Exam 1 Solutions - Phys 123 Winter 2011 - Colton

(17 pts) **Problem 1**: Multiple choice conceptual questions. Choose the best answer and fill in the appropriate bubble on your bubble sheet. You may also want to circle the letter of your top choice on this paper.

1.1. An extremely precise scale is used to measure an iron weight. It is found that in a room with the air sucked out, the mass of the weight is precisely 2.000000 kg. If you add the air back into the room, will the scale reading increase, decrease, or stay the same?

a. increase
b. decrease
c. stay the same

The buoyant force from the air now helps support some of the weight

1.2. Three cubes of the same size and shape are put in water. They all sink. One is lead, one is steel and one is a dense wood (ironwood). $\rho_{lead} > \rho_{steel} > \rho_{ironwood}$. On which cube is the buoyant force the greatest?

a. lead
b. steel
c. wood
d. same buoyant force

same size → same buoyant force

1.3. Water flows from a little pipe into a big pipe with no friction or height change. The volume flow rate (m^3/s) in the little pipe will be _______ in the big pipe.

a. greater than
b. the same as
c. less than

As long as fluid is incompressible, VFR = constant

1.4. As an airplane flies horizontally at a constant elevation, the pressure above a wing is ___________ the pressure below the wing.

a. larger than
b. smaller than
c. the same as

That's a major component in how airplanes fly!
The pressure difference above & below wing creates "lift"

1.5. A grandfather clock is controlled by a swinging brass pendulum. It keeps perfect time at 20°C. If the temperature drops to 10°C, does the clock run fast, slow, or the same?

a. runs fast
b. runs slow
c. runs the same

shorter pendulum → faster "ticks"

1.6. You have two jars of gas: helium and neon. Both have the same volume, same pressure, same temperature. Which jar contains the greatest number of gas molecules? (The mass of a neon molecule is greater than the mass of a helium molecule.)

a. jar of helium
b. jar of neon
c. same number

PV = nRT → if P, V, T are the same then n must be same.

1.7. The probability density function for the Maxwell-Boltzmann velocity distribution is: $f(v) = 4\pi\left(\frac{m}{2\pi k_B T}\right)^{3/2} v^2 e^{-mv^2/2k_BT}$.

What would be the appropriate integral to calculate how many molecules have speeds between 200 and 250 m/s?

a. $N_{tot} \times \int_{200}^{250} f(v)dv$

(See class discussion on histograms)

b. $N_{tot} \times \int_{200}^{250} v \cdot f(v)dv$

c. $N_{tot} \times \int_{200}^{250} v^2 \cdot f(v)dv$

d. $N_{tot} \times \sqrt{\int_{200}^{250} v^2 \cdot f(v)dv}$

1.8. What would be the appropriate integral to calculate the average speed (not rms speed) of all molecules?

a. $\int_0^\infty f(v)dv$

b. $\int_0^\infty v \cdot f(v)dv$ (Again, see class discussion on histograms)

c. $\int_0^\infty v^2 \cdot f(v)dv$

d. $\sqrt{\int_0^\infty v^2 \cdot f(v)dv}$

1.9. Slabs of metal 1 (thermal conductivity k_1) and metal 2 (k_2) are placed together so that their flat surfaces are in contact. The two slabs have identical dimensions. Metal 1 is in thermal contact with a reservoir at T_1 and metal 2 is in contact with a reservoir at T_2. What is the temperature at the interface between the metals?

a. $(k_1+k_2)(k_1T_1+k_2T_2)$

b. $(k_1+k_2)(k_1T_2+k_2T_1)$

c. $\dfrac{k_1T_1+k_2T_2}{k_1+k_2}$

d. $\dfrac{k_1T_2+k_2T_1}{k_1+k_2}$

e. $\left(\dfrac{1}{k_1}+\dfrac{1}{k_2}\right)^{-1}(k_1T_1+k_2T_2)$

f. $\left(\dfrac{1}{k_1}+\dfrac{1}{k_2}\right)^{-1}(k_1T_2+k_2T_1)$

$$\left(\frac{Q}{time}\right)_{slab1} = \left(\frac{Q}{time}\right)_{slab 2}$$

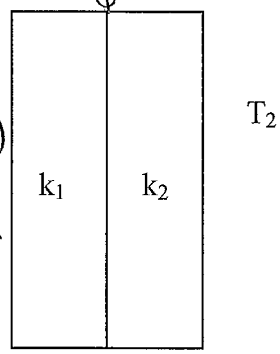
$$\frac{k_1 \cancel{A}(T_{middle}-T_1)}{\cancel{\ell}} = \frac{k_2\cancel{A}(T_2-T_{middle})}{\cancel{\ell}}$$

$$k_1T_m - k_1T_1 = k_2T_2 - k_2T_m$$

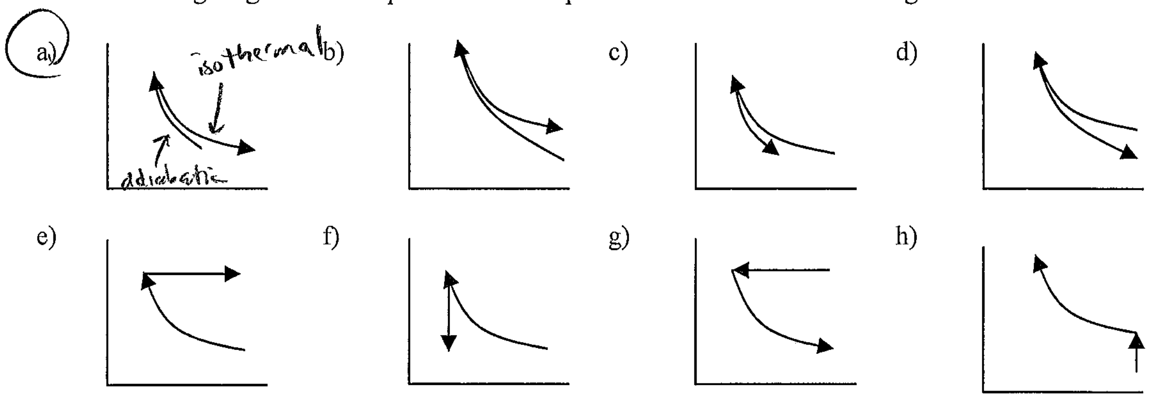
$$T_m(k_1+k_2) = k_1T_1+k_2T_2$$

$$\boxed{T_m = \frac{k_1T_1+k_2T_2}{k_1+k_2}}$$

This is a weighted average.

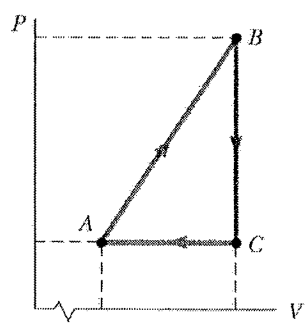


1.10. First, a gas is compressed adiabatically. This heats up the gas, while its pressure increases to 2.5× the original value. Next, more heat is added to the gas, this time as it is expanded isothermally, until it returns to its original pressure. Which of the following diagrams best represents the two processes on a standard P-V diagram?



1.11. For the next three problems, consider the cyclic process described by the figure.
For A to B: does the internal energy increase, decrease, or stay the same?

a. Increase $T_B > T_A$ because it's "higher up mountain"
b. Decrease
c. Stays the same ($\Delta E_{int} = 0$)



1.12. For B to C: is heat added or taken away from the gas?

a. Added
b. Taken away
c. Neither ($Q_{added} = 0$)

$\Delta E_{int} = Q + \underbrace{W_{on}}_{0}$ because no area

$Q = \Delta E_{int}$ → $T_C < T_B$ so this is negative

1.13. For C to A: is $W_{on\ gas}$ positive, negative, or zero?

a. Positive
b. Negative
c. Zero

Volume is decreasing, so work is being done on the gas

1.14. A heat engine performs x joules of work in each cycle and has an efficiency of e. For each cycle of operation, how much energy is absorbed by heat?

a. x
b. x/e
c. xe
d. $(1-x)$
e. $(1-x)/e$
f. $(1-x)e$

$e = \frac{|W|}{Q_h} \rightarrow Q_h = \frac{|W|}{e}$

1.15. If you flip 10 coins, what is the probability of getting exactly 8 heads?

a. $\frac{8!}{2!\,8!\,2^{10}}$
b. $\frac{8!}{2!\,10!\,2^{8}}$
c. $\frac{8!}{10!\,2^{10}}$
d. $\frac{8!\,2!}{2!\,2!\,2^{8}}$
e. $\frac{8!\,2!}{2!\,10!\,2^{8}}$
f. $\frac{8!\,2!}{10!\,2^{10}}$
g. $\frac{10!}{2!\,2!\,2^{8}}$
h. $\frac{10!}{2!\,8!\,2^{10}}$
i. $\frac{10!}{8!\,8!\,2^{8}}$

$P = \frac{\text{\# microstates in "8 heads" macrostate}}{\text{total \# microstates}} = \frac{\binom{10}{8}}{2^{10}} = \left(\frac{10!}{2!\,8!}\right)\left(\frac{1}{2^{10}}\right)$

1.16. As I'm taking data in my lab, I typically average data for 1 second per point. Suppose I decide to average the data for 2 seconds per point instead. How much better is my signal-to-noise ratio likely to be? (If you like, you can consider the "signal" to be 3 volts and the "noise" to be a standard deviation of 0.112 V, like in the homework problem.)

a. the same
b. $\sqrt{2}$ times better
c. 2 times better
d. 4 times better
e. 8 times better

fluctuations increase by $\sqrt{2}$
but signal increases by 2
signal/noise increases by $\frac{2}{\sqrt{2}} = \sqrt{2}$

1.17. Suppose an atom has only two available energy levels, which are at these energies: state 1 = 0 J; state 2 = 2×10^{-20} J. If the temperature is 300 K, what is the probability that the atom is in the higher energy state?

a. $1+e^{+2\cdot10^{20}/1.38\cdot10^{-23}\cdot300}$
b. $1+e^{-2\cdot10^{20}/1.38\cdot10^{-23}\cdot300}$
c. $\frac{1}{1+e^{-2\cdot10^{20}/1.38\cdot10^{-23}\cdot300}}$
d. $\frac{1+e^{-2\cdot10^{20}/1.38\cdot10^{-23}\cdot300}}{e^{-2\cdot10^{20}/1.38\cdot10^{-23}\cdot300}}$
e. $\frac{1+e^{-2\cdot10^{20}/1.38\cdot10^{-23}\cdot300}}{e^{+2\cdot10^{20}/1.38\cdot10^{-23}\cdot300}}$
f. $\frac{1+e^{+2\cdot10^{20}/1.38\cdot10^{-23}\cdot300}}{e^{-2\cdot10^{20}/1.38\cdot10^{-23}\cdot300}}$
g. $\frac{e^{-2\cdot10^{20}/1.38\cdot10^{-23}\cdot300}}{1+e^{-2\cdot10^{20}/1.38\cdot10^{-23}\cdot300}}$
h. $\frac{e^{-2\cdot10^{20}/1.38\cdot10^{-23}\cdot300}}{1+e^{+2\cdot10^{20}/1.38\cdot10^{-23}\cdot300}}$
i. $\frac{e^{+2\cdot10^{20}/1.38\cdot10^{-23}\cdot300}}{1+e^{-2\cdot10^{20}/1.38\cdot10^{-23}\cdot300}}$

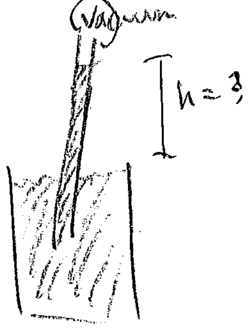
BF

$E=2e\text{-}20$ $\quad e^{-2e\text{-}20/kT}$

$E=0 \leftarrow \quad e^{0}\ (=1)$

$P(\text{upper state}) = \frac{e^{-2e\text{-}20/kT}}{1+e^{-2e\text{-}20/kT}}$

(8 pts) **Problem 2.** (a) What is the upper limit to how far soda (density of soda ≈ density of water) can be sucked through a vertical straw, assuming you can obtain a perfect vacuum in your mouth?



Pressure here = 1 atm

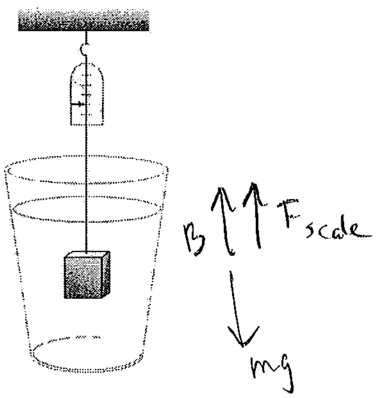
Pressure here also = 0 atm + $\rho g h$

so 1 atm = $\rho g h$

$$1.01\cdot 10^5\ Pa = \left(1000\ \frac{kg}{m^3}\right)\left(9.8\ \frac{m}{s^2}\right) h$$

$$\boxed{h = 10.3\ m}$$

(b) A 13 kg block of metal is suspended from a scale and immersed in water as in the figure. The dimensions of the block are 12 cm × 10 cm × 9 cm. The 12 cm dimension is vertical, and the top of the block is 7 cm below the surface of the water. What is the reading of the spring scale (in Newtons)?



$$\Sigma F = 0$$

$$B + F_{scale} - mg = 0$$

$$F_{scale} = mg - B$$

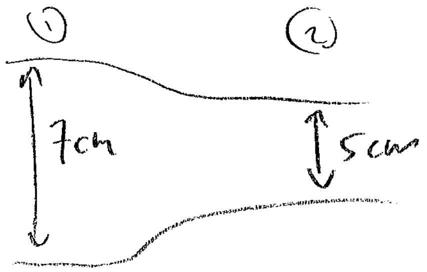
$$= mg - \rho_{fl} g V$$

$$= (13\,kg)\left(9.8\frac{m}{s^2}\right) - \left(1000\frac{kg}{m^3}\right)\left(9.8\frac{m}{s^2}\right)\left(.12 \times .1 \times .09\ m^3\right)$$

$$= \boxed{116.8 N}$$

(9 pts) **Problem 3**. A horizontal pipe 7 cm in diameter has a smooth reduction to a pipe 5 cm in diameter. If the pressure of the water in the larger pipe is 120 kPa and the pressure in the smaller pipe is 85 kPa, how fast (m/s) does water flow through the pipes? Hint: how does the velocity in the second section relate to the velocity in the first section?

Ambiguous. I'll solve for both v_1 and v_2 but either is ok.



Bernoulli: $P_1 + \cancel{\rho g h_1} + \frac{1}{2}\rho v_1^2 = P_2 + \cancel{\rho g h_2} + \frac{1}{2}\rho v_2^2$

unknowns

Need to relate V_2 to V_1

Use "garden hose" equation:

$$A_1 v_1 = A_2 v_2$$

$$v_2 = v_1\left(\frac{A_1}{A_2}\right) = v_1\left(\frac{\pi d_1^2/4}{\pi d_2^2/4}\right) = v_1\left(\frac{d_1}{d_2}\right)^2$$

Plug into Bernoulli:

$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_1^2\left(\frac{d_1}{d_2}\right)^4$$

$$P_1 - P_2 = \frac{1}{2}\rho v_1^2\left[\left(\frac{d_1}{d_2}\right)^4 - 1\right]$$

$$v_1 = \sqrt{\frac{2(P_1 - P_2)}{\rho}\,\frac{1}{(d_1/d_2)^4 - 1}} = \boxed{4.96\ \frac{m}{s}}$$

then $v_2 = v_1\left(\frac{d_1}{d_2}\right)^2 = 4.96\left(\frac{7}{5}\right)^2$

from above

$$= \boxed{9.73\ \frac{m}{s}}$$

$c = 900\ \text{J/kg°C}, \quad \alpha = 24\times10^{-6}\ /°C$

(10 pts) **Problem 4.** (a) An aluminum rod is exactly 20 cm long at 20°C, and has a mass of 300 g. If 11 kJ of energy is added to the rod by heat, what will be the change in length of the rod? (Hint: the heat causes a change in temperature, which in turn causes a change in length.)

$$Q = mc\Delta T \rightarrow \Delta T = \frac{Q}{mc}$$

$$= \frac{11000\ \text{J}}{(.3\ \text{kg})(900\ \text{J/kg°C})} = \underline{40.74\ °C}$$

$$\Delta L = \alpha L \Delta T = (24\times10^{-6}\ \tfrac{1}{°C})(.2\ \text{m})(40.74°C)$$

$$= \boxed{1.956\times10^{-4}\ \text{m}}$$

$\Delta L =$ ______________ m

(b) A Styrofoam box has a surface area of 0.8 m^2 and a wall thickness of 2 cm. The temperature of the inner surface is 0°C, and the outside temperature is 22°C. If it takes 3 hours for 1 kg of 0°C ice to melt in the container, determine the thermal conductivity of the Styrofoam.

$\rightarrow Q = mL, \quad L = 333000\ \text{J/kg}$

$$\frac{Q}{t} = \frac{kA\Delta T}{\ell} \longrightarrow k = \frac{Q}{t}\frac{\ell}{A\Delta T}$$

$$= \frac{mL\ \ell}{t\ A\Delta T}$$

$$= \frac{(1\ \text{kg})(333000\ \tfrac{\text{J}}{\text{kg}})(.02\ \text{m})}{(3\ \text{hrs}\times 3600\ \tfrac{\text{s}}{\text{hr}})(.8\ \text{m}^2)(22°C)}$$

$$= \boxed{3.50\times10^{-2}\ \frac{\text{J}}{\text{s m °C}}}$$

$k_{Styrofoam} =$ ______________ J/s·m·°C

(10 pts) **Problem 5.** (a) How many nitrogen molecules (N_2) are required to fill a spherical balloon to a diameter of 35 cm at a temperature of 290K? Take the pressure to be exactly 1 atm.

$$PV = N k_B T \rightarrow N = \frac{PV}{k_B T}$$

$$= \frac{(1.01\times10^5\ Pa)\left(\frac{4}{3}\pi\left(\frac{.35m}{2}\right)^3\right)}{(1.38\times10^{-23}\ J/K)(290K)}$$

$$= \boxed{5.666\times10^{23}\ \text{molecules}}$$

molecules = ___________

(b) In one of the liquid nitrogen demos that I did, a small volume of liquid nitrogen turned abruptly to gas and expanded. The expanding gas was then trapped by a giant thin blue cylindrical plastic bag-type thing. Find the ratio of the volume of the expanded gas (now at 300K, 1 atm) to the original volume of liquid.

Plan: gas: use ideal gas law
liquid: use density $\rho = 808\ kg/m^3$

$$\frac{V_{gas}}{V_{liquid}} = \frac{nRT/P}{m/\rho_{liquid}}$$

can simplify, since $n = \frac{m}{MM} \rightarrow \frac{n}{m} = \frac{1}{MM}$

$$= \frac{RT\rho_{liquid}}{MM\ P} = \frac{(8.31\ J/mol\,K)(300K)(808\ kg/m^3)}{(.028\ kg/mol)(1.01\times10^5\ Pa)}$$

$$= \boxed{712}$$

→ it's actually somewhat higher in Provo, because atmospheric pressure here is $< 1.01\cdot10^5$ Pa

V_{gas}/V_{liquid} = ____________________

(10 pts) **Problem 6.** (a) Find the average kinetic energy and rms speed of individual nitrogen molecules at 300K.

↳ diatomic, 5 d.o.f.

$$KE = (\# d.o.f) \times \frac{k_B T}{2}$$

$$= \frac{5}{2}(1.38 \times 10^{-23}\ J/K)(300K) = \boxed{1.035 \cdot 10^{-20}\ J}$$

trans KE only (no rotational) is $\frac{3}{2} k_B T$

also $= \frac{1}{2} m v^2$

$$\frac{3}{\cancel{2}} k_B T = \frac{1}{\cancel{2}} m v^2$$

$$v^2 = \frac{3 k_B T}{m}$$

$$V_{rms} = \sqrt{\frac{3 k_B T}{m}} = \sqrt{\frac{3(1.38 \cdot 10^{-23})(300)}{.028 / 6.022 \cdot 10^{23}}} = \boxed{516.8\ \frac{m}{s}}$$

$$m = \frac{MM}{N_A}$$

KE_{ave} = ______________ J

v_{rms} = ______________ m/s

(b) According to the Dulong-Petit law, what should the specific heat of aluminum be? Note that the molar heat capacity C, in J/mol·°C, is related to the specific heat c, in J/kg°C, via the molar mass. Hint: You can compare your answer to the measured specific heat of aluminum, given on pg 1 of this exam.

6 d.o.f, as per lowest problem

$$\Delta E_{int} = \frac{6}{2} nR\Delta T$$

No appreciable volume change, so $Q = \Delta E_{int}$

$$Q = 3nR\Delta T$$

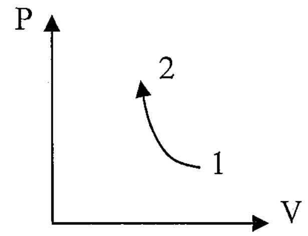
$$C_V = 3R\ \frac{\text{Joules}}{\text{mol}\cdot°C} \times \frac{1\ \text{mole}}{.02698\ kg} = \boxed{924.5\ \frac{J}{kg\,°C}}$$

Molar mass from front page of exam

= "c", now, the specific heat

c_{Al} = __________________ J/kg·°C

(6 pts) **Problem 7.** In the "cotton burner" demo, Wayne compressed a volume of air by a factor of about 10. That is, the final volume was 1/10 of the original volume. Treating air as an ideal diatomic gas, find the final temperature of the gas (assuming it started at 300K).



$\gamma = 7/5$

$$P_1 V_1^{\gamma} = P_2 V_2^{\gamma}$$

$$\left(\frac{\cancel{nR}T_1}{V_1}\right) V_1^{\gamma} = \left(\frac{\cancel{nR}T_2}{V_2}\right) V_2^{\gamma}$$

$$T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$$

$$T_2 = T_1 \left(\frac{V_1}{V_2}\right)^{\gamma-1}$$

$$= 300\text{K} \, (10)^{0.4}$$

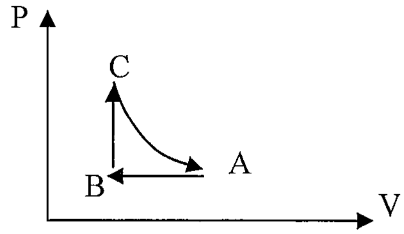
$$= \boxed{753.6\ \text{K}}$$

$T_f =$ ______________ K

$\to C_v = \frac{5}{2}R,\ C_p = \frac{7}{2}R,\ \gamma = 7/5$

(12 pts) **Problem 8.** An engine using 0.0401 moles of a **diatomic** ideal gas is driven by this cycle: starting from state A, the gas is cooled at constant pressure until it reaches state B. Then, ~~the~~ gas is heated at constant volume until it reaches state C. Finally, the gas is expanded isothermally back to the original state. The pressures, volumes, and temperature of all three states are given in the table.

	P (kPa)	V (m^3)	T (K)
A	100	0.003	900
B	100	0.001	300
C	300	0.001	900



(a) Find the heat added to the gas during each of the three legs.

A-B constant pressure

$$Q = nC_p\Delta T$$
$$= (.0401)\left(\tfrac{7}{2}\cdot 8.31\right)(300-900) = \underline{-700\ \text{J}}$$

B-C constant volume

$$Q = nC_v\Delta T$$
$$= (.0401)\left(\tfrac{5}{2}\cdot 8.31\right)(900-300) = \underline{500\ \text{J}}$$

C-A isothermal

$\Delta E_{int} = Q + W_{on} \to \Delta E_{int} = 0$ so $Q = -W_{on} = W_{by}$

$Q = nRT \ln V_f/V_0$ (work done for isothermal, obtained by $\int P\,dV$)

$$= (.0401)(8.31)(900)\ln 3 = \underline{329.5\ \text{J}}$$

(b) How much net work is done by the gas each cycle?

$Q_h = 500 + 329.5 = 829.5\ \text{J}$

$|Q_c| = 700\ \text{J}$

$W_{net} = Q_h - |Q_c| = \boxed{129.5\ \text{J}}$

(c) What is the efficiency of the engine?

$$e = \frac{W_{net}}{Q_h} = \frac{129.5}{829.5} = \boxed{15.6\%}$$

(d) What is the maximum theoretical efficiency for an engine operating between the same minimum and maximum temperatures?

$$e_{max} = 1 - T_c/T_h = 1 - \frac{300}{900} = \boxed{66.7\%}$$

(10 pts) **Problem 9**. (a) Given the same gas and the same three states as in the last problem, calculate the entropy change of the gas in the isothermal change from C to A.

C→A: $\Delta S = \int \frac{dQ}{T} = \frac{1}{T}\int dQ = \frac{Q}{T}$ ← Q_{CA} found in last problem

↖ take out of integral since T = constant

$$\Delta S = \frac{329.5\ J}{900\ K} = \boxed{.366\ J/K}$$

$\Delta S_{C\text{-}A}$ = __________________

(b) Calculate the entropy change from C to B, and then from B to A, then add them together and show you get the same ΔS as found in part (a). (Note that C-B and B-A are opposite the direction used by the gas in the previous problem; otherwise you'd get an answer that was the negative of the answer found in part (a) instead of the same as that answer.)

C to B: constant volume

$$\Delta S = \int \frac{dQ}{T} = \int \frac{nC_v\,dT}{T} = nC_v \ln T_B/T_C$$

$$= (.0401)(\tfrac{5}{2}\cdot 8.31)\ln(\tfrac{1}{3}) = \boxed{-.915\ J/K}$$

B to A: constant pressure

$$\Delta S = \int \frac{dQ}{T} = \int \frac{nC_p\,dT}{T} = nC_p \ln \frac{T_A}{T_B}$$

$$= (.0401)(\tfrac{7}{2}\cdot 8.31)\ln(3) = \boxed{1.281\ J/K}$$

Add together: $-.915 + 1.281 = \underline{.366\ J/K}$ ✓

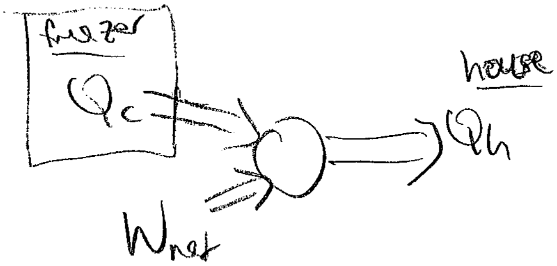
Yes, same as part (a)!

$\Delta S_{C\text{-}B}$ = __________________

$\Delta S_{B\text{-}A}$ = __________________

268K 296 K

(8 pts) **Problem 10.** (a) Suppose you want to keep the inside of your freezer at a temperature of –5°C when your house is at 23°C. What is the maximum possible coefficient of performance for a refrigerator operating between those two temperatures?



$$COP = \frac{Q_c}{W_{net}} = \frac{Q_c}{Q_h - Q_c}$$

$$COP_{max} = \frac{T_c}{T_h - T_c} = \frac{268}{296-268} = \boxed{9.57}$$

COP_{max} = ___________

(b) If 400 J of heat leak from the environment into your freezer each second, what is the minimum theoretical power that your freezer will consume to keep the temperature inside the freezer at –5°C.

$$400\,\frac{J}{s} = \frac{Q_c}{time}$$

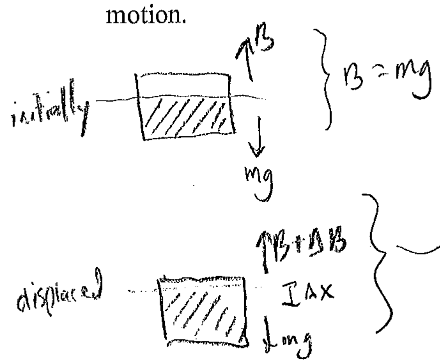
$$W_{net} = \frac{Q_c}{COP}$$

$$P = \frac{W_{net}}{time} = \frac{Q_c/time}{COP} = \frac{400}{9.57} = \boxed{41.8\,W}$$

P_{min} = _____________ W

(5 pts, no partial credit) **Extra Credit.** You may pick one of the following extra credit problems to do. (If you work more than one, only the first one will be graded.)

(a) A wooden cube, side length x and density ρ ($\rho < \rho_{water}$), bobs up and down in the water in simple harmonic motion. While bobbing, somehow it always continues to sit square in the water. In terms of x, ρ, ρ_{water}, and g, find the period of the harmonic motion.



$\Sigma F = B + \Delta B - mg$

$= \cancel{B+} \rho_W (\Delta V) g \cancel{- mg}$

$= \rho_W (x^2 \Delta x) g$

$= \text{constant} \times \Delta x$

"spring constant" $= \rho_W x^2 g$

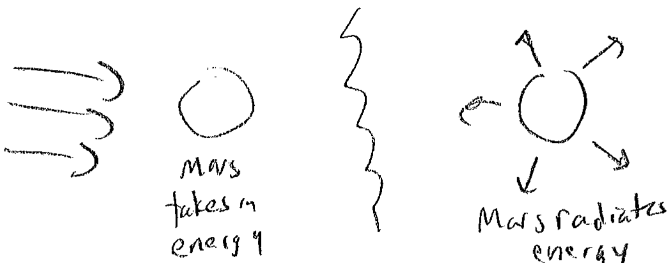
Then $T = 2\pi\sqrt{\dfrac{m}{K}}$

$T = 2\pi\sqrt{\dfrac{m}{\rho_W x^2 g}}$

$= 2\pi\sqrt{\dfrac{\rho \cdot x^3}{\rho_W x^2 g}}$

$= \boxed{2\pi\sqrt{\dfrac{\rho x}{\rho_W g}}}$

(b) Make an estimate of the surface temperature of Mars the same way you did for Earth in HW problem 7-4: The light from the sun reaches Mars's orbit with an intensity of 580 W/m^2. The radius of Mars is 3390 km. Wikipedia says the actual surface temperature varies between –87°C in the winter to –5°C in the summer; you should get an answer in between those two temperatures.



$P_{in} = \pi R^2 \times 580$

$P_{out} = e\sigma(4\pi R^2)T^4$

$e \approx 1$, $\sigma = 5.67 \cdot 10^{-8}$

Set equal for steady-state

$\cancel{\pi R^2}\, 580 = (5.67 \cdot 10^{-8})\, 4 \cancel{\pi R^2}\, T^4$

$T = \left(\dfrac{580}{5.67 \cdot 10^{-8} \cdot 4}\right)^{1/4} = \boxed{224.9\,K}$

$= \boxed{-48°C}$

almost exactly in middle of given temps!

(c) 400 g of ice at –20°C is added to 50 g of steam at 160°C in a thermally-insulated container. Ignoring the heat capacity of the container itself, find the final temperature of the combination. If it turns out that not all the ice melts, or not all the steam condenses, then give the amount of ice that melts (or steam that condenses) instead.

$Q_{\text{gained by ice}} = Q_{\text{lost by steam}}$

1) Assume all ice melts.

$m_i c_i (0 - -20) + m_i L_{melt} + m_i c_W (T_f - 0) = m_s c_{st} (160 - 100) + m_s L_{condense} + m_s c_W (100 - T_f)$

$(.4)(2090)(20) + .4(333000) + (.4)(4186)(T_f) = (.05)(2010)(60) + (.05)(2.26 \cdot 10^6) + (.05)(4186)(100 - T_f)$

$16720 + 133200 + 1674.4\, T_f = 6030 + 113000 + 20930 - 209.3\, T_f$

T_f = negative → not possible!

conclude: not all ice melts and final temp is 0°C

2) Amount of ice that melts is "m"

$(.4)(2090)(20) + m(333000) = (.05)(2010)(60) + (.05)(2.26 \cdot 10^6) + (.05)(4186)(100)$

$m = \boxed{.37\text{ kg}}$ → possible! (since we started out with 0.4 kg)

Assumption was correct.