Initial Laboratory Experiments to Validate a Phase and Amplitude Gradient Estimator Method for the Calculation of Acoustic Intensity from Broadband Sources

Darren K. Torrie

A senior thesis submitted to the faculty of

Brigham Young University

In partial fulfillment of the requirements for the degree of

Bachelor of Science

Dr. Kent L. Gee, Advisor

Dr. Tracianne B. Neilsen, Co-advisor

Department of Physics and Astronomy

Brigham Young University

April 20, 2016

ABSTRACT

Initial Laboratory Experiments to Validate a Phase and Amplitude Gradient Estimator Method for the Calculation of Acoustic Intensity from Broadband Sources

Darren K. Torrie

Department of Physics and Astronomy

Bachelor of Science

A recently developed phase and amplitude gradient estimator (PAGE) method for calculating acoustic intensity from multiple pressure measurements [Thomas et al., J. Acoust. Soc. Am. 134, 4058 (2013)] has been tested via anechoic laboratory measurements of the radiation from broadband sources. The measurements determine that the effective frequency bandwidth of valid acoustic intensity calculations can be substantially increased when using the PAGE method over the traditional cross-spectral approaches. Preliminary results are shown for two probe sizes and multiple broadband source configurations.

Table of Contents

List of Figures

1 Introduction

Acoustic intensity is vector quantities that show the direction of propagation of the energy from a propagating sound wave[1]. A new method for estimating acoustic intensity has been developed that overcomes limitations of traditional methods and extends the usable bandwidth[2]. This method is a phase and amplitude gradient estimator (PAGE) method for pressure microphone intensity probes. Preliminary applications to high-amplitude, broadband sources, such as jet[3,4,5] and rocket[6] noise show promise[7]. Anechoic laboratory experiments have been performed to validate the PAGE method in a controlled setting and under rigorous conditions. Results of laboratory experiments indicate that, in general, the PAGE method extends the usable bandwidth of acoustic intensity, but care must be taken if deep interference nulls are present in the acoustic field.

1.1 Acoustic Intensity

Time-averaged acoustic intensity is the rate of energy flux in the direction normal to the unit area, given in units of W/m^2 . It is calculated as follows in the time domain:

$$\bar{\boldsymbol{I}} = \frac{1}{T}\int_0^T \boldsymbol{p}(\boldsymbol{t})\boldsymbol{u}(\boldsymbol{t})\boldsymbol{dt}, \qquad [\ 1\ ]$$

where p is the pressure and u is the particle velocity[8]. In the frequency domain, equation [1] is recast as

$$\bar{\boldsymbol{I}}(f) = \frac{1}{2}\mathrm{Re}\{\boldsymbol{p}(f)\boldsymbol{u}^*(f)\}. \qquad [\ 2\ ]$$

The frequency-dependent complex spectrum, $\boldsymbol{p}(f)$, is obtained from the Fourier transform of the pressure waveform recorded by a microphone and $\boldsymbol{u}^*(f)$ can be measured with a specialized sensor or estimated from the pressure.

While sensors are available to measure the acoustical particle velocity directly, it is more common to approximate it. The pressure is measured using a microphone, but the particle velocity is approximated from the pressure from closely spaced microphone due to difficulties in accurately measuring the particle velocity directly. This work will focus comparing the acoustic intensity from the traditional and PAGE methods of anechoic, laboratory experiments using broadband noise sources.

The acoustic particle velocity is estimated from two microphone measurements that are made with a separation distance of x. The particle velocity estimate in the direction of the axis through two microphones is obtained from Euler's equation of motion

$$\frac{\partial p(t)}{\partial x} + \rho\frac{\partial u_x(t)}{\partial t} = 0. \qquad [\ 3\ ]$$

By rearranging Euler's equation and approximating the gradient, where the gradient of the pressure is approximate by a finite difference, results in the particle velocity being determined as the following:

$$\hat{u}_a(t) = -\frac{1}{\rho}\int_{-\infty}^{t}\frac{p_2(\tau)-p_1(\tau)}{a}\,d\tau. \qquad [\ 4\ ]$$

Here p_1 and p_2 are the pressure measurements from each microphone, and τ is a dummy variable. This expression is an approximation of the true particle velocity[9].

The most common method of estimating acoustic intensity is the *p-p* method, which will be referred to as the traditional method[10]. This method involves taking the gradient of the complex pressure between two microphones placed a measured distance from each other. However, as the microphone spacing becomes large relative to a wavelengths of the incident waves, the gradient estimation is no longer reliable and errors are introduced. This limitation has been sufficiently studied, the appropriate bandwidth has been defined for a given microphone spacing, and has been the topic of many standards[11].

The new PAGE method, developed at BYU, is superior to the traditional method. The PAGE method uses the amplitude and phase components of the complex pressures which vary more predictably than the real and imaginary parts, to approximate the particle velocity. Both the theory and numerical implementations of the PAGE method indicate robustness that extends the usable bandwidth of the intensity calculations[12].

Initial application of the PAGE method to laboratory-scale jet noise also support this same claim[5]. Anechoic laboratory experiments have been conducted to further evaluate the performance of the PAGE method and validate the claim that the PAGE method can accurately calculate the intensity at frequencies beyond the limitations of the traditional method.

1.2 Thesis scope and outline

This thesis will cover the experimental validation of the PAGE method and is laid out in the following manner. First, the two methods, namely the traditional and PAGE method, for calculating acoustic vector intensity are defined. Next, a computational model is described that establishes a benchmark for the anechoic laboratory measurements. These are followed by a description of the laboratory experiment set-up and equipment in Chapter 3. Then the measured results at various frequencies using both the traditional and PAGE methods are presented along with individual components of specific intensity vector in the measured vector field, specifically the magnitude and direction. Finally in Chapter 4, the conclusions about the reliability and applications of the PAGE method are outlined, as well as ideas for future research.

2 Methods

The methods used in this thesis have been developed in other works and will be introduced in this section. First a two-dimensional probe developed at BYU, designed specifically for taking intensity measurements will be presented. Next the principles and theories behind the traditional and Page method will be established. Each method has a different approach at approximation of the particle velocity that makes them uniquely different. Lastly a computational model used to create a benchmark for the anechoic laboratory measurements is introduced.

2.1 Probe

Each measurement in the anechoic laboratory experiments is taken using the same intensity probe. This probe is in a two-dimensional configuration that was developed at Brigham Young University[13]. It consists of four 6.35mm diameter GRAS 40BD pressure microphones, as shown

Figure 1 - A computational simulation of one baffled circular pistons radiating with a broadband noise signal completely out of phase from each other. The vectors represent the resulting intensity of at each grid point in front of the sources and the color map shows the resulting magnitude of the intensity across the field.

in Figure 1. The chosen probe geometry is an equilateral triangle with a microphone at the center. The central microphone is lowered so that all the microphones are in the same plane. Each of the outer microphones is placed a distance r from the center microphone in Figure 2.

Two sizes of intensity probes are used in the measurements: r = 1 in and 2 in. The position of each microphone, relative to the center microphone, can be expressed in terms of the radius of

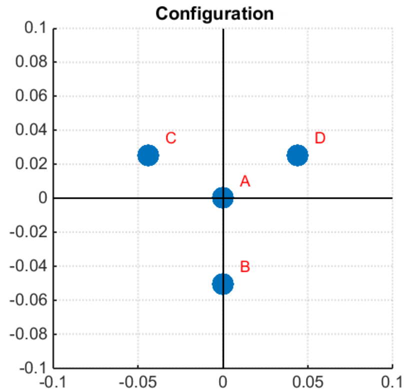


Figure 2- Two-dimensional representation in meters of intensity probe used in this work with specific radius, r, of 2 inches and four 6.35mm diameter GRAS 40BD pressure microphones. This is the same orientation relative to the sources used in all of the measurements in Chapter 4.

the circle that circumscribes the probes, r. The positions of each of the microphones relative to the center of the probe are

$$\boldsymbol{r}_A = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \qquad \boldsymbol{r}_B = a\begin{bmatrix} 0 \\ -1 \end{bmatrix},$$

$$\boldsymbol{r}_C = \frac{a}{2}\begin{bmatrix} \sqrt{3} \\ 1 \end{bmatrix}, \qquad \boldsymbol{r}_D = -\frac{a}{2}\begin{bmatrix} \sqrt{3} \\ -1 \end{bmatrix},$$

where r_i represents each microphone in the probe and is displayed graphically in Figure 2.

2.2 Traditional Cross-Spectral Method

The traditional method for calculating acoustic intensity involves cross-spectral analysis of pressure measurements between multiple phase-matched microphones. Due to the fact that these pressure microphones are in different locations, the pressure wave reaches each microphones at different instances in time. This makes it possible to estimate the gradient of the pressure between different sets of microphones[14], corresponding to the particle velocity of the wave.

For the most general of case using a two microphones probe with a fixed spacing, the acoustic intensity estimate in the frequency domain relies on this estimated pressure gradient:

$$\hat{\boldsymbol{I}}_c(f) = \frac{1}{\rho_0 \omega} p_0 \widehat{\nabla p}^*. \quad [5]$$

Here ω is the angular frequency, ρ_0 is the density of air, the average pressure, p_0, is the pressure estimated halfway between microphones for the two microphones and $\widehat{\nabla p}^*$, conjugate of the estimated complex pressure gradient. The magnitude of the intensity vector is expected to closely mimic the overall sound pressure level. In the case of the multi-microphone intensity probe from section 2, this pressure is measured from the microphone located at the center of mass of probe[2].

The accuracy of the gradient estimation in the intensity calculation is vital to having quality results. It is important to note that the physical spacing of the microphones, the actual wavelength of the incident wave effects the gradient estimation in two way. First, when the wavelength becomes small relative to the microphone spacing, the gradient is underestimated which introduces errors into the estimation. When the wavelength is large relative to the microphone spacing, the difference in the phase of the wave received at the two microphones is relatively small and can be comparable to the phase mismatch between the microphones. Thus the need for closely space microphones to get reliable high-frequency intensity estimates competes with the need to have the microphones spaced farther apart to get reliable intensity at lower frequencies. These two limitations cause the intensity estimates to be frequency bandwidth limited when using traditional method[15].

This limitation of the traditional method creates difficulty when taking measurements of broadband noise sources such as rockets. In order to be able to accurately model the rocket source at all frequencies requires that multiple probe configurations need to be employed with different spacing to cover the broad range of frequencies. More equipment means the cost of the measurements is higher, and the likelihood for equipment malfunction and complication increases as well. In most cases intensity measurements are difficult to conduct due to theses high costs and complex setups needed for broadband intensity measurements with the traditional method.

For the given probe configuration the complex intensity estimations are given by x and y components as follows

$$\hat{\boldsymbol{I}}_c^{FD} = \frac{-j}{6\rho_0 \omega r}\begin{bmatrix} \sqrt{3}(G_{31} - G_{41}) \\ 2G_{21} - G_{31} - G_{41} \end{bmatrix} \quad [\ 6\ ]$$

where $G_{ij} = 2p_i^* p_j$, r is the probe spacing, ω is the wave number and ρ_0 is the density of air**Error! Bookmark not defined.**. Equation [5] will be used to calculate the traditional method for all of the measurements taken in Chapter 4.

2.3 Phase and Amplitude Gradient Estimator (PAGE) Method

The PAGE method takes a different approach to estimating acoustic intensity by using manipulating equation [1] so that the pressure and particle velocity are now the amplitude and phase components of the complex pressures of the multiple phase-matched microphones[16]. The active acoustic intensity is now written as

$$\hat{\boldsymbol{I}}_a(f) = \frac{1}{\omega \rho_0} P^2 \nabla \varphi \quad [\ 7\ ]$$

ω is the angular frequency, ρ_0 is the density of air, P for the given probe geometry is the pressure from the center microphone, and φ is the phase from each microphone pair[17].

The amplitude estimation is simple because the amplitude is smoothly varying and will not effect by microphone spacing. The gradient of the phase however is a discontinuous function that alternates between ±π. An unwrapping algorithm can be applied to this phase component of the pressure so that it is now a continuous functions. This allows the PAGE method to take a reliable gradient of the phase without limitations so long as the function continues to smoothly vary.

For the given probe configuration in Figure 2 the complex intensity estimations are given by x and y components as follows

$$\hat{\boldsymbol{I}}_a^{\mathrm{PAGE}} = \frac{G_{11}}{6a\omega\rho_0}\begin{bmatrix} \sqrt{3}\arg\{H_{CD}\} \\ \arg\{H_{BC}\} + \arg\{H_{BD}\} \end{bmatrix}. \quad [\ 8\ ]$$

G_{AA} is the autospectrum of the center microphone, and H_{ij} are the transfer functions between the corresponding microphone pairs indicated[2]. This will be used to calculate the PAGE method for all of the measurements taken in Chapter 4.

3 Laboratory Experiment

In an anechoic environment a loudspeaker array is configured to emit broadband white noise. The two dimensional probe from Section 2.1 is used to scan the plane in front of the array with 3 different configurations. First with only one loudspeaker emitting broadband white noise. Second, two loudspeakers emitting coherent white noise completely out of phase from each other. Third, two loudspeakers emitting incoherent broadband white noise.

3.1 Experimental Setup

The anechoic laboratory experiments were conducted at BYU in the large anechoic chamber. This chamber is 8.71 m by 5.66 m by 5.74 m and is anechoic for the frequencies of 80 - 20,000 Hz. These conditions are necessary to ensure that for all of the measurements, the signal being measured was originating directly from the source, without any reflections effecting the intensity vectors. The anechoic chamber is equipped with a precision three dimensional positioning system. This allows an intensity probe to be positioned anywhere in the chamber. The established model discussed in Chapter 2 is in two dimensions, so the positioning system was configured to limit the

Figure 3 - The experimental setup in the corner of the large anechoic chamber at Brigham Young University. The intensity probe is suspended upside down form the boom arm that is hanging from the 3D precision positioning system. The plan to be scanned is directly in front of the loudspeaker array parallel to the floor.

movement to only the x-y plane of the chamber for measurements. This plane is oriented paralleled to the floor of the chamber.

An array consisting of four 2.5 inch loudspeakers spaced 7 inches from the adjacent speakers was used as the sources to be measured. This array allows for one to four simultaneous sources with varying phases to be measured. For this experiment only the center two loudspeakers will be utilized. A broadband white noise signal generated by the BYU Acoustic Field Recorder in LabVIEW, is fed into an amplifier. From the amplifier the signal is then routed to the speaker array. Each loudspeaker has an independent input and are passive, and unpowered. This allows the signal to be put out of phase by simply swapping the two input wires on the back of the speaker.

The two-dimensional probe discussed in Chapter 2 was used take measurements of the loudspeaker array. The probe was mounted to the scanning system and positioned such that the plane of the probe was parallel to the floor and set at a height at the consistent with the center of the loudspeakers on the array. The x-axis is parallel to the front face of the speaker array, and the y-axis comes out perpendicular front face of the speaker array.

The origin of the x-y plane was then set to be on the surface of the speaker box halfway up and directly in between the two speakers in the x-direction for a two source configuration and at the center of one of the speakers for the single source case. This places the speakers themselves directly along the x-axis. Due to the probe size, measurements actually start 3 inches from the surface of the speaker box. This allows ample room for the probe while still coming sufficiently close to the sources.

A grid of 41 points by 41 points was defined in the x-y plane in front of the loudspeaker array. The spacing between each point is .25 cm with one point at 0 cm, forming a square 1 meter by 1 meter plane to be scanned. Note that the origin is located in between the speakers and measurements will be taken half meter away from the speakers in the positive and negative x-directions and one meter away in the y-direction. The resultant grid has a total of 1681 points, or locations that the scanning system will move the probe to while the speakers are emitting broadband white noise to take measurements. Each measurement location will correspond to an intensity vector in Chapter 4.

These points are input into the motion recording plug-in of the BYU ARG Field Recorder, and set to record for 10 seconds at each point. The velocities of each motor on the scanning system were set to minimal levels to reduce swinging of the probe mounted from the boom arm. At each point the scanning system motors are disabled to eliminated any positional electrical noise that my leak in the microphone measurements. There is electrical noise present in chamber at 60 Hz and 10 kHz respectively due to a ground loop yet to be resolved. Each scan took an on average of 9 hours to complete.

The configurations that were measure are as follows: 1.) A single loudspeaker radiating broadband a white-noise measured using a probe with a 2 inch spacing 2.) Two loudspeaker radiating the same broadband white-noise signal but completely out of phase from each other measured using a probe with a 2 inch spacing 3.) Two loudspeaker radiating the different unique broadband white-noise signals measured using a probe with a 2 inch spacing 4.) Two loudspeaker radiating the different unique broadband white-noise signals measured using a probe with a 1 inch spacing.

3.2 Models

An analytical model of radiated acoustic vector intensity is very important to establishing a benchmark for the laboratory studies. This will be especially important at high frequencies, for the probe spacing used beyond 2000 Hz for the 2 inch spacing and above 3000 Hz for the 1 inch spacing. Beyond these limits of the traditional method the model will give insight into the validity of the PAGE method. The experiments are conducted with either a single loudspeaker or two loudspeakers that are modeled as baffled circular pistons. The single loudspeaker case is shown first followed by two, closely spaced loudspeakers emitting first coherent, out-of-phase noise then incoherent noise.

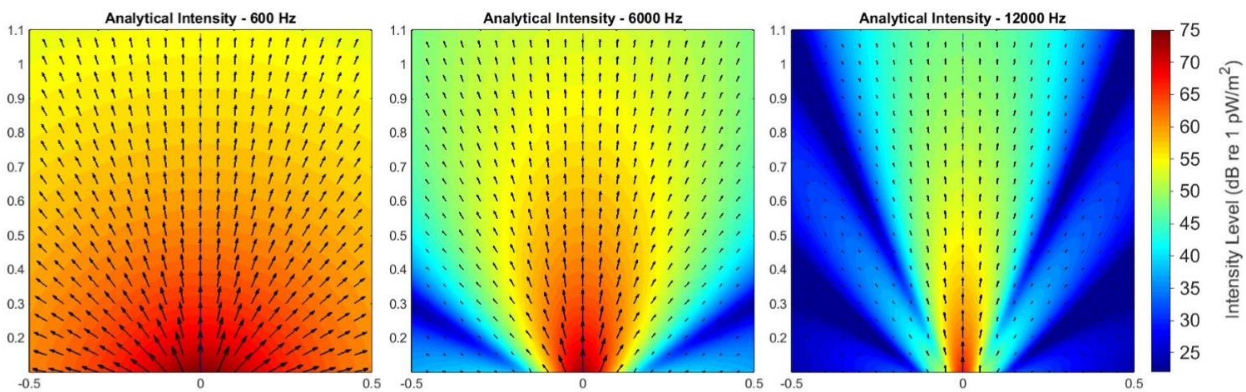


Figure 4 – A computational simulation of one baffled circular pistons radiating with a broadband noise signal completely out of phase from each other. The vectors represent the resulting intensity of at each grid point in front of the sources and the color map shows the resulting magnitude of the intensity across the field.

A baffled circular piston model was developed in MATlab to simulate the intensity field from a broadband noise source. An example of the resulting intensity vector field at 600, 6000, and 12000 Hz is shown in Figure 3. The intensity vectors calculated at designated grid locations in front of the source. Each vector has a scaled magnitude, creating a symmetric vector field relative to each other, as represented by the size of the arrow as well as the color of the plot behind the arrow. The phase is represented by the direction that the arrow is pointing relative to the source.

The intensity from a single baffled circular piston is frequency dependent. At 600 Hz the source looks very much like a monopole, but at 6 kHz the source becomes much more directional. At sufficiently high frequencies, as is evident at 12 kHz, there is one main lobe with minor side lobes. These patterns are typical for a baffled circular piston source at high frequencies.

The next model is expanded to incorporate two baffled circular piston sources the same size and strength and that are each emitting the same broadband noise signal 180° out of phase from each other. In this model there are several unique features exhibited as seen in Figure 4 First, an interference null forms directly in between the two sources. Here the intensity effectively goes to zero. This null is present for all frequencies. The radiation formation at 600 Hz, corresponds to that of a dipole. At 12 kHz, there are multiple interference nulls that form out at various angles from the source. The intensity also drops off on the sides of the source as a result of the source being more directional at 12 kHz.

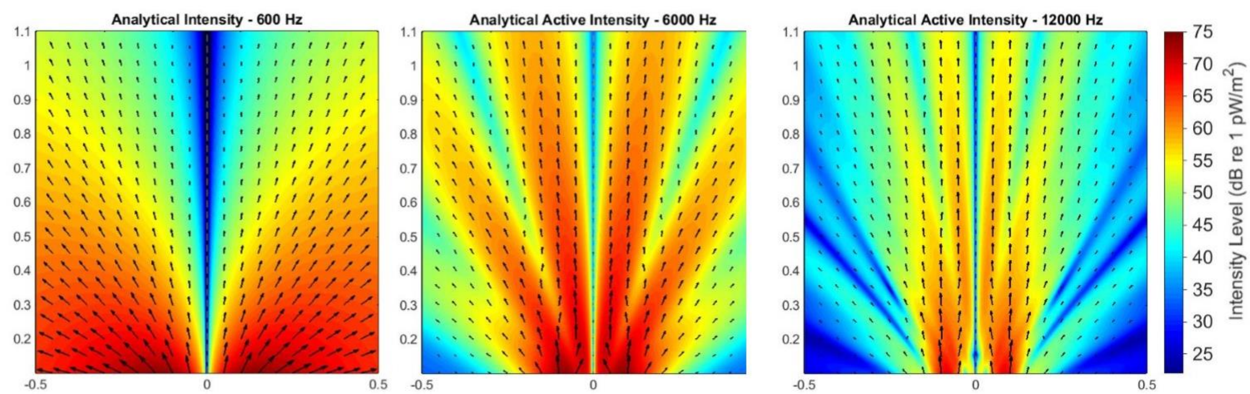


Figure 5 – A computational simulation of two baffled circular pistons radiating the same broadband noise signal completely out of phase from each other. The vectors represent the resulting intensity of at each grid point in front of the sources and the color map shows the resulting magnitude of the intensity across the field.

The third and final model configuration consists of two baffled circular pistons that are each radiating a unique or incoherent broadband signal in Figure 5. This creates an intensity field that looks very comparable to the single source model at 600 Hz. At 12000 Hz, the two sources are more apparent and are radiating independently of each other without causing any interference nulls where the two fields are over lapping.

The measurements taken and presented in Chapter 4 are all of loudspeakers emitting broadband white noise. The acoustic vector intensity from the loudspeakers for these three source configurations should nearly resemble the model for the designated frequencies.

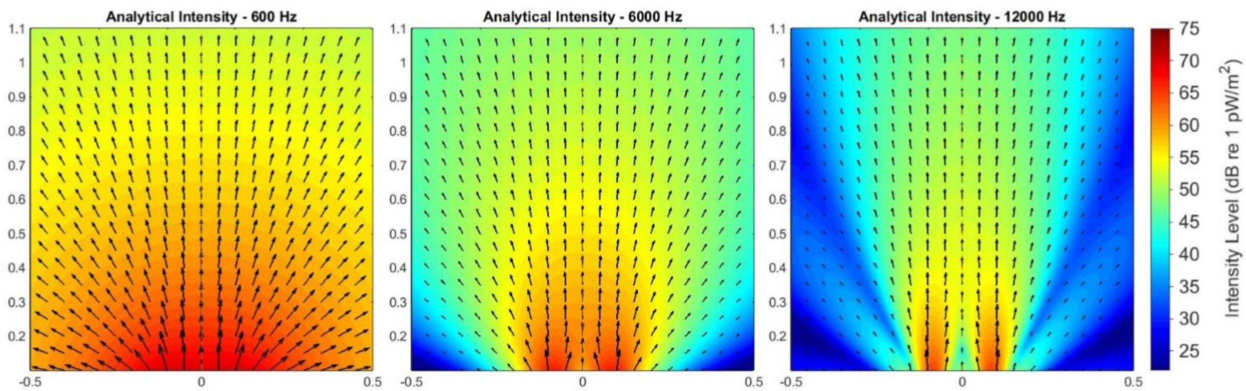


Figure 6 - A computational simulation of two baffled circular pistons each radiating a unique broadband noise signal. The vectors represent the resulting intensity of at each grid point in front of the sources and the color map shows the resulting magnitude of the intensity across the field.

4 Results and Analysis

For the purposes of comparing the traditional and PAGE methods, only a three frequencies will be examined in this section, specifically 600, 6000, and 12000 Hz, and for the three different loudspeaker configurations. All of the following plots used the two-dimensional probe configuration established in Section 2.2. The comparison of all these plots are with the model that was presented in Section 2.1. Then the magnitude and phase components of the intensity vector will be examined for a given probe location as a function of frequency. The location on the grid given as an x-y coordinate (15, 15) or in spatial location, in meters, on the intensity field map is at (-0.15 , 0.45). These plots will give greater inside into at what frequency each method begins to break down so a frequency bandwidth can be determined.

4.1 Data Processing

At each measurement location in the grid, an acoustic intensity vector was calculated. A Mat LAB script has been developed that calculates the intensity vector using both the traditional and PAGE method separately, at each of these locations. This then saves each method's final values in a multi-dimensional array that are then used to plot the resulting intensity field with both the traditional and PAGE method. A vector that represents the magnitude by the size of the arrow, and shows the direction that the energy is flowing in relation to the source by the arrows direction is plotted from every measurement location. Each plot also has a p-color map underneath the vector field, with the color representing the magnitude of the intensity at the given point. The plots are limited to showing the intensity at given frequency. Multiple plots are necessary to accurately compare these two methods and determine the effective bandwidth of each.

Further analysis was also done by probe position. At any given recording location, the two components of the intensity vector, the magnitude, and direction, are examined as functions of frequency. These plots give better intuitions as to the starting frequency of the underestimations and the overall frequency bandwidth of each method, and how the probe configuration and size impacts these methods.

The results will vary between the traditional and PAGE method for all of the results. It will be very clear though at 600 Hz, the difference will be very minimal since this frequency is still within the limitations of the traditional method. However for 6,000 and 12,000 Hz the traditional method will begin have underestimations. These plots will not me physically possible and will not mimic analytical models for these frequencies. The PAGE method will then relay more on the model of what the resultant vector fields will look like. The line plots show when the methods begin to deviate. A threshold of 1dB difference between each method and the sound pressure level is used to define when that magnitude of each method is no longer accurate.

4.2 Single Source 2 Inch Probe

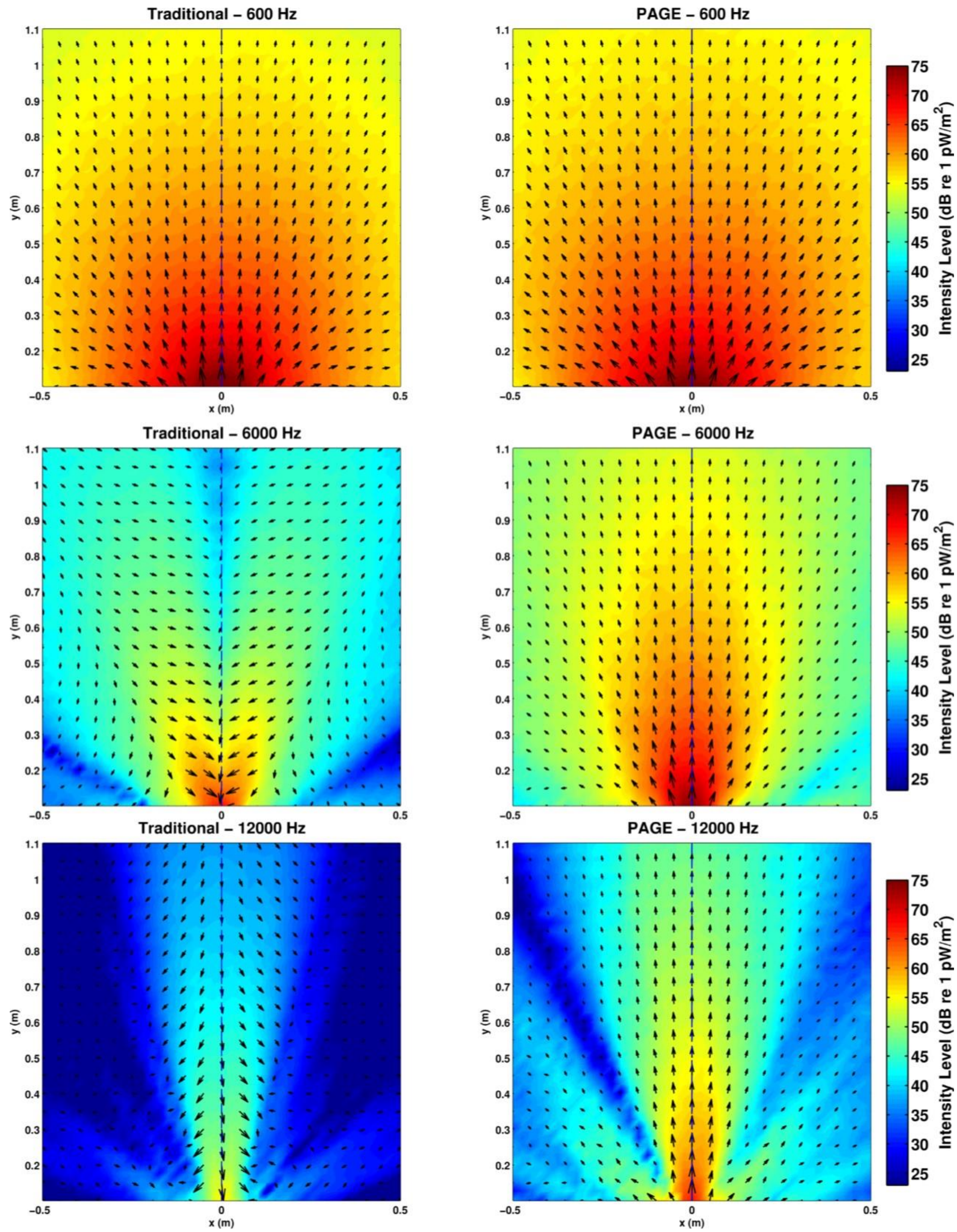
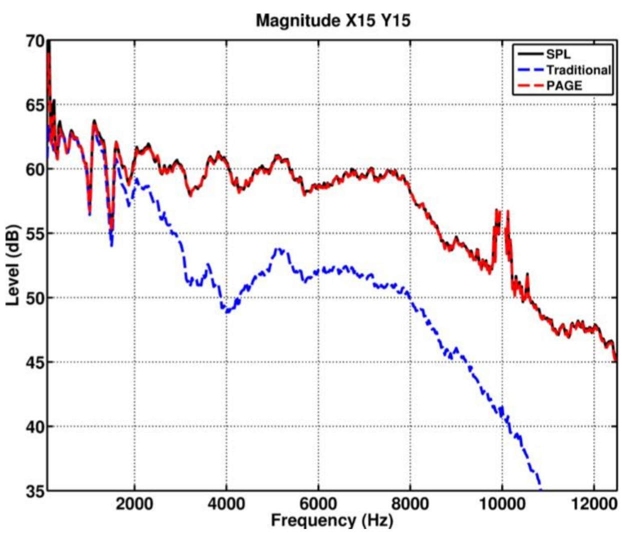


Figure 7 - Measurements taken using the precision scanning system and 2 dimensional intensity probe with radius of 2 inches. The source is a single loud speaker emitting broadband white noise. The left-hand column is the resulting intensity vector field calculated using the traditional method and the right hands is the same calculated using the PAGE method for the three frequencies of 600 Hz, 600 Hz, and 12000 Hz.

This set of measurements was taken of a single loudspeaker on the array radiating broadband white-noise. The y-axis is centered on the loudspeaker. At 600 Hz, both methods have intensity fields similar to that of a monopole, radiating out from the speaker in all direction. Any frequencies below this rapidly begin to deteriorate because the microphones used in the probe are not phase matched. This does match the model of a baffled circular piston at 600 Hz very closely.

At 6000 Hz, the speaker begins to become more directional, as seen by the magnitude rolling off on the sides. The traditional method however has already begun to deteriorate by this frequency. The arrows are pointing back towards the source, but the magnitudes across the field are all lower than those of the model. The PAGE method still has a coherent energy propagation direction across the field. While the traditional method falls off by this point, the PAGE continues to give results that are comparable to the analytical model.



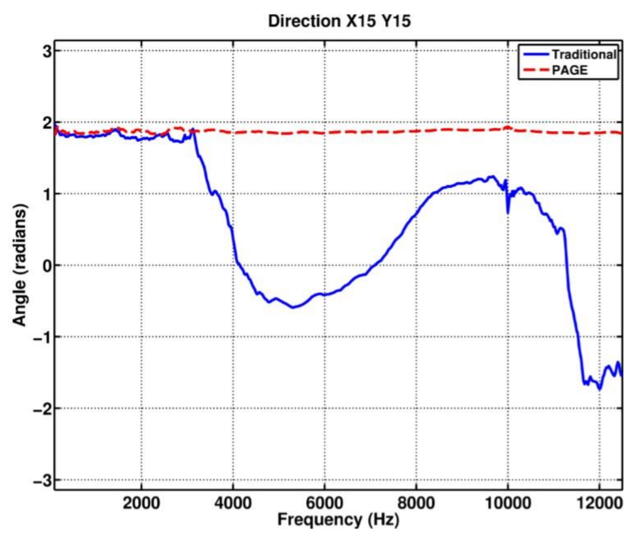


Figure 8 – The resulting magnitude on the left and direction of the right of the intensity vector at the gird location of (15, 15) and at the spatial location, in meters, of (-0.15, 0.45) for a single loudspeaker radiating broadband white-noise. Measurements taken using a two dimensional probe with radius of 2 inches.

The 12,000 Hz plot continue to get more directional and lobes and null are present in the field as is expected as determined by the analytical model at 12,000 Hz for a single source. The magnitude begins to rolls off in the PAGE method since the higher frequencies won't propagate nearly as far, but are still strong near to the source. The traditional method not only has incorrect directions but by this point has a magnitude that is far too low.

It is expected that the magnitude of the intensity will mimic the sound pressure level (SPL) for all frequencies. In the magnitude plots we can see that the PAGE method follows the SPL from the center microphone of the probe very closely. It is very obvious however that the traditional method rolls off and has difference of great than 1 dB at 6000 Hz and continues to drop off as the frequency gets higher.

As for the direction of the intensity, in the case of the monopole this close to the center of the source, the direction of propagations should remain constant for all frequencies. This is true for the PAGE method which remains constant all the way up to 12,000 Hz. The traditional method however begins to flip after 3,000 Hz and never recovers to propagate in the expected or even in a physically probably direction.

4.3 Two Sources Coherent 2 Inch Probe

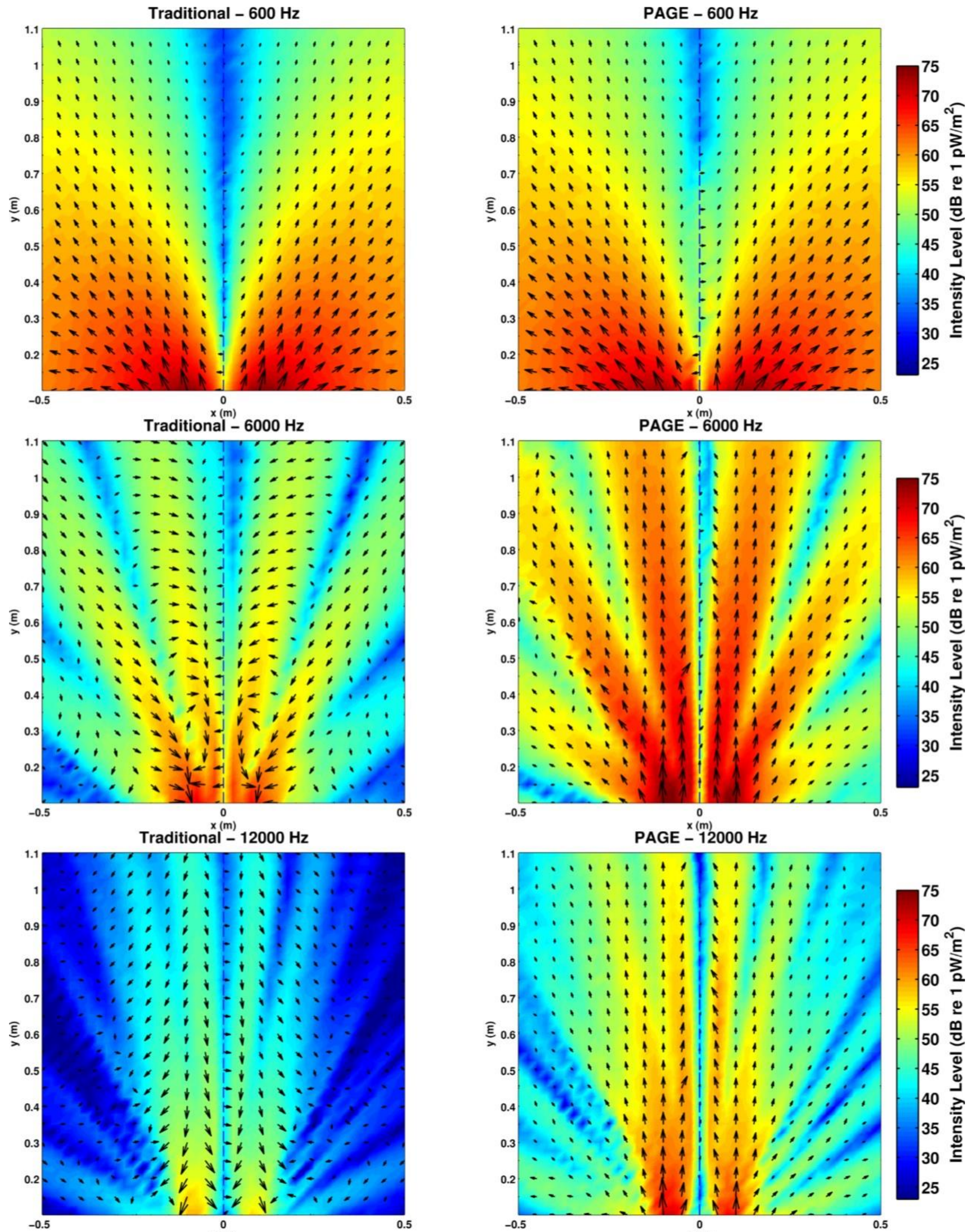


Figure 9- Measurements taken using the precision scanning system and 2 dimensional intensity probe with radius of 2 inches. The sources are two loudspeakers emitting broadband white noise completely out of phase from each other. The left-hand column is the resulting intensity vector field calculated using the traditional method and the right hands is the same only calculated using the PAGE method for the three frequencies of 600 Hz, 600 Hz, and 12000 Hz.

In this case the middle two loudspeakers are radiating the same broadband white-noise signal. The wires going into the loudspeaker are reversed on the second loudspeaker so that they are radiating completely out of phase from each other. The y-axis is positioned directly in-between the two loudspeaker sources.

At 600 Hz, the two speakers are obviously radiating outward with an interference null forming between the two sources corresponding to the y-axis. This null is not as prominent in the measurements near the sources at this frequency, as it is in the analytical model. The radiation pattern is very similar to that of a dipole and very nearly resembles the formation of the model from Chapter 3.

Continuing to 6000 Hz, the distinct null in between the speakers forms more clearly. The traditional method's magnitude is beginning to roll off and the vectors are pointing in a direction clearly opposite to direction of propagation. The PAGE method has several clearly defined nulls and lobes forming on either side of both speakers. The directionality of the loudspeaker at these higher frequencies has become very apparent. There are also some anomalies that occur in the

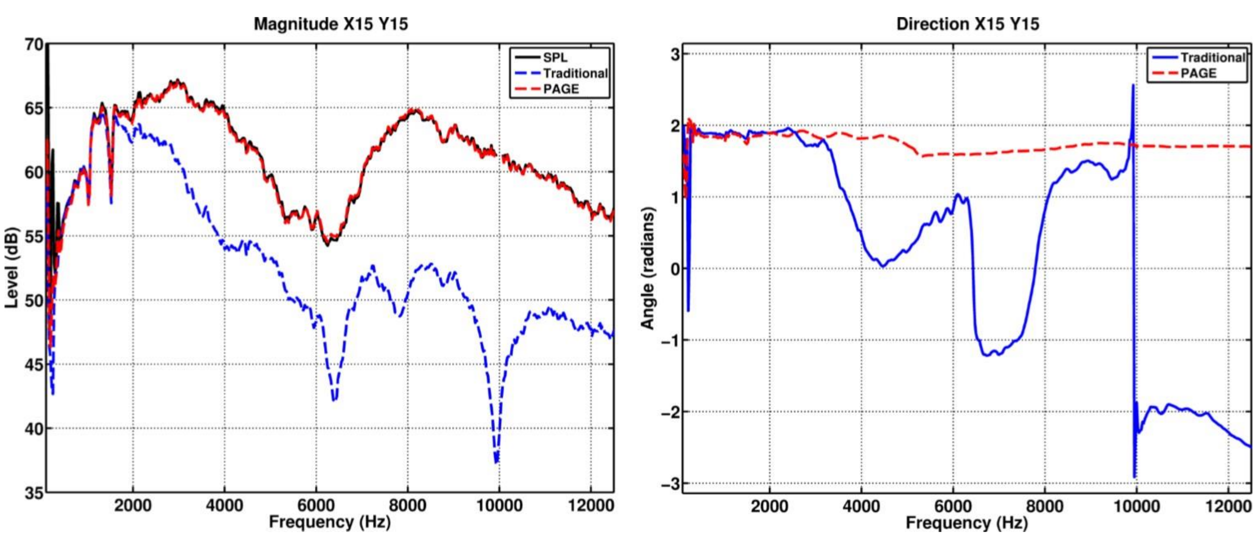


Figure 10 - The resulting magnitude on the left and direction of the right of the intensity vector at the gird location of (15, 15) and at the spatial location, in meters, of (-0.15, 0.45) for a two loudspeakers emitting the same broadband white noise signal completely out of phase from each other. Measurements taken using a two dimensional probe with radius of 2 inches.

PAGE method, mostly on the left hand side of the vector field and fewer on the right hand side. From inspection, the anomalies occur more persistent on the left because of the depth of the null is more significant at this location. However these are not present in the single source case or any of later measurements. The anomalies being at 3000 Hz, and are present at all higher frequencies.

For 12,000 Hz, the traditional method is clearly wrong in both the magnitude and direction. The PAGE method however has the magnitudes rolling off on the sides of the speakers as a result of the source being more directional at this high frequency relative to the loudspeaker size. At frequencies that are this high, propagation distance is expected to be minimal. This mimics the model of two coherent sources at 12,000 kHz.

In the magnitude plots we can see that the PAGE method follows the SPL of the center microphone of the probe very closely all the way up to 12,000 Hz. The dip in both the SPL and the PAGE method are due to the formation of a null at the probes location that reaches its minimum at just above 6,000 Hz. The traditional method begins rolls off and has a difference of greater than 1 dB at 2,000 Hz and continues to drop off as the frequency gets higher.

The direction of the intensity at this location, and in the source configuration should remain unchanged for most frequencies except for in the presence of a null which occurs around 6,000 Hz. The PAGE method does remains the same with only a slight variation through the null at 6,000 Hz, and then on up to 12,000 Hz. The traditional method however begins to flip again before 3,000 Hz and never recovers, leaving the direction to be unreliable.

4.4 Two Sources Incoherent 2 Inch Probe

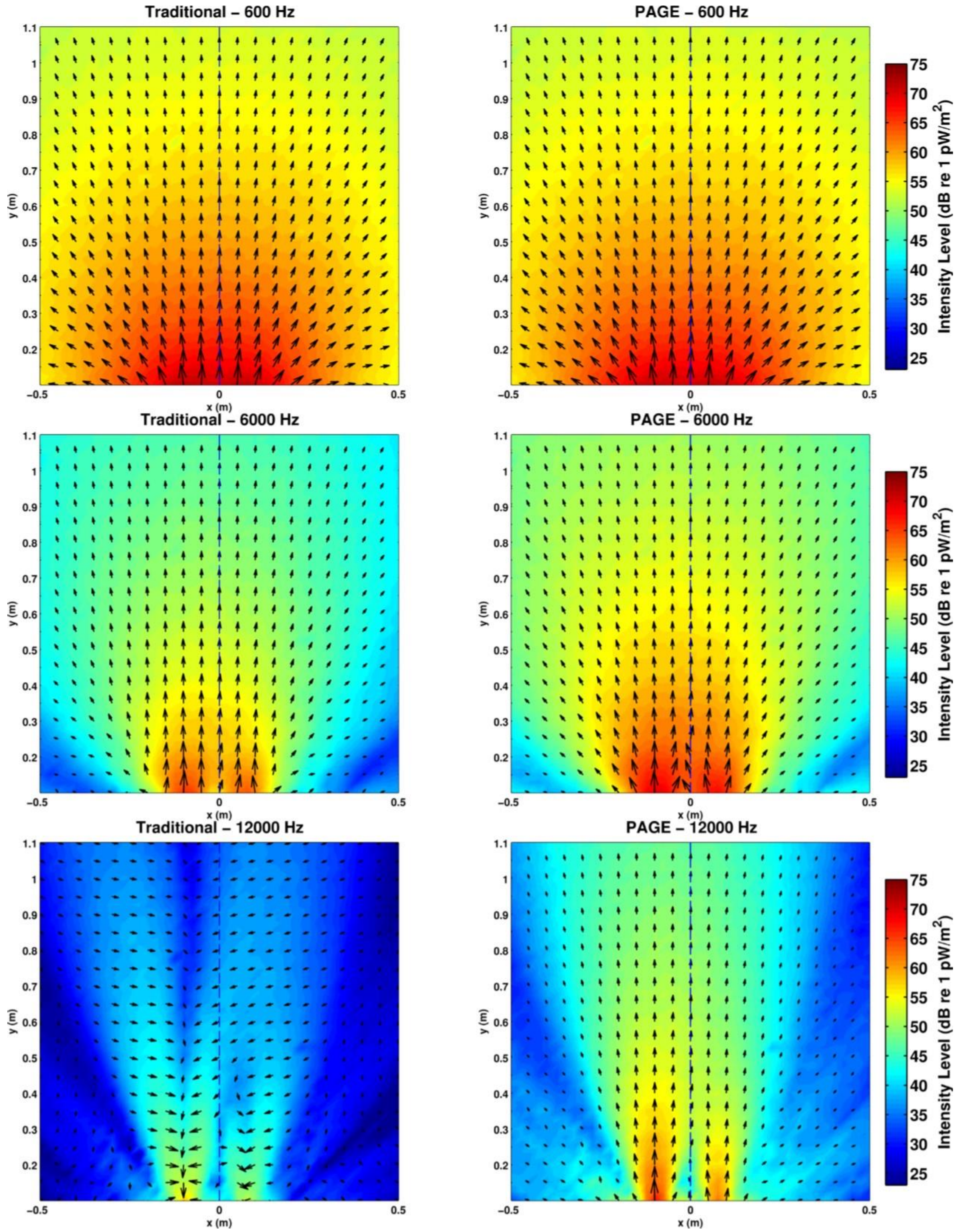


Figure 11 - Measurements taken using the precision scanning system and 2 dimensional intensity probe with radius of 2 inches. The sources are two loudspeakers each emitting a unique broadband white noise signal. The left-hand column is the resulting intensity vector field calculated using the traditional method and the right hands is the same only calculated using the PAGE method for the three frequencies of 600 Hz, 600 Hz, and 12000 Hz.

The middle two loudspeakers are each radiating a unique broadband white-noise signal and measurements are taken using a probe with a 2 inch radius. The y-axis is positioned directly in between the two loudspeaker sources. This configuration is less demanding than the previous case and should lend insight into the effective bandwidth of each method without interference nulls skewing the results.

Beginning at 600 Hz, the direction of propagations is clearly away from the two sources and the resulting field looks very similar to that of the single source. The model with two incoherent sources indicates that the field would be uniform with no interference nulls. The measurements are qualitatively similar to the model and also are very similar to the radiation pattern seen in Figure 7 at 600 Hz.

At 6000 Hz, as far as direction is concerned both methods perform very well. The speakers are more directional at this frequencies and it is evident in the vector field that there are in fact two distinct sources. The magnitudes however of the traditional method however begin to roll off due to the microphone spacing being large relative to the wavelength at this frequency.

For the 12,000 Hz plot, the traditional method magnitude has drops off significantly and the vectors are beginning to point perpendicular to the direction that the speaker is radiating, which is incorrect. The PAGE method however has the magnitudes rolling off on the sides of the speakers and it is clear from this plot that the left loudspeaker is radiating more efficiently at this high frequency than the loudspeaker on the right. This is very similar to the results of the model for 2 incoherent sources at 12,000 Hz.

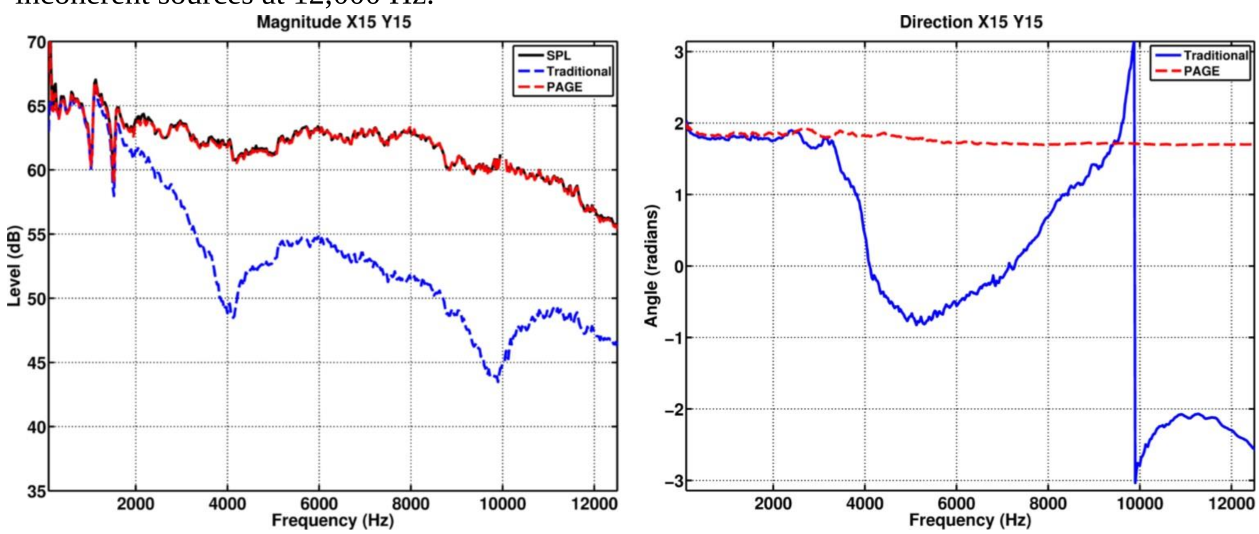


Figure 12 - The resulting magnitude on the left and direction of the right of the intensity vector at the gird location of (15, 15) and at the spatial location, in meters, of (-0.15, 0.45) for two loudspeakers each emitting a unique broadband white noise signal. Measurements taken using a two dimensional probe with radius of 2 inches.

The magnitude plots show that the PAGE method follows the SPL of the center microphone of the probe very closely all the way up to 12,000 Hz. There are no dips in the magnitude because there are no inference nulls forming in the configuration. The traditional method begins rolls off and has difference of great than 1 dB at 2,000 Hz and is underestimated for all frequency beyond this this point.

The direction of the intensity at this location should remain steady for all frequencies given this configuration. The direction of the PAGE method does remains the same all the way up to 12,000 Hz. The traditional method however begins break down at 3000 Hz and never recovers, leaving the direction to be nonphysical. These results are very similar to the previous case.

4.5 Two Sources Incoherent 1 inch Probe

The middle two loudspeakers of the array are each radiating a unique broadband white-noise signal and measurements are taken using a probe with a 1 inch spacing. The y-axis is positioned directly in between the two loudspeaker sources.

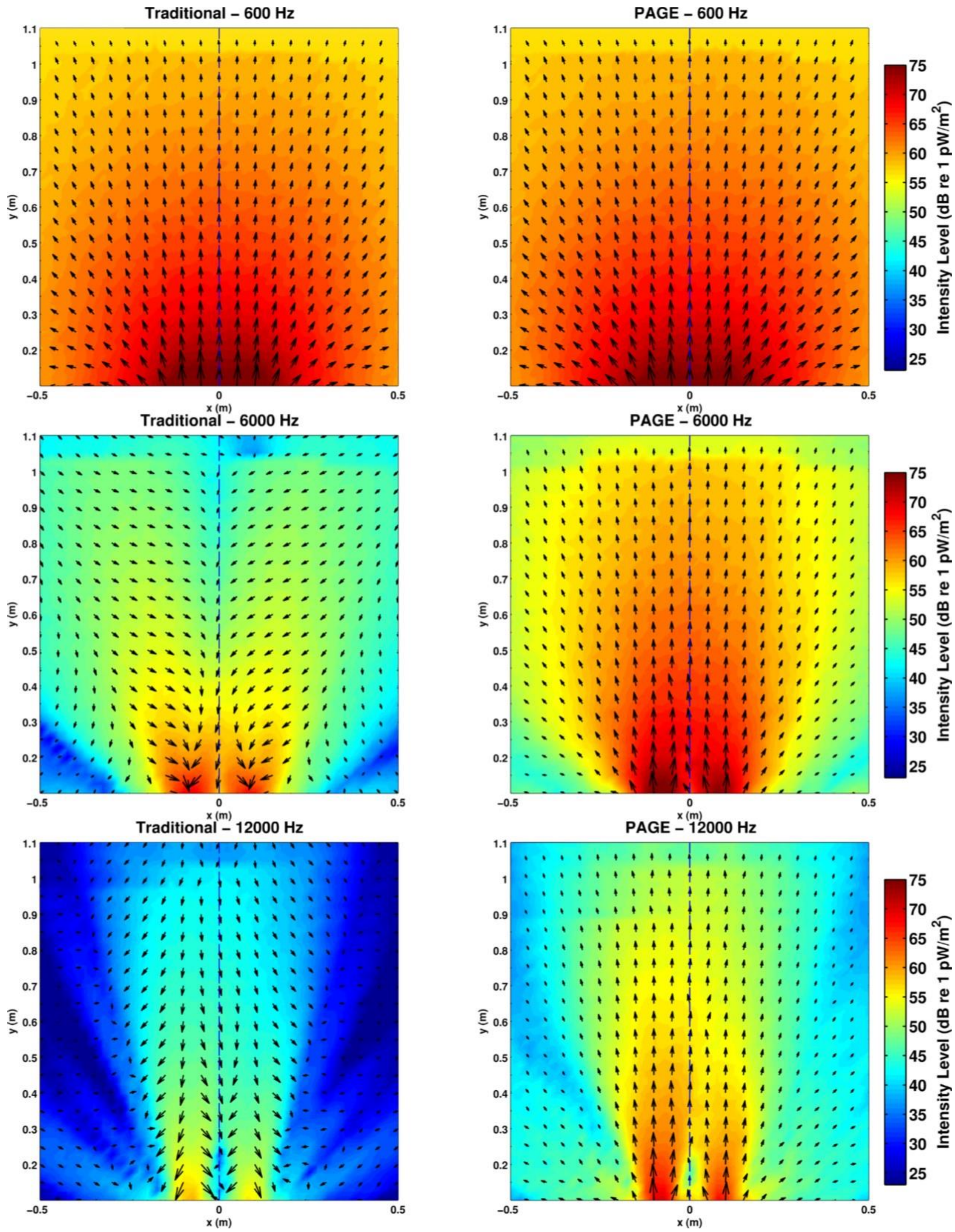


Figure 13 - Measurements taken using the precision scanning system and 2 dimensional intensity probe with radius of 1 inch. The sources are two loudspeakers each emitting a unique broadband white noise signal. The left-hand column is the resulting intensity vector field calculated using the traditional method and the right hands is the same only calculated using the PAGE method for the three frequencies of 600 Hz, 600 Hz, and 12000 Hz.

Beginning at 600 Hz, the direction of propagations is clearly away from the two sources and the resulting field looks very similar to that of the single source and that of the same configuration using the probe both the 2 inch and 1 inch spacing. The magnitude as seen in the p-color plot is clearly higher for the 1 inch probe case than the 2 inch probe case by an average of about 4 dB. The direction of the vector on the both plots are nearly identical propagating out uniformly at all angles form the speakers.

At 6000 Hz, the magnitude difference between the one 1 inch and 2 inch probes remains the same. The magnitude is beginning to roll off though in both plots as seen by the formation of the lobes in from of each speaker. The prominent feature is that the sources are now more directional at this location. Both have vectors point in directions similar to the 6000 Hz plot in the analytical model as shown in Figure 6.

For the 12,000 Hz plot, both the 1 and 2 inch probe have both loudspeaker being very directional. The lobes are narrower and only overlap slightly in the middle of the ploy. Both lobes are radiating out directly form the loudspeakers with very low levels off to the left and right sides.

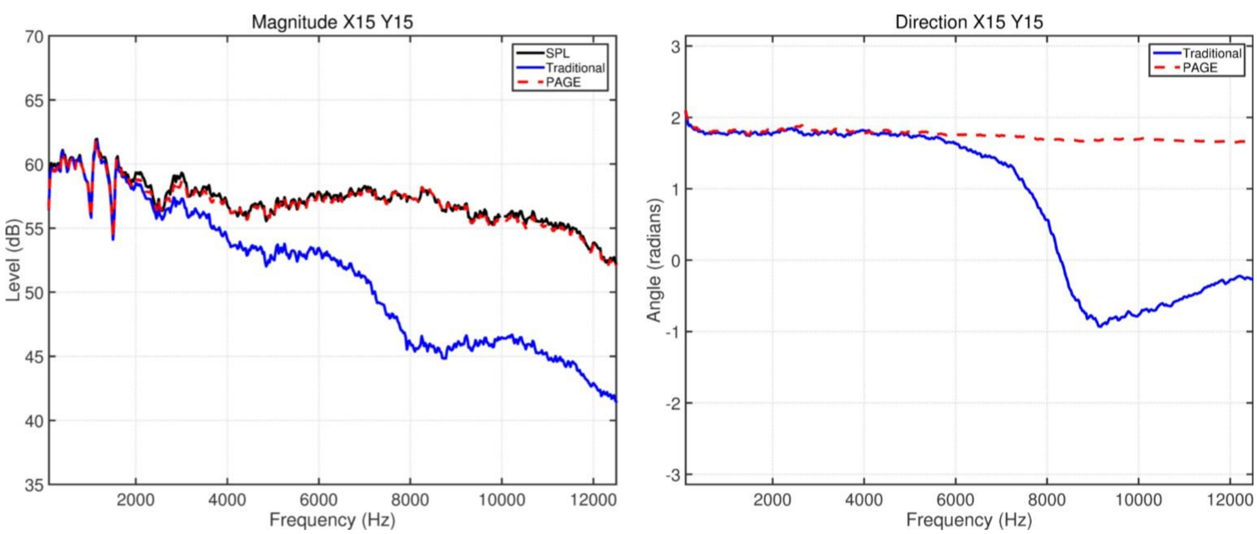


Figure 14 - The resulting magnitude on the left and direction of the right of the intensity vector at the gird location of (15, 15) and at the spatial location, in meters, of (-0.15, 0.45) for two loudspeakers each emitting a unique broadband white noise signal. Measurements taken using a two dimensional probe with radius of 1 inch.

This mimics the model for 2 incoherent sources at 12,000 Hz. For both probe sizes the loudspeaker on the left is radiating more efficiently than the speaker on the right side. However the overall levels of the plots on for the 2 inch probe versus the 1 inch probe are probably due to the amplifier levels being set differently for each scan.

The magnitude plots show that the PAGE method with 1 inch probe follows the SPL of the center microphone of the probe very closely all the way up to 12,000 Hz. The traditional method begins rolls off and has difference of great than 1 dB at 3,000 Hz and is underestimated for all frequency beyond this this point.

The direction of the vector using PAGE method does remains the constituent all the way up to 12,000 Hz. The traditional method however begins to deviate at 6,000 Hz and never recovers, leaving the direction to be nonphysical for all frequencies above this frequency.

5 Conclusions

The results support the conclusions that were formed by Thompson and Christensen. The PAGE method does improve the frequency bandwidth of calculated acoustic vector intensity for a broadband source. For the given probe geometry with 2 inch spacing, the magnitude estimate improves from an upper frequency limit 2,000 Hz with the traditional method to 12,000 Hz with the PAGE method, and the direction of the vector improves from an upper limit of 3,000 Hz with the traditional method to 12,000 Hz with the PAGE method. For the same probe geometry with 1 inch spacing, the magnitude upper frequency limit improves from 3,000 Hz with the traditional method to 12,000 Hz with the PAGE method, and the direction of the vector improves from an upper limit of 6,000 Hz with the traditional method to 12,000 Hz with the PAGE method.

Despite these improvements in frequency bandwidth, there are several anomalies that have arisen that indicate that the PAGE method does have limitations when measuring in a demanding sound field such as those of this paper. It has been observed that on both sides of the plots there are regions that once a strong null has swept through a given frequency that the resulting intensity at any frequency beyond this point are effected in both magnitude and direction. This was present for both sizes of the probe and the erroneous vector were more prevalent on the left side which corresponds to deeper null formations.

These erroneous points need to be further examined to find the cause of the errors and either find a solution to improve the calculations under the unique and demanding conditions that cause it or define limitation to the PAGE method. It is important to note that at the frequency when the PAGE method had these errors arise the traditional method had already broken down in both magnitude and direction and was by this point unreliable.

This experiment created the most extreme and difficult to calculate of intensity fields, with the coherent sources forming deep interference nulls. In application to real world acoustics, both methods perform significantly better when dealing with incoherent broadband sources. Further research is being done to further investigate and define the limits of the PAGE method, as well as apply its fundamental theory to other energy based acoustical estimations.

In conclusion the results of laboratory experiment indicate that the usable bandwidth of acoustic vector intensity is extended considerably in general when using the PAGE method, compared to the traditional method regardless of the microphone spacing of the probe. Care must be taken if deep interference nulls are present in the acoustic field.

Acknowledgments

Grant from the National Science Foundation supported this research.

Kent L. Gee

Traci B. Neilsen

Benjamin Y. Christensen

Eric J. Whiting

Michael T. Rose

Index

Works Cited

[1] F. Jacobsen, "Sound intensity measurements," in Handbook of Noise and Vibration Control, edited by M. J. Crocker (Wiley, Hoboken, NJ, 2007), pp. 534–548.
[2] Thomas, Derek C., Benjamin Y. Christensen, and Kent L. Gee. "Phase and amplitude gradient method for the estimation of acoustic vector quantities." The Journal of the Acoustical Society of America 137.6 (2015): 3366-3376.
[3] Whiting, Eric B., et al. "Analytical intensity calculated from a wavepacket model and comparison to intensity measurements near a high-performance military aircraft." The Journal of the Acoustical Society of America 138.3 (2015): 1917-1917.
[4] Gee, Kent L., et al. "Near-field acoustical array measurements of an impinging supersonic jet." The Journal of the Acoustical Society of America 138.3 (2015): 1891-1891.
[5] Gee, Kent L., et al. "Application of a phase and amplitude gradient estimator to intensity-based laboratory-scale jet noise source characterization."
[6] Gee, Kent L., et al. "Near-field vector intensity measurements of a small solid rocket motor." The Journal of the Acoustical Society of America 128.2 (2010): EL69-EL74.
[7] Stout, Trevor A., et al. "Acoustic intensity near a high-powered military jet aircraft." The Journal of the Acoustical Society of America 138.1 (2015): EL1-EL7.
[8] M. J. Crocker, "Measurement of sound intensity," in Handbook of Acoustical Measurements and Noise Control, edited by C. M. Harris (McGraw-Hill, New York, 1991), pp. 14.1–14.17.
[9] F. Jacobsen, "Measurements of Sound intensity," in Springer Handbook of Acoustics, edited by T. D. Rossing (Springer, New York, 2007), pp. 1058–1068
[10] IEC 1043:1993, "Electroacoustics—Instruments for the measurement of sound intensity—Measurement with pairs of pressure sensing microphones" (International Electrotechnical Commission, Geneva, Switzerland, 1993).
[11] ANSI/ASA S1.9-1996, Instruments for the Measurement of Sound Intensity (Acoustical Society of America, Melville, NY, 1996).
[12] J. A. Mann III, J. Tichy, and A. J. Romano, "Instantaneous and time-averaged energy transfer in acoustic fields," J. Acoust. Soc. Am. 82, 17–30 (1987).
[13] Wiederhold, Curtis P., et al. "Comparison of multimicrophone probe design and processing methods in measuring acoustic intensity." The Journal of the Acoustical Society of America 135.5 (2014): 2797-2807.
[14] Chung, J. Y. "Cross-spectral method of measuring acoustic intensity without error caused by instrument phase mismatch." The Journal of the Acoustical Society of America 64.6 (1978): 1613-1616.
[15] Thompson, J. K., and D. R. Tree. "Finite difference approximation errors in acoustic intensity measurements." Journal of Sound and Vibration 75.2 (1981): 229-238.
[16] J. A. Mann III and J. Tichy, "Acoustic intensity analysis: Distinguishing energy propagation and wave-front propagation," J. Acoust. Soc. Am. 90, 20–25 (1991).

[17] J. A. Mann III and J. Tichy, “Near-field identification of vibration sources, resonant cavities, and diffraction using acoustic intensity measurements,” J. Acoust. Soc. Am. 90, 720–729 (1991).