Final Exam

Physics 471 ~~Exam 2~~ Winter 2008
Instructor: John Colton

Barcode Here

Name: Solutions

I promise I do not have any "illegal" constants/formulas stored in my calculator:

(signed) ______________________

Instructions: Closed book. 3 hour time limit, 1% penalty per minute over. Calculators permitted. **Show your work.** Include units where appropriate. If additional space is needed, you may use the backs of pages.

Formulas:

$$\omega_p = \frac{Nq^2}{m\varepsilon_0}$$

Lorentz model: $\vec{r}_o = \dfrac{q_e}{m_e}\dfrac{\vec{E}_o}{\omega_o^2 - \omega^2 - i\omega\gamma}$ and

$$\chi = \frac{\omega_p^{\ 2}}{\omega_0^{\ 2} - \omega^2 - i\omega\gamma}$$

Metals: $\chi = \dfrac{\omega_p^{\ 2}}{-\omega^2 - i\omega\gamma}$

Fresnel Eqns: $\alpha = \dfrac{\cos\theta_2}{\cos\theta_1}$, $\beta = \dfrac{n_2}{n_1}$

(p-polarization) $r = \dfrac{\alpha - \beta}{\alpha + \beta}$, $t = \dfrac{2}{\alpha + \beta}$

(s-polarization) $r = \dfrac{1 - \alpha\beta}{1 + \alpha\beta}$, $t = \dfrac{2}{1 + \alpha\beta}$

Jones vectors

general, standard form: $\begin{pmatrix} A \\ Be^{i\delta} \end{pmatrix}$

RCP: $\dfrac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ e^{i(-90^\circ)} \end{pmatrix}$

LCP: $\dfrac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ e^{i(+90^\circ)} \end{pmatrix}$

Jones matrices

linear pol: $\begin{pmatrix} \cos^2\theta & \sin\theta\cos\theta \\ \sin\theta\cos\theta & \sin^2\theta \end{pmatrix}$

$\lambda/4$:

$$\begin{pmatrix} \cos^2\theta + i\sin^2\theta & \sin\theta\cos\theta - i\sin\theta\cos\theta \\ \sin\theta\cos\theta - i\sin\theta\cos\theta & \sin^2\theta + i\cos^2\theta \end{pmatrix}$$

$\lambda/2$: $\begin{pmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{pmatrix}$

$$\frac{1}{n^2} = \frac{u_x^{\ 2}}{n^2 - n_x^{\ 2}} + \frac{u_y^{\ 2}}{n^2 - n_y^{\ 2}} + \frac{u_z^{\ 2}}{n^2 - n_z^{\ 2}}$$

Uniaxial, optic axis $\perp$ to surface:

$$n = n_o, \frac{n_o n_e}{\sqrt{n_o^{\ 2}\sin^2\theta_2 + n_e^{\ 2}\cos^2\theta_2}}$$

$\tan\theta_2 = \dfrac{n_e}{n_o}\dfrac{\sin\theta_1}{\sqrt{n_e^{\ 2} - \sin^2\theta_1}}$ (p-polar)

$\tan\phi' = \dfrac{n_o}{n_e}\dfrac{\sin\theta_1}{\sqrt{n_e^{\ 2} - \sin^2\theta_1}}$ (p-polar)

Two interfaces:

$$t_{13} = \frac{t_{12}t_{23}}{e^{-ik_2 d\cos\theta_2} - r_{21}r_{23}e^{ik_2 d\cos\theta_2}}$$

$$T_{13} = \frac{n_3}{n_1}\frac{\cos\theta_3}{\cos\theta_1}|t_{13}|^2 = \frac{T_{\max}}{1 + F\sin^2\frac{\Phi}{2}}$$

$$T_{\max} = \frac{n_3}{n_1}\frac{\cos\theta_3}{\cos\theta_1}\frac{|t_{12}|^2|t_{23}|^2}{(1 - |r_{21}||r_{23}|)^2} = \frac{T_{12}T_{23}}{\left(1 - \sqrt{R_{21}}\sqrt{R_{23}}\right)^2}$$

$$F=\frac{4|r_{21}||r_{23}|}{\left(1-|r_{21}||r_{23}|\right)^2}$$

$$\Phi=2k_2 d\cos\theta_2+\delta_{21}+\delta_{23}$$

$$\Delta\Phi_{FWMH}=\frac{4}{\sqrt{F}}$$

$$\Delta\lambda_{FWMH}=\frac{\lambda^2}{\pi n_2 d\cos\theta_2\sqrt{F}}$$

$$\Delta\lambda_{FSR}=\frac{\lambda^2}{2n_2 d\cos\theta_2}$$

Multilayers:

$t_{13}=1/a_{11}$

$r=a_{21}/a_{11}$

$\beta_j=k_j l_j\cos\theta_j$

p-polar:

$$M_j=\begin{pmatrix}\cos\beta_j & \dfrac{-i\sin\beta_j\cos\theta_j}{n_j}\\ \dfrac{-in_j\sin\beta_j}{\cos\theta_j} & \cos\beta_j\end{pmatrix}$$

$$A=\frac{1}{2n_0\cos\theta_0}\begin{pmatrix}n_0 & \cos\theta_0\\ n_0 & -\cos\theta_0\end{pmatrix}\left(\prod_{j=1}^{N}M_j\right)\begin{pmatrix}\cos\theta_{N+1} & 0\\ n_{N+1} & 0\end{pmatrix}$$

s-polar:

$$M_j=\begin{pmatrix}\cos\beta_j & \dfrac{-i\sin\beta_j}{n_j\cos\theta_j}\\ -in_j\cos\theta_j\sin\beta_j & \cos\beta_j\end{pmatrix}$$

$$A=\frac{1}{2n_0\cos\theta_0}\begin{pmatrix}n_0\cos\theta_0 & 1\\ n_0\cos\theta_0 & -1\end{pmatrix}\left(\prod_{j=1}^{N}M_j\right)\begin{pmatrix}1 & 0\\ n_{N+1}\cos\theta_{N+1} & 0\end{pmatrix}$$

Linear dispersion:

$$v_g\approx\left(\frac{d}{d\omega}\mathrm{Re}(k)\Big|_{\omega=\omega_0}\right)^{-1}$$

$$t'\approx\frac{d}{d\omega}\mathrm{Re}(\vec{k})\Big|_{\omega=\omega_0}\cdot\Delta\vec{r}$$

$$I\propto e^{-2\vec{k}_{imag}(\omega_0)\cdot\Delta\vec{r}}\left|E(t-t',\vec{r}_0)\right|^2$$

Quadratic dispersion:

$$k=k_0+\frac{1}{v_g}(\omega-\omega_0)+\alpha(\omega-\omega_0)^2$$

$$\frac{1}{v_g}=\frac{1}{c}(n'\omega+n)\Big|_{\omega=\omega_0}$$

$$\alpha=\frac{1}{2c}(n''\omega+2n')\Big|_{\omega=\omega_0}$$

Gaussian wavepacket, through thickness z:

$\Phi=2\alpha z$

$T=\tau\sqrt{1+\Phi^2}$

$$E(t,z)=\frac{E_0e^{i(kz-\omega_0t)}}{\left(1+\Phi^2\right)^{1/4}}e^{\frac{i}{2}\tan^{-1}\Phi-\frac{i\Phi}{2T^2}\left(t-\frac{z}{v_g}\right)^2}e^{-\frac{1}{2T^2}\left(t-\frac{z}{v_g}\right)^2}$$

Michelson:

$I_{det}(\tau)=2I_0(1+\cos\omega\tau)$ (single ω)

Band of ω's:

$$\varepsilon_0=\int_{-\infty}^{\infty}I(\omega)d\omega$$

$$\gamma(\tau)=\frac{1}{\varepsilon_0}\int_{-\infty}^{\infty}I(\omega)e^{-i\omega\tau}d\omega$$

$$\int_{-\infty}^{\infty}I_{det}(\tau)dt=2\varepsilon_0\left(1+\mathrm{Re}\,\gamma(\tau)\right)$$

$$\langle I_{det}(\tau)\rangle=2\langle I\rangle\left(1+\mathrm{Re}\,\gamma(\tau)\right)$$

$$\frac{FT(NormSig)}{\sqrt{2\pi}}=2\varepsilon_0\delta(\omega)+I(\omega)+I(-\omega)$$

Young:

$I_{det}(h)=2I_0\left(1+\cos\left(\frac{kyh}{D}+\Delta\phi\right)\right)$ (pt source)

Extended source:

$$\varepsilon_0=\int_{-\infty}^{\infty}I(y')dy'$$

$$\gamma(h)=\frac{e^{-\frac{ikyh}{D}}}{\varepsilon_0}\int_{-\infty}^{\infty}I(y')e^{-\frac{ikhy'}{R}}dy'$$

$$\langle I_{det}(h)\rangle=2\langle I_{oneslit}\rangle\left(1+\mathrm{Re}\,\gamma(h)\right)$$

Rays: $\nabla R(\vec{r})=n(\vec{r})\hat{s}(\vec{r})$

ABCD Matrices:

Translation $\begin{pmatrix}1 & d\\ 0 & 1\end{pmatrix}$

Flat surface refraction $\begin{pmatrix}1 & 0\\ 0 & \frac{n_1}{n_2}\end{pmatrix}$

Curved surface refraction $\begin{pmatrix}1 & 0\\ \frac{1}{R}\left(\frac{n_1}{n_2}-1\right) & \frac{n_1}{n_2}\end{pmatrix}$

R = positive for convex, negative for concave

Spherical mirror/thin lens $\begin{pmatrix} 1 & 0 \\ -1/f & 1 \end{pmatrix}$

$$f_{lens} = \left[\left(\frac{n_2}{n_1}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)\right]^{-1}$$, R = positive for curving away; negative for curving towards

$$f_{mirror} = \frac{R}{2},$$ R = positive for concave

$p_1 = (1\text{–}D)/C$, $p_2 = (1\text{–}A)/C$

Cavity stability: $-1 < \dfrac{A+D}{2} < 1$

Diffraction formulas

Fresnel-Kirchoff:

$$E(x,y,z=d) = -\frac{i}{\lambda}\iint_{aperture} E(x',y',z=0)\frac{e^{ikR}}{R}dx'dy'$$

Fresnel:

$$E(x,y,d) = -\frac{ie^{ikd}e^{i\frac{k}{2d}(x^2+y^2)}}{\lambda d}\iint_{aperture} E(x',y',0)e^{i\frac{k}{2d}(x'^2+y'^2)}e^{-i\frac{k}{d}(xx'+yy')}dx'dy'$$

Fraunhofer:

$$E(x,y,d) = -\frac{ie^{ikd}e^{i\frac{k}{2d}(x^2+y^2)}}{\lambda d}\iint_{aperture} E(x',y',0)e^{-i\frac{k}{d}(xx'+yy')}dx'dy'$$

Fourier Transforms: (without factors of sqrt(2π))

Comb function (N total deltas): $\dfrac{\sin\left(\frac{N\omega t_0}{2}\right)}{\sin\left(\frac{\omega t_0}{2}\right)}$

1D

Single slit: $a\,\mathrm{sinc}(k_x a/2)$

Top hat: $\pi a^2\left(\dfrac{2J_1(k_\rho a)}{k_\rho a}\right)$

Spectrometer: $\lambda = \dfrac{xh}{md}$, $\Delta\lambda = \dfrac{\lambda}{mN}$

$$\theta_{min} = \frac{1.22\lambda}{l}$$

Gaussian Beams:

$$E(x,y,z) = E_0\frac{w_0}{w}e^{\frac{-\rho^2}{w^2}}e^{ikz+\frac{ik\rho^2}{2R}-i\tan^{-1}\left(\frac{z}{z_0}\right)}$$

with $z_0 = \dfrac{kw_0^{\,2}}{2}$, $R = z + \dfrac{z_0^{\,2}}{z}$, $w = w_0\sqrt{1+\dfrac{z^2}{z_0^{\,2}}}$

$$q_2 = \frac{Aq_1+B}{Cq_1+D};\ q = z + iz_0$$

Plank: $\rho_f = \dfrac{8\pi hf^3}{c^3\left(e^{hf/k_BT}-1\right)}$

$\sigma = 5.6696\cdot10^{-8}$ W/m²·K⁴

Integrals:

$$\int_{-\infty}^{\infty} e^{i\omega(t-t_0)}d\omega = 2\pi\delta(t-t_0)$$

$$\int_{-\infty}^{\infty} e^{-Ax^2+Bx+C}dx = \sqrt{\frac{\pi}{A}}\,e^{\frac{B^2}{4A}+C},\ \mathrm{Re}(A) > 0$$

Final Exam – 110 total points possible

True/False. Please circle the correct answer. (2 pts each)

1. T or F: T or F: When light is incident upon a material interface at Brewster's angle, only one polarization can transmit. only one can reflect, but both can transmit
2. T or F: Aside from a constant factor, the Fourier transform of a convolution is the convolution of the Fourier transforms of the individual functions. it's the product of the FTs
3. T or F: Spherical waves of the form $\frac{A}{r}\cos(kr-\omega t)$ are exact solutions to Maxwell's equations.
4. T or F: The resolving power of a spectrometer used in a particular diffraction order depends on the number of lines illuminated, but not on the wavelength or grating period.
5. T or F: The central peak of the Fraunhofer diffraction from two narrow slits separated by *a* (slit width << *a*) has the same width as the central diffraction peak from a single slit of width *a* (width of the central peak being measured from first zero on left to first zero on right).
   $\text{sinc}^2 x$ has twice the width of $\cos^2 x$ (around $x = 0$)
6. T or F: The function $J_0(x)$ crosses zero at π, 2π, 3π, etc.

Multiple Choice. Please circle the letter of the correct answer for full credit (2 pts each).

7. If sunlight is unpolarized coming from the sun, after it reflects off of a smooth surface when the sun is in front of (and above) you, it will probably be:
   a. more horizontally-polarized
   b. more vertically -polarized
   c. still unpolarized

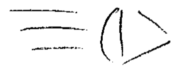
Anywhere close to Brewster's angle, p-polarized won't reflect much, so it will be more s-polarized than p.
S-polarized = horizontal to you.

8. In the analysis of a three-layer system, the electric field at the right side of the middle layer was connected to the electric field at the left side of the middle layer via:
   a. changing polarization
   b. the Fresnel coefficients
   c. a phase factor
   d. a rubber band

9. The approximation made to derive the eikonal equation was
   a. short wavelengths
   b. long wavelengths

10. "Optical path length" depends on something besides length. That other thing is:
    a. index of refraction
    b. angle of incidence
    c. frequency

11. If you want to focus a laser beam to a tight spot, spherical aberration can be partially corrected by placing a plano-convex lens:
    a. so the light strikes the flat side
    b. so the light strikes the curved side



12. Whose law says that the amount of emitted blackbody radiation is proportional to T^4?
    a. Curie & Weiss's
    b. Planck's
    c. Rayleigh & Jeans'
    d. Sommerfeldt's
    e. Stefan & Boltzmann's

13. Which of these processes was not considered in the "Einstein A and B coefficients" analysis?
    a. spontaneous absorption   Is there such a thing?
    b. stimulated absorption
    c. spontaneous emission
    d. stimulated emission

Problems. Please answer the following questions/solve the following problems.

14. (5 pts) At a certain frequency a material has $n = 3$ and $\kappa = 4$. (a) Find the complex susceptibility χ. (b) Find the phase of the polarization relative to the phase of the electric field.

(a) $\tilde{n} = \sqrt{\tilde{\epsilon}_r}$

$= \sqrt{1 + \tilde{\chi}_e}$

(see → if not memorized!)

$D = \epsilon_0 E + P$
$= \epsilon_0 E + \epsilon_0 \chi E$
$= \epsilon_0 (1 + \chi) E$
$= \epsilon_0 \epsilon_r E$

So $\tilde{n}^2 - 1 = \tilde{\chi}_e$

$\tilde{\chi}_e = (3 + 4i)^2 - 1$

$= 9 + 24i - 16 - 1$

$\boxed{\tilde{\chi}_e = -8 + 24i}$

(b) $\tilde{\chi}_e = 25.3\, e^{i108.4°}$ in polar form

$\tilde{P} = \epsilon_0 \tilde{\chi}_e \tilde{E}$

$\tilde{P} = \epsilon_0 \cdot 25.3 \left(e^{i108.4°}\right) \tilde{E}$

$\boxed{\text{phase of } \tilde{P} = \text{phase of } \tilde{E} + 108.4°}$

15. (8 pts) Suppose you have a laser that is vertically polarized. What optical element could you use to turn it into a right circularly polarized laser beam? Be as specific as possible.

From experience and/or class discussion I know that a quarter wave plate is needed, at either 45° or −45°

But which one?

Initial Jones vector = $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$

$\frac{1}{4}\lambda$ at $+45°$ matrix = $\begin{pmatrix} \frac{1}{2}+\frac{1}{2}i & \frac{1}{2}-\frac{1}{2}i \\ \frac{1}{2}-\frac{1}{2}i & \frac{1}{2}+\frac{1}{2}i \end{pmatrix}$

Light after $\frac{1}{4}\lambda$ at $+45°$ = $\begin{pmatrix} & \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix}$

$= \begin{pmatrix} \frac{1}{2}-\frac{1}{2}i \\ \frac{1}{2}+\frac{1}{2}i \end{pmatrix} = C\begin{pmatrix} e^{i(-45°)} \\ e^{i45°} \end{pmatrix}$

$= C e^{i(-45°)} \begin{pmatrix} 1 \\ e^{i90°} \end{pmatrix}$

$= \underline{\underline{LCP}}$

$\frac{1}{4}\lambda$ at $-45°$ matrix = $\begin{pmatrix} \frac{1}{2}+\frac{1}{2}i & -\frac{1}{2}+\frac{1}{2}i \\ -\frac{1}{2}+\frac{1}{2}i & \frac{1}{2}+\frac{1}{2}i \end{pmatrix}$

Light after $\frac{1}{4}\lambda$ at $-45°$ = $\begin{pmatrix} & \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix}$

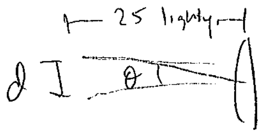
$= \begin{pmatrix} -\frac{1}{2}+\frac{1}{2}i \\ \frac{1}{2}+\frac{1}{2}i \end{pmatrix} = C\begin{pmatrix} e^{i135°} \\ e^{i45°} \end{pmatrix}$

$= C e^{i135°} \begin{pmatrix} 1 \\ e^{i(-90°)} \end{pmatrix}$

$= \underline{\underline{RCP}}$ ✓

Need $\boxed{\frac{1}{4}\lambda \text{ Waveplate at } -45°}$ to get RCP

16. (5 pts) The first lens of a telescope has a diameter of 30 cm, which is the only place where light is clipped. You wish to use the telescope to examine two stars in a binary system. The stars are approximately 25 light-years away. (One light-year is $9.4605 \cdot 10^{15}$ m.) How far apart need the stars be (in the perpendicular sense) for you to distinguish them in the visible range of $\lambda = 500$ nm?



Rayleigh: $\theta_{min} = \frac{1.22\lambda}{d}$ also $= \frac{d}{(25 \cdot 9.4605 \cdot 10^{15})m}$ by figure $\sin\theta \simeq \tan\theta \simeq \theta$

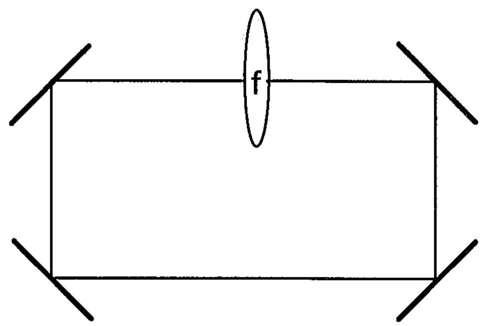
$$d = \frac{(25 \cdot 9.4605 \cdot 10^{15})(1.22)(500 \cdot 10^{-9})}{.3}$$

$$\boxed{d = 4.809 \cdot 10^{11} \text{ m}}$$

17. (6 pts) A laser cavity is formed with four flat mirrors and a lens of focal length f (see figure). Let the full path around the cavity be L.

   a. What is the round-trip ABCD matrix for the cavity? Please start by having the light first go through the lens.

   b. What are the possible values for L if the cavity is to be stable? Let the focal length of the lens be 1 meter.



Important: the mirrors basically do nothing $\begin{pmatrix} A & B \\ C & D \end{pmatrix}$ = identity

(a) $\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} \text{distance} \\ L \end{pmatrix} \begin{pmatrix} \text{lens} \end{pmatrix}$

$$= \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -1/f & 1 \end{pmatrix} = \boxed{\begin{pmatrix} 1 - L/f & L \\ -1/f & 1 \end{pmatrix}}$$

(b) stable. $-1 < \frac{A+D}{2} < 1$

$-1 < \frac{2 - L}{2} < 1$ (if $f = 1$)

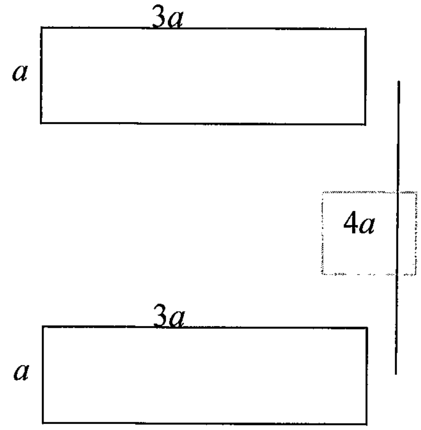
$-1 < 1 - \frac{L}{2}$ and $1 - \frac{L}{2} < 1$

$\frac{L}{2} < 2$ $\quad 0 < \frac{L}{2}$

$L < 4$ $\quad L > 0$

$$\boxed{0 < L < 4\text{m}}$$

18. (6 pts) Derive the Fraunhofer intensity pattern for the two identical rectangular apertures as shown at the right, whose centers are separated by $4a$. Put your answer in terms of I_0, the intensity at the center of the pattern. *Hint*: you can start with the FT of a single slit aperture function.



1 slit width a : $FT = a \operatorname{sinc} \frac{k_x a}{2}$

1 rectangle as shown : $FT = 3a^2 \operatorname{sinc} \frac{3k_x a}{2} \operatorname{sinc} \frac{k_y a}{2}$

convolved with 2 delta functions $\left(\begin{smallmatrix}\uparrow\\\uparrow\end{smallmatrix}\right) \to FT\left(\begin{smallmatrix}\uparrow\\\uparrow\end{smallmatrix}\right) = \int \left(\begin{smallmatrix}\uparrow\\\uparrow\end{smallmatrix}\right) e^{iky}\,dy$

$$= e^{ik_y 2a} + e^{-ik_y 2a} = 2\cos 2k_y a$$

FT of total aperture $= FT(\square) \cdot FT\left(\begin{smallmatrix}\uparrow\\\uparrow\end{smallmatrix}\right)$

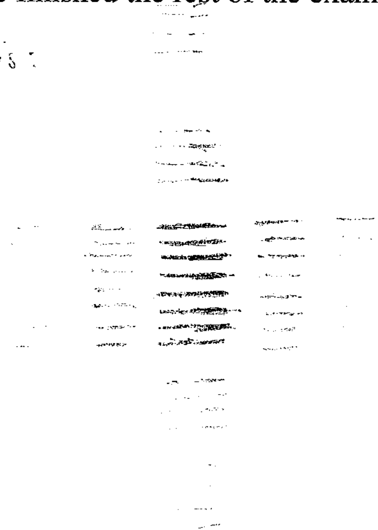
$$= C \operatorname{sinc} \frac{3k_x a}{2d} \operatorname{sinc} \frac{k_y a}{2d} \cos \frac{2k_y a}{d}$$

$I \sim |FT|^2$

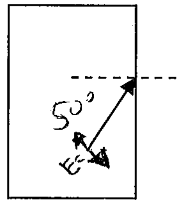
$$\boxed{I = I_0 \operatorname{sinc}^2\left(\frac{3k_x a}{2d}\right) \operatorname{sinc}^2\left(\frac{k_y a}{2d}\right) \cos^2\left(\frac{2k_y a}{d}\right)}$$

Extra Credit (3 pts) Use your Fourier transform intuition and/or an analysis of your above answer to sketch what the Fraunhofer pattern would look like. *Advice*: It's probably wisest to save this for after you've finished the rest of the exam.

something like this:



19. (14 pts) p-polarized light travelling inside a diamond ($n = 2.4$) strikes a surface going to air ($n = 1$) at an angle of 50° with respect to the perpendicular. (a) Draw the direction of the light's oscillating electric field on the picture on the paper. (b) Find the reflection and transmission coefficients, r and t, as well as the reflectance and transmittance, R and T.



P-polarized

$r = \frac{\alpha - \beta}{\alpha + \beta}$

$\alpha = \frac{\cos\theta_2}{\cos\theta_1} = \frac{1.5428i}{\cos 50^\circ} = 2.4001i$

$\beta = \frac{n_2}{n_1} = \frac{1}{2.4} = .41667$

Snell: $n_1 \sin\theta_1 = n_2 \sin\theta_2$

$2.4 \sin 50^\circ = 1 \sin\theta_2$

$\sin\theta_2 = 1.8385$

$\rightarrow \cos\theta_2 = \sqrt{1 - \sin^2\theta_2}$

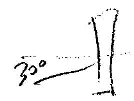
$= 1.5428i$

$r = \frac{2.4001i - .41667}{2.4001i + .41667} = \boxed{.9415 + .3371i = 1e^{i\,19.70^\circ}}$

$R = |r|^2 = \boxed{1}$

$T = 1 - R = \boxed{0}$

$t = \frac{2}{\alpha + \beta} = \frac{2}{2.4001i + .41667} = \boxed{.1404 - .8089i = .8210e^{i(-80.15^\circ)}}$



$n_1 \sin\theta_1 = n_2 \sin\theta_2$

$1 \sin 30° = 1.8 \sin\theta_2$

$\theta_2 = 16.13°$

20. (14 pts) A thin glass film is suspended in air, with index $n = 1.8$ and thickness w. It has s-polarized 633 nm light incident on it at an angle of 30° from the perpendicular.
 a. What is the smallest non-zero thickness w that will give a maximum in the transmission?
 b. Evaluate T_{13} for $w = 0.5$ µm.

$$T_{13} = \frac{T_{max}}{1 + F \sin^2\frac{\Phi}{2}} \qquad T_{max} = \frac{T_{12}T_{23}}{\left(1+\sqrt{R_{21}R_{23}}\right)^2} = 1$$

$$F = \frac{4|r_{21}||r_{23}|}{(1-|r_{21}||r_{23}|)^2}$$

$$\Phi = 2k_2 d\cos\theta_2 + \underbrace{\delta_{21} + \delta_{23}}$$

↳ assume these are 0 for now (check with r_{21} below)

(a) Max in transmission when $\sin^2\frac{\Phi}{2} = 0 \rightarrow \frac{\Phi}{2} = \pi$

$$\frac{\cancel{2}\left(\frac{2\pi n_2}{\lambda}\right) d\cos\theta_2}{\cancel{2}} = \pi$$

$$d = \frac{\lambda}{2n_2\cos\theta_2} = \frac{633\cdot 10^{-9}}{(2)(1.8)(\cos 16.13°)} = \boxed{\begin{aligned}&1.83\cdot 10^{-7}\text{ m}\\ =\ &.183\ \mu\text{m}\end{aligned}}$$

(b) $r_{21} = r_{23} = \frac{1-\alpha\beta}{1+\alpha\beta}$ for s-polar

$$\alpha_{21} = \frac{\cos\theta_1}{\cos\theta_2} = .9015$$

$$\beta_{21} = \frac{n_1}{n_2} = \frac{1}{1.8} = .5556$$

$$r_{21} = \frac{1-(.9015)(.5556)}{1 + \text{''}\ \text{''}} = .33259$$

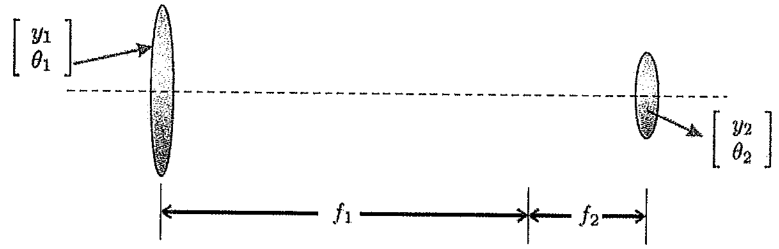
yes, no phase shift!

$$F = \frac{4(.33259)(.33259)}{(1-(.33259)(.33259))^2} = .5594$$

For 0.5 µm: $\Phi = 2\left(\frac{2\pi\cdot 1.8}{633\cdot 10^{-9}}\right)(.5\cdot 10^{-6})\cos 16.13° = 17.16\text{ rad} = 983.4°$

$$T_{13} = \frac{1}{1+.5594\sin^2\left(\frac{983.4°}{2}\right)} = \boxed{76.23\%}$$

21. (14 pts) A telescope is formed with two thin lenses separated by the sum of their focal lengths f_1 and f_2. Rays from a given far-away point all strike the first lens with essentially the same angle θ_1. "Angular magnification," M_θ, is defined as θ_2/θ_1 and quantifies the telescope's purpose of enlarging the apparent angle between points in the field of view.



Use ABCD-matrix formulation to derive M_θ of this system in terms of f_1 and f_2.

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} = (\text{lens } 2)(\text{distance } f_1+f_2)(\text{lens } 1)$$

$$= \begin{pmatrix} 1 & 0 \\ -1/f_2 & 1 \end{pmatrix} \underbrace{\begin{pmatrix} 1 & f_1+f_2 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ -1/f_1 & 1 \end{pmatrix}}_{\begin{pmatrix} 1-\frac{f_1+f_2}{f_1} & f_1+f_2 \\ -\frac{1}{f_1} & 1 \end{pmatrix}}$$

$$1 - \frac{f_1+f_2}{f_1} = 1-1-\frac{f_2}{f_1} = -\frac{f_2}{f_1}$$

$$= \begin{pmatrix} -f_2/f_1 & f_1+f_2 \\ +\frac{1}{f_1}-\frac{1}{f_1} \to 0 & \left(-\frac{f_1+f_2}{f_2}+1\right) \end{pmatrix}$$

$$-\frac{f_1+f_2}{f_2}+1 = -\frac{f_1}{f_2}$$

$$\begin{pmatrix} y_2 \\ \theta_2 \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix}\begin{pmatrix} y_1 \\ \theta_1 \end{pmatrix} \rightarrow \theta_2 = C y_1 + D\theta_1$$

$\theta_2 = D\theta_1$ since $C=0$

$$M_\theta = \frac{\theta_2}{\theta_1} = D$$

$$\boxed{M_\theta = \frac{-f_1}{f_2}}$$

For Problem 22 Solns, see

HW 24 #1