y component of one particle does not have a similar effect. The expectation value of the product of the spins of the first three particles is:

$$\langle \Psi | (n_1 \bullet \sigma_1)(n_2 \bullet \sigma_2)(n_3 \bullet \sigma_3) | \Psi \rangle$$

$$= \langle c(\downarrow\downarrow\uparrow\uparrow)|\,(n_1\bullet\sigma_1)(n_2\bullet\sigma_2)(n_3\bullet\sigma_3)\,|c(\downarrow\downarrow\uparrow\uparrow)\rangle$$
$$- \langle c(\uparrow\uparrow\downarrow\downarrow)|\,(n_1\bullet\sigma_1)(n_2\bullet\sigma_2)(n_3\bullet\sigma_3)\,|c(\downarrow\downarrow\uparrow\uparrow)\rangle$$
$$- \langle c(\downarrow\downarrow\uparrow\uparrow)|\,(n_1\bullet\sigma_1)(n_2\bullet\sigma_2)(n_3\bullet\sigma_3)\,|c(\uparrow\uparrow\downarrow\downarrow)\rangle$$
$$+ \langle c(\uparrow\uparrow\downarrow\downarrow)|\,(n_1\bullet\sigma_1)(n_2\bullet\sigma_2)(n_3\bullet\sigma_3)\,|c(\uparrow\uparrow\downarrow\downarrow)\rangle$$
$$= \langle c|\,(-\cos\theta_1)(-\cos\theta_2)(\cos\theta_3)(1)\,|c\rangle$$
$$- \langle c|\,(\sin\theta_1 e^{-i\phi}{}_1)(\sin\theta_2 e^{-i\phi}{}_2)(\sin\theta_3 e^{i\phi}{}_3)(0)\,|c\rangle$$
$$- \langle c|\,(\sin\theta_1 e^{i\phi}{}_1)(\sin\theta_2 e^{i\phi}{}_2)(\sin\theta_3 e^{-i\phi}{}_3)(0)\,|c\rangle$$
$$+ \langle c|\,(\cos\theta_1)(\cos\theta_2)(-\cos\theta_3)(1)\,|c\rangle$$

$$= \langle c|c\rangle\ (\cos\theta_1)(\cos\theta_2)(\cos\theta_3)$$
$$- \langle c|c\rangle\ (\cos\theta_1)(\cos\theta_2)(\cos\theta_3)$$
$$= 0 \qquad (10)$$

A result similar to (10) applies for any three particles. There are no three particle interference effects. Finally, the expectation value of the product of the spins of all of the particles is:

$$\langle\Psi|\,(n_1\bullet\sigma_1)(n_2\bullet\sigma_2)(n_3\bullet\sigma_3)(n_4\bullet\sigma_4)\,|\Psi\rangle$$
$$\langle c(\downarrow\downarrow\uparrow\uparrow - \uparrow\uparrow\downarrow\downarrow)|\,(n_1\bullet\sigma_1)(n_2\bullet\sigma_2)(n_3\bullet\sigma_3)(n_4\bullet\sigma_4)\,|c(\downarrow\downarrow\uparrow\uparrow - \uparrow\uparrow\downarrow\downarrow)\rangle$$
$$= \langle c(\downarrow\downarrow\uparrow\uparrow)|\,(n_1\bullet\sigma_1)(n_2\bullet\sigma_2)(n_3\bullet\sigma_3)(n_4\bullet\sigma_4)\,|c(\downarrow\downarrow\uparrow\uparrow)\rangle$$
$$- \langle c(\uparrow\uparrow\downarrow\downarrow)|\,(n_1\bullet\sigma_1)(n_2\bullet\sigma_2)(n_3\bullet\sigma_3)(n_4\bullet\sigma_4)\,|c(\downarrow\downarrow\uparrow\uparrow)\rangle$$
$$- \langle c(\downarrow\downarrow\uparrow\uparrow)|\,(n_1\bullet\sigma_1)(n_2\bullet\sigma_2)(n_3\bullet\sigma_3)(n_4\bullet\sigma_4)\,|c(\uparrow\uparrow\downarrow\downarrow)\rangle$$
$$+ \langle c(\uparrow\uparrow\downarrow\downarrow)|\,(n_1\bullet\sigma_1)(n_2\bullet\sigma_2)(n_3\bullet\sigma_3)(n_4\bullet\sigma_4)\,|c(\uparrow\uparrow\downarrow\downarrow)\rangle$$

$$= \langle c|\,(-\cos\theta_1)(-\cos\theta_2)(\cos\theta_3)(\cos\theta_4)\,|c\rangle$$
$$- \langle c|\,(\sin\theta_1 e^{-i\phi}{}_1)(\sin\theta_2 e^{-i\phi}{}_2)(\sin\theta_3 e^{i\phi}{}_3)(\sin\theta_4 e^{i\phi}{}_4)\,|c\rangle$$

$$- \langle c| (\sin\theta_1 e^{i\phi_1})(\sin\theta_2 e^{i\phi_2})(\sin\theta_3 e^{-i\phi_3})(\sin\theta_4 e^{-i\phi_4}) |c\rangle$$

$$+ \langle c| (\cos\theta_1)(\cos\theta_2)(-\cos\theta_3)(-\cos\theta_4) |c\rangle$$

$$= \langle c|c\rangle \ (\cos\theta_1)(\cos\theta_2)(\cos\theta_3)(\cos\theta_4)$$

$$- \langle c| e^{-i\phi_1 - i\phi_2 + i\phi_3 + i\phi_4} |c\rangle \ (\sin\theta_1)(\sin\theta_2)(\sin\theta_3)(\sin\theta_4)$$

$$- \langle c| e^{+i\phi_1 + i\phi_2 - i\phi_3 - i\phi_4} |c\rangle \ (\sin\theta_1)(\sin\theta_2)(\sin\theta_3)(\sin\theta_4)$$

$$+ \langle c|c\rangle \ (\cos\theta_1)(\cos\theta_2)(\cos\theta_3)(\cos\theta_4)$$

$$= \cos\theta_1 \cos\theta_2 \cos\theta_3 \cos\theta_4$$

$$- \cos(\phi_1 + \phi_2 - \phi_3 - \phi_4) \sin\theta_1 \sin\theta_2 \sin\theta_3 \sin\theta_4 \qquad (11)$$

I will now demonstrate the discrepancy between (11), and the hidden variable theories. According to hidden variable theories, the particles are too far apart to influence each other at the time of measurement. This implies that each particles wave function collapse is influenced only by the immediate measuring apparatus, and some set of hidden variables which were determined before the particles separated. If this were true, then the product of the outcomes of the spins of the various particles will be determinable from the product of some functions $A_1(\theta_1, \phi_1) A_2(\theta_2, \phi_2) A_3(\theta_3, \phi_3) A_4(\theta_4, \phi_4)$, where the functional forms of the A's are set by hidden variables, and the θ's and ϕ's are determined by the measuring apparatus.

The GHZ authors point out that if hidden variable theories are compatible with quantum mechanics, then we should be able to find functional forms of the A's which will give the same results as the quantum mechanical expectation values for cases in which the expectation values are either +1 or -1.

When $\langle\Psi|\,(n_1\bullet\sigma_1)(n_2\bullet\sigma_2)(n_3\bullet\sigma_3)(n_4\bullet\sigma_4)\,|\Psi\rangle = +1$ or -1, then

$$A_1(\theta_1,\phi_1)A_2(\theta_2,\phi_2)A_3(\theta_3,\phi_3)A_4(\theta_4,\phi_4) = \cos\theta_1 \cos\theta_2 \cos\theta_3 \cos\theta_4 - \cos(\phi_1 + \phi_2 - \phi_3 - \phi_4) \sin\theta_1 \sin\theta_2 \sin\theta_3 \sin\theta_4. \quad (12)$$

Consider the solutions of (12) in which all of the θ's = $\pi/2$.

$$A_1(\pi/2,\phi_1)A_2(\pi/2,\phi_2)A_3(\pi/2,\phi_3)A_4(\pi/2,\phi_4) = -\cos(\phi_1 + \phi_2 - \phi_3 - \phi_4)$$

We can simplify this expression by dropping the $\pi/2$'s.

$$A_1(\phi_1)A_2(\phi_2)A_3(\phi_3)A_4(\phi_4) = -\cos(\phi_1 + \phi_2 - \phi_3 - \phi_4) \quad (13)$$

Now consider the following possible choices of ϕ's:

$$A_1(0)A_2(0)A_3(0)A_4(0) = -\cos(0 + 0 - 0 - 0) = -1, \quad (14)$$

$$A_1(\phi)A_2(0)A_3(\phi)A_4(0) = -\cos(\phi + 0 - \phi - 0) = -1, \quad (15)$$

$$A_1(\phi)A_2(0)A_3(0)A_4(\phi) = -\cos(\phi + 0 - 0 - \phi) = -1, \quad (16)$$

$$A_1(2\phi)A_2(0)A_3(\phi)A_4(\phi) = -\cos(2\phi + 0 - \phi - \phi) = -1, \quad (17)$$

$$A_1(\phi+\pi)A_2(0)A_3(\phi)A_4(0) = -\cos(\phi + \pi - \phi) = +1. \quad (18)$$

From (14) and (15): $A_1(0)A_3(0) = A_1(\phi)A_3(\phi)$. (19)

From (14) and (16): $A_1(0)A_4(0) = A_1(\phi)A_4(\phi)$. (20)

By combining (19) and (20):

$$A_1(\phi)A_3(\phi)A_1(0)A_4(0) = A_1(\phi)A_4(\phi)A_1(0)A_3(0),$$

$$A_3(\phi)/A_4(\phi) = A_3(0)/A_4(0).$$

but $A_i(\phi) = +1$ or -1, so the functions equal their inverses.

$$A_3(\phi)A_4(\phi) = A_3(0)A_4(0). \tag{21}$$

Applying (21) to (17): $A_1(2\phi)A_2(0)A_3(0)A_4(0) = -1.$ (22)

From (22) and (14): $A_1(2\phi) = A_1(0).$ (23)

Equality (23) implies that a measurement of the spin of the first particle anywhere in the x-y plane, will always yield the same result regardless of the direction of the measurement. This is in direct conflict with (18) and (15) from which:

$$A_1(\phi) = -1 = -A_1(\phi+\pi). \tag{24}$$

The discrepancy between equations (23) and (24) demonstrate one possible choice of measurements, for which quantum mechanical and the hidden variable predictions disagree.

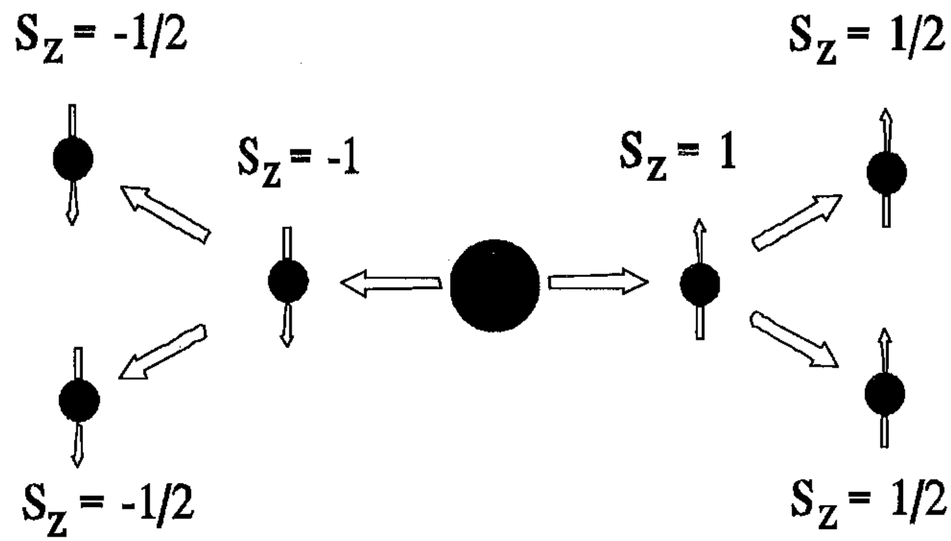


Figure 4-1. One possible four particle spin arrangement.

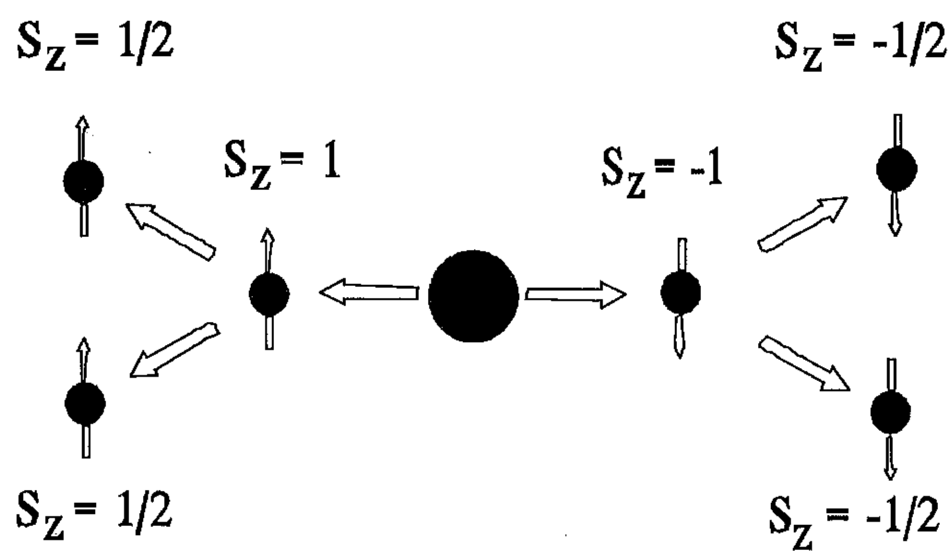


Figure 4-2. The other four particle spin arrangement.

4.2 A Date With Quantum Reality

There is one other possible interpretation of the collapse of the wave function which is worth mentioning. I will explain it by relating an experience an acquaintance of mine had a few years ago. I knew a girl named Susan, who over the course of a week was asked on 5 different dates by 5 different individuals all on the same night. The perspective dates were all attractive and interesting people. This caused Susan much anxiety as she mused over her options. She was so troubled by this dilemma, that she wasn't able to eat or sleep much that entire week. Eventually, her distress became so severe that it created a rift in reality and she suddenly divided into 5 possible Susans. Each Susan attended a different event with a different individual. During the course of the night, she was able to select her favorite date, at which time the wave function collapsed and all of the Susans but one disappeared.

This story may seem unbelievable, because if the wave function were to suddenly collapse, then all of the Susans would disappear but one. One date would now enjoy the real Susan, while all of the other dates are left alone. I also had difficulty believing this story at first, because I couldn't imagine that anyone would be impolite enough to disappear in the middle of a date leaving their companion deserted. However, it all made sense when Susan corrected my understanding about the collapse of the wave function.

She admitted that when the wave function collapsed, all of the possible Susans but one disappeared. But how they disappeared is important. They didn't disappear leaving their companions deserted, but rather disappeared entirely from existence. After the collapse of the wave function, it was as if those other dates had never happened. The collapse of the wave function had a retroactive effect in time, erasing the other possible Susans so that they had never existed. Only one date occurred with one happy couple. The other possible dates were erased from time.

I was much relieved by this explanation, and by knowing that she hadn't left 4 eligible bachelors to sink in their sorrow. I was still troubled by one point though: I have always believed that it is impossible to change the past. Once words are spoken and acts done, they remain and can not be changed. In her usually warm and friendly manner, Susan also eased my mind on this point. She agreed that we can not change the past. Words spoken and acts done do remain. But possible realities are not acts done. Possible realities are acts possibly done. Sometimes, a point of indecision is reached in which no choice is made. This is the case in the double slit experiment. Nature is unable to decide which slit the particle should go through, so the decision is deferred until later. It may be that the decision is deferred indefinitely, in which case both slits contribute to an interference pattern, or it may be that the wave function collapses and retroactively causes the particle to have gone through only one of the slits.

I have included this story to illustrate a different way of thinking about the collapse of the wave function. This brings the total number of explanations to three. One explanation considered in section 1.7 was particle-wave separability as illustrated in figure 1-23. In this explanation, the wave and particle are separate entities which interact with each other but exist independently. The weakness of this explanation, is that it leads to an EPR type hidden variables theory in which the particles represent hidden variables in nature which ultimately determine the location of the quantum entity.

The second explanation in section 1.7 is possible particles. In this theory, as illustrated in Figures 1-24 and 1-25, possible particles exist until the collapse of the wave function, at which time all of the possible particles except one disappear. This theory also has weaknesses when applied to Bell's experiments. As discussed in sections 3 and 4.1, the collapse of the wave functions of coupled systems must be correlated with respect to certain values such as momentum, spin, and time. This correlation requires instantaneous communication known as *action at a distance*. But instantaneous communication is forbidden by special relativity.

A third theory is represented in Susan's story in which the collapse of the wave function has a retroactive effect in time. When the wave function collapses, the other possible worlds are entirely erased from existence.

Epilogue: Deity and The Quantum Entity.

I will add a few theological thoughts as they relate to the previous discussion of the Quantum Entity. It is my opinion that the scientific world has largely been moving in an atheistic direction for the past two to three hundred years. I believe this is partially a result of Newtonian science. The Newtonian laws were so well formulated, that the existence of a supreme being was no longer required. All of the forces of nature could be explained in terms of the four forces of physics, and everything that happens is the result of some natural process which occurs in spite of the existence or nonexistence of Deity.

With these ideas in mind, the scientific existence of God has become dubious. From my limited experience, I have encountered three post-Newtonian teleological reasons for including Deity in a contemporary world view. One is the creation. We may argue that no scientific theory is sufficient to explain how the universe was created. In this theory, Deity is needed to begin the process and to place all of the laws of physics in order. Once the creation is completed, then the system will precede on its own without any further assistance.

A second reason stems from miracles such as those mentioned in the Bible. These miracles represent certain events which are beyond scientific explanation and are believed to have been produced by a divine being. This teleological basis for belief in Deity requires an active God who after the creation continues to be involved in human affairs as opposed to the inactive God who retired after creating the universe.

A third teleological basis for belief in Deity is supported in L.D.S. literature. This is the claim that the natural state of the universe is chaotic. The laws of physics are not innate to matter, but are enforced by a Divine Being. This idea may be the inspiration for such L.D.S. scripture as D&C 88:12-13 "Light proceedeth forth from the presence of God to fill the immensity of space--- The light which is in all things, which giveth life to all things, which is the **law** by which all things are governed, even the **power** of God who sitteth upon his throne, who is in the bosom of eternity, who is in the midst of all things."

In addition to these three, I believe quantum mechanics provides a possible fourth need for Deity. Quantum Mechanics is a probabilistic theory. Rather then predicting definite outcomes of an experiment, quantum mechanics gives the probabilities of which things are more likely and less likely to happen. This aspect of quantum mechanics provides an open door for Deity to play a more active role. It may be that these probabilities are not left to chance, but each one is individually decided by a Supreme Being. Each collapse of a wave function may be an act of God, in which one reality is selected and the others are dispensed with.

In addition to defining a more active role for Deity, this would also be a source of immense power. It turns out that almost anything is possible in quantum mechanics. The quantum physicist seldom says that anything is impossible, but rather that it is unlikely to happen within the age of the universe. Things like stopping the rotation of the earth, parting the red sea, turning water into wine, and walking on water may all have some infinitely small quantum probability of happening on their own without divine intervention. However, if the collapse of the wave function were under the control of Deity, then any quantum possibility, no matter how unlikely, could be realized. This would provide ample means to accomplish all of the known miracles and many more.

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