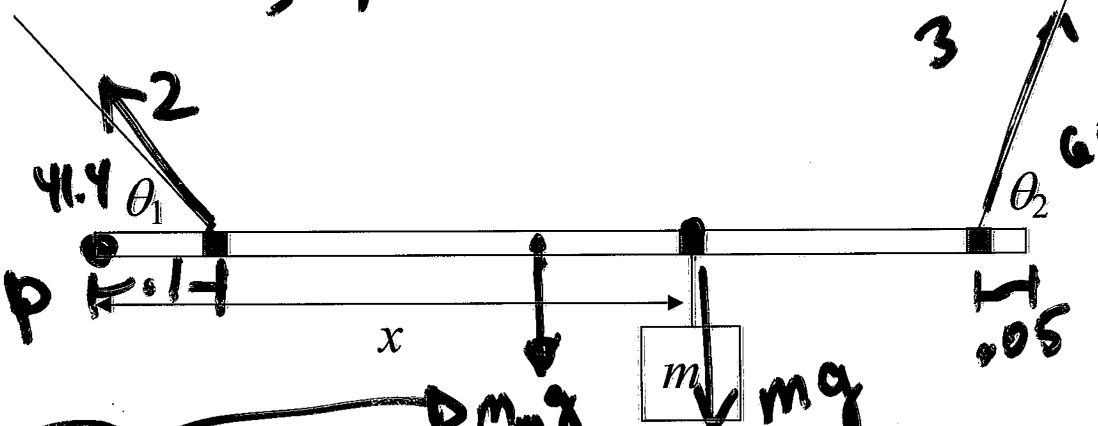
a. less than 2 ← "1-2"
b. 2-3

Problem: c. 3-4



A 0.4 kg meterstick is suspended from support pillars (not shown) via two strings at θ_1 =41.4° and θ_2 = 60°, with tensions of 2 N and 3N. The strings are attached at 10 cm and 5 cm from the two ends of the meterstick. The stick is *not* in equilibrium until an additional mass is hung from a point in the middle. Find the unknown x and m.

$\Sigma F_x = 0$ $\quad$ $\Sigma F_y = 0$ $\quad$ $\Sigma \tau_p = 0$

$\boxed{+3\cos 60 - 2\cos 41.4 = 0}$

$+2\sin 41.4 + 3\sin 60 - \overset{(.4)}{m_m}(9.8) - m(9.8) = 0$

↳ solve for m

$m = .171\text{ kg}$?

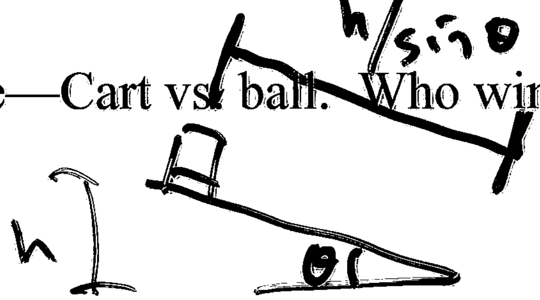
$+(2\sin 41.4)(.1) + (3\sin 60)(.95) - (.4)(9.8)(.5) - \overset{.171}{m}\overset{9.8}{g}x = 0$

Solve for x

Answers: 0.171 kg, 38.2 cm

Rotational kinetic energy

Demo: Race—Cart vs. ball. Who wins? (Like warmup)



Review:

How fast is cart going at bottom? (Energy)

$$PE_i = KE_f + \text{Rotational energy}$$

$$\cancel{m}gh = \frac{1}{2}\cancel{m}v^2 + RE$$

$$v_f = \sqrt{2gh - RE)} \qquad \boxed{\text{less}}$$

How long did it take to get there? (Kinematics)

$$v_0 = 0$$

$$d = h/\sin\theta$$

You can figure it out.

→ What's different about the ball?

$v = \omega r$

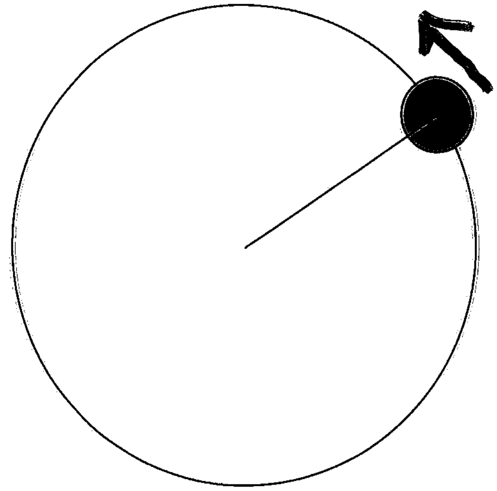
Kinetic energy of a "**point object**" rotating in a circle:

$KE = \frac{1}{2} m v^2$

$KE_{rot} = \frac{1}{2} m (\omega r)^2$

Write in terms of ω:

$= \frac{1}{2} \underbrace{(mr^2)} \omega^2$



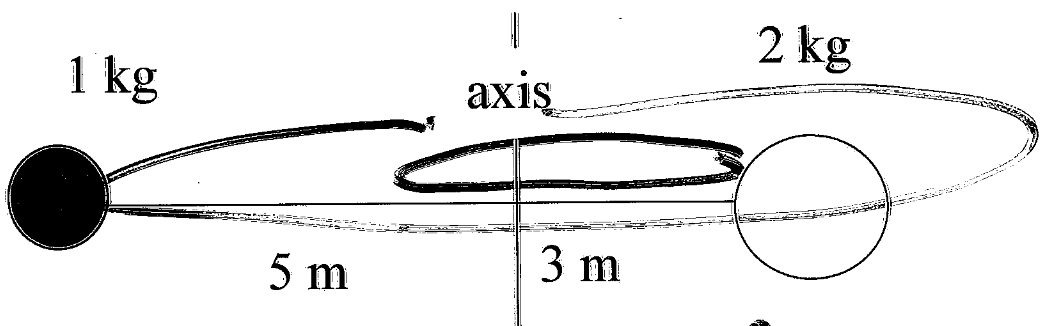
"Moment of inertia" I

$\boxed{I_{pt\ mass} = mr^2}$ (rotating in a circle)

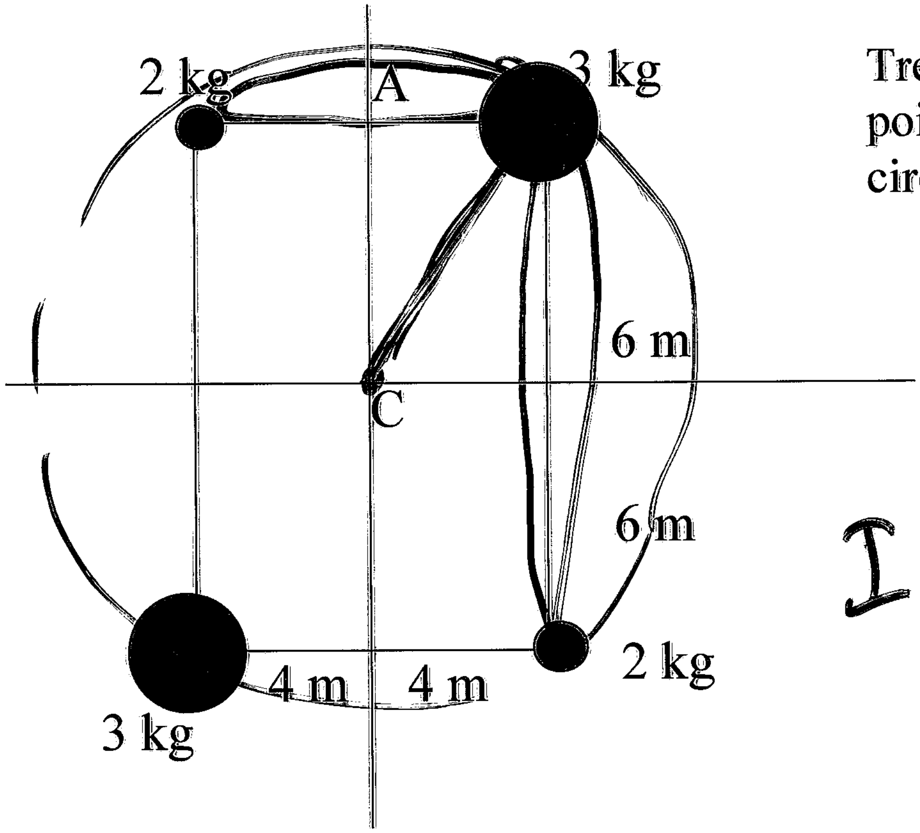
Kinetic energy in terms of I and ω:

$\boxed{KE_{rot} = \frac{1}{2} \underbrace{I}_{\text{like } m} \underbrace{\omega^2}_{\text{like } v}}$

Moment of inertia for two mass? (connected with a rod)



$\boxed{I = I_1 + I_2 + \ldots}$ $I = m_1 r_1^2 + m_2 r_2^2$



Treat each mass as a point if its rotating circle > size of object

$I = mr^2$

Clicker quiz: Does *I* change when you rotate about axis A vs. axis B?

a. About axis A has larger *I*

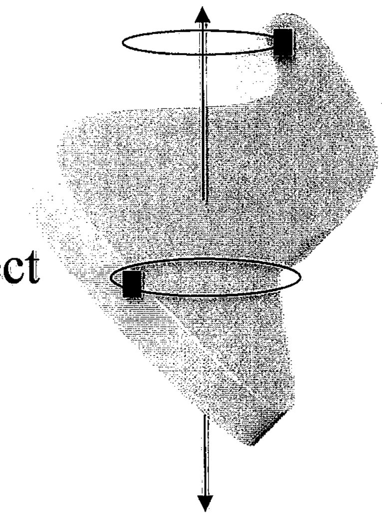
b. About axis B has larger *I*

c. They have the same *I*

What about axis C? (C is into the page) I bigger still

Demo: variable "I-rotator"

Rotating "extended" objects



Must add up mr^2 *for each bit of mass* in the object

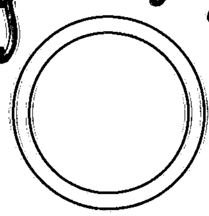
Which bits of mass contribute the most to I?

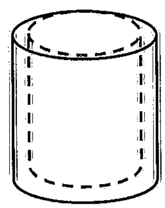
bigger circle

Which objects have the largest I?

more bits going in big circle

Hoop/cylindrical shell

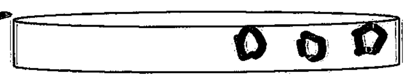


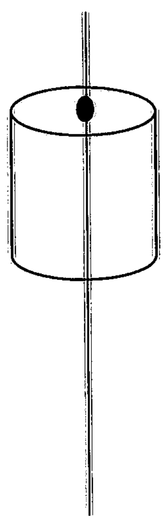


large I

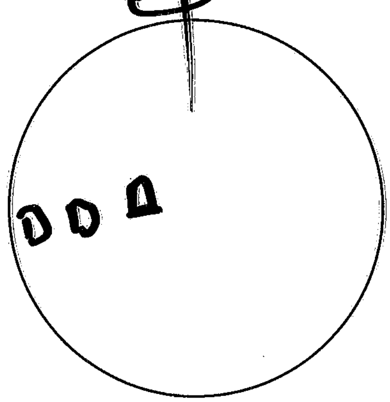
Solid disk/cylinder

small I





Solid sphere



smaller I

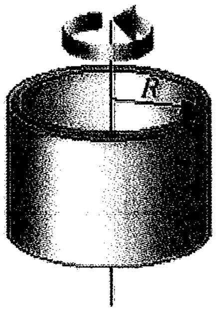
Demo: Long & short "I-bars"

TABLE 8.1

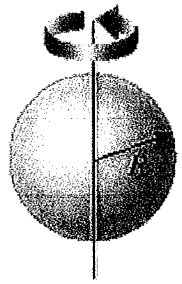
Moments of Inertia for Various Rigid Objects of Uniform Composition

Hoop or thin cylindrical shell
$I = MR^2$

$I = 1mr^2$



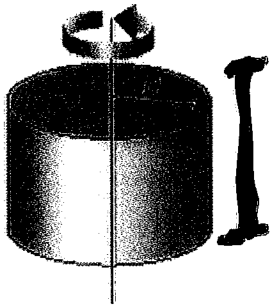
Solid sphere
$I = \frac{2}{5} MR^2$



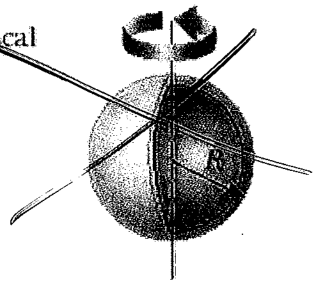
$\frac{2}{5} mr^2$

Solid cylinder or disk
$I = \frac{1}{2} MR^2$

$\frac{1}{2} mr^2$

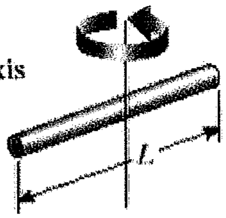


~~Thin spherical shell~~
$I = \frac{2}{3} MR^2$

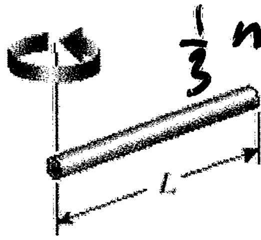


Long thin rod with rotation axis through center
$I = \frac{1}{12} ML^2$

$\frac{1}{12} mL^2$



Long thin rod with rotation axis through end
$I = \frac{1}{3} ML^2$



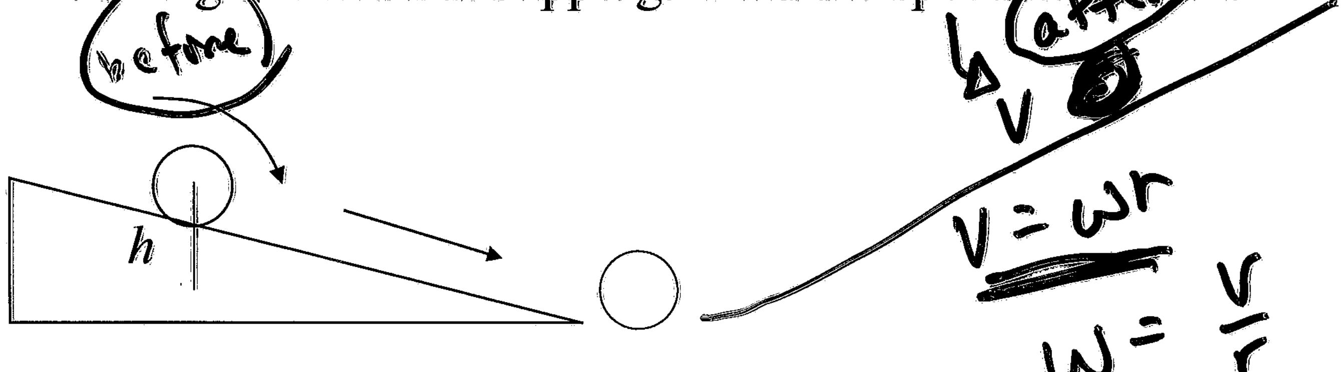
$\frac{1}{3} mL^2$

Clicker quiz: Which kind of rolling object will be moving the **fastest** at bottom of an incline? (Which will get there first?)

a. Hoop
b. Solid disk
c. Sphere
d. It depends on size

Demo: racing objects down incline

Worked Problem: An object with moment of inertia I rolls down a height h without slipping. Find the speed at the bottom.



$v = \omega r$

$\omega = \frac{v}{r}$

$E_{bef} = E_{aft}$

$PE_i = KE_f + KE_{rot,f}$

$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$

$2gh = v^2 + \frac{I}{m}\left(\frac{v}{r}\right)^2$

$2gh = v^2\left(1 + \frac{I}{mr^2}\right)$

$$\boxed{v = \sqrt{\frac{2gh}{1 + \frac{I}{mr^2}}}}$$

Answer: $v = \sqrt{\dfrac{2gh}{1 + I/mR^2}}$

Clicker quiz: If they continue on, which would go the farthest up a hill on the other side?

a. Hoop
b. Solid disk
c. Sphere
d. All the same height at the end

Newton's second law for rotation

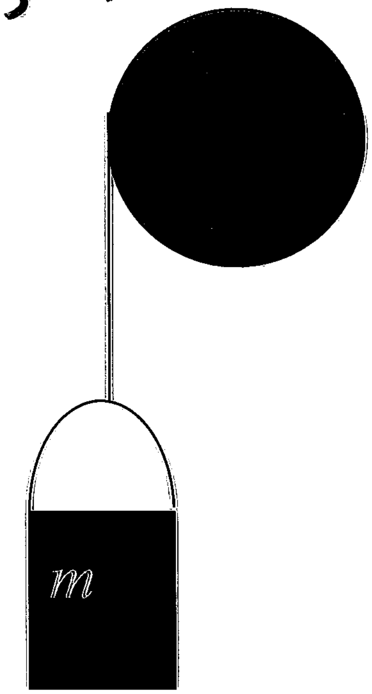
$$\boxed{\sum \tau = I\alpha}$$ still also have $\sum \vec{F} = m\vec{a}$

rotation around a pivot

(but accel. of what?)

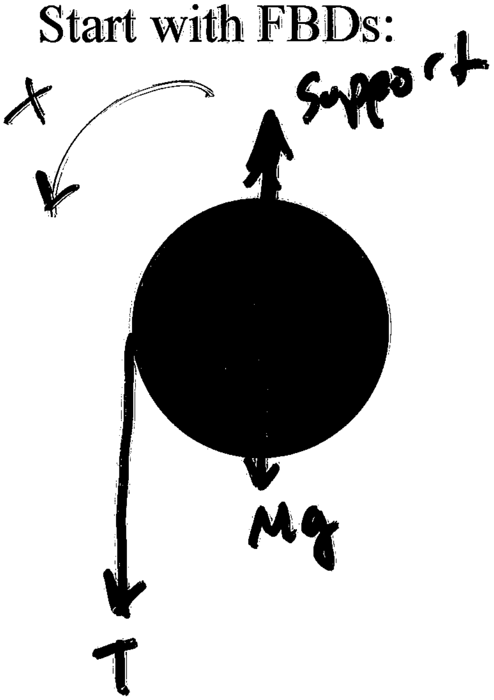
center of mass

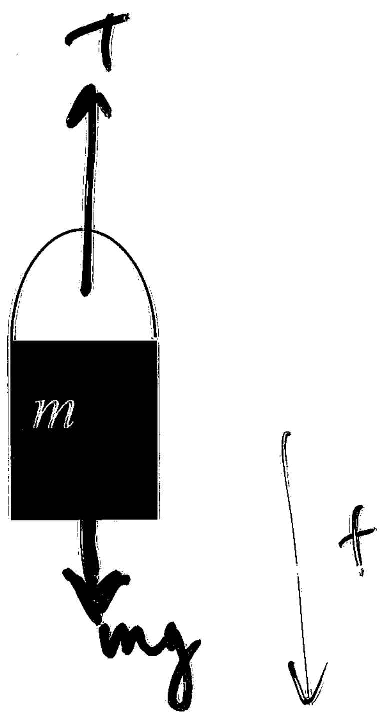
$I = \frac{1}{2}MR^2$



Worked problem: A falling mass starts a cylinder rotating (<u>not</u> a "massless pulley"). What is the acceleration of *m*?

Start with FBDs:





cylinder

Write equations…

$$\Sigma \tau_p = I\alpha$$

$$(T)(R) = \frac{1}{2}MR^2\alpha$$

$$(T)R = \frac{1}{2}MR^2\frac{a}{\cancel{R}}$$

pail

$$\Sigma F = \text{[redacted]}\ ma$$

$$mg - T = ma$$

$$T = mg - ma$$

Make a connection between α and *a*: $a = \alpha R$

$$\alpha = \frac{a}{R}$$

$$(mg - ma)R = \frac{1}{2}MR^2\frac{a}{\cancel{R}}$$

solve for a

Clicker quiz: Mary and Fred are rolling a large tire down a hill. Mary says it will go faster if Fred gets inside the tire as shown and rolls down with it. Fred's not sure. What do you think?

a. It will go faster
b. It will go slower
c. It will take the same time