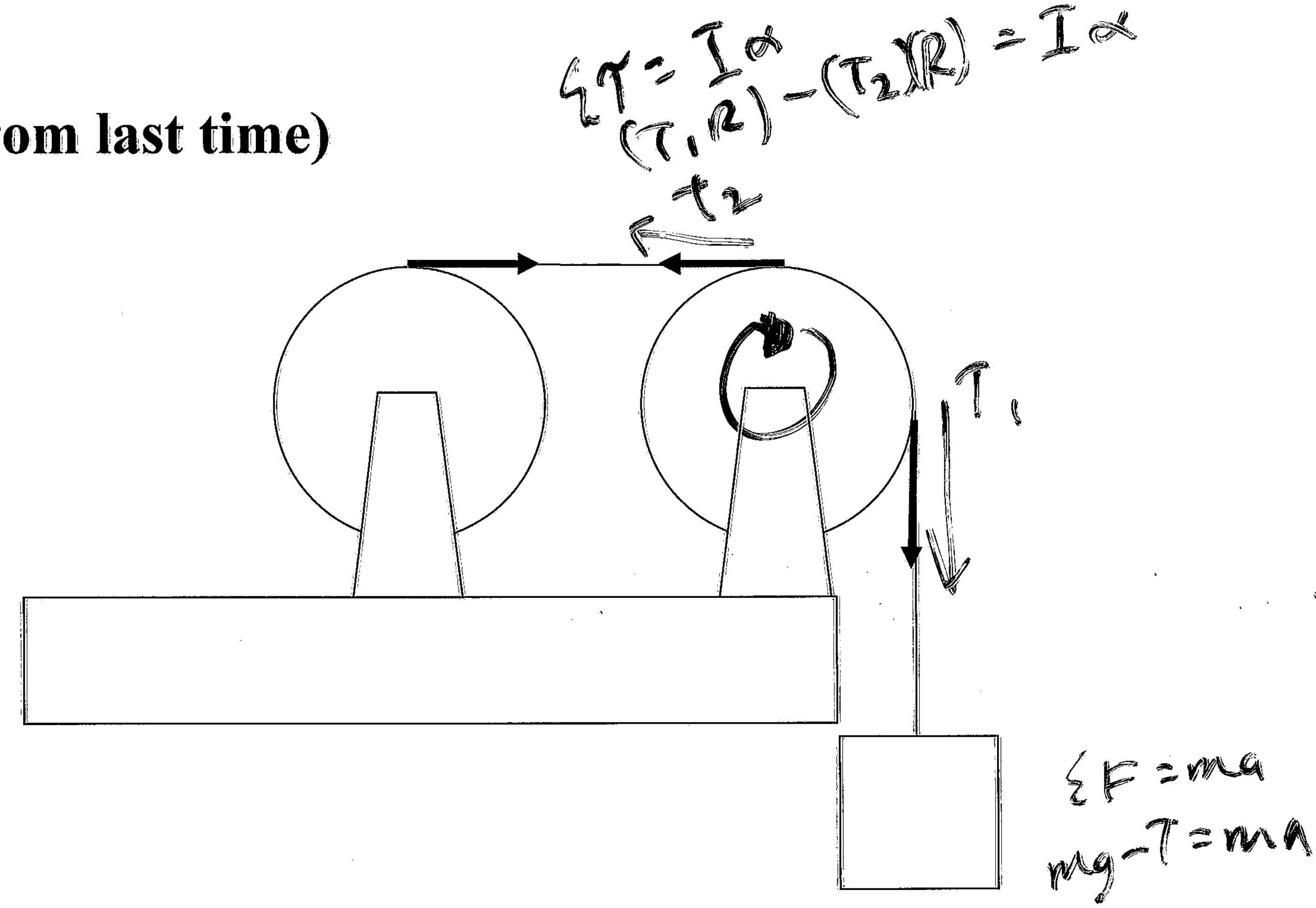
(From last time)



The left disk has a rope wrapped around its edge and the rope passes over a second disk. The two disks are identical and their **mass is significant**. As the system accelerates there is **no slipping of the rope on either wheel and both wheels accelerate the same**. The tension in the rope is

a. Largest between the disks
b. Largest above the mass
c. The same in both places.

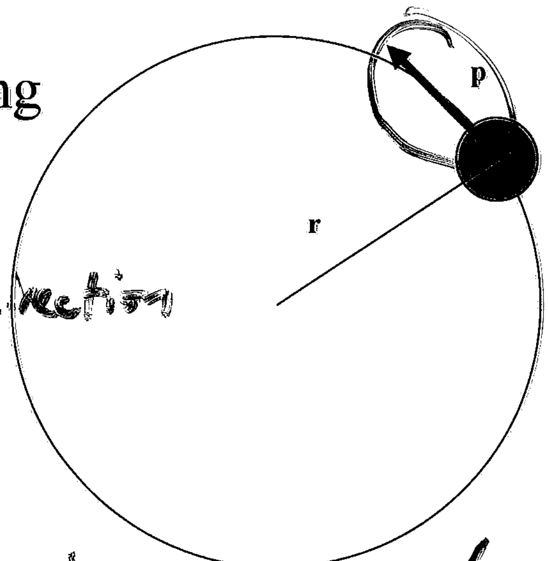
What's different with our old "massless pulleys"?

$\Sigma\tau = I\alpha$, $I = 0$

$\Sigma\tau = 0$

Angular momentum

Imagine a mass *m* on a thin rod moving in a circle, with constant speed $\vec{v}$. It has linear momentum $\vec{p} =$ mv.



Is $\vec{p}$ constant? No – changing direction

Is $|p|$ constant? yes

What do we need to change $|p|$?

Apply F tangent to circle : $\vec{F} \perp \vec{r}$ = torque!

<u>Force-momentum relationship</u>

Start with Newton 2:

$$\sum \vec{F} = m\vec{a}$$

$$\Sigma \vec{F} = m \frac{\Delta \vec{v}}{\Delta t}$$

$$\Sigma \vec{F} = \frac{\Delta(\vec{p})}{\Delta t}$$

$= 0$

If no net external forces, no change in momentum

<u>Torque-ang. mom. relationship</u>

$$\sum \tau = I\alpha$$

$$\Sigma \tau = I \frac{\Delta\omega}{\Delta t} = \frac{\Delta(I\omega)}{\Delta t}$$

$$\Sigma \tau = \frac{\Delta L}{\Delta t}$$

Define $\boxed{L = I\omega}$

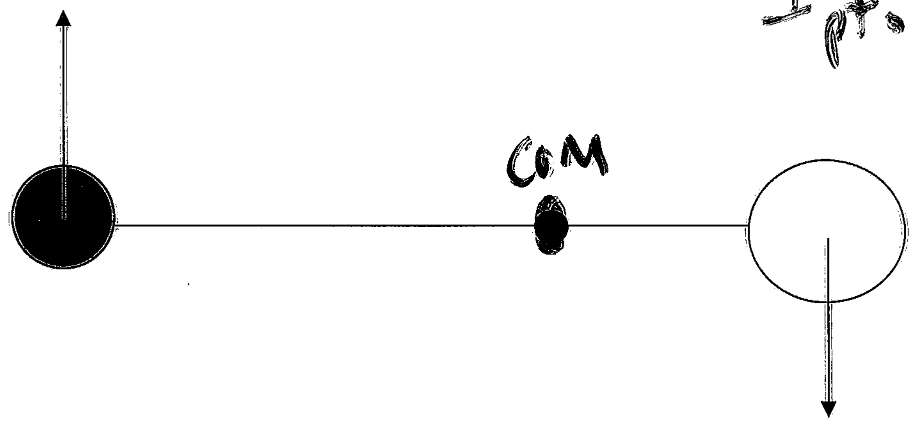
$= 0$

If no net external *torques*, no L change in *angular* momentum

Conservation of Angular momentum

$\boxed{\Sigma L_{bef} = \Sigma L_{aft}}$ → if and only if *no net external torques*

$I_{pt\ object} = mR^2$



Imagine two space stations connected by a cable. They are rotating about their center of mass.

Someone in the blue station pulls the cable in so they are each closer to the center of rotation. What happens? speeds up.

$$\left[(I_1 + I_2)\omega\right]_{initial} = \left[(\underbrace{I_1 + I_2}_{\text{decreases}})\underbrace{\omega}_{\text{increases}}\right]_{final}$$

Clicker quiz: Is rotational kinetic energy conserved? The total energy afterwards is:

a. more
b. less
c. the same

$E_{bef} + W = E_{aft}$

Hint: is there any non-conservative work done?

Skaters spinning

(frictionless ice)

Clicker quiz: When an ice skater brings her arms close to her body during a spin, her moment of inertia

a. decreases
b. increases
c. stays the same

Clicker quiz: When an ice skater brings her arms close to her body during a spin, her angular momentum

a. decreases
b. increase
c. stays the same

$L_{bef} = L_{aft}$

$I_1 \omega_1 = I_2 \omega_2$

I decreases, ω increases

Demo: Hoberman sphere
Demo: spinning chair
Videos: train on circular track, pocket watch

Worked Problem: A skater has an initial ω of 2 rad/s and $I = 30\ \text{kg}\cdot\text{m}^2$. When she brings in her arms, $I = 10\ \text{kg}\cdot\text{m}^2$. What is the final ω?

bef

$\omega_1 = 2\ \text{rad/s}$

$I_1 = 30\ \text{kgm}^2$

aft

$\omega_2 = ?$

$I_2 = 10\ \text{kgm}^2$

$$\Sigma L_{bef} = \Sigma L_{aft}$$

$$I_1\omega_1 = I_2\omega_2$$

$$\omega_2 = \frac{I_1\omega_1}{I_2}$$

$$= \frac{(30\ \text{kgm}^2)(2\ \text{rad/s})}{(10\ \text{kgm}^2)}$$

$$\omega_2 = 6\ \text{rad/s}$$

How much energy did it take to do this?

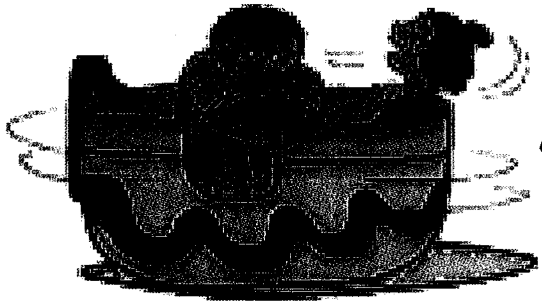
$$KE_i = \tfrac{1}{2} I_1\omega_1^2 = \tfrac{1}{2}(30\ \text{kgm}^2)(2\ \tfrac{\text{rad}}{\text{s}})^2 = 60\text{J}$$

$$KE_f = \tfrac{1}{2} I_2\omega_2^2 = \tfrac{1}{2}(10)(6)^2 = 180\text{J}$$

$$\Delta KE = 120\text{J}$$

Answers: 6 rad/s, 120 J

Is L conserved in these cases?

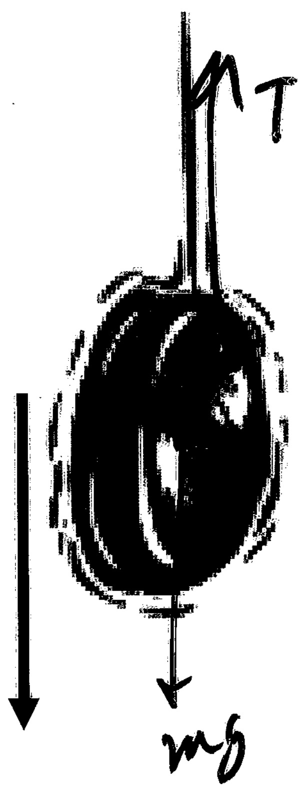


No

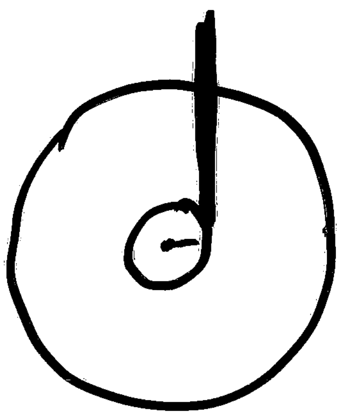
"Teacups": Hands-on-post is connected to the platform floor

Yes

$\Sigma \tau_p = I\alpha$



No

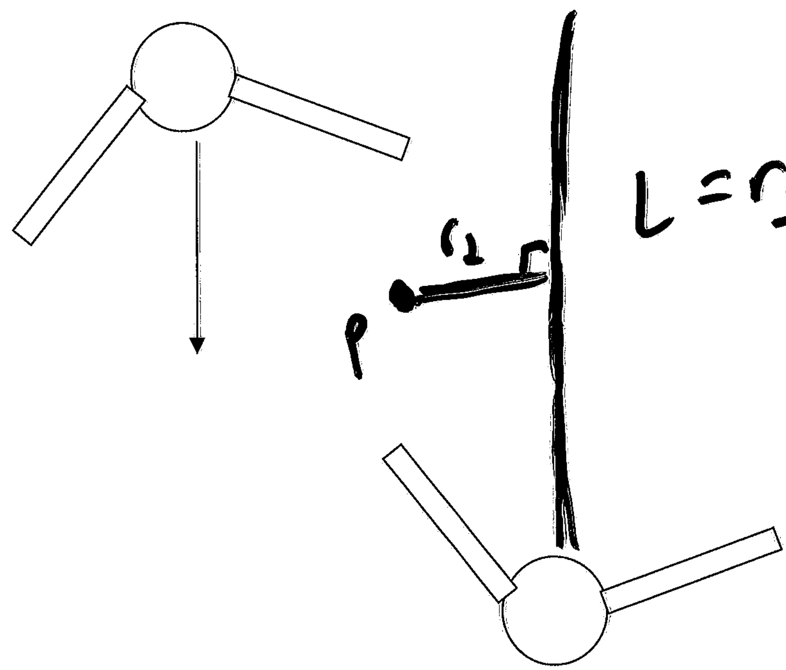


Demo: gyroscopes

System = both skaters

Food for thought: two skaters joining hands

Before

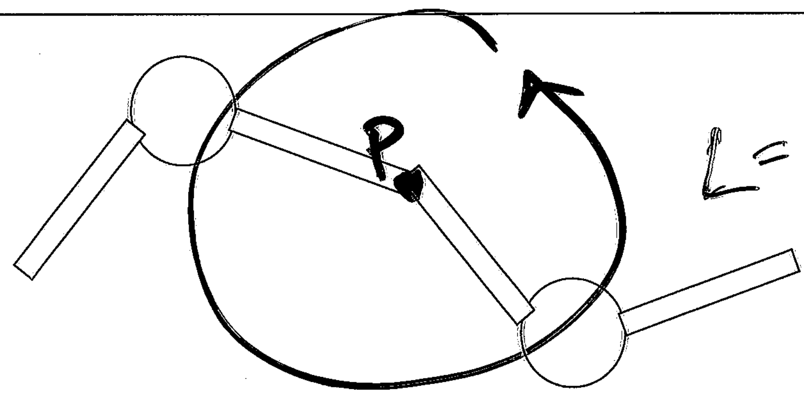


$L = r_\perp p$

Any L?

Yes!

After



$L = I\omega$

Any L?

Yes

Is angular momentum conserved? Yes

Yes if there are no net external torques

Clicker quiz: Was there an external torque here?

a. Yes

b. No

$p = mv \rightarrow \underline{L = I\omega}$

$\tau = r_\perp F$

$L = r_\perp p$?

Another expression for L…

Start with

$$\tau = r_\perp F$$

$$= r_\perp \left(\frac{\Delta p}{\Delta t}\right)$$

Remember

$$F_{net} = \Delta p / \Delta t$$

$$\tau_{net} = \Delta L / \Delta t$$

$\tau = \frac{\Delta(r_\perp p)}{\Delta t}$ but also $\tau = \frac{\Delta L}{\Delta t} \Rightarrow \underline{L = r_\perp p}$

Result: $\boxed{L = r_\perp p}$ $\quad (= r p_\perp = r p \sin\theta)$

Worked Problem: Skaters on previous page have 0.7 m arms and are each 62 kg. They come together at 3.5 m/s. How fast (rad/s) are they turning afterwards?

$$\Sigma L_{bef} = \Sigma L_{aft}$$

$$r_{\perp 1} p_1 + r_{\perp 2} p_2 = I_1 \omega_1 + I_2 \omega_2$$

$$(.7\text{m})(62\text{kg})(3.5\tfrac{\text{m}}{\text{s}}) + \text{same} = (I_1 + I_2)\,\omega_f$$

$$= (mR^2 + mR^2)\,\omega_f$$

$$= 2(62)(.7)^2 \cdot \omega_f$$

$$\underline{\omega_f = 5\ \text{rad/s}}$$

Answer: 5 rad/s

Comment on vectors… (aka L has a direction!)

Does ω have a direction? CCW vs CW

Therefore L has direction!

Thus with **no external torques**…

magnitude of $\vec{L}$ = constant

direction of $\vec{L}$ = constant

Demos: gyro

With external torque weird things happen!

Demos: briefcase, bicycle wheel (Ralph's question)