Announcements

1. **Grades so far:**

Grade	% with this grade (both classes)
A	8.6
A-	16.9
B+	12.9
B	15.6
B-	9.6
C+	7.7
C	15.8
C-	6.6
D+	3.4
D	1.5

Angular Correspondences

Kinematics

Distance: x | Angle: θ

Velocity: v | Angular velocity: ω

Acceleration: a | Angular accel.: α

$x = x_0 + v_0 t + \frac{1}{2}at^2$ | $\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$

$v = v_0 + at$ | $\omega = \omega_0 + \alpha t$

$v^2 = v_0{}^2 + 2a(x - x_0)$ | $\omega^2 = \omega_0{}^2 + 2\alpha(\theta - \theta_0)$

Mass

Mass: m | Moment of inertia: I

Force/Newton's 2nd Law

Force: F | Torque: τ $(= r_\perp \times F)$

$\sum \vec{F} = m\vec{a}$ | $\sum \tau = I\alpha$

Energy

$KE_{trans} = \frac{1}{2}mv^2$ | $KE_{rot} = \frac{1}{2}I\omega^2$

Momentum…

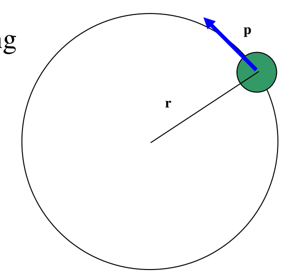
Momentum: p $(= mv)$ | Angular momentum??

Angular momentum

Imagine mass m on a thin rod moving in a circle, with constant speed v. It has linear momentum $\vec{p}$ =_______.

Is $\vec{p}$ constant?_______

Is $|p|$ constant?_______



What do we need to change $|p|$?

Start with Newton 2:

$\sum \vec{F} = m\vec{a}$ $\qquad$ $\sum \tau = I\alpha$

Define $\boxed{L = I\omega}$

If no external forces, no change in momentum

If no external *torques*, no change in *angular* momentum

Conservation of Angular momentum

$\boxed{\Sigma L_{bef} = \Sigma L_{aft}}$ → if and only if *no external torques*

Imagine two space stations connected by a cable. They are rotating about their center of mass.

The blue station pulls the cable in so they are separated less.

What happens?

Q4. Is rotational kinetic energy conserved? The total energy afterwards is:

a. more
b. less
c. the same

Hint: is any work done by the blue station?

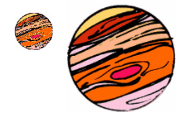
Skater and spinning

Demo: spinning chair

Worked Problem:
A skater has an initial ω of 2 rad/s and I = 30 kg·m^2. When he brings in his arms, I = 5 kg·m^2. What is the final ω?

Demo: conical pendulum (pull string)

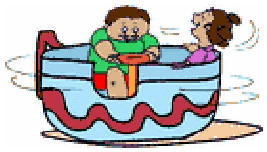
Worked problem:
A gas planet has a 10 hour day. It then heats up due to a nuclear reaction in its core, and expands to *three times* the original radius. It keeps the same mass. Consider it to be a uniform solid sphere.



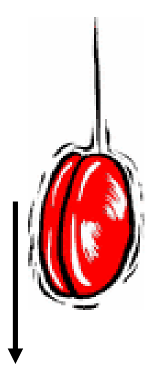
Q5. After the expansion, ω
- a. increases
- b. decreases
- c. stays the same

Q6. After the expansion, the "day" will be_____hours
- a. < 3 hours
- b. 3-9 hours
- c. 9-20 hours
- d. 20-50 hours
- e. > 50 hours

Is L conserved in these cases?



"Teacups": Hands-on-post is connected to the platform floor



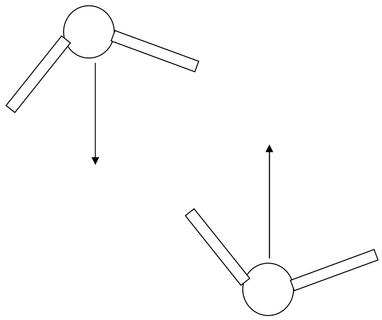
Q7. What can divers change after they leave the board?
- a. ω
- b. L
- c. I
- d. ω and L
- e. ω and I

Demo: lazy-susan twist

Two skaters joining hands

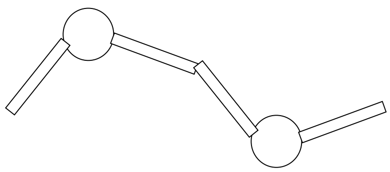
Before

Any L?



After

Any L?



Is angular momentum conserved?

Yes if there are no external torques.

Was there an external torque here?

Another expression for L…

Start with

$\tau = r_{\perp} F$

Remember

$F_{net} = \Delta p/\Delta t$

$\tau_{net} = \Delta L/\Delta t$

Result: $\boxed{L = r_{\perp}\, p}$ $(= r\, p_{\perp} = r\, p \sin\theta)$

Comment on vectors… (aka L has a direction!)

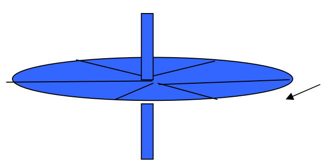
Demo: gyro, bicycle wheel

With **no external torques**… the rotation axis (and L) is________________

With external torque…..

Demo: bicycle wheel, briefcase

José sits on frictionless ice, holding a spinning bicycle wheel. View from above it is going **clockwise** (CW).



Neglect external friction.

Q8. If he grabs on to the wheel edge firmly and "stops" it he will then be

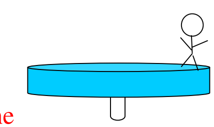
a. turning CW (viewed from the top)
b. turning CCW
c. not turning

Q9. If instead of stopping it he flips the wheel over, so it is going CCW (viewed from the top), he will be

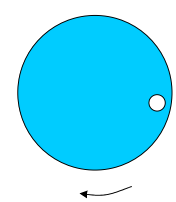
a. turning CW, but slower than in the previous problem
b. turning CCW, slower than in the previous problem
c. turning CW, faster than in the previous problem
d. turning CCW, faster than in the previous problem

Demo: rotating chair

A girl is on a spinning merry-go round.



What will happen to the rotational speed ω of the merry-go-round if she…



HINT: Sometimes it's easier to think of the **forces (torques)** she put on the merry-go-round to change, rather than conservation of L.

Q10. Walks towards the center?

a. it slows down
b. it stays same speed
c. it speeds up

Q11. Runs opposite to the spinning so she is at rest vs the ground?

Q12. Slips off when she steps on a frictionless icy part?

Q13. She jumps so that she is moving radially outward versus the ground?

Q14. Jumps tangentially off the merry-go-round so that she goes faster relative to the ground than she was before she jumped?

Q15. Did you discuss at least half of the discussion quiz questions today with a neighbor?

a. Yes b. No