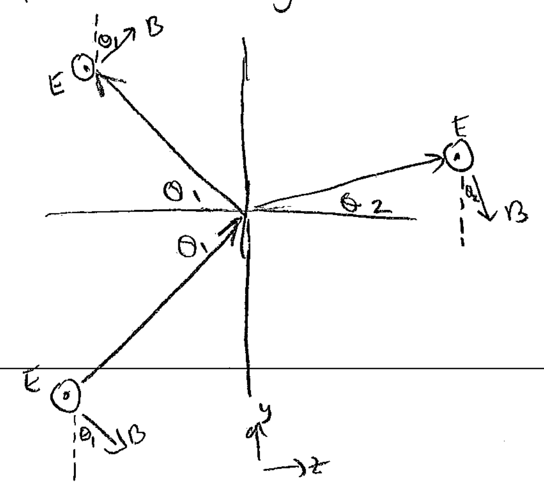
Optics - Deriving Fresnel Eqns for s-polarization by Dr Colton

(using method of Griffiths)



($\hat{x}$ = into paper)

s-polarization → E = perp. to interface

Directions of $\vec{B}$ chosen to make $\vec{E}\times\vec{B}$ be in direction of propagation

incident
$$\tilde{E}_I = \tilde{E}_{0I}\, e^{i(\vec{k}_I\cdot\vec{r}-\omega t)}\,(-\hat{x})$$
$$\tilde{B}_I = \frac{1}{v_1}\tilde{E}_{0I}\, e^{i(\vec{k}_I\cdot\vec{r}-\omega t)}\,(-\cos\theta_1\,\hat{y}+\sin\theta_1\,\hat{z})$$

reflected
$$\tilde{E}_R = \tilde{E}_{0R}\, e^{i(\vec{k}_R\cdot\vec{r}-\omega t)}\,(-\hat{x})$$
$$\tilde{B}_R = \frac{1}{v_1}\tilde{E}_{0R}\, e^{i(\vec{k}_R\cdot\vec{r}-\omega t)}\,(+\cos\theta_1\,\hat{y}+\sin\theta_1\,\hat{z})$$

transmitted
$$\tilde{E}_T = \tilde{E}_{0T}\, e^{i(\vec{k}_T\cdot\vec{r}-\omega t)}\,(-\hat{x})$$
$$\tilde{B}_T = \frac{1}{v_2}\tilde{E}_{0T}\, e^{i(\vec{k}_T\cdot\vec{r}-\omega t)}\,(-\cos\theta_2\,\hat{y}+\sin\theta_2\,\hat{z})$$

BC1: $(E_\parallel)_1 = (E_\parallel)_2$ this is x-component

$$\tilde{E}_{0I}\,e^{i(\;)} + \tilde{E}_{0R}\,e^{i(\;)} = \tilde{E}_{0T}\,e^{i(\;)}$$

exponentials must all be equal, so cancel them out

Eqn(1) $$\boxed{\tilde{E}_{0I} + \tilde{E}_{0R} = \tilde{E}_{0T}}$$

BC2: $\frac{1}{\mu_1}(B_\parallel)_1 = \frac{1}{\mu_2}(B_\parallel)_2$ this is y-component

Non magnetic, so $\mu_1 = \mu_2 = \mu_0$. Cancel them.

Cancel exponentials again.

Eqn (2) $$\boxed{\frac{1}{v_1}\tilde{E}_{0I}(-\cos\theta_1) + \frac{1}{v_1}\tilde{E}_{0R}(+\cos\theta_1) = \frac{1}{v_2}\tilde{E}_{0T}(-\cos\theta_2)}$$

Let $\alpha = \frac{\cos\theta_2}{\cos\theta_1}$, $\beta = \frac{v_1}{v_2}$ $\left(= \frac{n_2}{n_1}\right)$

multiply Eqn 2 by $v_1/\cos\theta_1$ on both sides,
call fields I, R, T for simplicity

$$\boxed{\begin{aligned} &\text{Eqn (1)} \quad I + R = T \\ &\text{Eqn (2)} \quad -I + R = -\alpha\beta T \end{aligned}}$$

Some algebra... subtract eqns

$$2I = (1+\alpha\beta)\,T$$

$$\frac{T}{I} = \boxed{t = \frac{2}{1+\alpha\beta}}$$

"transmission coefficient" for s
plug back in for α and β, and this is obviously far right hand side of Eqn 3.19

more algebra... multiply top eqn by $\alpha\beta$, then add

$$\alpha\beta I + \alpha\beta R = \alpha\beta T$$

$$\text{add} \quad -I + R = -\alpha\beta T$$

$$(\alpha\beta - 1) I + (1+\alpha\beta) R = 0$$

$$\frac{R}{I} = \boxed{r = \frac{1-\alpha\beta}{1+\alpha\beta}}$$

"reflection coefficient" for s
plug back in for α and β, and this is obviously far right hand side of Eqn 3.18

TOPS
35500