GENERATING ORDER 2 AND 4 FREE-FERMIONIC NAHE-BASED HETEROTIC MODELS

by

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ABSTRACT

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A short overview of string phenomenology is presented which motivates the search of the free-fermionic heterotic string models. The process of generating these models is discussed along with the use of group theory in describing the Standard Model. All possible order 2 and 4 boundary vectors (BV's) are generated with the constraint that they can be added to the NAHE set and still form a consistent MSSM string model. Combining the BV's with the NAHE set results in new string models which will be analyzed in group theoretic terms to classify them according to their individual Standard Models.

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Introduction

1.1 Group Theory

Symmetries exist everywhere in nature. Anything from the arrangement of atoms in a molecule to the petals of a flower exhibit some kind of physical symmetry. Group theory is the mathematical study of symmetry and it is useful in helping us to describe physical systems. A group is any collection of objects that follow a specific set of axioms. That set of axioms are as follows: Let G be a group and $a, b, c \in G$ and $*$ be a binary operation defined on G.

1. Closure: $a * b \in G \;\; \forall\, a, b \in G$
2. Identity: $\exists\; e \in G$ such that $e * a = a * e = a \;\; \forall\, a \in G$
3. Inverse: $\exists\; a^{-1} \in G$ such that $a^{-1} * a = a * a^{-1} = e \;\; \forall\, a \in G$
4. Associativity: $a * (b * c) = (a * b) * c \;\; \forall\, a, b, c \in G$

Some informative examples (and non-examples) of groups are the integers under the operation of addition, $\mathbb{Z}^{+}$, but not the integers under multiplication, $\mathbb{Z}^{\times}$, since not all elements have an inverse. Including the inverse would require the use of the rationals, $\mathbb{Q}$, which are a group under addition or multiplication. It is interesting to note that some subsets of a group also follow the above axioms and are also groups. A group contained within a larger group is called a subgroup.

The special unitary group of order two, $SU(2)$ (the collection of 2x2, unitary, determinant 1 matrices), is a more physically motivated group which can be used to represent spin and angular momentum in quantum mechanics and is generated by Pauli matrices. The Lorentz transformations from special relativity also form a group. These are just two examples of how groups can be used to represent symmetries of a given system.

1.1.1 Symmetries

In discussing how groups relate to symmetries, I have failed to define what a symmetry actually is. A symmetry is defined as some action that leaves a system invariant. Consider the dihedral group of order four, D_4. It is a group of eight elements which are used to represent the symmetries of the square. The elements of the group correspond to actions performed on the square and consist of one identity, 90°, 180°, 270° rotations and 4 flips: horizontal, vertical and two diagonal. These are visualized in Fig. 1.1. As can be shown, these actions all leave the square invariant and so they represent all independent symmetries of the square. In the previous section, quantum mechanical spin and Lorentz transformations were given as examples of groups. Specifically, these groups are known as Lie groups. Lie groups have elements that are infinitesimally separated in terms of a mathematically continuous parametrization. It is this parametrization that allows Lie groups to be used to represent systems with continuous symmetries. Many other important symmetries exist in the physical universe and now I will highlight a set of symmetries that pertain to particles.

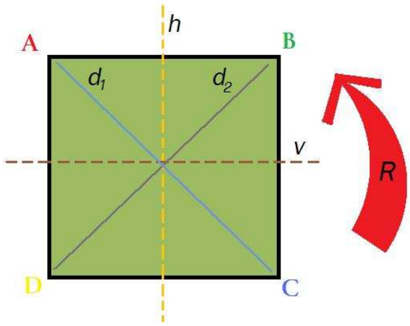

Figure 1.1 The dihedral group of order 4, has the same symmetries of the square and contains, as elements, three rotations (R) and four flips (h,v,d_1,d_2).

1.2 Particle Physics

1.2.1 Particles, Forces and the Standard Model

According to our current understanding of the universe, four forces govern the interactions between matter particles. These four forces are gravity, electromagnetism and the weak and strong nuclear forces. In terms of particle physics, each of these forces has a corresponding particle constituent. When matter particles exchange this 'force' particle, the matter particles experience this force between each other.

All force-carrying particles are bosons. A boson is a particle with integer spin and does not obey the Pauli exclusion principle. All force-carrying particles have spin 1 or 2. The graviton mediates the force of gravity while the photon, represented symbolically by γ, is the force particle for the electromagnetic force. The strong nuclear force is carried by eight bosons called gluons and the weak nuclear force is mediated by the W and Z bosons: W^+, W^- and Z.

The Standard Model of particle physics is one of the most inclusive and well-verified theories of modern physics. The theory helps to explain three of the four forces: electromagnetism and the strong and weak nuclear forces. Through the Standard Model these three forces can be unified into a grand unified theory, which is used to understand the condition of the universe just after the big bang. It has been proposed that as the universe expanded and cooled the single unified force splintered into the four forces that we see today.

1.2.2 The Groups of the Standard Model

Three of the four forces in nature have been shown to come from a single force by the same grand unified theory. This is much like how a group can contain many subgroups. In fact the forces do have a group theoretic representation, below I list the Lie group associated with the corresponding force.

Lie Group		Force
$SO(10)$	$\Rightarrow$	Grand Unified Theory (GUT)
$SU(3)$	$\Rightarrow$	Strong Nuclear (Gluons)
$SU(2) \times U(1)$	$\Rightarrow$	Electroweak (EM coupled with Weak)

The Lie groups associated with the Standard Model are also called gauge groups. I will use the terms interchangeably.

1.3 String Phenomenology

1.3.1 String Theory

You will recall that in our discussion of the Standard Model only three of the four forces were represented. Gravity is not part of the Standard Model. Due to the success of the Standard Model in unifying three forces, it is logical to look for a way to unify all four forces into one theory, a "Theory of Everything." String theory offers a possible theory of everything. Therefore, it has great promise in describing our universe. In its current state, string theory has been shown to yield gravity, the standard model, and even dark matter/energy.

In its most fundamental form, string theory describes fundamental particles (quarks, electrons, etc.) as one dimensional, vibrating strings. These strings are identical and the vibrational modes that propagate on the string determine what type of particle we see it as. These loops can be either closed or open and this necessitates various boundary conditions for the string vibrations. To make the theory mathematically consistent, string theory requires the existence of anywhere between 6 (in the case of supersymmetry) or 22 (in the non-supersymmetric case) extra dimensions above the four spacetime dimensions required by Einsteinian relativity. The six extra dimensions are represented by mathematical objects called Calabi-Yau manifolds.

String theory is a generalized name given to a set of five string theories: Types I, IIA, IIB, Heterotic $E_8 \times E_8$ and Heterotic SO(32). There has been much work on investigating the differences between the string theories and it is now believed that they are all low-energy forms of an 11-dimensional theory called M-theory, for which 11-D supergravity is also a low energy form. If that is the case then M-Theory is truly the "Theory of Everything."

1.3.2 String Models

String theory requires the existence of extra dimensions above the four extended dimensions we experience everyday. This means that the extra dimensions of string theory must be very small and curled up (or compactified). Consider compactifying two extra dimensions of a four dimensional spacetime. An extra compactified dimension can be thought of as a loop and when another dimension is added a second loop is allowed to traverse the first loop creating a sphere or a torus for the two dimensional surface. (See Fig. 1.3.) As the number of extra dimensions increases there are

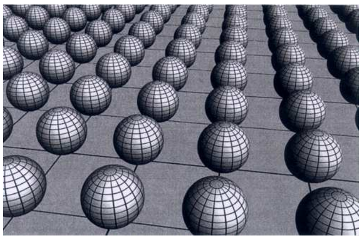

Figure 1.2 Extra dimensions can be visualized by considering the 4-D spacetime to be a plane and then at every point on that plane there would exist a surface created by the extra dimensions. Here the extra dimensions yield a sphere.

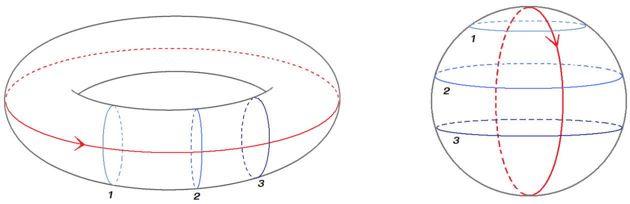

Figure 1.3 Adding one compactified dimension to another can be visualized by identifying points on both loops and then allowing those points to travel around the one of the loops. To make a torus one must identify only one point one each loop. Whereas to make a 2-sphere, one must identify two points on each loop.

many more ways to combine the compactified dimensions (loops) together. In the work which we will be considering here, the number of extra dimensions is six.

It has been shown that there exist a minimum of 10^{500} to 10^{1000} string models, called the "string landscape" [1]. One arrives at this number based on taking the number of Calabi-Yau manifolds (approximately 100 billion), the manifolds on which the six extra dimensions are compactified [2], and then adding possible electromagnetic-like fluxes to the manifolds (there are anywhere from 10^{500} to 10^{1000} flux combinations per manifold) [3]. Each of these string models corresponds to a different universe, each with its own physics and corresponding gauge groups. To test string theory as a quantum theory of gravity, it is necessary to find out if any of these string models

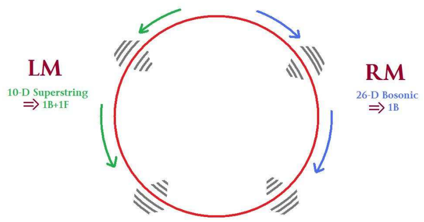

Figure 1.4 A heterotic closed string has both left-moving and right-moving waves. The LM live in a 10-D superstring theory while the RM live in a 26-D bosonic string theory.

contain our universe as a solution. This requires that we must begin a systematic check of these models (or at least regions of them).

1.3.3 Heterotic String Theory

String phenomenology is the process of modeling specific string theories and investigating what gauge groups and matter are produced. This matter and these gauge groups come about through different compactifications of the extra dimensions above four. In a superstring theory there is one bosonic wave and one fermionic wave for each dimension while in bosonic string theory there is only a bosonic wave for each dimension. There are varying methods used to describe the compactification process. We use the free-fermionic approach, which is a method that describes compactifications in terms of fermionic fields. Real fermionic fields can be paired to represent bosonic waves. Alternate bosonic methods of compactification appear more physical, as they tend to describe compactifications in terms of geometry while the free-fermionic is an abstract mathematical approach. However, the free-fermionic approach is more manageable for systematic computer-based model building.

1.3.4 NAHE Model Generating

Since there has been shown to be a duality [4] between the 5 main string theories: heterotic $E_8 \times E_8$, heterotic $SO(32)$, IIa, IIb and I, we are also free to choose the most convenient interpretation in order to make our modeling calculations easiest. Thus we have chosen to use the free fermionic language of the heterotic $E_8 \times E_8$ approach.

In this approach, closed strings are endowed with the above described fermionic fields. These fields move in either a clockwise (called right-moving, RM) or a counter-clockwise (known as left-moving, LM) direction. What makes the heterotic theories unique is that the RM waves can be thought of as living in a 26 dimensional, purely bosonic string theory while the LM waves live in a 10 dimensional superstring theory.(see Fig. 1.4) In a null (also called light cone) gauge, the LM have 8 transverse degrees of freedom. Since these are part of a 10 dimensional superstring theory, each dimension contributes one bosonic and one fermionic field. In a four dimensional theory, two of the light cone dimensions remain uncompactified and each dimension corresponds to a LM bosonic field, a LM fermionic field and a RM bosonic field. The two LM spacetime bosonic fields produce only Planck-scale massive modes and the two RM spacetime bosonic fields only occur for graviton or gravitino particles. We will effectively ignore those fields since we are solely concerned with the standard models of each string model we investigate and not with gravity. Letting the other bosonic fields be replaced by two fermionic fields, much like a Cooper pair, we find 20 fermionic LM fields. Since the RM waves are all bosonic in nature, we get 44 fermionic fields from the 22 compact dimensions. This yields 64 degrees of freedom for each free-fermionic heterotic model. We associate with these fields a 64-component vector, called a boundary vector (BV), $\overrightarrow{\alpha}$. Since these fermionic fields represent waves, we can characterize them by a phase factor, which is the number of times the closed string is traversed before a given wave comes back into its original state. This gives us a range for the components of the BV, $-1 < \alpha_i \leq 1$ where $i = 1$ to 64. The BV are not freely specifiable but have been shown to be constrained according to [5,6], We call these the ABK/KLT constraints:

$$N_{i,j}\overrightarrow{V_i} \cdot \overrightarrow{V_j} = 0 \ (mod\, 4) \tag{1.1a}$$

$$N_i\overrightarrow{V_i} \cdot \overrightarrow{V_i} = 0 \ (mod\, 8) \tag{1.1b}$$

$\overrightarrow{V_i}, \overrightarrow{V_j}, \overrightarrow{V_k}$ must have an even

number of periodic real fermions. (1.1c)

Where $\overrightarrow{V_i}$ is the i^{th} BV, N_i is defined to be the lowest integer such that $N_i\overrightarrow{V_i} = \overrightarrow{0} \ (mod\, 2)$ (called the order of $\overrightarrow{V_i}$) and $N_{i,j}$ is the least common multiple of N_i and N_j. Also note that the dot products in equations (1.1a) and (1.1b) are actually Lorentz dot products where the RM products are subtracted from the LM products. The periodicity constraint means that there must be an even number of value 1 components shared between all three vectors. These constraints make the BV consistent with conformal (Lorentz) invariance of the worldsheet (spacetime).

An important set of consistent BV's is called the NAHE (Nanopoulos, Antoniadis, Hagelin, Ellis) [7] set. They are:

$$\begin{aligned}
V_I &= (1_{64}) \\
V_{II} &= (1_2, (1,0,0)_6 \| 0_{44}) \\
V_{III} &= (1_2, (1,0,0)_2, (0,1,0)_4 \| 1_{12}, 0_6, 1_4, 0_{22}) \\
V_{IV} &= (1_2, (0,1,0)_2, (1,0,0)_2, (0,0,1)_2 \| \\
&\quad\ 1_{10}, 0_2, 1_2, 0_2, 1_2, 0_8, 1_2, 0_{16}) \\
V_V &= (1_2, (0,0,1)_4, (1,0,0)_2 \| \\
&\quad\ 1_{10}, 0_4, 1_2, 0_6, 1_4, 0_{18})
\end{aligned} \tag{1.2}$$

Where $X_n = X, X, ..., X, X$ (n copies of X). This set is of order 2 since the $\alpha_i \in \{0, 1\}$. There are symmetries of the components found in the NAHE set and this constrains the allowable wave phases. So given a BV, when the first two components have 1's (denoting BV's producing spacetime fermions), the next 18 slots are divided into sets of three and must have an odd number of 1's in each of those sets of three with 0's elsewhere. That takes care of the LM. The RM must be divided up into pairs of components with the exception of α_i with $i \in [37, 48]$ remaining possibly unpaired among themselves. In that case, those unpaired RM α's must be paired with the corresponding LM α's. With that in mind, the first 6 must be the same followed by the next 4 being the same to ultimately produce the standard model gauge group, $SU(3)_c \times SU(2)_L \times U(1)_Y$. The remainder of the BV consists of a set of 6 paired and 12 possibly unpaired followed by 16 paired components, the last producing the E_8 hidden sector gauge group. The entire RM is freely specifiable within order N. Order N means that the elements are composed of $\alpha_i \in \left\{0, \pm\frac{1}{\frac{N}{2}}, \pm\frac{2}{\frac{N}{2}}, \cdots, \pm\frac{\frac{N}{2}-1}{\frac{N}{2}}, +\frac{\frac{N}{2}}{\frac{N}{2}}\right\}$ [8] (note: that I use $\alpha_i \in \{0, 1, ..., N\}$ for the discussion in this paper). These form the first set of constraints. When used by itself, the NAHE set yields a Grand Unified Theory. In terms of gauge groups this is: $SO(10) \times SO(6)^3$. Because of this result, many semi-realistic models are formed by trying to break the $SO(10)$ symmetry of the NAHE set. This is done by adding more BV's. Eventually we intend to add to this set more BV's that satisfy the ABK/KLT constraints outlined by [5, 6] and the component symmetries consistent with the work of Nanopoulos, Antoniadis, Hagelin and Ellis.

One of the main problems presented by heterotic model building is the wide range of string models that can be generated. It then becomes important to classify the different models. In the end it would be profitable to know the characteristics of each of the string models and especially to know what subset of the models contains our universe's physics. One way to classify these string models is by their individual Standard Models. In an effort to categorize the heterotic string models in this way, we first must generate sets of BV's consistent with the ABK/KLT constraints and the NAHE BV's. We generate our BV's computationally thus ensuring a complete set of order 2 and 4 BV's. The program we wrote to perform this calulation was designed to generate the BV's combinatorically as will be explained in Chapter 2.

This paper focuses on just the generation of the BV's but the next step in categorizing heterotic string models would be to combine a single BV, of order 2 or 4, to the NAHE set. We add the BV to the NAHE set so as to break the $SO(10)$ gauge symmetry into smaller gauge groups like $SU(3)$, $SU(2)$ and $U(1)$. This would yield string models with Standard Models potentially similar to the Standard Model in our own universe. We would then analyze the resultant string model in terms of it's Standard Model and classify it according to the Lie algebras associated with its constituent particles and forces. Adding more than one BV to the NAHE set and performing the same analysis would then be the next step. In lieu of these goals, we will move onto describing the process of generating BV's.

Chapter 2

Methods and Revisions

Generating string models in the free-fermionic heterotic language is equivalent to generating 64-component linear arrays called boundary vectors, BV's. In Chapter 1, we discussed some of the previous work in creating the BV's and the different forms the components can take. Each of these vectors is split into two pieces representing the left moving (LM) and right moving (RM) waves that exist on a heterotic string.

2.1 Language

According to the component symmetries listed in Chapter 1 we can calculate an upper limit on the number of vectors that can be used to represent string models. Combinatorically, there are 224 possible LM's after requiring $\alpha_3 = \alpha_6, \alpha_9 = \alpha_{12}, \alpha_{15} = \alpha_{18}$ with either one pair equal to unity and the rest zero or all three pairs equal to unity [7]. In addition, there are 22 paired RM fermions, each ranging through the values specified by the order of the components, N. This yields N^{22} possible RM's. Combining these results we have an upper limit on the number of BV's that could be produced. Even for $N = 2$ this upper limit is very large, $224 \times N^{22} = 939\,524\,096$.

Therefore, we have employed CASPER's RUSH computing cluster located at Baylor University for our calculations. Since the main purpose of this investigation is to construct these 64-component vectors, we used MATLAB to write the bulk of the BV generating program. We did so because MATLAB is a powerful matrix analysis language and has relatively simple syntax for loops.

2.2 Programming Method

Our first task was to write a code to generate all 256 LM's. This part of the code was written in MATHEMATICA and consisted of generating unique permutations (not combinations) of (1,0,0),(0,1,0),(0,0,1), and (1,1,1). In addition, every LM begins with (1,1) to represent that the waves living on the heterotic string are fermionic. Next, we ran a series of embedded *do* loops to create the permutations of the four previously listed 3-vectors. These were appended to a DAT file to be referenced by the RM generating program.

Once we realized the difficulty in programming with MATHEMATICA, we switched to MATLAB to write the RM generating program. A basic outline of the program is as follows:

i. Input Order, N
ii. Generate RM
iii. Associate RM with a LM
iv. Test the BV against ABK/KLT
v. Repeat iii and iv 256 times
vi. Return to ii until no RM's are left

After specifying an order, N, our original code was setup to generate a RM according to the NAHE symmetries [7] using a large number of *for* loops. That RM was then paired with all 256 LM's one by one, thus creating a BV. Each one of these candidate BV's were tested against the ABK/KLT constraints [5, 6]. Coding the constraint equations *(see section 1.3.4)* was relatively simple. We used MATLAB's native dot product command while adapting the RM's elements to turn the products into Lorentz products. A Lorentz product is similar to a dot product except that when certain elements are squared they receive a minus sign instead of a positive sign for the sum. In the case of the BV's, the RM elements receive this minus sign. The periodicity constraint was tested by adding the candidate BV and two NAHE vectors together. There must be an even number of components of this new vector that equal $(3+(N-2)/2)$. We also removed any BV's generated that were not order N. This is a concern because BV's with order equal to all factors of N will be produced by our program. For example, order 2 is a subset of order 4 so we have designed our program to remove all order 2 BV from the $N = 4$ database. Once passed, a candidate vector is appended to a DAT file for later use. We ran the program for $N = 2$ and 4.

2.3 Programming Setbacks

Upon investigating the initial output of the program, Gerald Cleaver noticed that another essential symmetry was neglected when building the new BV's. This symmetry has to do with the constraints of which fermionic waves are allowed to be paired together. In programming terms this equates to ensuring that certain BV elements match. This change necessitated a rewriting of the MATLAB script. In order to get the fermionic waves paired correctly (also called charge pairing), I had to create a new LM generator and to adapt the original RM generator. Before discussing the programming changes it is important to overview how the charges can be paired.

First, here is the generalized form of a RM:

Set 1:$\{y_1, y_2, w_5, w_6, \overline{y_1}, \overline{y_2}, \overline{w_5}, \overline{w_6}\}$

Set 2:$\{y_3, y_4, y_5, y_6, \overline{y_3}, \overline{y_4}, \overline{y_5}, \overline{y_6}\}$

Set 3:$\{w_1, w_2, w_3, w_4, \overline{w_1}, \overline{w_2}, \overline{w_3}, \overline{w_4}\}$

Table 2.1 Using the notation of the generalized LM's and RM's discussed in this section, here are the three sets of charges to be paired.

$$RM[i] = [a, a, a, a, a, a, b, b, b, b, \overline{y}_1, \overline{w}_1, \overline{y}_2, \overline{w}_2, \overline{y}_3, \overline{w}_3, \overline{y}_4, \overline{w}_4, \overline{y}_5, \overline{w}_5, \overline{y}_6, \overline{w}_6,$$
$$c, c, d, d, e, e, f, f, g, g, h, h, i, i, j, j, k, k, l, l, m, m]$$

The generalized LM we are working with takes the form:

$$LM[j] = [1, 1, x_1, y_1, w_1, x_2, y_2, w_2, x_3, y_3, w_3, x_4, y_4, w_4, x_5, y_5, w_5, x_6, y_6, w_6]$$

Here we see that certain elements are already paired ($\{a, b, ..., l, m\}$) and will range over $\{0, 1\}$ for order 2 and $\{0, 1, 2, 3\}$ for order 4. The charge pairing that we did not take into account before was between the x, y, w and the $\overline{x}, \overline{y}, \overline{w}$. These components can be paired amongst themselves and also between the sets. Here are the three sets of charges that can be paired together:

There are two possible pairing within each set. Take for example: Set 3 (see table 2.1); we can pair up all the LM's with LM's or we can only pair up two of the LM's. In any case, the RM's must be paired up in the same manner. If $w_1 = w_2$ then we are free to pair up any two of the RM's as well. Let's call the paired RM's $\overline{w}_1$ and $\overline{w}_2$ then. The case where only two LM's are paired requires that the remaining two LM's are paired with RM's. For convenience we label the RM's in the same manner as the LM's, i.e. if w_3 and w_4 are to be paired with RM's then they will be paired with charges which we call $\overline{w}_3$ and $\overline{w}_4$, respectively. It is important to remember that LM's can still only have the values: $\{0, 1\}$ and that RM's can still range over $\{0, 1, ..., N\}$. The only restriction to this is when the RM's are paired with LM's. Back to our

	w_1, w_2, w_3, w_4	$\overline{w_1}, \overline{w_2}, \overline{w_3}, \overline{w_4}$
case i:	a, a, b, b	$\alpha, \alpha, \beta, \beta$
	a, b, a, b	$\alpha, \beta, \alpha, \beta$
	a, b, b, a	$\alpha, \beta, \beta, \alpha$
case ii:	a, a, b, d	$\alpha, \alpha, \beta, \delta$
	a, b, a, d	$\alpha, \beta, \alpha, \delta$
	a, b, d, a	$\alpha, \beta, \delta, \alpha$
	b, a, a, d	$\beta, \alpha, \alpha, \delta$
	b, a, d, a	$\beta, \alpha, \delta, \alpha$
	b, d, a, a	$\beta, \delta, \alpha, \alpha$

Table 2.2 Here are the possible charge pairings for set three in a symbolic form.

example, then if $w_3 = 0$, $\overline{w_3}$ must also be zero. However, if $w_3 = 1$, $\overline{w_3}$ must be equal to $N/2$.

Now in terms of combinatorics, the first case (where LM's are paired only with LM's) has a total of three combinations. While case two yields six different combinations. We only need to generate these few unique pairings because we will allow each of the pairs to range over all allowed values; giving us every possible pairing. Symbolically these are:

In the table of Set 3 pairings, (table 2.2), the pairs are allowed to range over the values discussed above. It is of interest to note that even though I have used set three as an example, there is no difference with the other sets and the combinations allowed therein.

In allowing for this correction, I had to change the code to first produce all the possible pairings for any given set. This is done by taking the allowed pairings for a set and then, with *for* statements, letting the elements range over the correct values for the given order. This matrix composed of LM and RM elemtents is then permuted column-wise to generate all the remaining possiblities. Another program I wrote takes that ouput and uses it to make a LM with the appropriate RM elements. This program uses three *for* loops (one for each set: 1,2,3) to combine the sets in every possible way. These potential LM's are tested using the rules for LM's discussed in 2.1. This output of LM's (plus RM elements) is then combined with the set of all possible remaining RM elements, which creates a new BV to test according to 2.2.

Chapter 3

Results and Discussion

In Chapter 2, I gave the rules for and a general outline on how to generate BV's. In this chapter, I list some of the BV's that were produced and present a brief discussion on how these BV's will be used eventually.

3.1 Results

Due to the enormity of the program's output, I will not list all the BV's generated. Instead I list below a few example BV's that satisfy the ABK/KLT constraints and that produce a consistent string model when individually combined with the NAHE set. In theses examples I have used the original component range convention which allows the RM components to be $\alpha_i \in \left\{0, \pm\frac{1}{\frac{N}{2}}, \pm\frac{2}{\frac{N}{2}}, \cdots, \pm\frac{\frac{N}{2}-1}{\frac{N}{2}}, +\frac{\frac{N}{2}}{\frac{N}{2}}\right\}$ as described in 1.3.4.

3.1.1 Sample Output: Order 2

$$(1_2, (1,0,0)_2, (0,0,1)_4 \| 0_{44})$$

$$(1_2, (1,1,1)_6 \| 0_{40}, 1_4)$$

$$(1_2, (0,0,1)_2, (1,0,0)_4 \| 0_{28}, 1_2, 0_4, 1_2, 0_4, 1_4)$$

$$(1_2, (0,1,0), (0,0,1), (1,1,1), (1,0,0), (0,0,1), (0,1,0)|$$

$$|0_{26}, 1_6, 0_2, 1_2, 0_6, 1_2)$$

$$(1_2, (0,0,1)_2, (1,1,1), (1,0,0), (0,1,0), (0,0,1)|$$

$$|0_{24}, (1,0,0)_2, 1_6, 0_2, 1_2, 0_4)$$

$$(1_2, (0,0,1)_2, (0,1,0)_2, (1,0,0)_2|$$

$$|0_{10}, 1_2, 0_4, 1, 0, 1, 0_2, (1,0,1,0)_2, 0_5, 1_2, 0_2, 1_6)$$

$$(1_2, (0,0,1)_2, (1,1,1)_2, (0,0,1)_2|$$

$$|0_{10}, 1_2, 0_4, 1, 0, 1, 0, (0,1,0,1)_2, 0_8, (1,1,0,0)_2)$$

$$(1_2, (0,1,0)_2, (0,0,1), (0,1,0), (1,0,0), (1,1,1)|$$

$$|0_{10}, 1_2, 0_2, 1_2, 0, 1_2, 0_2, 1, 0, 1_3, 0_2, 1_{16})$$

3.1.2 Sample Output: Order 4

$(1_2, (0,0,1), (0,1,0), (1,0,0), (1,1,1), (0,1,0), (0,0,1) \| (\text{-}1/2)_{40}, 0_4)$

$(1_2, (1,0,0)_2, (0,0,1)_4 \| (\text{-}1/2)_{40}, 0_2, 1_2)$

$(1_2, (1,0,0)_2, (0,1,0)_4 \| (\text{-}1/2)_{30}, 0_2, 1_2, (\text{-}1/2)_2, (1/2)_2, 0_4, 1_2, 0_2)$

$(1_2, (1,1,1)_6 \| (\text{-}1/2)_{28}, 1_{12}, (1/2)_4)$

$(1_2, (1,1,1)_6 \| (\text{-}1/2)_{26}, (1/2)_4, (\text{-}1/2)_2, 0_2, (\text{-}1/2)_2, (1/2)_4, 1_2, (\text{-}1/2)_2)$

$(1_2, (1,1,1), (1,0,0), (1,1,1)_4|$

$|(\text{-}1/2)_{25}, (1/2), (\text{-}1/2), (1/2)_3, 1_2, (1/2)_2, (\text{-}1/2)_4, 1_2, (1/2)_2, (\text{-}1/2)_2)$

$(1_2, (1,1,1)_2, (1,0,0), (1,1,1), (1,0,0), (1,1,1)|$

$|(\text{-}1/2)_{24}, 0_2, 1_6, (1/2)_2, (\text{-}1/2)_2, (1/2)_2, 0_2, (\text{-}1/2)_2, 1_2)$

$(1_2, (0,1,0)_2, (0,0,1), (0,1,0), (1,0,0), (1,1,1)|$

$|(\text{-}1/2)_{24}, 0_2, 1_2, (\text{-}1/2)_2, (1/2)_2, (\text{-}1/2)_2, 0_6, (\text{-}1/2)_2, 0_2)$

Since the generated BV's are at most order 4, all components must lie in the range -2 to 2. For physical significane the components represent the phase of the propagating fields. This phase is the number of times the wave must move around the string to come back to its original state. Note the use of subscripts to denote the number of times an element (or set of elements) is successively repeated and the double bars to separate the LM's from RM's.

3.2 Discussion and Future Work

Since these BV's correspond to compactifications of the extra dimensions, they also are related to symmetries of the string models. It is these symmetries that yield

$\alpha(21:27)$ versus $\alpha(28:31)$	$SO(10) \rightarrow$
Same, $\alpha_i \in \{0, 1\}$	unchanged
Different, $\alpha_i \in \{0, 1\}$	$SO(6) \times SO(4)$
Same, $\alpha_i \in \{non-integer, \mathbb{Q}\}$	$SU(5) \times U(1)$

Table 3.1

the unique particle inhabitants of each string model. Because particles arise with gauge group symmetries, it is necessary to find the group-theoretic description of each string model. This symmetry analysis is performed by relating some choices of the BV components, α_i, to generators of a specific subalgebra in an adjoint representation [9]. These subalgebras then correspond to the algebras of gauge groups and therefore to the string model's standard model. A program designed and written by Gerald Cleaver, called FMG (Free-fermionic Model Generator), has been developed to do this and our output is formatted to be utilized by this program for categorization.

For a brief analysis of the process, consider the first 10 real components of the RM BV's. These components correspond to the $SO(10)$ sector of the gauge group, so depending on the values for the α_i in those slots we can see different gauge group breaking. Table 3.1 lists the gauge group breaks.

By adding BV's with different symmetries for the $SO(10)$ sector (components 21-31 of $\overrightarrow{\alpha}$), we can cause different gauge groups to arise in the string model. This symmetry breaking is done by letting the values of α_{21} through α_{27} be different from α_{28} through α_{31}. Note: By combining the last two entries of Table 3.1 we can break the $SO(10)$ into $SU(3) \times U(1) \times SU(2) \times U(1)$, a standard model like the one in our own universe.

The database I have constructed is of two orders: 2 and 4. These vectors were computed so that any one of them could be added to the NAHE set. Now this set of BV's can be used by the CASPER string group to find acceptable linear combinations of NAHE, order 2 BV's and order 4 BV's to categorize more string models. The process by which these new combinations will be organized is beyond the scope of this report, but comparison to analytically derived known models will be forthcoming.

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