Physics 121 Math reference

Mathematical Constants

Use a calculator to get values of constants and conversions such as π, e, $\sqrt{2}$, $\ln(10)$, $1\,\text{rad} = 180°/\pi = 57.296°$, etc.

Scalar Algebra

$(a+b)(c+d) = ac + ad + bc + bd$

$(a \pm b)^2 = a^2 \pm 2ab + b^2$

$a^2 - b^2 = (a-b)(a+b)$

$a^2 + b^2 = (a - bi)(a + bi)$

$a^x \times a^y = a^{x+y}$. $a^{-x} = \dfrac{1}{a^x}$. $\dfrac{a^x}{a^y} = a^{x-y}$.

$(a^x)^y = a^{xy}$. If $a \neq 0$, $a^0 = 1$. $a^{\frac{1}{x}} = \sqrt[x]{a}$

If $ax^2 + bx + c = 0$, $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

If $a = b$ and $c = d$ then $ac = bd$ and $\dfrac{a}{c} = \dfrac{b}{d}$.

$e^{\ln(x)} = x$, $\ln(1) = 0$.

$\ln(ab) = \ln a + \ln b$, $\ln(a/b) = \ln a - \ln b$.

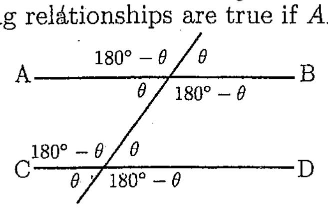
Geometry

Circle of radius r:
circumference $C = 2\pi r$; area $A = \pi r^2$.

Parallelogram with base b and altitude h:
area, $A = bh$.

Sphere with radius r:
area, $A = 4\pi r^2$; volume, $V = \frac{4}{3}\pi r^3$.

The following relationships are true if $AB \parallel CD$:



Trigonometry

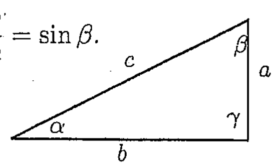
For a right triangle:

($\gamma = 90° = \pi/2$ radians)

$\sin\alpha = \dfrac{a}{c} = \cos\beta$. $\cos\alpha = \dfrac{b}{c} = \sin\beta$.

$\tan\alpha = \dfrac{a}{b} = \dfrac{\sin\alpha}{\cos\alpha}$

$a^2 + b^2 = c^2$



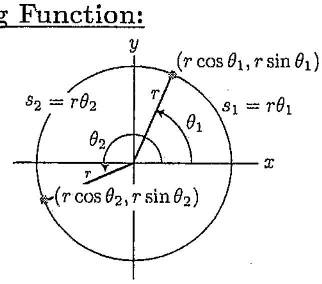
General Definitions of Trig Function:

$\theta \equiv s/r$ radians,

$\cos\theta \equiv x/r$

$\sin\theta \equiv y/r$,

$\tan\theta \equiv y/x$.



Trig Relations:

$\sin^2\theta + \cos^2\theta = 1$

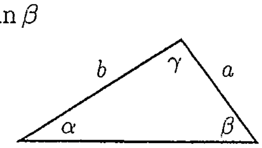
$\cos(\theta \pm \frac{\pi}{2}) = \mp \sin(\theta)$

$\sin(\theta \pm \frac{\pi}{2}) = \pm \cos(\theta)$

$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$

$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$

$\lim_{\theta \to 0}\left(\dfrac{\sin\theta}{\theta}\right) = 1$



For an oblique triangle:

$\alpha + \beta + \gamma = 180° = \pi$ radians.

Law of sines: $\dfrac{a}{\sin\alpha} = \dfrac{b}{\sin\beta} = \dfrac{c}{\sin\gamma}$

Law of cosines: $c^2 = a^2 + b^2 - 2ab\cos\gamma$

Calculus

u, v are functions of t; b, m are constants.

$\dfrac{dt}{dt} = 1$ $\qquad \displaystyle\int dt = t$

$\displaystyle\int \frac{dv}{dt}dt = v$ $\qquad \displaystyle\int_a^b \frac{dv}{dt}dt = v(b) - v(a) = \Delta v$

$\Delta v \approx \dfrac{dv}{dt}\Delta t$ for 'small' Δt

$\dfrac{d}{dt}(bv) = b\dfrac{dv}{dt}$ $\qquad \displaystyle\int bv dt = b\int v dt$

$\dfrac{d}{dt}u^m = mu^{m-1}\dfrac{du}{dt}$ $\qquad \displaystyle\int u^m \frac{du}{dt}dt = \frac{u^{m+1}}{m+1}$ $(m \neq 1)$

$\dfrac{d}{dt}(u+v) = \dfrac{du}{dt} + \dfrac{dv}{dt}$ $\qquad \displaystyle\int (u+v)dt = \int u dt + \int v dt$

$\dfrac{d}{dt}(uv) = u\dfrac{dv}{dt} + v\dfrac{du}{dt}$ $\qquad \displaystyle\int u\frac{dv}{dt}dt = uv - \int v\frac{du}{dt}dt$

$\dfrac{d}{dt}\sin u = \cos u\dfrac{du}{dt}$ $\qquad \displaystyle\int \cos u\frac{du}{dt}dt = \sin u$

$\dfrac{d}{dt}\cos u = -\sin u\dfrac{du}{dt}$ $\qquad \displaystyle\int \sin u\frac{du}{dt}dt = -\cos u$

$\dfrac{d}{dt}e^u = e^u\dfrac{du}{dt}$ $\qquad \displaystyle\int e^u\frac{du}{dt}dt = e^u$

$\dfrac{d}{dt}\ln u = \dfrac{1}{u}\dfrac{du}{dt}$ $\qquad \displaystyle\int \frac{1}{u}\frac{du}{dt}dt = \ln u$

$\displaystyle\int \frac{1}{\sqrt{A^2 - x^2}}dx = \sin^{-1}\left(\frac{x}{A}\right)$, or $-\cos^{-1}\left(\dfrac{x}{A}\right)$

$\dfrac{du}{dt} \equiv \lim\limits_{\Delta t \to 0}\dfrac{\Delta u}{\Delta t}$ $\qquad \displaystyle\int_a^b u\,dt \equiv \lim_{\Delta t \to 0}\sum_a^b u\Delta t$

Mathematical Notation

$a = b$	a is equal to b
$a \overset{\text{law}}{=} b$	a is equal to b by experiment
$a \approx b$ or $a \simeq b$	a is approximately equal to b
$a \overset{\text{def}}{=} b$ or $a \equiv b$	a is defined to be b
$a \neq b$	a is not equal to b
$a > b$	a is greater than b
$a \gg b$	a is much greater than b
$a \geq b$	a is greater than or equal to b
$a < b$	a is less than b
$a \ll b$	a is much less than b
$a \leq b$	a is less than or equal to b
Δx	$(x_{final} - x_{initial})$, the change in x
δx	a small amount of x
$\pm x$	plus or minus x
$\mp x$	minus or plus x
$\|x\|$	the absolute value of x
$\overline{x} = \langle x \rangle$	the average value of x
ΣW	the sum of all W's
$\vec{a}$	the vector $\vec{a}$
$\ln x$	the natural (base e) logarithm of x
$\log x$	logarithm (base 10) of x
i	$\sqrt{-1}$
$AB \parallel CD$	AB is parallel to CD
$AB \perp CD$	AB is perpendicular to CD
$\dfrac{dx}{dt}$	the derivative of x
$\int x dt$	the indefinite integral of x
$\int_a^b x dt$	the definite integral of x

Metric Prefixes

prefix	*symbol*	*factor*	*factor*	*symbol*	*prefix*
tera-	T	10^{12}	10^{-12}	p	pico-
giga-	G	10^{9}	10^{-9}	n	nano-
mega-	M	10^{6}	10^{-6}	μ	micro-
kilo-	k	10^{3}	10^{-3}	m	milli-

Vectors

Notation:

$\vec{a}$: vector $\vec{a}$

$a = |\vec{a}|$: magnitude of $\vec{a}$

(a, θ): 2-d polar form of $\vec{a}$
θ measured counterclockwise from the positive x axis.

$(a_x, a_y, a_z) = a_x\hat{\imath} + a_y\hat{\jmath} + a_z\hat{k}$: rectangular form of $\vec{a}$ in 2 or 3 dimensions

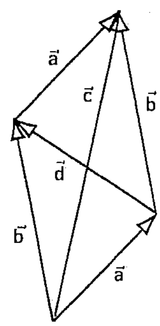
$a \equiv \sqrt{a_x^2 + a_y^2 + a_z^2}$ magnitude of $\vec{a}$ using rectangular coordinates

Equality:

If $\vec{a} = \vec{b}$ then
$a_x = b_x$, $a_y = b_y$, and $a_z = b_z$

Addition:

In the figure,
$\vec{c} = \vec{a} + \vec{b}$ and
$\vec{d} = \vec{b} - \vec{a}$.



Using rectangular components:
$\vec{a} \pm \vec{b} \equiv (a_x \pm b_x)\hat{\imath} + (a_y \pm b_y)\hat{\jmath} + (a_z \pm b_z)\hat{k}$

Polar $\Longleftrightarrow$ rectangular conversion, 2-d:

$\vec{a} = (a, \theta) = (a\cos\theta, a\sin\theta) = a\cos\theta\,\hat{\imath} + a\sin\theta\,\hat{\jmath}$

$(a_x, a_y) = a_x\hat{\imath} + a_y\hat{\jmath} = (\sqrt{a_x^2 + a_y^2}, \tan^{-1}\left(\frac{a_y}{a_x}\right))$

scalar times **vector** $s\vec{a}$:

$s\vec{a} \equiv (sa, \theta) \equiv (sa_x, sa_y, sa_z) \equiv sa_x\hat{\imath} + sa_y\hat{\jmath} + sa_z\hat{k}$

The scalar product $\vec{a}\cdot\vec{b}$:

$\vec{a}\cdot\vec{b} \equiv a_xb_x + a_yb_y + a_zb_z \equiv ab\cos\theta = ab_{\parallel} = a_{\parallel}b$

$a^2 \equiv \vec{a}\cdot\vec{a} \equiv a_xa_x + a_ya_y + a_za_z.$

The vector product $\vec{A} \times \vec{B}$:

If $\vec{A} \times \vec{B} = \vec{C}$, $\quad |\vec{C}| \equiv AB\sin\theta = AB_{\perp} = A_{\perp}B$
$\vec{C}$ is perpendicular to $\vec{A}$ and $\vec{B}$ in the direction given by the *right-hand rule.*

In rectangular components, .

$$\vec{A} \times \vec{B} \equiv \begin{matrix} (A_yB_z - A_zB_y)\hat{\imath} \\ +(A_zB_x - A_xB_z)\hat{\jmath} \\ +(A_xB_y - A_yB_x)\hat{k} \end{matrix} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

$$= -\vec{B} \times \vec{A}$$

Physical Consants

free fall acceration	g_e	$9.8\,\mathrm{m/s^2}$
speed of light	c	$3.00 \times 10^8\,\mathrm{m/s}$
gravitational constant	G	$\frac{2}{3} \times 10^{-10}\,\mathrm{N{\cdot}m^2/kg^2}$

Physical Quantities and their Units

Quantity		SI Unit		
Name	Symbol	Name	Symbol	units
length	L, l	meter	m	m
position	$\vec{r}$	meter	m	m
mass	m	kilogram	kg	kg
time	t	second	s	s
angle	θ	radian	rad	1 m/m
velocity	$\vec{v}$			m/s
speed	v			m/s
acceleration	$\vec{a}$			$\mathrm{m/s^2}$
force	$\vec{F}$	newton	N	$\mathrm{kg{\cdot}m/s^2}$
work	W	newton-meter	N·m	1 N·m= 1 J
energy	E, K, U	joule	J	$\mathrm{kg{\cdot}m^2/s^2}$
power	P	watt	W	J/s
momentum	$\vec{p}$, $\vec{P}$			kg·m/s
frequency	f	hertz	Hz	$\mathrm{s^{-1}}$
angular frequency	ω			rad/s
angular velocity	$\vec{\omega}$			rad/s
angular acceleration	$\vec{\alpha}$			$\mathrm{rad/s^2}$
torque	$\vec{\tau}$	newton-meter	N·m	1 N·m = 1 J/rad
rotational inertia	I			$\mathrm{kg{\cdot}m^2}$
angular momentum	ℓ, $\vec{L}$			$\mathrm{kg{\cdot}m^2/s}$

Conversion Factors

Mass
- 1 kg = 1000 g
- 1 u = 1.66×10^{-27} kg

Length
- 1 m = 100 cm = 3.28 ft
- 1 mi = 1.61 km = 5280 ft

Time
- 1 day = 86.400 ks
- 1 year = 31.6 Ms

Angle
- 1 rev = 360°= 2π rad

Force
- 1 N = 0.225 pound
- 1 pound = 4.45 N

Energy
- 1 eV = 1.60×10^{-19} J
- 1 kcal = 1 Cal = 10^3 cal = 4.19 kJ
- 1 kW·h = 3.60 MJ = 3.60×10^6 J

Power
- 1 hp = 746 W