Honors 225 – Study Guide/Chapter Summaries for Exam 1
***Roots* Chapters 1-7**

Introduction - "Conventions, Abbreviations, Symbols"

Units. Standard metric units that I would expect you to know are as follows:

- Length – meter (m)
- Mass – kilogram (kg)
- Time – second (s)
- Energy – joule (J)
- Force – newton (N)
- Electric potential – volt (V)
- Temperature – kelvin (K)
- Power – watt (W), which is energy per time (J/s)

Convert between these standard units and other units by using *conversion factors*. I will give you any conversion factors you may need to know, perhaps in a list like this:

- 1 inch = 2.54 cm
- 1 foot = 0.3048 m
- 1 mile = 1.609 km
- 1 mi/hr = 1 mph = 0.44704 m/s
- 1 cal = 4.186 J
- 1 eV = 1.602×10^{-19} J

Powers of 10. Standard metric prefixes indicate powers of ten; I would expect you to know the following:

- Giga (G) – 10^{9}
- Mega (M) – 10^{6}
- Kilo (k) – 10^{3}
- Centi (c) – 10^{-2}
- Milli (m) – 10^{-3}
- Micro (μ) – 10^{-6}
- Nano (n) – 10^{-9}

If any other metric prefixes show up in a problem (like "femto", etc.), I would give you their meaning.

Short-cut "e" notation can be used to mean powers of 10, for example 3.58×10^{13} is 3.58e13.

Fundamental constants. I will give you the following list of fundamental physical constants that you might need to know, perhaps in a list like this:

- c = speed of light, 2.998×10^{8} m/s
- e = charge of an electron (magnitude), 1.602×10^{-19} C
- h = Plank's constant, 6.626×10^{-34} J·s
- k = Boltzmann's constant, 1.381×10^{-23} J/K. Not to be confused with the wavenumber k, described below.
- m = mass of an electron, 9.109×10^{-31} kg
- σ = Stefan-Boltzmann constant, 5.670×10^{-8} W/(m^{2}·K^{4})
- Wien's law constant, 2.898×10^{-3} meters · K

Basic math. I will expect you to be able to do algebra and basic trigonometry, as well as know what the mathematical constants *e* and π refer to. You should have a basic scientific calculator that can do mathematical functions such as sin, cos, tan, and exp.

Chapter 1: Themes, Appendix DIR Paul Dirac

These are good topics to set the stage for the rest of the book, but nothing that would be explicitly tested on.

Chapter 2: O Light Divine; Appendix FRE Frequency (the Fourier Analysis appendix gets its own section below)

Basic wave properties. Although Newton pictured light as a stream of particles, other scientists who were his successors (and in some cases even his contemporaries) soon discovered that light demonstrated features of waves. Wave properties are characterized by:

- *wavelength*, λ (lambda) – This is the distance over which a wave repeats itself, if for example the wave were frozen in time in a photograph. That is, how many meters between wave crests.
- *period*, *T* – This is the time over which a wave repeats itself, as the wave passes through a particular point in space. That is, how many seconds between wave crests.
- *frequency*, *f* (book symbol ν, nu) – This is the inverse of the period, $f = \frac{1}{T}$. That is, how many wave crests pass through a particular point in space each second.
- *angular frequency*, ω (omega) – This is 2π times the frequency, $\omega = 2\pi f$, and represents how many *radians* of the wave pass by per second. (There are 2π radians in a complete oscillation.) It's also related to the period through $\omega = \frac{2\pi}{T}$.
- *wavenumber*, *k* – This is 2π divided by the wavelength, $k = \frac{2\pi}{\lambda}$, and represents how many radians of the wave exist per meter.

The speed of a wave is related to the wavelength and frequency through this equation:

$$\lambda f = v$$

For light waves, the wave speed is equal to *c*, the fundamental constant given above.

Sine waves. We often use sine functions to represent waves for reasons that are explained in the next section. They could have the following mathematical forms, where *y* is whatever quantity that is oscillating and *A* is the amplitude of the oscillation.

$y = A\sin(kx)$ – a wave that has no time dependence (like in a photograph)
$y = A\sin(\omega t)$ – a wave that has no spatial dependence (like if you're restricted to a single point in space)
$y = A\sin(kx - \omega t)$ – a *traveling wave* with both time and space dependence

Since sine equals 0 when its argument is 0, all of those forms implicitly require a wave which starts at zero. If a particular wave does not start at zero, one can take that into account by adding a *phase shift* to the wave, commonly symbolized as ϕ (phi). For example:

$y = A\sin(kx + \phi)$ – a wave with no time dependence, that is shifted toward negative *x* by ϕ radians (where there are 2π radians in a complete oscillation).
$y = A\sin(\omega t + \phi)$ – a wave with no spatial dependence, that is shifted toward negative *t* by ϕ radians

When waves come together, they add together. This is called *interference*. Interference can be *constructive* if they are in phase or *destructive* if they are out of phase.

More properties of light include the following; you should review them in the book if needed:

- *Reflection*
- *Refraction*
- *Dispersion*
- *Diffraction*
- *Transverse* vs. *longitudinal* oscillations
- *Polarization*
- *Coherence*

Chapter 2 Appendix FOU Fourier Analysis

All waves can be represented as a sum of sinusoidal functions at different frequencies. This is called Fourier analysis. Including more and more frequencies gives a more and more accurate representation of the wave.[1]

This was demonstrated in the book using an example of a square pulse having a duration ("pulse width") of 1 ms and a pulse repetition period of 4 ms. The book called this the RECT function. The function and its Fourier representation are given in Fig A2.2.

One can depict a wave either as a function as time or as its Fourier representation in terms of the amplitude of the frequency components that make up the wave. The frequency representation is called the "frequency spectrum"; if the squares of the amplitudes are plotted this is called the "frequency power spectrum", or often just the "power spectrum" for short (because the energy or power contained in a sinusoidal wave depends on its amplitude squared). The power spectrum for the RECT function is given in Fig A2.3.

In class I depicted power spectra of a few other functions similar to RECT but with different pulse widths and repetition periods. The three most important take aways from that discussion were:

- The shape of the power spectrum depends only on the shape of pulse. All of the RECT-type functions had the same basic power spectrum shape as Fig A2.3.
- The spacing between frequency points is inversely proportional to the period *T*. Doubling the period (for example) will result in the power spectrum points being spaced together 2× more closely.
    - To be even more specific, the spacing between points is *equal* to 1/*T*. One can say: $period \times frequency\ spacing = 1$
    - As period goes to infinity, the frequency points become infinitely close together.
- The width of the power spectrum is inversely proportional to the pulse width. Doubling the pulse width (for example) will result in the power spectrum being half as wide, while keeping the same overall shape.

[1] We are considering now only waves that are functions of time, although similar things can be done by for waves that are functions of space by talking about "spatial frequencies".

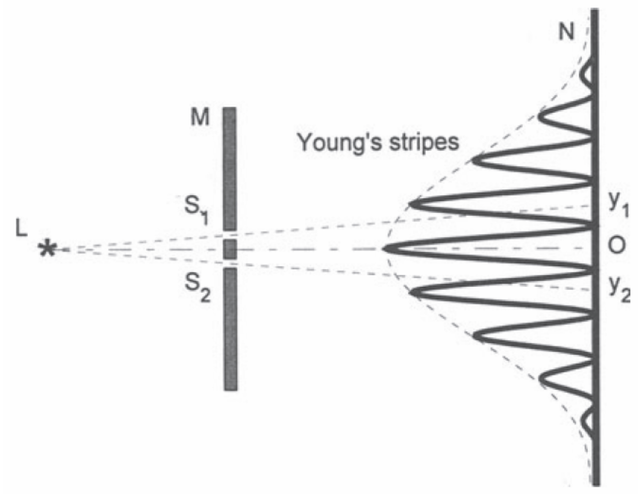
- To be even more specific, the width of the power spectrum is equal to a constant C (a small positive number), divided by the pulse width. One can say:
  $width\ of\ pulse \times width\ of\ power\ spectrum = C$

That last fact is given in the book as Eq. A2 in Appendix FOU. The value of the constant *C* depends on the details of the pulse shape along with how you specifically define the power spectrum width. For the RECT pulse function, and with the power spectrum width defined as the "distance" (in Hz) from the power spectrum minimum point just to the right of 0 minus the minimum point just to the left of 0, it turns out that the constant $C = 2$. As mentioned in a homework problem, for the usual definition of power spectrum width (which is a little different than what we did for RECT, but which I won't go into here), it can be mathematically proven that the smallest that *C* can ever be is $C = 1/(4\pi)$.

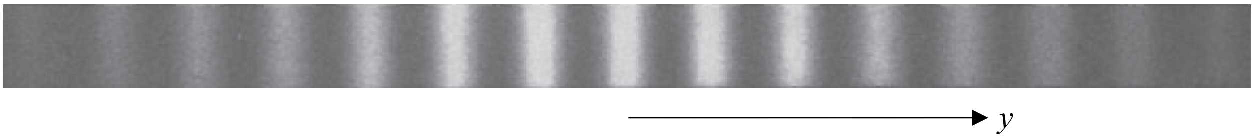
The third bullet point is the first major instance of *uncertainty* in the physics of this class. If you want a wave with a well-defined frequency (narrow frequency power spectrum), it must by necessity be long in duration. If you want a wave with a well-defined arrival time (short pulse), it must by necessity have a wide frequency power spectrum. You cannot simultaneously have a wave with a well-defined frequency and also a well-defined arrival time.

Chapter 3 - The Young Experiment (1801)

Young's "two slit experiment" showed unambiguously that light waves interfere with each other, and hence (people thought at the time) light must be a wave. Here's the setup, from Fig. 3.3b.



L is the light source. M stands for a "mask", which contains two slits, S_1 and S_2. The light passes through the mask and gets projected onto a screen labeled N. The stripes on the figure indicate spots of varying intensity or brightness as shown below. The letters y_1 and y_2 indicate positions on the screen relative to the middle of the pattern (origin O).



Using the symbols d and D to refer to the distance between the two slits, and the distance between mask and screen, respectively, the path length difference (PD) for light rays traveling through S_2 vs. S_1 is:

$$PD = \frac{yd}{D}$$

The bright fringes occur when the path difference is zero, one wavelength, two wavelengths, etc., because that will result in the waves being in phase. The equation for constructive interference is thus:

$$\frac{yd}{D} = n\lambda$$

In that equation n represents any integer and y now means the specific distance on the screen from the origin to the center of the "n^{th}" fringe. One can solve for y for a given n, to find out where a given fringe will appear. The fringe separation distance (which I will call Δy) is constant, and is given by the distance from the origin to the y-value for $n = 1$: $\Delta y = \lambda D/d$

Chapter 4 – A Whiff of Ether (1887)

Some basic definitions

- Velocity, v – how fast an object changes position: $v = \frac{\text{change in position}}{\text{time it takes for the change}}$
- Acceleration, a – how fast an object's velocity changes: $a = \frac{\text{change in velocity}}{\text{time it takes for the change}}$
- Mass, m – a measure of how much matter is present in an object
- Force, F – the strength of a push or a pull on an object

Newton's laws of motion.

1) If F = 0 on an object, then there is no acceleration. An object with no forces acting on it will remain at rest or at constant velocity.
2) If F ≠ 0, then there will be an acceleration, and the amount of acceleration is given by $a = F/m$. Or, as it's usually written, $F = ma$.
    a. Note in laws 1 and 2 we're talking about the *net* force, since (for example) forces applied in opposite directions can cancel out.
3) If there's a force between two objects, then the force of object 1 on object 2 will always be equal and opposite to the force of object 2 on object 1.
    a. Note that this means that forces always comes in pairs. If (for example) I hit someone's face with my hand, their face will always be applying a force on my hand in the opposite direction at the same time (which can injure my hand!).

Example of forces.

- Gravity, $F \sim 1/r^2$
- Electromagnetic, $F \sim 1/r^2$
- Strong and weak nuclear (they are very short ranged, and very complicated)

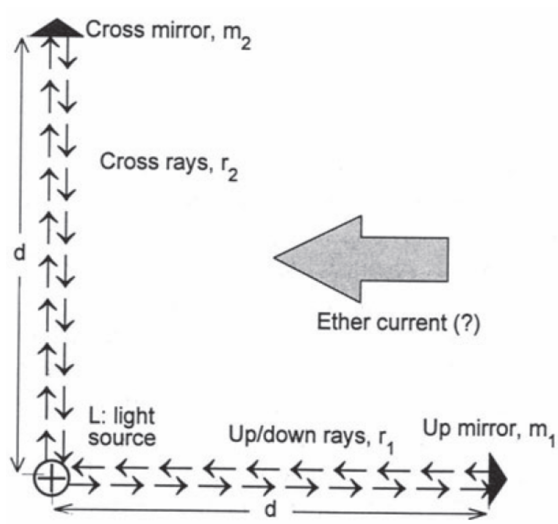
Gravity and electric forces are inverse square laws: double the distance between the objects and the force goes down by a factor of 4.

Inertial vs. non-inertial observers. Observers must be *inertial* for them to agree that Newton's laws are valid. Inertial means "at rest or moving at constant velocity" (like in the book's cargo hold example). Under those conditions everything will be normal. *Non-inertial* observers are ones in a frame of reference that is accelerating or rotating; under those conditions there will be strange effects which objects don't seem to obey Newton's laws. One example of this is the *Coriolis force* or *Coriolis effect* where objects in flight appear to be pushed to the right (in the Northern hemisphere) even though there is no force on them; this is really a result of the earth rotating underneath the object as it is in the air.

Chapter 4 - continued

The second half of chapter 4 deals with the speed of light, which has the symbol c and has been measured to be 3×10^8 m/s. (A more accurate value is given above.) This value exactly matched the predicted speed of electromagnetic waves by Maxwell's laws of electricity and magnetism in 1864, and the agreement between theory and experiment was seen as a great triumph of electromagnetic wave theory and incontrovertible evidence of light being a wave. Incidentally, the speed of light was measured with a fair degree of accuracy as early as 1849 (not discussed in book).

Waves require a medium in which to travel, so the *Michaelson-Morley experiment* was done to try to measure properties of the *luminiferous ether* which was thought to be the medium, but which we now know does not exist. The experiment measured interference between light waves starting at the same spot but going back and forth from there along two different paths. The set-up of the experiment (assuming equal paths) is given in Fig. 4.5a, which assume an ether current or "wind" coming from the right due to the Earth's motion through the ether:



Using v to indicate the speed of the ether wind, we worked out in class the time for light to travel along the round-trip horizontal and vertical paths:

$$t_{horiz} = \frac{2dc}{c^2 - v^2}$$
$$t_{vert} = \frac{2d}{\sqrt{c^2 - v^2}}$$

The two equations can be combined in a ratio which shows that the horizontal time is larger (because the denominator of the ratio is obviously smaller than the numerator):

$$\frac{t_{horiz}}{t_{vert}} = \frac{c}{\sqrt{c^2 - v^2}}$$

By measuring the differences in the two times, which Michaelson and Morely proposed to do via phase shifts and interference fringes, one can then calculate *v*. The result: there WAS no difference in times, and hence *v* = 0, and hence there is no ether. Since there is no ether, a necessary and confusing consequence is that light emitted by sources will be measured by all observers as traveling at *c*, regardless of any motion by the sources or observers. This was eventually resolved by Einstein through his theory of relativity in 1905, but (sadly?) is not a topic of this course.

Chapter 5 – Professor Plank is Desperate (1901); Appendix PLA Max Plank

The random motion of atoms and molecules is a form of energy. The hotter the object, the more vigorous the random motion. Like other forms of energy, this is measured in the metric systems in joules. Heat is the transfer of this random *thermal energy*. The three main ways thermal energy can be transferred are conduction, convection, and radiation. We'll focus on radiation: heat transfer through light energy.

Blackbody radiation is the "glow" of hot objects, such as incandescent lights, electric burners, lava, stars, etc. The glow carries away energy from the hot object, which then can be given to cooler objects surrounding it. The three important laws governing blackbody radiation are:

Stefan-Boltzmann law: $P = \epsilon\sigma AT^4$

- P is the radiant power (in watts), which is the radiant energy emitted per time; in the book the equation is given as $RE = \epsilon\sigma T^4$ where RE = power per surface area of the source.
- Epsilon, ϵ, is called the *emissivity*, and describes how different the actual radiation is from radiation of a perfect blackbody. It's always between 0 and 1 but has a different value for each radiating object.
- Sigma, σ, is the "Stefan-Boltzmann constant", a fundamental constant given above.

Wien's law: $\lambda_{peak} = \frac{2.898\times10^{-3}\text{ meters}\cdot\text{K}}{T}$

- λ_{peak} is the peak wavelength of the radiant emission (in meters)
- For some reason the constant in the numerator isn't typically given a special name/symbol, although Wikipedia uses the letter *b*. I'll just call it the "Wien's law constant".

Plank's law: $SRE = \frac{2\pi hc^2}{\lambda^5\left(\exp\left(\frac{hc}{\lambda kT}\right)-1\right)}$

- SRE is the "spectral radiant emission", which is the radiant power per surface area, per wavelength.
- *h* is a new fundamental constant, called "Plank's constant"; value given above.
- *k* is an older constant, called "Boltzmann's constant", value given above, which had previously been discovered in the thermodynamic studies of gases (if you care: it's the ideal gas law constant *R* in metric units, divided by Avogadro's number).
- As expected, *λ* is wavelength (in meters) and *c* is the speed of light.

Caution: In all three equations the temperature must be given in kelvin.

Plank's law actually predicts the Stefan-Boltzmann law and Wien's law (although we didn't do that in class), and perfectly explains the measured shape of blackbody radiation spectra.

Plank derived this equation theoretically via *quantized oscillators*, which means that although light can come in different frequencies, for a given frequency f the light can only come in bundles of energy which are multiples of hf.

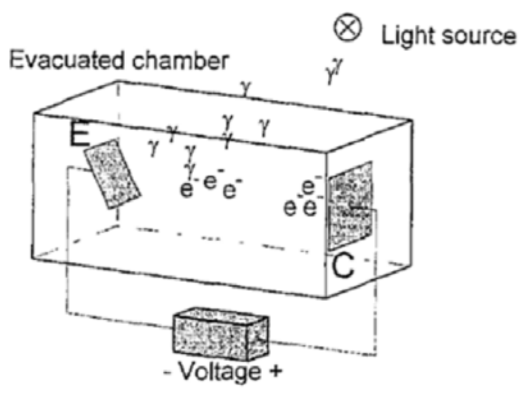
This is reminiscent of *standing waves*, such as the waves I made in class by oscillating a section of rubber tubing, where the waves are forced to be *nodes* (unmoving spots) at the edges. Increasing the frequency of oscillation results in special situations where you get successive "humps" (called *antinodes*) in the middle. Here's the table we made in class:

	Number of antinodes	Wavelength, in terms of length of tubing, L	Frequency, in terms of L and wave speed v (using $f = v/\lambda$)	Frequency, in terms of the first mode, f_1
1st mode	1	$2L$	$v/(2L)$	f_1
2nd mode	2	L	v/L	$2 \times f_1$
3rd mode	3	$(2/3)\ L$	$v/((2/3)L) = 3v/(2L)$	$3 \times f_1$
nth mode	n	$2L/n$	$v/(2L/n) = nv/(2L)$	$n \times f_1$

You can see that standing waves come in quantized frequencies, where only multiples of f_1 are allowed.

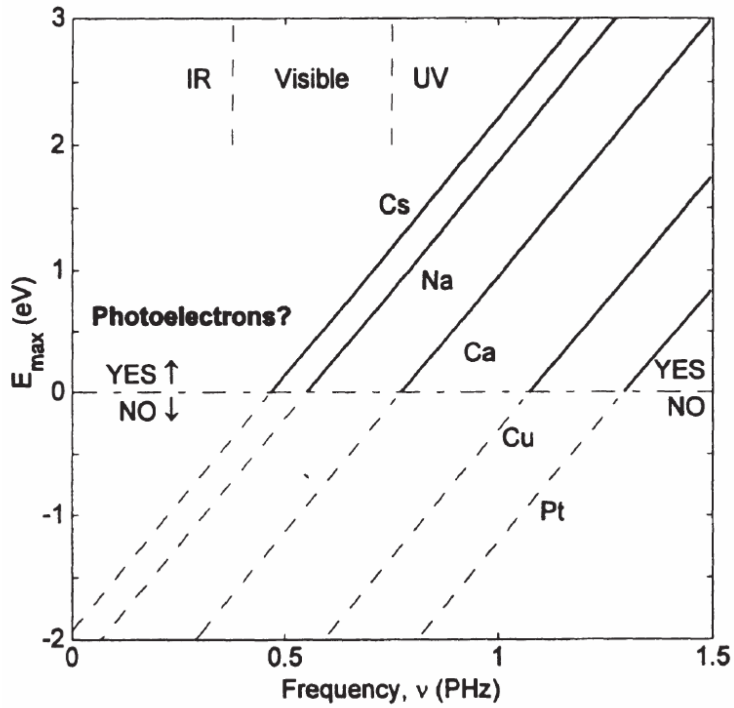
Chapters 6 and 7 – The Photoelectric Effect (1902) and Dr Einstein's Light Arrows (1905)

In some materials, electrons can be made to jump off of a surface and be attracted to a nearby positive electrode (electrons are negatively charged, and opposites attract). Light helps this process. Whether or not an electron can be made to escape the surface depends on the *spectrum* of the light, but not on the *intensity* of the light. This was studied in the *photoelectric effect* experiments done by Lenard and others, as depicted in Fig. 6.2:



The γ symbols refer to "quantized oscillations of electromagnetic energy", which we (thankfully) now call *photons*, coming from the light source; the e^- symbols refer to electrons which have left the electrode on the left and are attracted to the electrode on the right. The electrons can come off of the left electrode with varying amounts of energy.

The most important results of a photoelectric effect experiment are summarized by making a plot of the maximum energy of the electrons vs. the frequency of the applied light, for the material under study. This is given for several materials Fig. 7.2; the plots show that no electrons are produced until the frequency reaches some threshold (which depends on the material), and then the maximum energy of the electrons increases linearly with frequency with a slope that is equal to Plank's constant:



The equation describing the result is this:

$$E_{max} = hf - Wf$$

The *work function* (Wf) is different for each material, and is a measure of how deep the "well" is that the electrons must escape from in order to leave the material. The *y*-intercept of the graph for a given material, multiplied by –1, is the work function for that material.

<u>Caution</u>: When using that equation, E_{max} and Wf must be given in joules, so to make a plot like the one above, you would need to first convert joules to eV.

The results of these experiments were explained by Einstein thusly in 1905: the energy of a light wave comes in bundles, called photons. Einstein's genius was to give the explanation with only preliminary results, i.e. prior to the full results shown in the figure (those results were from 1916). In J.J. Thomson's words, the photons in a wave front are like "bright sparkles on a dark background". Each individual photon has energy $E = hf$, or in terms of the wavelength, $E = \frac{hc}{\lambda}$. Einstein's photons explain both the photoelectric effect and also the "quantized oscillators" of Plank's radiation law.