

Physics 471 Exam 1 Winter 2008
 Instructor: John Colton

Name: Solutions

I promise I do not have any "illegal" constants/formulas stored in my calculator:
 (signed) _____

Instructions: Closed book. 3 hour time limit, 1% penalty per minute over. **Show your work.**
 All calculators permitted.

Formulas:

$$\omega_p = \frac{Nq^2}{m\epsilon_0}$$

$$\text{Lorentz model: } \vec{r}_o = \frac{q_e}{m_e} \frac{\vec{E}_o}{\omega_o^2 - \omega^2 - i\omega\gamma} \text{ and } \chi = \frac{\omega_p^2}{\omega_o^2 - \omega^2 - i\omega\gamma}$$

$$\text{Metals: } \chi = \frac{\omega_p^2}{-\omega^2 - i\omega\gamma}$$

$$\text{Fresnel Eqns: } \alpha = \frac{\cos\theta_2}{\cos\theta_1}, \beta = \frac{n_2}{n_1}$$

$$\text{(p-polarization) } r = \frac{\alpha - \beta}{\alpha + \beta}, t = \frac{2}{\alpha + \beta}$$

$$\text{(s-polarization) } r = \frac{1 - \alpha\beta}{1 + \alpha\beta}, t = \frac{2}{1 + \alpha\beta}$$

Jones vectors

$$\text{general, standard form: } \begin{pmatrix} A \\ Be^{i\delta} \end{pmatrix}$$

Jones matrices

$$\text{linear pol: } \begin{pmatrix} \cos^2\theta & \sin\theta\cos\theta \\ \sin\theta\cos\theta & \sin^2\theta \end{pmatrix}$$

$$\lambda/4: \begin{pmatrix} \cos^2\theta + i\sin^2\theta & \sin\theta\cos\theta - i\sin\theta\cos\theta \\ \sin\theta\cos\theta - i\sin\theta\cos\theta & \sin^2\theta + i\cos^2\theta \end{pmatrix}$$

$$\lambda/2: \begin{pmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{pmatrix}$$

True/False. Please circle the correct answer. (2 pts each)

1. T of F: The real part of the refractive index cannot be less than one. *see eg Fig 2.4*
2. T or F: *s*-polarized and *p*-polarized light experience different phase shifts upon reflection from a material with a complex index of refraction. *The Fresnel coefficient eqns are different*
3. T of F: When light is incident upon a material interface at Brewster's angle, only one polarization can transmit. *Only one reflects, but they both transmit.*
4. T of F: Faraday's Law is a special case of Ampere's Law.
5. T of F: The Fresnel coefficients, r and t , must add up to 1. *R and T do, but not r and t.*

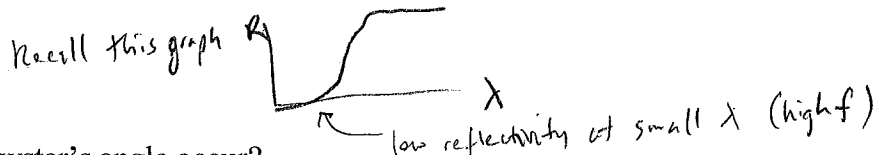
Multiple Choice. Please circle the letter of the correct answer for full credit (2 pts each).

6. In the Lorentz model, if ω is below resonance and you decrease ω , n will _____ and κ will _____.

- a. increase, increase
- b. decrease, decrease *see Fig 2.4a*
- c. increase, decrease
- d. decrease, increase
- e. stay the same, stay the same

7. In a conductor, the most transparent region is likely to be at _____ frequencies

- a. low
- b. high
- c. intermediate



8. For which polarization does Brewster's angle occur?

- a. p-polarization
- b. s-polarization
- c. both

9. For which polarization does total internal reflection occur?

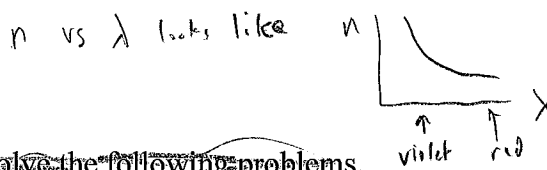
- a. p-polarization
- b. s-polarization
- c. both

10. In a material with electrons that have different resonant frequencies, the strength of each transition's contribution to the optical properties is called the _____ strength.

- a. frequency
- b. mixed
- c. oscillator
- d. transition

11. In transparent glass, which travels faster: red ($\lambda=650$ nm) or violet light ($\lambda=400$ nm)?

- a. red light
- b. violet light
- c. both travel at the same speed



smaller n \rightarrow faster v

~~Problems~~ Please answer the following questions/solve the following problems.

Problems. Please answer the following questions/solve the following problems.

- 5
 12. (8 pts) (a) Write down the four fundamental Maxwell equations. (b) Which of the four are typically modified for use inside materials? (No need to actually modify them, just specify the ones that would be modified.)

$$\begin{aligned} \nabla \cdot \vec{E} &= \rho / \epsilon_0 \leftarrow \\ \nabla \times \vec{E} &= -\frac{\partial \vec{B}}{\partial t} \\ \nabla \cdot \vec{B} &= 0 \\ \nabla \times \vec{B} &= \mu_0 \vec{J} + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t} \leftarrow \end{aligned}$$

the two that are modified for use inside materials

13. (5 pts) Use complex number techniques to find the amplitude and phase of:

$$f(t) = 2 \cos \omega t + 3 \cos(\omega t + 30^\circ)$$

$$= 2 \angle 0^\circ + 3 \angle 30^\circ$$

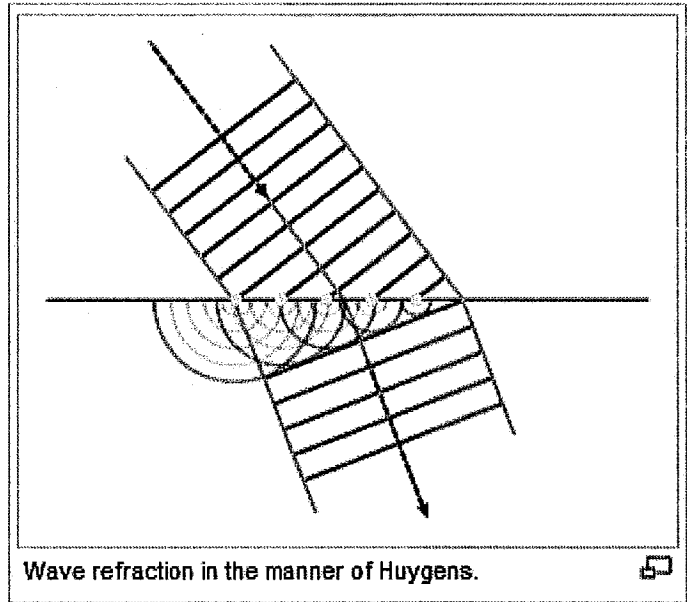
$$= 2 + 3 \cos 30^\circ + 3i \sin 30^\circ$$

$$= 4.5981 + 1.5i$$

$$= 4.8366 \angle 18.068^\circ$$

$$= \underline{\underline{4.8366 \cos(\omega t + 18.068^\circ)}}$$

14. (5 pts) In the Huygens' principle homework problem that used the figure to the right, why did I say/how did I know that "the green lines should be spaced slightly more closely together"? (The green lines, probably not green in the xeroxed picture, are on the lower right.)



Wave refraction in the manner of Huygens.

Two acceptable reasons

- (1) You can tell that the lower material has a higher index, because the ray bends towards \perp .
Higher $n \rightarrow$ smaller λ .
Smaller $\lambda \rightarrow$ wavefronts closer together

- (2) The smaller λ is also evident by the wavelength of the circular waves (ie that is smaller than the blue wavefronts' λ).
The green wavefronts should be spaced the same as the circular wavefronts.

15. (5 pts) A 10 mW continuously-on laser beam has a wavelength of 633 nm. Assume that the laser beam is 1 mm in diameter and has uniform intensity throughout the beam profile (the cross-section area). What are the magnitudes of the electric and magnetic fields present in the beam?

$$I = \langle S \rangle = \frac{1}{2} \frac{E_0^2}{\mu_0 c} \quad \left(\text{from } \vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B} \text{ and } |B| = \frac{|E|}{c}, \text{ time averaged} \right)$$

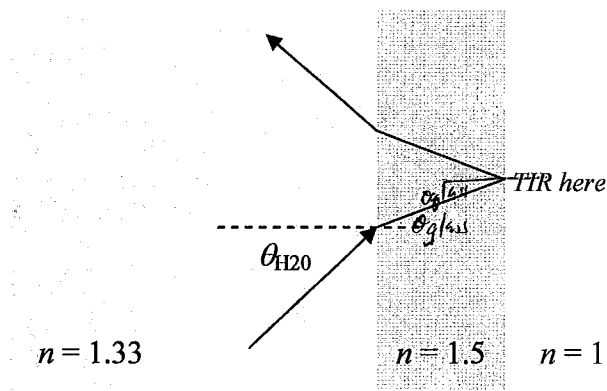
$$E_0 = \sqrt{2 I \mu_0 c}$$

$$= \sqrt{2 \left(\frac{0.01 \text{ W}}{\pi (0.0005 \text{ m})^2} \right) (4\pi \cdot 10^{-7}) (3 \cdot 10^8)}$$

$$= \boxed{3098.4 \text{ V/m}}$$

$$B_0 = \frac{E_0}{c} = \boxed{1.033 \cdot 10^{-5} \text{ T}}$$

- 12
16. (10 pts) In the Richards building there is a swimming pool with an underwater window so that it is possible to see into a side room from beneath the water. Consider total internal reflection which might occur when a swimmer attempts to look through the window. (a) Find the minimum angle θ_{H2O} for which TIR occurs at the glass-air surface. (b) How does this minimum angle compare to the case of a water-to-air interface with no glass in-between (e.g. looking upwards at the water's smooth surface)?



$$\text{TIR: } n_{\text{glass}} \sin \theta_{\text{glass}} = n_{\text{air}} \sin(90^\circ)$$

$$\sin \theta_{\text{glass}} = \frac{n_{\text{air}}}{n_{\text{glass}}}$$

(a) Snell's Law: $n_{H2O} \sin \theta_{H2O} = n_{\text{glass}} \sin \theta_{\text{glass}}$

$$n_{H2O} \sin \theta_{H2O} = n_{\text{glass}} \left(\frac{n_{\text{air}}}{n_{\text{glass}}} \right)$$

$$\sin \theta_{H2O} = \frac{n_{\text{air}}}{n_{H2O}}$$

$$\theta_{H2O} = \sin^{-1} \left(\frac{1}{1.33} \right) = 48.75^\circ$$

(b) exactly the same!

critical angle for water-air given by

$$n_{H2O} \sin \theta_{H2O} = n_{\text{air}} \sin(90^\circ)$$

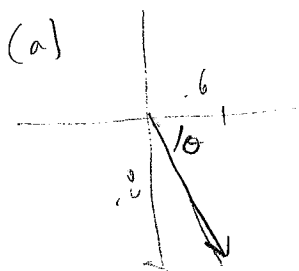
$$\theta_{H2O} = \sin^{-1} \left(\frac{n_{\text{air}}}{n_{H2O}} \right)$$

$$= 48.75^\circ$$

14

17. (12 pts) An initial light state is given by Jones vector: $\begin{pmatrix} 0.6 \\ -0.8 \end{pmatrix}$. (a) Describe the initial state

of the light in as much detail as reasonably possible. (b) The light strikes a quarter-waveplate with fast axis along the y-direction, and then a linear polarizer that transmits at -45 degrees from the +x axis. Find the Jones vector of the light that emerges from the linear polarizer, in standard form. (c) How much of the original intensity is transmitted?



linear polarized at $\theta = -\tan^{-1}\left(\frac{-0.8}{0.6}\right) = -53.13^\circ$

(b) final pol = $\begin{pmatrix} M_{-45^\circ} \end{pmatrix} \begin{pmatrix} M_{\text{quarter wave}} \end{pmatrix} \begin{pmatrix} 0.6 \\ -0.8 \end{pmatrix}$
 (use pg 1 formulas to get $M_{\text{quarter wave}}$)

$$= \begin{pmatrix} \frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} \end{pmatrix} \begin{pmatrix} i & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 0.6 \\ -0.8 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} \end{pmatrix} \begin{pmatrix} 0.6i \\ -0.8 \end{pmatrix}$$

$$= \begin{pmatrix} 0.3i + 0.4 \\ -0.3i - 0.4 \end{pmatrix}$$

$$= \begin{pmatrix} 0.5 \angle 36.87^\circ \\ 0.5 \angle 216.87^\circ \end{pmatrix}$$

$$= 0.5 \angle 36.87^\circ \begin{pmatrix} 1 \\ 1 \angle 180^\circ \end{pmatrix}$$

$$= 0.5 \angle 36.87^\circ \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

in standard form

$$36.87^\circ = 0.6435 \text{ rad}$$

(c) $\frac{I}{I_0} = (0.5)^2 [1^2 + 1^2]$ since $I \propto E^2$

$$= 0.5 \quad \text{50\%}$$

18. (16 pts) For the following, assume that E and B are complex plane waves that exist in a material with a complex k , $k = k_{real} + ik_{imag}$ (due to e.g. a complex index of refraction). You can assume the coordinate axes have been chosen such that the wave vector is in the z -direction and E_0 is in the x -direction. (a) Quickly derive the equation:

$$\tilde{E} = \tilde{E}_0 e^{-k_{imag}z} e^{i(k_{real}z - \omega t)} \hat{x}. \quad (b) \text{ Using one or more of Maxwell's equations and the previous}$$

result, show that $\tilde{B} = \frac{1}{\omega} (k_{real} + ik_{imag}) \tilde{E}_0 e^{-k_{imag}z} e^{i(k_{real}z - \omega t)} \hat{y}$. (c) How do the phases of E and B compare? Be as specific as possible.

$$(a) \quad \tilde{E} = \tilde{E}_0 e^{i(k \cdot r - \omega t)} \hat{x}$$

$$= \tilde{E}_0 e^{i[(k_{real} + ik_{imag})z - \omega t]} \hat{x}$$

$$\tilde{E} = \tilde{E}_0 e^{-k_{imag}z} e^{i(k_{real}z - \omega t)} \hat{x}$$

$$(b) \quad \nabla \times \tilde{E} = -\frac{\partial \tilde{B}}{\partial t}$$

$$\begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ \tilde{E}_x & 0 & 0 \end{vmatrix} = \omega \tilde{B} \hat{y}$$

$\frac{\partial \tilde{B}}{\partial t}$ will give you $(-i\omega) \tilde{B}$
if we assume \tilde{B} is a complex plane wave like \tilde{E}

$$\hat{y} \left(\frac{\partial \tilde{E}_x}{\partial z} \right) = \omega \tilde{B} \hat{y}$$

only non-zero term from curl

$$\hat{y} \left(i(k_{real} + ik_{imag}) \right) \tilde{E}_0 e^{-k_{imag}z} e^{i(k_{real}z - \omega t)} = \omega \tilde{B} \hat{y}$$

$$\tilde{B} = \frac{1}{\omega} (k_{real} + ik_{imag}) \tilde{E}_0 e^{-k_{imag}z} e^{i(k_{real}z - \omega t)} \hat{y}$$

$$(c) \quad \tilde{B}_0 = \frac{1}{\omega} \tilde{k} \tilde{E}_0 \quad \text{in magnitude (different directions)}$$

therefore if \tilde{k} is complex, \tilde{B}_0 will have different phase than \tilde{E}_0

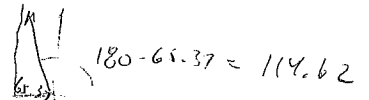
$$\text{let } \tilde{k} = |\tilde{k}| e^{i\phi} \quad \text{where } \phi = \tan^{-1} \left(\frac{k_{imag}}{k_{real}} \right) \quad (\text{assuming not a strange quadrant})$$

$$\text{then } \tilde{B}_0 = \frac{|\tilde{k}|}{\omega} e^{i\phi} \tilde{E}_0$$

$$\text{phase of } \tilde{B}_0 = \text{phase of } \tilde{E}_0 + \phi = \tan^{-1} \left(\frac{k_{imag}}{k_{real}} \right)$$

19. (16 pts) At a certain frequency a material has $n = 1.5$ and $\kappa = 2$. (a) Find the complex susceptibility χ . (b) Find the phase of the polarization, \vec{P} , relative to \vec{E} . (c) If the frequency is $\omega = 3 \times 10^{14}$ rad/sec, what is the absorption depth? (the distance light penetrates before the intensity drops by a factor of $1/e$; assume normal incidence) (d) If p-polarized light strikes the material from air with $\theta_i = 30^\circ$, what is the phase shift upon reflection, and what fraction of the intensity is reflected?

(a) $\tilde{n}^2 = 1 + \tilde{\chi} \rightarrow \tilde{\chi} = \tilde{n}^2 - 1 = (1.5 + 2i)^2 - 1$
 $= 2.25 + 6i - 4 - 1$
 $= \boxed{-2.75 + 6i}$
 $= 6.6062 \angle 114.62^\circ$



(b) $\vec{P} = \epsilon_0 \tilde{\chi} \vec{E} \rightarrow \vec{P} \text{ phase is } \vec{E} \text{ phase} + \tilde{\chi} \text{ phase}$
 $\vec{P} \text{ phase is } \vec{E} \text{ phase} + 114.62^\circ$

(c) $\alpha = 2k \text{imag}$ $\alpha = \text{rate of intensity decay}$

absorption depth = $1/\alpha$

because
 $I \propto E^2$

$\frac{\omega}{k} = \frac{c}{n} \rightarrow k = \frac{\omega}{c} \tilde{n}$

$k \text{imag} = \frac{3 \cdot 10^{14}}{2 \cdot 10^8} \cdot 2 \text{imag} = 2 \cdot 10^6 \text{ m}^{-1}$

abs. depth = $\frac{1}{2(2 \cdot 10^6)} = \boxed{.25 \mu\text{m}}$

(d) $n_1 \sin \theta_1 = n_2 \sin \theta_2$
 $1 \sin 30^\circ = (1.5 + 2i) \sin \theta_2$

$\theta_2 = 11.88 - 1.604i$

$\cos \theta_2 = 1.00577 + 0.019090i$

$r = \frac{\cos \theta_2}{\cos \theta_1} = \frac{1.00577 + 0.019090i}{\cos 30^\circ}$

$r = \boxed{1.1614 + 0.22046i}$

$r_p = \frac{n_2}{n_1} = 1.5 + 2i$

$r = \frac{r_p}{r_s} = \frac{-0.43268 - 0.40991i}{.600392 \angle 136.912^\circ}$

phase shift = -136.912°

$R = |r|^2 = (.600392)^2 = .360471$

36.0471% of intensity is reflected