Honors 225 – Physics Study Guide/Chapter Summaries for Final Exam; *Roots* **Chapters 15-18**

Chapter 15 – Collapsing the Wave

If a particle is in a quantum superposition state (that is, a superposition of eigenfunctions each with its own coefficient), when a measurement is done the wave function will abruptly change, or "collapse", to one of the eigenfunctions. Which eigenfunction gets chosen is randomly determined with a probability given by its normalized coefficient squared. One example of this is an electron wave packet that hits a barrier with some chance of reflection and some chance of transmission. As was discussed in chapter 14, the wave function splits into two parts: one that reflects and one that transmits. When a measurement is made the particle is found to be either reflected or transmitted, and the wave function collapses to one of those two options. Another example of this is an electron in an atom that is in a superposition of energy states. When it emits a photon the electron collapses to an eigenstate, so the photons are only emitted at energies corresponding to differences between eigenstates.

What does it mean to say that a "measurement is done"? My view is that it means the quantum system has interacted with a macroscopic system. I also believe (as I suspect most physicists do), but perhaps this is not provable, that if the macroscopic system could be described in quantum terms then there would be no abrupt wave function collapse but rather a smooth evolution in the combined wave function. The collapse is an artifact of not being able to describe the macroscopic system in quantum terms because there are too many interacting particles and the combined wave function is too complicated to possibly be worked out.

This chapter also mentions Heisenberg's matrix mechanics which was developed in 1925, the year before Schrödinger's wave equation was formulated. It is an alternate way of doing quantum mechanics compared to the wave equation. The mathematics is different and in some ways more complicated so we won't go into it, but sometime between 1926 and 1939 (I couldn't locate the exact date) Schrödinger and Dirac proved that the two methods were mathematically equivalent. As a side note, a third method for doing quantum mechanics was developed by Richard Feynman in 1948, and this method was *also* proven mathematically equivalent to Schrödinger's and Heisenberg's methods; each method is easier for some problems and harder for others, so each method is still used by physicists today.

Chapter 16 – Copenhagen Takes Over (1925-?)

The discussions of Niels Bohr and those who visited the Institute for Theoretical Physics of the University of Copenhagen where he worked (renamed the Niels Bohr Institute in 1965 in his honor) led to this prevailing view of what quantum mechanics is, how to use it, and what it means. It is called the "Copenhagen Interpretation" and includes the following points:

1) The quantum states of particles are described by wave functions, labeled Ψ (Schrödinger).
   a. The eigenfunctions are special solutions to Schrödinger's equation that oscillate at well-defined frequencies.
   b. If they are interacting, or "entangled" particles, then Ψ represents the combined system rather than an individual particle.
2) Quantum mechanical particles exhibit both particle-like and wave-like behavior, which is called the principle of "complementarity" (Bohr).
3) $|\Psi|^2$ represents a probability function. It should be normalized to 100% (Max Born).
4) Ψ can include parts of multiple eigenfunctions; if so, it's called a "superposition" state.
5) During an "observation", Ψ collapses to one of the eigenfunctions.
   a. One cannot directly measure Ψ because the act of measuring it affects the state.
6) Ψ represents all that can be known about the system prior to the observation.

7) Not all properties of the system can be completely known simultaneously; for example position and momentum are "conjugate pairs" and can't both simultaneously be known to high precision (Heisenberg's Uncertainty Principle).
8) Quantum mechanics will reproduce classical physics in the limit of large quantum numbers (i.e. for large orbits/energies); this is called the "correspondence principle" (Bohr and Heisenberg).

More on the Uncertainty Principle: since particles are described by wave functions, Fourier analysis is relevant to these waves. Fourier analysis tells us that a wave which is localized in time must contain many frequency components. If Δt represents the width (or uncertainty) of the pulse in time and Δf the width (or uncertainty) of the pulse in terms of frequencies, as was discussed back in Chapter 2 Appendix FOU, then graphically the situation is like this:
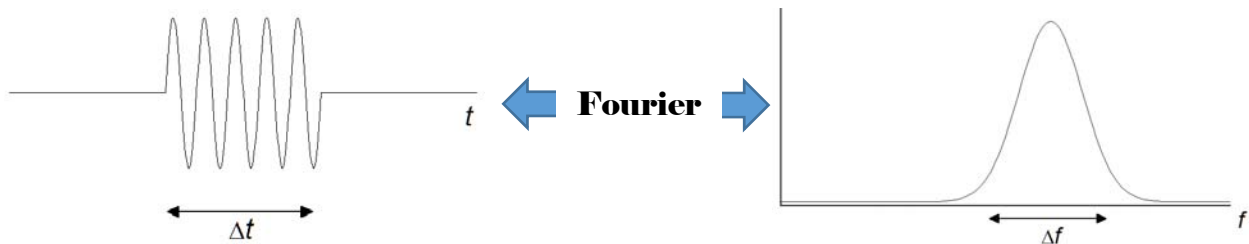
Even though it *looks* like the wave on the left might have a well-defined frequency because it has a few well-defined oscillations, due to its finite extent in time it actually does *not*. The question of "what frequency does that wave have" has no answer—*many* sinusoidal waves, each with its own frequency, are required to produce that localized wave. Fourier analysis leads to this relationship:

$$\Delta t \Delta f \geq \frac{1}{4\pi}$$

Since the frequency of oscillation of an eigenfunction relates to its energy via $E = hf$ (same as the energy-frequency relation of a photon) we can write this in terms of energy E:

$$\Delta t \Delta E \geq \frac{\hbar}{2}$$

Mathematically speaking, waves in space of the form $\sin kx$ are identical to waves in time of the form $\sin \omega t$, so an identical analysis for position x and wavenumber k ($= 2\pi/\lambda$, rad/m) can be done using de Broglie's equation ($p = \frac{h}{\lambda} = \hbar k$). This leads to:

$$\Delta x \Delta p \geq \frac{\hbar}{2}$$

Those last two centered equations are each called Heisenberg's Uncertainty Principle, the first being the "energy-time uncertainty principle" and the second being the "position-momentum uncertainty principle". Note that *Roots* leaves out the factor of 2 in the denominator but it's present if you use the normal definition of width or uncertainty (which is the standard deviation, for those with some statistical background) so I will leave it in.

Heisenberg's microscope. Heisenberg demonstrated in a thought experiment with a microscope that uses only a single photon of light to observe an electron, that measuring the position of an electron will invariably affect its momentum. A more and more precise measurement of position will give rise to more

and more uncertainty of momentum, in just the same way that the uncertainty principle predicts. I refer the reader to this excellent video on the topic: https://www.youtube.com/watch?v=dgoA_jmGIcA

Einstein and Bohr Solvay debates (1927 and 1930). In these two famous conferences in Brussels, Belgium, Einstein made several attempts to prove that quantum mechanics and the Copenhagen Interpretation were wrong, or at least incomplete. Bohr thwarted all his attempts. See this excellent video on the topic: https://www.youtube.com/watch?v=8PUi0I02Mdc, although note that with regards to Einstein's 1930 challenge, the video refers to the "mass" and the "weight" of a photon. That's incorrect. Photons have no mass nor weight. The way it's described in Roots on page 432 is better, which is that the *energy* of the departing photon creates a loss of measured mass of the box through Einstein's $E = mc^2$ equation.

Chapter 17 – EPR (1935) and EPR-B

In 1935 Einstein, Podolsky, and Rosen published the famous "EPR" paper objecting to quantum mechanics, specifically to entangled states. Their objection can be summarized by the following: if two particles are entangled, then collapsing the wave function of one will necessarily instantaneously collapse the wave function of the other one, *no matter how far apart the two particles may be*. On its surface this appears to violate Einstein's theory of relativity which holds that nothing can travel faster than the speed of light. Their solution was to propose that some properties (or "variables") exist that govern the situation to create what *looks* to be a wave function collapse (but isn't really), but that these variables are hidden so we don't notice them. The variables always existed before they interacted with the macroscopic system, though, so nothing really collapsed faster than the speed of light (in their view). This view is antithetical to quantum mechanics, and for a time people didn't know which was correct, the EPR view or the quantum mechanical one.

In 1951 David Bohm expanded on this a bit, elucidating the EPR position while at the same time defending the quantum mechanical one, in what *Roots* calls the EPR-B paper. And in 1964 John Bell wrote a seminal paper proposing an experiment to test whether the EPR view was correct or whether the quantum mechanical view was correct. Since most tests of Bell's theorem involve polarization of photons, that's what we'll discuss.
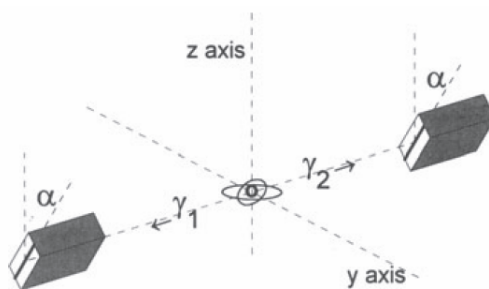
Figure 17.3. Bohm configuration. Each polarization analyzer is tilted at α to the vertical.

Here two photons are produced from a single source in such a way that their quantum states are entangled. This is typically done experimentally by exciting calcium atoms with a particular wavelength of light that causes two photons to be emitted in response. They go out in opposite directions, and hit two polarizers that the experimenter has positioned at the same angles, α. In this chapter and the next let's always assume the left photon hits its polarizer first. If the right hand one hits first then all of the 1's and 2's are just reversed and nothing is really changed.

Roots has a whole slew of polarization facts, or "P-facts", in this chapter and the next, but I will mention just a few facts about polarization here.

- Polarizations rotated by 180° are equivalent. I.e. there is no difference between a polarization of, say, 12.4° and a polarization of 192.4°.
- A polarizer acting on a polarized beam of photons will pass $\cos^2 \theta$ of them (expressed as a fraction or percentage), where θ is the angle between the polarizer and the polarization of the beam. Similarly, one can say:
  - (for the beam) $I = I_0 \cos^2 \theta$. This is called Malus's law. I_0 is the intensity of the beam before the polarizer and I is the intensity after.
  - (for individual photons) Each individual photon's chance of passing is given by $\cos^2 \theta$.
- The photons that make it through a polarizer pick up the polarization angle of the polarizer.

Quantum mechanics says that no matter what the state of the left photon before it hits polarizer 1, hitting that polarizer will cause its wave function to collapse into either a polarization state of α, in which case it passes, or a polarization state 90° away from that, in which case it is blocked (fails to pass). Since the two photons are entangled, collapsing photon 1 to α (or 90° – α) will immediately cause photon 2 to collapse to α (or 90° – α). It will then either pass or fail its polarizer in exactly the same way photon 1 did with *its* polarizer. In other words, the two photons will always have the same fate. In the language of *Roots*, 100% of these **SameTilt** runs are **SameFate** and 0% of them are **DiffFate**.

What happens when the experiment is done? It's exactly as quantum mechanics predicts:

<p align="center">100% of the SameTilt runs are SameFate</p>

Can this be explained within the EPR framework of hidden variables? Yes! However, there is only one possible explanation. Or at least, only one that anyone has come up with so far. As Grometstein writes in the next chapter on page 497, "if you know of a different explanation for [that result], please shout it from the rooftops." The EPR explanation is that photons carry a "code", which can be visualized as a table or a disk, that tells the photon what to do when it meets a polarizer… and that both of the entangled photons in the experiment are created with the exact same code. It might look like this, for example:

| polarizer angles	what to do
0-23.999°	pass
24-34.999°	fail
35-70.999°	pass
71-123.999°	fail
etc.	etc.
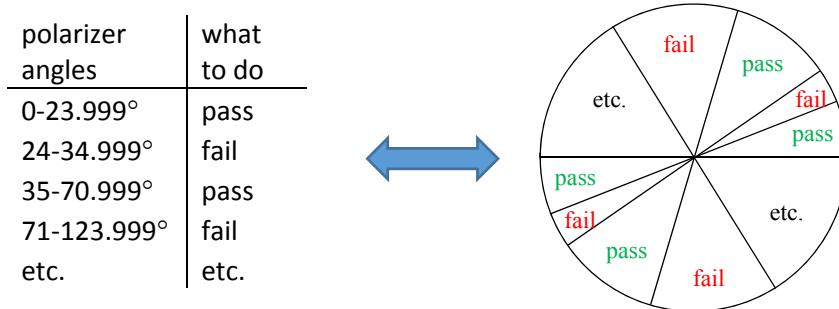
If both photons get produced with the same code, then that naturally leads to them both responding the identical way to polarizers at the same angle.

Chapter 18 – Bell's Thunderbolt (1964) – part 1

Bell's paper in 1964 asked the question, "What happens if the two polarizers are at different angles?" He showed that when the polarizers are at different angles, the EPR view and the quantum mechanics view

lead to different predictions. This is then an experiment that can be done to see whose view was correct, Einstein (EPR) or Bohr (quantum mechanics). The result? Einstein was wrong and Bohr was right.

This chapter examines two different configurations of polarizer 1 vs. polarizer 2. Here's the first one:
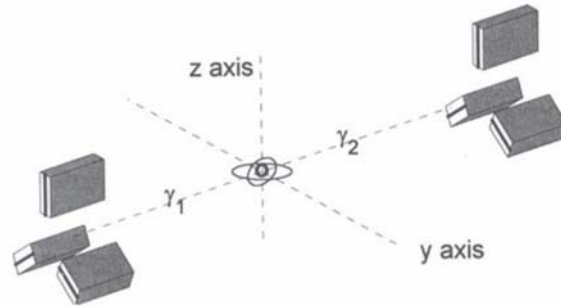
Figure 18.3. Bell triplets. The analyzers are spaced at 120°.

The figure makes it look like there are three different polarizers on each side, but what it really means is there are three different *options* for polarizers: 0°, 120°, and –120° (which is the same as 60°). There are thus 9 different combinations for the two polarizer locations that can be set up; the experimentalist choses between them randomly during the experiment. We will analyze this situation using both the EPR and the quantum mechanical views, and see which matches the actual experiments.

First, the EPR point of view. This uses the idea of hidden variables described in code tables or disks. Since there are only three different polarizer settings for a given photon, the infinite number of potential codes can be broken down into only 8 different possibilities based on whether a code says "pass" or "fail" for each of the three angles. We label those FFF, FFP, FPF, FPP, PFF, PFP, PPF, and PPP. For example, FFF means a photon will fail at 0°, 120°, and also –120°. "PFF" means a photon will pass at 0° but fail at 120° and–120°. Etc. Those eight possibilities will all have equal chances to be realized. All together then, there are 9 × 8 = 72 different possibilities of polarizer configurations and photon codes. To analyze this we can make a large table that describes what will happen for each of the 72 cases. Here ✓ means pass and ✗ means fail, and S and D refer to **SameFate** and **DiffFate** results, respectively.

polari zer 1	polari zer 2	PPP code phot on 1	phot on 2	same /diff	PPF phot on 1	phot on 2	same /diff	PFP phot on 1	phot on 2	same /diff	PFF phot on 1	phot on 2	same /diff	FPP phot on 1	phot on 2	same /diff	FPF phot on 1	phot on 2	same /diff	FFP phot on 1	phot on 2	same /diff	FFF phot on 1	phot on 2	same /diff
0	0	✓	✓	S	✓	✓	S	✓	✓	S	✓	✓	S	✗	✗	S	✗	✗	S	✗	✗	S	✗	✗	S
0	120	✓	✓	S	✓	✓	S	✓	✗	D	✓	✗	D	✗	✓	D	✗	✓	D	✗	✗	S	✗	✗	S
0	-120	✓	✓	S	✓	✗	D	✓	✓	S	✓	✗	D	✗	✓	D	✗	✗	S	✗	✓	D	✗	✗	S
120	0	✓	✓	S	✓	✓	S	✗	✓	D	✗	✓	D	✓	✗	D	✓	✗	D	✗	✗	S	✗	✗	S
120	120	✓	✓	S	✓	✓	S	✗	✗	S	✗	✗	S	✓	✓	S	✓	✓	S	✗	✗	S	✗	✗	S
120	-120	✓	✓	S	✓	✗	D	✗	✓	D	✗	✗	S	✓	✓	S	✓	✗	D	✗	✓	D	✗	✗	S
-120	0	✓	✓	S	✗	✓	D	✓	✓	S	✗	✓	D	✓	✗	D	✗	✗	S	✓	✗	D	✗	✗	S
-120	120	✓	✓	S	✗	✓	D	✓	✗	D	✗	✗	S	✓	✓	S	✗	✓	D	✓	✗	D	✗	✗	S
-120	-120	✓	✓	S	✗	✗	S	✓	✓	S	✗	✗	S	✓	✓	S	✗	✗	S	✓	✓	S	✗	✗	S

There are 48 **SameFate** cases out of the 72 possibilities (48/72 = 66.7%), so EPR predicts:

SameFate = 66.7% (EPR prediction)

Or, the way *Roots* describes it, all of the middle cases have five out of nine **SameFate** results (5/9 = 55.6%), and the first and last two columns will only increase that number, so **SameFate ≥ 55.6%**.

Now the quantum mechanical point of view. Suppose the left polarizer is in the 0° position. The left photon will either pass or fail.

If the left photon passes, then that's because it has collapsed to 0°, and because of entanglement the right photon also collapses to 0°. If the right polarizer is set to 0° (1/3 of the time), the right photon will pass and we'll get **SameFate** 100% of the time. If the right polarizer is set to either 120° or –120° (2/3 of the time), the right photon will pass $\cos^2 120° = 25\%$ of the time. So overall for the "left photon passes" case we'll get **SameFate** (1/3)×100% + (2/3)×25% = 50% of the time.

If the left photon fails, then that's because it has collapsed to 90°, and therefore the right photon also collapses to 90°. If the right polarizer is set to 0° (1/3 of the time), the right photon will pass 0% of the time, will therefore fail 100% of the time, and we'll get **SameFate** 100% of the time. If the right polarizer is set to either 120° or –120° (2/3 of the time), both of which are 30° away from 90° and/or –90°, the right photon will pass $\cos^2 30° = 75\%$ of the time, will therefore fail 25% of the time, and we'll get **SameFate** 25% of the time. So just like before we'll get **SameFate** (1/3)×100% + (2/3)×25% = 50% of the time.

If the left polarizer is in the 120° or –120° positions, the math works out exactly the same. All of the various cases work out to give exactly a 50% chance for **SameFate** results, so overall we have:

<p style="text-align:center;">**SameFate = 50%** (quantum mechanics prediction)</p>
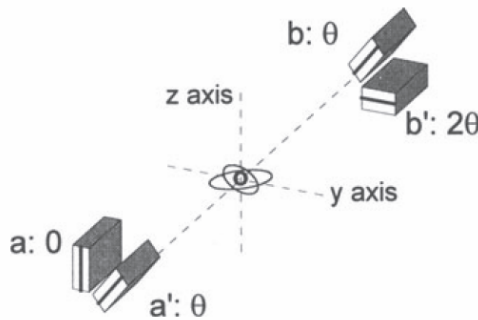
What do the experiments show? They are simply stated:

<p style="text-align:center;">**SameFate = 50%** (experimental results)</p>

The experimental results match the prediction of quantum mechanics and not the EPR view.

Chapter 18 – Bell's Thunderbolt (1964) – part 2

The second configuration of polarizer 1 vs. polarizer 2 is this; each polarizer has only two basic options:

Figure 18.4. Bell configuration 2. Analyzer **a** is vertical. Analyzers **a′** and **b** have same tilt: θ. Analyzer **b′** has twice the tilt: 2θ.

Polarizer 1 can be either 0° or some arbitrary angle θ; polarizer 2 can be either θ or 2θ, where θ is the same angle used for 1. Those four settings are referred to as a, a′, b, and b′, respectively.

We'll analyze this configuration in terms of a *correlation function*: c is the correlation between polarizers 1 and 2 for a given run, defined as $c = +1$ if **SameFate** and $c = -1$ if **DiffFate**. For a large number of runs, we can talk about the average correlation which I will call C_{ave} (it's just called C in *Roots*). As with the previous section, we will analyze this situation using both the EPR and the quantum mechanical views, and see which matches the actual experiments.

First, the quantum mechanical point of view. There are four possible combinations for the two polarizers: ab, ab′, a′b′, and a′b. We need to consider the four configurations separately. The average correlation for a given configuration will be given by $C_{ave} = +1\times$(chance of **SameFate**) $+ -1\times$(chance of **DiffFate**). Note: *Roots* neglects the second term and says $C_{ave} =$ chance of **SameFate** but that is an error.

Suppose we have combination ab, namely polarizer 1 at $0°$ and polarizer 2 at θ. Suppose photon 1 passes its polarizer; that must be because it collapsed to $0°$ and therefore photon 2 also collapses to $0°$. When photon 2 hits its polarizer it has a $\cos^2 \theta$ chance of passing and a $\sin^2 \theta$ chance of failing (since the chance of failing is 100% – the chance of passing, and since $1 - \cos^2 \theta = \sin^2 \theta$). The chance of **SameFate** is therefore $\cos^2 \theta$ and the chance of **DiffFate** is $\sin^2 \theta$, so $C_{ave} = \cos^2 \theta - \sin^2 \theta$.

If, however, photon 1 fails, then it has collapsed to $90°$ and photon 2 also collapses to $90°$. Photon 2 then has a $\cos^2(90° - \theta) = \sin^2 \theta$ chance of passing its polarizer and a $1 - \sin^2 \theta = \cos^2 \theta$ chance of failing. This leads to the same **SameFate** and **DiffFate** chances as previously, and therefore the same C_{ave}. Either way, then, for this combination we have:

$$\text{Combination ab: } C_{ave} = \cos^2 \theta - \sin^2 \theta$$

The other combinations lead to very similar equations for the exact same reasons, just with the angle θ in that equation replaced by the relative angles between polarizers 1 and 2.

$$\text{Combination ab′: } C_{ave} = \cos^2 2\theta - \sin^2 2\theta$$

$$\text{Combination a′b′: } C_{ave} = \cos^2 \theta - \sin^2 \theta$$

$$\text{Combination a′b: } C_{ave} = \cos^2 0° - \sin^2 0° = 100\%$$

For reasons that relate to the EPR analysis of the situation discussed below, we will put these four C_{ave} values together in a specific way to construct this function which Grometstein calls B in honor of Bell:
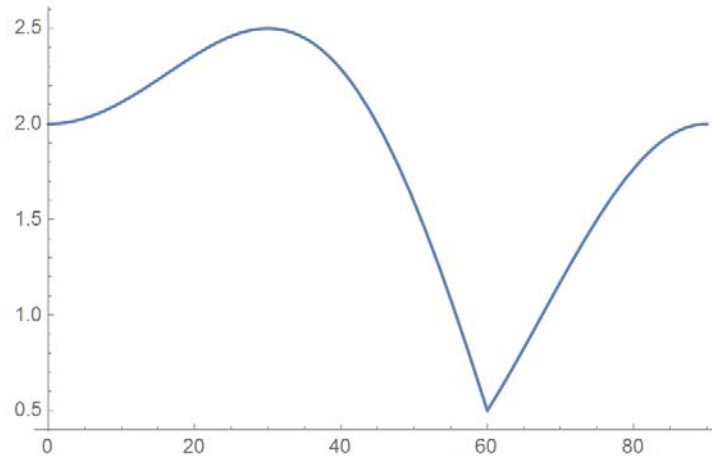
$$B = |C_{ave}(ab) - C_{ave}(ab′)| + |C_{ave}(a′b′) + C_{ave}(a′b)|$$

Plugging in the correlations we just worked out, here it is as a function of θ:

$$B = |(\cos^2 \theta - \sin^2 \theta) - (\cos^2 2\theta - \sin^2 2\theta)| + |(\cos^2 \theta - \sin^2 \theta) + 1|$$
$$\text{(quantum mechanics prediction)}$$

Here's a plot of B vs. θ (in degrees):

This is the same as the book's Fig. 18.5, except as mentioned above the book gets the C_{ave} definition slightly wrong and so its plot is slightly different. It still looks quite similar though.

Now the EPR point of view… OK, actually I will skip nearly all of this analysis because the math on page 503 is pretty horrible and *Roots* itself even gets some of it wrong. But here's the general idea. The EPR analysis involves λ_k, which stands for the kth photon polarization code, p_k, which is the probability of that code, and $c(a, b, \lambda_k)$, which is the correlation function for that specific code and polarizer setting ab. (Here a and b could be a' and/or b' as well.) The key step in the book's EPR analysis is to assume that $c(a, b, \lambda_k)$ is composed of separate functions of a and b, multiplied together. That's because in the EPR view, the setting of polarizer 1 cannot influence the results of polarizer 2, and vice versa. This is called the *locality assumption*. After the horrible math, the result is this:

$$B \leq 2 \text{ (EPR prediction, called "Bell's inequality")}$$

Clearly this disagrees with the quantum mechanical prediction because as you can see in the graph above, the quantum mechanical plot says that B should go above 2 for some of the angles.

What do the experiments show? Again we have a testable difference between quantum mechanics and the EPR view. So, what do the experimental results tell us? An overwhelming number of experiments show Bell's inequality to be violated and the quantum mechanical prediction to be fulfilled. *Roots* says this is the case for 5 out of 7 experiments, but the numbers since *Roots* was published are far more conclusive, with dozens and dozens additional experiments verifying the violation of Bell's inequality, i.e. validating quantum mechanics. In fact, the only experiments I'm aware of which did not validate quantum mechanics are the two early ones which *Roots* mentioned, and are generally considered now to be inaccurate. There may still be a little debate as to whether any of the experiments allow for tiny loopholes that still permit an EPR-type explanation, but generally speaking the experiments are now considered to be definitive.

So, what does this all mean? Well, it means that the EPR view of hidden variables is wrong. The polarization state of the photons is not predetermined in some sort of code at the moment of creation. The equations of quantum mechanics were proven to give the correct predictions. This does not, however, mean that the Copenhagen Interpretation is necessarily correct. Specifically, there may be other ways of interpreting the theory of quantum mechanics that don't involve the instantaneous collapse of a wave function. Some of these are covered in Chapter 19, and many more are listed in this Wikipedia article, https://en.wikipedia.org/wiki/Interpretations_of_quantum_mechanics, but since we have now reached the end of the semester you'll need to read more about those on your own.